Appendix 5M

Compound Mean and Variance

For a collective risk model,

$$S = \sum_{i=1}^{N} X_i, \qquad N = 1, 2, \ldots$$

The compound mean and variance can be calculated using Law of Total Expectation and Law of Total Variance.

$$\begin{aligned}
\mathbf{E}[S] &= \mathbf{E}[\mathbf{E}[S \mid N]] \\
&= \mathbf{E}\left[\mathbf{E}\left[\sum_{i=1}^{N} X_i \,\middle|\, N\right]\right] \\
&= \mathbf{E}\left[\sum_{i=1}^{N} \mathbf{E}[X_i \mid N]\right] \\
&= \mathbf{E}[N \cdot \mathbf{E}[X]] \\
&= \mathbf{E}[N]\,\mathbf{E}[X]
\end{aligned}$$

$$\begin{aligned}
\mathbf{Var}[S] &= \mathbf{E}[\mathbf{Var}[S \mid N]] + \mathbf{Var}[\mathbf{E}[S \mid N]] \\
&= \mathbf{E}\left[\mathbf{Var}\left[\sum_{i=1}^{N} X_i \,\middle|\, N\right]\right] + \mathbf{Var}\left[\mathbf{E}\left[\sum_{i=1}^{N} X_i \,\middle|\, N\right]\right] \\
&= \mathbf{E}\left[\sum_{i=1}^{N} \mathbf{Var}[X_i \mid N]\right] + \mathbf{Var}\left[\sum_{i=1}^{N} \mathbf{E}[X_i \mid N]\right] \\
&= \mathbf{E}[N \cdot \mathbf{Var}[X]] + \mathbf{Var}[N \cdot \mathbf{E}[X]] \\
&= \mathbf{E}[N]\,\mathbf{Var}[X] + \mathbf{Var}[N]\,\mathbf{E}[X]^2
\end{aligned}$$