Coinsurance

30M

Coinsurance is the portion of loss the **insurer** is responsible for. Suppose a policy has a coinsurance of α, where $0 < \alpha < 1$. For a loss of amount X, the insurer will pay αX while the policyholder will pay the rest, $(1-\alpha)X$.

Based on the expected value property covered in Section S2.1.4, the expected payment is simply the expected loss multiplied by the coinsurance.

$$\mathrm{E}[\alpha X] = \alpha \cdot \mathrm{E}[X]$$

In the case where a policy has both a **deductible** and **coinsurance**, there are two methods for handling the coinsurance: **before** or **after** the deductible is met.

By **default**, the coinsurance only comes into play **after** the deductible is met. The payment per loss variable is as shown below. For a policy with deductible d and coinsurance α,

$$Y^L = \begin{cases} 0, & X \le d \\ \alpha\,(X-d), & X > d \end{cases}$$

The policyholder pays the first d of the loss, and shares the excess amount with the insurer. Then, the average payment is

$$\mathrm{E}\left[Y^L\right] = \alpha\,(\mathrm{E}[X] - \mathrm{E}[X \wedge d])$$

However, questions may sometimes specify that the coinsurance is applied **before** the deductible. That means instead of applying the entire loss X against the deductible, the insurer will only apply the coinsured portion of the loss, αX. The policyholder will still be responsible for the remaining portion, i.e. $(1-\alpha)X$, plus the amount needed to meet the deductible.

The payment per loss variable is as shown below. For a policy with deductible d and coinsurance α,

$$Y^L = \begin{cases} 0, & \alpha X \leq d \\ \alpha X - d, & \alpha X > d \end{cases}$$

Notice the equation can be expressed such that we will get the exact same form as the case where coinsurance is applied **after** the deductible is met, if we define the deductible as $d^* = \frac{d}{\alpha}$ instead.

$$Y^L = \begin{cases} 0, & X \leq \dfrac{d}{\alpha} \\ \alpha\left(X - \dfrac{d}{\alpha}\right), & X > \dfrac{d}{\alpha} \end{cases}$$

$$= \begin{cases} 0, & X \leq d^* \\ \alpha\,(X - d^*), & X > d^* \end{cases}$$

In conclusion, if the question specifies that the coinsurance is applied **before** the deductible, adjust the deductible by dividing it by the coinsurance. Then, solve it the same way as the case where the coinsurance is applied **after**. Thus, the average payment is

$$\mathrm{E}\left[Y^L\right] = \alpha\left(\mathrm{E}[X] - \mathrm{E}[X \wedge d^*]\right)$$

where $d^* = \dfrac{d}{\alpha}$.

Coach's Remarks

Since only αX contributes to meeting the deductible, a loss of $X = \frac{d}{\alpha}$ is required to meet the deductible, i.e. $\alpha\left(\frac{d}{\alpha}\right) = d$.

In other words, $\frac{d}{\alpha}$ is the point that defines whether the loss does or does not meet the deductible. Therefore, only when the loss is greater than $\frac{d}{\alpha}$ will the insurer start sharing the cost.

Now that we have discussed each coverage modification individually, let's look at how they can be combined. There will be multiple examples at the end to demonstrate the most common cases.

Here is the general formula for the expected payment per loss when all of the coverage modifications we have covered are included. We will modify and apply it to each of the examples below.

$$\mathrm{E}\left[Y^L\right] = \alpha \cdot \{\mathrm{E}[X \wedge m] - \mathrm{E}[X \wedge d]\} \qquad \text{(S2.3.3.1)}$$

where

- X is the loss variable,
- u is the policy limit (set $u = \infty$ if policy limit doesn't apply),
- d is the deductible (set $d = 0$ if deductible doesn't apply),
- α is the coinsurance (set $\alpha = 1$ if coinsurance doesn't apply), and
- m is the maximum covered loss and is equal to $\dfrac{u}{\alpha} + d$.

The derivation of the formula is provided in the appendix at the end of this section.

Notice the first limited expected value is evaluated at the **maximum covered loss**, rather than the policy limit. Many students make this mistake.

The maximum covered loss is the loss amount above which the insurer pays the policy limit, i.e., when $X \geq m$, $Y^L = u$. As a result, there is a maximum covered loss only when there is a policy limit.

If a policy only has a policy limit, then

$$m = u$$

If a policy has a policy limit and a deductible, then

$$m = u + d$$

If a policy has a policy limit, a deductible, and coinsurance, then

$$m = \frac{u}{\alpha} + d$$

Coach's Remarks

$(X \wedge \infty)$ means that there is no cap on the loss X. Thus, the limited expectation is simply the expected loss.

$$\mathrm{E}[X \wedge \infty] = \int_0^\infty S(x)\,\mathrm{d}x = \mathrm{E}[X]$$

$(X \wedge 0)$ means the loss X is capped at 0. Thus, the limited expectation is 0.

$$\mathrm{E}[X \wedge 0] = \int_0^0 S(x)\,\mathrm{d}x = 0$$

Therefore, the expected payment for an insurance policy with deductible 0, $\mathrm{E}\left[(X-0)_+\right]$, equals the expected loss.

$$\begin{aligned} \mathrm{E}\left[(X-0)_+\right] &= \mathrm{E}[X] - \mathrm{E}[X \wedge 0] \\ &= \mathrm{E}[X] \end{aligned}$$

Example S2.3.3.1

For an insurance policy, you are given:

- The claim severity follows a Pareto distribution with parameters $\alpha = 2$ and $\theta = 1000$.
- The policy has an ordinary deductible of 200 and a policy limit of 2,000.

