STAT 2509B4

Assignment 3

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1. **Indicate whether or not each of the following models can be treated as an multiple linear regression (MLR) model:**

   (i) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \epsilon$, can be treated as MLR

   (ii) $y = (e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2^2})\epsilon$, cannot be treated as MLR

   (iii) $y = \beta_0 + \beta_1 x_1 + \beta_2 e^{x_1} + \epsilon$, can be treated as MLR

   (iv) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_1^3 + \beta_4 x_2 + \epsilon$, can be treated as MLR

   (v) $y = \beta_0 e^{\beta_1 x_1 + \beta_2 x_2} + \epsilon$, cannot be treated as MLR

2. **A medical study was conducted to study the relationship between infants' systolic blood pressure and two explanatory variables, age (days) and weight (kg). The data for 25 infants are given below..:**

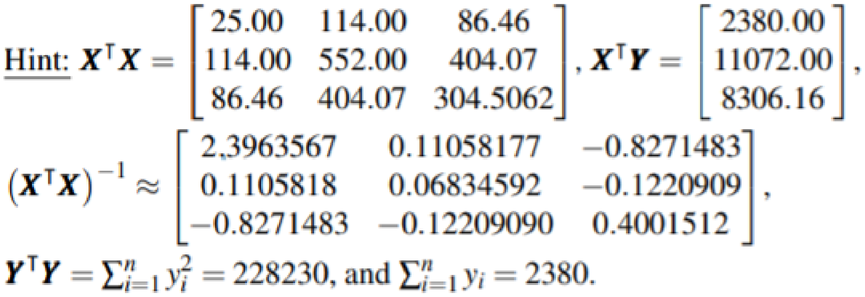
Age (x_1)	**Weight (x_2)**	**Systoli BP (y)**
3	2.61	80
4	2.67	90
5	2.98	96
6	3.98	102
3	2.87	81
4	3.41	96
5	3.49	99
6	4.03	110
3	3.41	88
4	2.81	90
5	3.24	100
6	3.75	102
3	3.18	86
4	3.13	93
5	3.98	101
6	4.55	103
3	3.41	86
4	3.35	91
5	3.75	100
6	3.83	105
3	3.18	84
4	3.52	91
5	3.49	95
6	3.81	104
6	4.03	107

   (a) **State all the assumptions that are necessary for the statistical inference under the MLR model.:**

Model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon, n = 25$
Assumptions

1. X_1, X_2 are observed without error
2. ϵ's are independently distributed
3. ϵ's have common mean 0 in other words $E(\epsilon) = 0$ for all X_1, X_2.
4. ϵ's have common/constant variance σ^2 meaning $Var(\epsilon) = \sigma^2$ for all X_1, X_2
5. $\epsilon \sim N(0, \sigma^2)$ for any value of X_1, X_2

(b) **Use matrices to compute the least-squares estimates of the population parameters β_0, β_1 and β_2, and obtain the fitted least-squares regression line:**



Hint: $\boldsymbol{X}^{\mathsf{T}}\boldsymbol{X} = \begin{bmatrix} 25.00 & 114.00 & 86.46 \\ 114.00 & 552.00 & 404.07 \\ 86.46 & 404.07 & 304.5062 \end{bmatrix}$, $\boldsymbol{X}^{\mathsf{T}}\boldsymbol{Y} = \begin{bmatrix} 2380.00 \\ 11072.00 \\ 8306.16 \end{bmatrix}$,

$(\boldsymbol{X}^{\mathsf{T}}\boldsymbol{X})^{-1} \approx \begin{bmatrix} 2{,}3963567 & 0.11058177 & -0.8271483 \\ 0.1105818 & 0.06834592 & -0.1220909 \\ -0.8271483 & -0.12209090 & 0.4001512 \end{bmatrix}$,

$\boldsymbol{Y}^{\mathsf{T}}\boldsymbol{Y} = \sum_{i=1}^{n} y_i^2 = 228230$, and $\sum_{i=1}^{n} y_i = 2380$.

