

STAT 2509B4

Assignment 1

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Winter 2020

1. **Verify that $f(x) = x^3 + 8x - 7$ has a root in $[0, 1]$. Use fixed-point iteration to find this root to 5 decimal places. Start with $x_0 = 0.75$, but verify that your chosen $g(x)$ will work before you begin.:**

First verify that $f(a) * f(b) < 0$. $f(0) = 0^3 + 8 * 0 - 7 = -7$, $f(1) = 1^3 + 8 * 1 - 7 = 2 \rightarrow f(a) * f(b) \rightarrow -7 * 2 < 0$. Therefore since there is a sign change there must also be a root in the interval $[0, 1]$.

$$f(x) = x^3 + 8x - 7 \rightarrow \frac{7-x^3}{8}. \text{ Take } g(x) = \frac{7-x^3}{8}$$

Since $|g'(x)| = |\frac{-3x^2}{8}| = |-\frac{3}{8}x^2| \leq \frac{3}{8} < 1$ on $[0, 1]$ then $x_{n+1} = g(x_n)$ will generate a convergent sequence.

$$x_0 = 0.75, x_1 = g(0.75) = \frac{7 - 0.75^3}{8} = 0.82227$$

$$x_2 = g(0.82227) = 0.80551$$

$$x_3 = g(0.80551) = 0.80967$$

$$x_4 = g(0.80967) = 0.80865$$

$$x_5 = g(0.80865) = 0.80890$$

$$x_6 = g(0.80890) = 0.80884$$

$$x_7 = g(0.80884) = 0.80885$$

$$x_8 = g(0.80885) = 0.80885$$

Check $f(0.80885) \simeq -0.00002$ therefore very close to 0 and the root is 0.80885.

2. **Use Newton's Method to find the point of intersection of the curves $y = x^3$ and $y = \cos(x)$ to 6 decimal places. Start with $x_0 = 1$:**

If $x^3 = \cos(x)$ then $f(x) = x^3 - \cos(x) = 0$. So $f'(x) = 3x^2 + \sin(x)$.

$$\text{So } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x^3 - \cos(x)}{3x^2 + \sin(x)} = \frac{2x^3 + x \sin(x) + \cos(x)}{3x^2 + \sin(x)}$$

$$x_0 = 1, x_1 = \frac{2 * 1^3 + 1 * \sin(1) + \cos(1)}{3 * 1^2 + \sin(1)} = 0.880333$$

$$x_2 = g(0.880333) = 0.865684$$

$$x_3 = g(0.865684) = 0.865474$$

$$x_4 = g(0.865474) = 0.865474$$

Check $f(0.865474) \simeq -9.96 * 10^{-8}$ which is very close to 0 therefore okay and the root solution to the intersection of the curves is $x = 0.865474$.

3. **Solve the following dynamical systems. Find and classify any equilibria.:**

(a) $a_{n+1} = (2/5)a_n, a_0 = 10$:

(b) $a_{n+1} = (3/5)a_n + 100, a_0 = 20$:

(c) $a_{n+1} = (-2/3)a_n + 500, a_0 = 25$:

(d) $a_{n+1} = 3a_n - 30, a_0 = 0$:

4. **Suppose that Owls have Mice for their primary food source in a wildlife sanctuary. If M_n is the Mouse population after n years and O_n is the Owl, the following model has been suggested:**

$$\begin{aligned} M_{n+1} &= 1.3M_n - 0.002O_nM_n \\ O_{n+1} &= 0.6O_n + 0.0004O_nM_n \end{aligned}$$

We know that when a_n is in the form $a_{n+1} = ra_n + b$

- (a) **What type of interaction is this ? How do the coefficients tell us ?:**
- (b) **Find the equilibrium values ?**
- (c) **Predict the long-term outcome if $M_0 = 1200$ and $O_0 = 100$**