MATH 3800 F

Assignment 3

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1. **Use Simpson's Rule with 4 subintervals to approximate $\int_0^1 \frac{2}{1+x^2}dx$ to 6 decimal places. Compare your result with the true value by calculating the simple error, ie $|true - approx|$.**

   4 subintervals thus $h = \frac{1-0}{4} = 0.25 \Rightarrow x_0 = 0, x_1 = 0.25, x_2 = 0.5, x_3 = 0.75, x_4 = 1$

   $\int_0^1 \frac{2}{1+x^2}dx \simeq \frac{h}{3}[\frac{2}{1+x_0^2} + 4\frac{2}{1+x_1^2} + 2\frac{2}{1+x_2^2} + 4\frac{2}{1+x_3^2} + \frac{2}{1+x_4^2}] = \frac{h}{3}[\frac{2}{1+0^2} + 4\frac{2}{1+0.25^2} + 2\frac{2}{1+0.5^2} + 4\frac{2}{1+0.75^2} + \frac{2}{1+1^2}] = 1.570784$

   True value of $\int_0^1 \frac{2}{1+x^2}dx = 2 * \arctan(1) - 2 * \arctan(0) = 1.570796$

   $Error = |true - approx| = |1.570796 - 1.570784| = 0.000012$ or 0.00076%

   Very close to 0.

2. **Use Gaussian Quadrature with 4 steps to approximate $\int_0^1 \cos^2 x dx$ to 6 decimal places**

   First change the interval from 0 to 1 $\Rightarrow -1$ to 1.

   $x = \frac{b-a}{2} * t + \frac{b+a}{2} \Rightarrow x = (1/2)t + 1/2 = (1/2)(t+1)$
   $dx = 1/2dt$

   $\int_0^1 \cos^2 x dx = \frac{1}{2}\int_{-1}^1 \cos^2(\frac{1}{2}(t+1))dt$
   $\simeq \frac{1}{2}\sum_{j=1}^4 A_j \cos^2(\frac{1}{2}(t_j+1)) = \frac{1}{2}[$
   $0.3478548451 \cos^2\left(\frac{1}{2}(0.8611363116+1)\right) +$
   $0.3478548451 \cos^2\left(\frac{1}{2}(-0.8611363116+1)\right) +$
   $0.6521451549 \cos^2\left(\frac{1}{2}(0.3399810436+1)\right) +$
   $0.6521451549 \cos^2\left(\frac{1}{2}(-0.3399810436+1)\right)] =$
   $0.727324318763 \simeq 0.727324$

   True value of $\int_0^1 \cos^2 x dx = \frac{\cos(1)\sin(1)+1}{2} - \frac{\cos(0)\sin(0)+0}{2} = 0.727324$

   $Error = |true - approx| = 0.727324 - 0.727324 = 0$. Excellent approximation.

3. **Use Monte Carlo simulation to approximate the probability of three tails occurring when four fair coins are flipped. Do a sufficient number of trials to have a meaningful result.**

   Python 3 code:

```
[7] import random

    total = 10000
    three_tails = 0
    for _ in range(total):
      coin1 = (random.getrandbits(1))
      coin2 = (random.getrandbits(1))
      coin3 = (random.getrandbits(1))
      coin4 = (random.getrandbits(1))
      if coin1 + coin2 + coin3 + coin4 == 3:
        three_tails += 1

    print(three_tails / total)
```

```
0.2498
```

The true probability of 3 tails occurring in four fair coins is $4/16 = 0.25$ and this simulation is very close to that value.

4. **Given the loaded (unfair) dice probabilities below, use Monte Carlo simulation to simulate the results of ten rolls of the dice**

result	**1**	**2**	**3**	**4**	**5**	**6**
die 1	**0.1**	**0.1**	**0.2**	**0.3**	**0.2**	**0.1**
die 2	**0.3**	**0.1**	**0.2**	**0.1**	**0.05**	**0.25**

Python 3 code:

```python
dist1 = (0.1, 0.1, 0.2, 0.3, 0.2, 0.1)
dist2 = (0.3, 0.1, 0.2, 0.1, 0.05, 0.25)

def roll(massDist):
    randRoll = random.random() # in [0,1)
    sum = 0
    result = 1
    for mass in massDist:
        sum += mass
        if randRoll < sum:
            return result
        result += 1
    return result

rolls10 = 0
for _ in range(total):
  for _ in range(10):
    rolls10 += roll(dist1)
    rolls10 += roll(dist2)


print(rolls10 / total)
```

```
69.5011
```

The average result of throwing each of the dies 10 times is 69.5011.

Given the distribution table $E(dice_1) = \sum_{i=1}^{6} P(dice_1 == i) * i = 1*0.1+2*0.1+3*0.2+4*0.3+5*0.2+6*0.1 = 3.7$

Similarly $E(dice_2) = 1 * 0.3 + 2 * 0.1 + 3 * 0.2 + 4 * 0.1 + 5 * 0.05 + 6 * 0.25 = 3.25$

Meaning if 10 rolls are done the expected value is $10 * 3.7 + 10 * 3.25 = 69.5$, very close to what was found in the simulation.