$$(X^TX)^{-1}X^TY = \begin{bmatrix} 2.3963567 & 0.11058177 & -0.8271483 \\ 0.1105818 & 0.06834592 & -0.1220909 \\ -0.8271483 & -0.12209090 & 0.4001512 \end{bmatrix} * \begin{bmatrix} 2380.00 \\ 11072.00 \\ 8306.16 \end{bmatrix} = \begin{bmatrix} 57.2642 \\ 5.80416 \\ 3.31649 \end{bmatrix} = \begin{bmatrix} \hat{\beta_0} \\ \hat{\beta_1} \\ \hat{\beta_2} \end{bmatrix} = \hat{\beta}$$

Therefore the fitted least squares regression line is $\hat{y} = 57.2642 + 5.80416x_1 + 3.31649x_2$

(c) **Set up the ANOVA table and test for significance of the model at the significance level of α = 0.05**

$TSS = Y^TY - \frac{(\sum_{i=1}^n y_i)^2}{n} = \sum_{i=1}^n y_i^2 - \frac{(\sum_{i=1}^n y_i)^2}{n} = 228230 - \frac{2380^2}{25} = 1654$

$SSR = \hat{\beta}^T(X^TY) - \frac{(\sum_{i=1}^n y_i)^2}{n} = \begin{bmatrix} 57.2642 & 5.80416 & 3.31649 \end{bmatrix} * \begin{bmatrix} 2380.00 \\ 11072.00 \\ 8306.16 \end{bmatrix} - \frac{2380^2}{25} = 1523.752098$

$SSE = TSS - SSR = 1654 - 1523.752098 = 130.247902$
$MSR = \frac{SSR}{k} = \frac{1523.752098}{2} = 761.876$
$MSE = \frac{SSE}{n-(k+1)} = \frac{130.247902}{22} = 5.920359$
$F = \frac{MSR}{MSE} = \frac{761.876}{5.920359} = 128.687$

Source	d.f	SS	MS	F
Regression	2	1523.752098	761.876	128.687
Error	22	130.247902	5.920359	
Total	24	1654		

$H_0 : \beta_1 = \beta_2 = 0$
H_a : at least one of the $\beta's \neq 0$
$\alpha = 0.05$
test-statistics: $F = \frac{MSR}{MSE} = 128.687$
Rejection region, we rejected H_0 if $F > F_{(k,n-(k+1)),\alpha} = F_{2,22;0.05} = 3.4434$
Since $F = 128.687 > 3.4434$, we reject H_0 and conclude that at a 5% level of significance there is evidence to say there is a linear relationship between age, weight and the systolic BP.

(d) **Test whether age (x1) contributes to explaining (or predicting) the systolic blood pressure (y) under the MLR model. Use t-test with $\alpha = 0.05$.**

$H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$
$\alpha = \rightarrow \alpha/2 = 0.025$

test statistics: $t = \frac{\hat{\beta_1}}{\sqrt{v_{11}MSE}} = \frac{5.80416}{\sqrt{0.0683459*5.920359}} = 9.1245$
Rejection region, we reject H_0 if $|t| > t_{n-(k+1),\alpha/2} = t_{22,0.025} = 2.07383$.
Since $t = 9.1245 > 2.07$, we reject H_0 and conclude that at a 5% level of significance there is evidence to say that the x_1 term contributes to the model.

(e) **Find the values of the coefficient of determination, r^2, and the adjusted r^2. Interpret their meanings in this problem**

$r^2 = \frac{SSR}{TSS} = \frac{1523.752098}{1654} = 0.9213 = 92.125\%$
In other words approximately 92.13% of the total variation in the data is explined by the regression line. The rest is due to error.
$r^2_{adj} = 1 - \frac{SSE/n-(k+1)}{TSS/n-1} = 1 - \frac{MSE}{TSS/n-1} = 1 - \frac{5.920359}{1654/24} = 0.9141 = 91.41\%$
Since both r^2 and r^2_{adj} are quite high, around 90%, and since the x_1 term does contribute to the model, we can conclude that the model is good.

