STAT 2509B4

Assignment 1

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1. **Verify that $f(x) = x^3 + 8x - 7$ has a root in $[0, 1]$. Use fixed-point iteration to find this root to 5 decimal places. Start with $x_0 = 0.75$, but verify that your chosen $g(x)$ will work before you begin.:**

   First verify that $f(a) * f(b) < 0$. $f(0) = 0^3 + 8 * 0 - 7 = -7, f(1) = 1^3 + 8 * 1 - 7 = 2 \to f(a) * f(b) \to -7 * 2 < 0$. Therefore since there is a sign change there must also be a root in the interval $[0, 1]$.

   $f(x) = x^3 + 8x - 7 \to \frac{7-x^3}{8}$. Take $g(x) = \frac{7-x^3}{8}$

   Since $|g\prime(x)| = |\frac{-3x^2}{6}| = | - \frac{1}{2}x^2| \leq \frac{1}{2} < 1$ on $[0, 1]$ then $x_{n+1} = g(x_n)$ will generate a convergent sequence.

$$
\begin{aligned}
x_0 = 0.75, x_1 = g(0.75) &= \frac{7 - 0.75^3}{8} = 0.82227 \\
x_2 = g(0.82227) &= 0.80551 \\
x_3 = g(0.80551) &= 0.80967 \\
x_4 = g(0.80967) &= 0.80865 \\
x_5 = g(0.80865) &= 0.80890 \\
x_6 = g(0.80890) &= 0.80884 \\
x_7 = g(0.80884) &= 0.80885 \\
x_8 = g(0.80885) &= 0.80885
\end{aligned}
$$

   Check $f(0.80885) \simeq -0.00002$ therefore very close to 0 and the root is 0.80885.

2. **Use Newton's Method to find the point of intersection of the curves $y = x^3$ and $y = cos(x)$ to 6 decimal places. Start with $x_0 = 1$:**

   If $x^3 = cos(x)$ then $f(x) = x^3 - cos(x) = 0$. So $f\prime(x) = 3x^2 + sin(x)$.

   So $x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)} = x_n - \frac{x^3 - cos(x)}{3x^2 + sin(x)} = \frac{2x^3 + x * sin(x) + cos(x)}{3x^2 + sin(x)}$

$$
\begin{aligned}
x_0 = 1, x_1 &= \frac{2 * 1^3 + 1 * sin(1) + cos(1)}{3 * 1^2 + sin(1)} = 0.880333 \\
x_2 = g(0.880333) &= 0.865684 \\
x_3 = g(0.865684) &= 0.865474 \\
x_4 = g(0.865474) &= 0.865474
\end{aligned}
$$

   Check $f(0.865474) \simeq -9.96 * 10^-8$ which is very close to 0 therefore okay and the root solution to the intersection of the curves is $x = 0.865474$.

3. **Solve the following dynamical systems. Find and classify any equilibria.:**

   (a) $a_{n+1} = (2/5)a_n, a_0 = 10$**:**

   (b) $a_{n+1} = (3/5)a_n + 100, a_0 = 20$**:**

(c) $a_{n+1} = (-2/3)a_n + 500, a_0 = 25$**:**

(d) $a_{n+1} = 3a_n - 30, a_0 = 0$**:**

4. **Suppose that Owls have Mice for their primary food source in a wildlife sanctuary. If M_n is the Mouse population after n years and O_n is the Owl, the following model has been suggested:**

$$M_{n+1} = 1.3M_n - 0.002O_nM_n$$
$$O_{n+1} = 0.6O_n + 0.0004O_nM_n$$

We know that when a_n is in the form $a_{n+1} = ra_n + b$ then $a_n = r^n(a_0 - a) + a, a = \frac{b}{1-r}$

(a) **What type of interaction is this ? How do the coefficients tell us ?:**

(b) **Find the equilibrium values ?** As taught in class when we have a system of equtions as follows:

$$x_{n+1} = ax_n - bx_ny_n$$
$$y_{n+1} = cy_n + dx_ny_n$$

Then the system has two fixed points (0,0) and $(\frac{1-c}{d}, \frac{a-1}{b})$.
In our case this gives equilibrium values of $(\frac{1-c}{d}, \frac{a-1}{b}) \to (\frac{1-0.6}{0.0004}, \frac{1.3-1}{0.002}) \to (1000, 150)$.

(c) **Predict the long-term outcome if $M_0 = 1200$ and $O_0 = 100$**