MATH 3800 F

Assignment 3

Krystian Wojcicki, 101001444

Winter 2020

1. **Use Simpson's Rule with 4 subintervals to approximate Z 1 $\int_0^1 \frac{2}{1+x^2}dx$ to 6 decimal places. Compare your result with the true value by calculating the simple error, ie $|true - approx|$.**

   $h = \frac{1-0}{4} = 0.25 \Rightarrow x_0 = 0, x_1 = 0.25, x_2 = 0.5, x_3 = 0.75, x_4 = 1$

   $\int_0^1 \frac{2}{1+x^2}dx \simeq \frac{h}{3}[\frac{2}{1+0^2} + 4\frac{2}{1+0.25^2} + 2\frac{2}{1+0.5^2} + 4\frac{2}{1+0.75^2} + \frac{2}{1+1^2}] = 1.570784$

   True value of $\int_0^1 \frac{2}{1+x^2}dx = 1.570796$

   $Error = |true - approx| = |1.570796 - 1.570784| = 0.000012$

   Very close to 0.

2. **Use Gaussian Quadrature with 4 steps to approximate $\int_0^1 \cos^2 x dx$ to 6 decimal places**

   $\int_0^1 \cos^2 x dx = \frac{1}{2}\int_{-1}^1 \cos^2(\frac{1}{2}(t+1))dt \simeq \frac{1}{2}\sum_{j=1}^4 A_j \cos^2(\frac{1}{2}(t_j+1) = \frac{1}{2}[0.3478548451 \cos^2\left(\frac{1}{2}(0.8611363116+1)\right)+$
   $0.3478548451 \cos^2\left(\frac{1}{2}(-0.8611363116+1)\right)+$
   $0.6521451549 \cos^2\left(\frac{1}{2}(0.3399810436+1)\right)+0.6521451549 \cos^2\left(\frac{1}{2}(-0.3399810436+1)\right)] = 0.727324318763 \simeq 0.727324$

   True value of $\int_0^1 \cos^2 x dx = 0.727324$

   $Error = |true - approx| = 0.727324 - 0.727324 = 0$. Excellent approximation.

3. **Use Monte Carlo simulation to approximate the probability of three tails occurring when four fair coins are flipped. Do a sufficient number of trials to have a meaningful result.**

4. **Given the loaded (unfair) dice probabilities below, use Monte Carlo simulation to simulate the results of ten rolls of the dice**

result	**1**	**2**	**3**	**4**	**5**	**6**
die 1	**0.1**	**0.1**	**0.2**	**0.3**	**0.2**	**0.1**
die 2	**0.3**	**0.1**	**0.2**	**0.1**	**0.05**	**0.25**