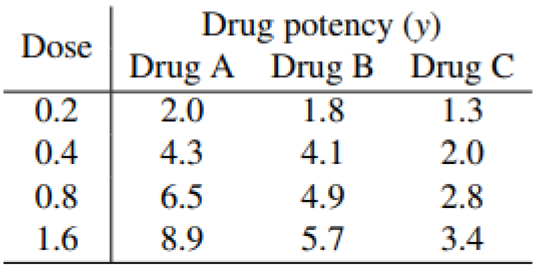
(f) **Run SAS to verify your answers to the above questions. In addition, use the SAS output to answer subquestion (d) using the partial F-test with $\alpha = 0.05$.** See attatched SAS output
$H_0 : \beta_1 = 0$
$H_a : \beta_1 \neq 0$
$\alpha = 0.05$

full model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$
reduced model: $y = \beta_0 + \beta_2 x_2 + \epsilon$

$SSR_f = 1521.53295$ with d.f $= 2$
$SSE_f = 132.46705$ with d.f $= 22$
$SSR_r = 1494.04975$ with d.f $= 1$
$SSE_r = 159.95025$ with d.f $= 23$

test statistics: $F_{part} = \frac{[SSR_f - SSR_r]/[df_{SSR_f} - df_{SSR_r}]}{SSE_f/df_{SSE_f}} = \frac{(1521.53295-1494.04975)/(2-1)}{132.46705/22} = 4.564$
Rejection Region, we reject H_0 if $F_{part} > F_{(1,22);0.05} = 4.3009$
Since $F_{part} = 4.565 > 4.3009$ we reject H_0 and conclude that at a 5% level of significance there is enough evidence to say that the X_1 (age) term contributes to the model.

3. **An experimenter wished to compare the potencies of three different drug products. To do this, 12 test tubes were inoculated with a culture of the virus under study and incubated for 2 days at 35C. Four dosage levels (0.2, 0.4, 0.8, and 1.6 mg per tube) were to be used from each of the three drug products (A, B and C), with only one dose-drug product combination for each of the 12 test-tube cultures. The data are shown in the following table**



Dose	Drug potency (y)		
	Drug A	Drug B	Drug C
0.2	2.0	1.8	1.3
0.4	4.3	4.1	2.0
0.8	6.5	4.9	2.8
1.6	8.9	5.7	3.4

Let
$x_1 = \ln(dose), x_2 = \begin{cases} 1, & \text{if drug B} \\ 0, & \text{otherwise} \end{cases}, x_3 = \begin{cases} 1, & \text{if drug C} \\ 0, & \text{otherwise} \end{cases}$ **and $y =$ drug potency. Consider the following**

MLR model. $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \epsilon$**. Run SAS to test whether the 3 lines corresponding to the effects of the 3 drugs are parallel (i.e. whether these 3 lines have the same slope). Use** $\alpha = 0.05$**.**

Full model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3 + \epsilon$

If drug A: $y = \beta_0 + \beta_1 x_1 + \epsilon$

If drug B: $y = \beta_0 + \beta_1 x_1 + \beta_2 + \beta_4 x_1 + \epsilon$

If drug C: $y = \beta_0 + \beta_1 x_1 + \beta_3 + \beta_5 x_1 + \epsilon$

Test if any of the 3 drug lines are parallel or have the same slope:
$H_0 : \beta_4 = \beta_5 = 0$
H_a : at least one of the β's $\neq 0$.
$\alpha = 0.05$

Reduced model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$

$SSR_f = 55.29350$ with d.f. $= 5$
$SSE_f = 0.68900$ with d.f. $= 6$
$SSR_r = 48.84417$ with d.f. $= 3$
$SSE_r = 7.13833$ with d.f. $= 8$

$F_{part} = \frac{[SSR_f - SSR_r]/[df_{SSR_f} - df_{SSR_r}]}{SSE_f/df_{SSE_f}} = \frac{(55.29350-48.84417)/(5-3)}{0.68900/6} = 28.081$

Rejection Region, we reject H_0 if $F_{part} > F_{(2,6);0.05} = 5.14$.

Since $F_{part} = 28.081 > 5.14$ we reject H_0 and conclude that at a 5% level of significance there is enough evidence to say that the slopes of the 3 drug lines are not parallel.