Calculate the expected insurance payment per loss.

Solution

Start by modifying (S2.3.3.1). Substitute $d = 200$, $u = 2{,}000$, and $\alpha = 1$ because there is no coinsurance. The modified formula is

$$\mathrm{E}\left[Y^L\right] = \mathrm{E}[X \wedge 2{,}200] - \mathrm{E}[X \wedge 200]$$

Look up the Pareto distribution's limited expectation formula in the exam table.

$$\begin{aligned} \mathrm{E}[X \wedge 2{,}200] &= \frac{1{,}000}{2-1}\left[1 - \left(\frac{1{,}000}{2{,}200 + 1{,}000}\right)^{2-1}\right] \\ &= 687.50 \end{aligned}$$

$$\begin{aligned} \mathrm{E}[X \wedge 200] &= \frac{1{,}000}{2-1}\left[1 - \left(\frac{1{,}000}{200 + 1{,}000}\right)^{2-1}\right] \\ &= 166.67 \end{aligned}$$

Calculate the expected payment per loss.

$$\begin{aligned} \mathrm{E}\left[Y^L\right] &= 687.50 - 166.67 \\ &= \mathbf{520.83} \end{aligned}$$

Coach's Remarks

The table below illustrates why the expected payment per loss expression above is correct.

	$X \wedge 2{,}200$	$X \wedge 200$	Y^L
$X \le 200$	X	X	0
$200 < X < 2{,}200$	X	200	$X - 200$
$X \ge 2{,}200$	2,200	200	2,000

Subtracting column 3 from column 2 will produce the same outcome as the payment per loss variable, i.e.

$$Y^L = (X \wedge 2{,}200) - (X \wedge 200)$$

Therefore, the expected payment per loss is

$$\mathrm{E}\left[Y^L\right] = \mathrm{E}[X \wedge 2{,}200] - \mathrm{E}[X \wedge 200]$$

Example S2.3.3.2

Losses follow an exponential distribution with mean 1,000.

For an insurance policy:

- Each claim is subject to an ordinary deductible of 200.
- The policy will reimburse 80% of the claims in excess of 200.
- The insurance payment is capped at 2,000.

Calculate the expected payment per loss.

Solution

Modify (S2.3.3.1) by setting $u = 2{,}000$, $d = 200$ and $\alpha = 0.8$. The resulting formula is

$$\mathrm{E}\left[Y^L\right] = 0.8 \cdot (\mathrm{E}[X \wedge 2{,}700] - \mathrm{E}[X \wedge 200])$$

We need to compute the limited expected values.

$$\begin{aligned}\mathrm{E}[X \wedge 2{,}700] &= 1{,}000\left(1 - e^{-2{,}700\,/\,1{,}000}\right) \\ &= 932.79\end{aligned}$$

$$\begin{aligned}\mathrm{E}[X \wedge 200] &= 1{,}000\left(1 - e^{-200\,/\,1{,}000}\right) \\ &= 181.27\end{aligned}$$

Substitute the limited expected values into the formula to calculate the final answer.

$$\begin{aligned}\mathrm{E}\left[Y^L\right] &= 0.8 \cdot (932.79 - 181.27) \\ &= \mathbf{601.22}\end{aligned}$$

Coach's Remarks

Let's confirm that the expected payment per loss expression is correct.

- For losses below 200, $Y^L = 0$.

- For losses between 200 and 2,700, Y^L is 80% of the amount in excess of 200, i.e. $Y^L = 0.8(X - 200)$.
- For losses greater than 2,700, Y^L is capped at the policy limit, i.e. 2,000.

	$\mathbf{0.8\,(X \wedge 2{,}700)}$	$\mathbf{0.8\,(X \wedge 200)}$	$\boldsymbol{Y^L}$
$\boldsymbol{X \le 200}$	$0.8X$	$0.8X$	0
$\boldsymbol{200 < X < 2{,}700}$	$0.8X$	$0.8(200)$	$0.8(X - 200)$
$\boldsymbol{X \ge 2{,}700}$	$0.8(2{,}700)$	$0.8(200)$	$2{,}000$

Subtracting column 3 from column 2 produces the last column, as intended.

Example S2.3.3.3

Losses follow a gamma distribution with parameters $\alpha = 2$ and $\theta = 100$.

An insurance policy covers the loss with a policy limit of 250. The insurance company is offering an alternative coverage to replace the policy limit with coinsurance c, which is the proportion of the loss paid by the insurance company, such that the expected insurance cost remains the same.

