Summary 10M

Transformation

1. Determine the CDF of X.
2. Restate CDF of Y using CDF of X.
3. Determine the PDF of Y.

Continuous Distributions

Only special properties are included here. The PDF, CDF, mean, and variance formulas are not listed because they are either provided on the exam table or can be derived from other quantities.

PARETO

The S-P Pareto distribution is a Pareto distribution shifted rightwards by θ. Thus, its mean is θ greater than the Pareto's mean. They have the same variance.

GAMMA

The exponential distribution is a special case of the gamma distribution with $\alpha = 1$.

The sum of n **i.i.d.** exponential random variables follows a gamma distribution with parameters $\alpha = n$ and θ.

The exponential distribution is memoryless.

BETA

A beta distribution with parameters $a = b = 1$ and θ is equivalent to a uniform distribution on the interval $[0,\ \theta]$.

NORMAL

Distribution	CDF	Mean	Variance	Percentile
Standard normal, Z	$\Phi(z)$	0	1	$z_p = \Phi^{-1}(p)$
Normal, Y	$\Phi\left(\dfrac{y-\mu}{\sigma}\right)$	μ	σ^2	$\mu + z_p\sigma$
Lognormal, X	$\Phi\left(\dfrac{\ln x-\mu}{\sigma}\right)$	$e^{\mu+0.5\sigma^2}$	$\mathrm{E}[X]^2\left(e^{\sigma^2}-1\right)$	$e^{\mu+z_p\sigma}$

A lognormal distribution is a log-transformed normal distribution:

$$X = e^Y$$

INVERSE DISTRIBUTIONS

If a random variable X follows a certain distribution that has a parameter θ and an inverse counterpart, then X^{-1} follows the inverse counterpart with the same parameters, except θ is inverted.

USEFUL SHORTCUTS

$\boldsymbol{X}$	$\boldsymbol{X-d \mid X>d}$
Pareto $(\alpha,\ \theta)$	Pareto $(\alpha,\ \theta+d)$
Exponential (θ)	Exponential (θ)
Uniform $(a,\ b)$	Uniform $(0,\ b-d)$

Linear Exponential Family

Any distribution with a probability function that can be expressed in the form below, where the support does **not** depend on the parameter θ, belongs to the linear exponential family:

$$f(x;\theta)=\frac{p(x)e^{\,r(\theta)\cdot x}}{q(\theta)}$$

Roles of parameters

All distributions listed in the exam tables that have a θ parameter (except inverse Gaussian) are parameterized such that θ is the scale parameter.

Most of the other parameters are shape parameters, which affect the general shape of the distributions.

SCALING

- To scale any continuous distribution listed on the exam tables except lognormal, inverse Gaussian and log-t, multiply the parameter θ by the scaling factor.
- To scale a normal distribution, multiply the mean and the standard deviation by the scaling factor.
- To scale a lognormal distribution, add the natural log of the scaling factor to the parameter μ.
- To scale all other distributions, use the transformation technique discussed at the beginning of this section.

Discrete Mixtures

A random variable Y is a discrete mixture of the random variables X_1, X_2, ..., X_n if its PDF is given by:

$$f_Y(y) = \sum_{i=1}^{n} w_i \cdot f_{X_i}(y),\ \text{where}\ \sum_{i=1}^{n} w_i = 1$$

The following equations are also true.

$$F_Y(y) = \sum_{i=1}^{n} w_i \cdot F_{X_i}(y)$$

$$S_Y(y) = \sum_{i=1}^{n} w_i \cdot S_{X_i}(y)$$

$$\mathrm{E}\left[Y^k\right] = \sum_{i=1}^{n} w_i \cdot \mathrm{E}\left[X_i^k\right]$$

Remember that a mixture is different from a linear combination of random variables!

BERNOULLI SHORTCUT

If X is a random variable that can only take on two values a and b, i.e.,

$$X = \begin{cases} a, & \text{Probability} = q \\ b, & \text{Probability} = 1 - q \end{cases}$$

then

$$\mathrm{Var}[X] = (a - b)^2 q(1 - q)$$