Math 448 Project 1B

1 Introduction

The purpose of this project is to use a binomial tree more realistically than what was done for the earlier homework assignment. That is to make a tree with many more nodes and to notice the improvements to the error that result from more nodes. You may use any programming language you want, or even do it in excel, but you must include your code/screen shots of excel commands with your answers to the questions at the end of the project.

2 European Option Prices according to the Black-Scholes solution

Before programming the binomial tree it is a good idea to program the Black-Scholes solution for the European call option. This will give us a a value to compare the binomial tree approximation against.

Consider

$$\begin{aligned} C &= \text{ current price of a European call option on a non-dividend-paying stock} \\ S_0 &= \text{ current stock price} \\ K &= \text{ strike price of option} \\ r &= \text{ constant risk-free interest rate} \\ \sigma &= \text{ constant volatility of the stock price} \\ T &= \text{ time to maturity.} \end{aligned}$$

Then according the The Black-Scholes formula[1],which will be covered later in our class in chapter 8, the price of a European call option is

$$C = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2),$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ d_2 &= \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \end{aligned}$$

Φ is the cumulative distribution function of the standard normal distribution. The price for the put option can be found in our book or you can just use the put-call parity to find it. We will cover this formula and where it comes from in class, but for the purposes of this project it is fine to think of it as a black box where you plug in parameter values and it spits out the option price.

[1]Black, Fisher S.; Scholes, Myron S.; The Pricing of Options and Corporate Liabilities, Journal of Political Economy, Vol. 81, No. 3 (May-June 1973), pp. 637-654

3 Question

1. Using the definition of the percentage up movement $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$ and

$$\begin{aligned} K &= 10 \\ r &= 0.02 \\ \sigma &= 0.25 \\ T &= 0.25 \\ S_0 &= 10 \end{aligned}$$

Perform the following steps.

(a) Use the binomial tree method to compute the price, C, for number of time steps $N = 10,\ 100,\ 1,000$, and 10,000.

(b) Use the Black-Scholes formula to compute the price, C.

(c) Let $|E|$=|Difference Between Binomial Tree and Black-Scholes Solutions| and use the results from parts (a) and (b) to fill in the table below.

Number of Timesteps	Binomial Tree Solution	$\lvert E\rvert$
N=10		
N=100		
N=1.000		
N=10.000		

(d) Plot the $\ln|E|$ vs $\ln N$ and find the least squares line for the data points,

$$\ln|E| = A\ln N + B$$

you may use R, excel or any other program to compute the formula for the line for you.

(e) In the regression formula found in part (d) A and B are consants determined by the data from the table in part(c). You may algebraically manipulate the regrssion to arrive at

$$|E| = e^{A\ln N + B} = e^B e^{A\ln N} = e^B e^{\ln N^A} = e^B \cdot N^A = \frac{e^B}{N^{-A}}.$$

Now $|E| = \frac{e^B}{N^{-A}}$ with e^B being constant while N^A varies for different values of N. This means your the regression coefficient A is determining how quickly the difference between the Black-Scholes solution and and the Binomial Solution converge as N increases. We call $-A$ the convergence rate of the Binomial Tree Method. For example if $-A = 1$ that would mean that doubling the number of timesteps would halve the error of our method while a $-A = 2$ would mean that doubling the number of timesteps would quarter the error of our method. State the apparent convergence rate of the Binomial Tree method from the regression coefficient found in the previous part.