Calculate c.

Solution

Let X represent the losses.

$$X \sim \text{Gamma}\ (2,\ 100)$$

For the coverage with a policy limit of 250, modify (S2.3.3.1) by setting $u = 250$, $d = 0$ and $\alpha = 1$:

$$\mathrm{E}[X \wedge 250]$$

For the alternative coverage with coinsurance c, modify $(\mathrm{S2.3.3.1})$ by setting $u = \infty$, $d = 0$ and $\alpha = c$:

$$c \cdot \mathrm{E}[X]$$

Equate the two expected payments. The goal is to solve for c.

$$c \cdot \mathrm{E}[X] = \mathrm{E}[X \wedge 250]$$

First, calculate the mean.

$$\begin{aligned}\mathrm{E}[X] &= 2(100)\\ &= 200\end{aligned}$$

Next, calculate the limited expected value. The formula is provided in the exam table. Substitute $k = 1$ into the formula to get:

$$\mathrm{E}[X \wedge u] = \alpha\theta\Gamma(\alpha + 1;\, u/\theta) + u\left[1 - \Gamma(\alpha;\, u/\theta)\right]$$

Substitute the parameters and policy limit into the formula. We will need to evaluate incomplete gamma functions.

$$\mathrm{E}[X \wedge 250] = 2(100)\Gamma(3;\, 2.5) + 250\left[1 - \Gamma(2;\, 2.5)\right]$$

To evaluate the incomplete gamma functions, recall the shortcut introduced in Section S2.2.4. Start by defining a Poisson variable.

$$N \sim \text{Poisson}\left(\lambda = \frac{250}{100} = 2.5\right)$$

Then,

$$\begin{aligned}\Gamma(3;\ 2.5) &= 1 - \Pr(N < 3) \\ &= 1 - [p_N(0) + p_N(1) + p_N(2)] \\ &= 1 - \left[e^{-2.5} + e^{-2.5}(2.5) + \frac{e^{-2.5}(2.5)^2}{2}\right] \\ &= 0.4562\end{aligned}$$

$$\begin{aligned}\Gamma(2;\ 2.5) &= 1 - \Pr(N < 2) \\ &= 1 - [p_N(0) + p_N(1)] \\ &= 1 - \left[e^{-2.5} + e^{-2.5}(2.5)\right] \\ &= 0.7127\end{aligned}$$

Substitute the calculated values into the limited expected value formula.

$$\begin{aligned}\mathrm{E}[X \wedge 250] &= 2(100)(0.4562) + 250(1 - 0.7127) \\ &\approx 163\end{aligned}$$

Lastly, solve for c.

$$\begin{aligned}c(200) &\approx 163 \\ c &\approx \mathbf{81.5\%}\end{aligned}$$

Coach's Remarks

The incomplete gamma functions can also be evaluated by integrating the gamma PDF. Integration by parts is required. Knowing the Poisson

Example S2.3.3.4

An insurance company will provide a financial incentive to its insurance agents if the total incurred losses for the year are less than 500.

The bonus will be 15% of the amount by which the total incurred losses are under 500.

The total losses in a year follow a Pareto distribution with parameters $\alpha = 3$ and $\theta = 400$.

Calculate the expected bonus.

Solution

Start by defining the bonus random variable.

Let X be the incurred loss amount.

If the incurred losses are less than 500, the bonus is 15% of the difference between the incurred losses and 500.

If the incurred losses are greater than 500, no bonus is paid.

Thus, the bonus variable is

$$B = \begin{cases} 0.15(500 - X), & X < 500 \\ 0, & X \geq 500 \end{cases} = 0.15(500 - X)_+$$

The expected bonus is

$$\begin{aligned} \mathrm{E}[B] &= 0.15\,\mathrm{E}\left[(500 - X)_+\right] \\ &= 0.15\left(\mathrm{E}[500] - \mathrm{E}[500 \wedge X]\right) \end{aligned}$$

$(500 \wedge X)$ can be expressed as $\min(500,\ X)$. The order of the items in the parentheses does not matter. Thus, it can also be expressed as $\min(X,\ 500)$, or $(X \wedge 500)$, which means

$$\begin{aligned}
\mathrm{E}[B] &= 0.15\left(\mathrm{E}[500] - \mathrm{E}[X \wedge 500]\right) \\
&= 0.15\left\{500 - \frac{400}{3-1}\left[1 - \left(\frac{400}{500+400}\right)^{3-1}\right]\right\} \\
&= \mathbf{50.93}
\end{aligned}$$

Coach's Remarks

There is not one specific type of ***bonus*** question. Although they are usually similar to this example, they can take many different forms. But don't fret! As long as the bonus random variable is defined correctly, the expected bonus will boil down to computing limited expected values.

On a similar note, although we only focused on ordinary and franchise deductibles, the concepts surrounding expected payment are also applicable to other types of deductibles. Like with this example, the key to successfully calculating the expected payment is by correctly defining the payment structure.