Bornhuetter-Ferguson Method

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A reserving actuary often estimates ultimate losses using several methods. Then, by rationalizing the differences among the results, the actuary arrives at the final value. The ***Bornhuetter-Ferguson method*** is a product of this practice. It is able to find a balance between the chain-ladder method, which relies on developing the loss triangle, and the expected loss ratio method, which only considers losses that have fully developed.

To see how this is done, first recall that under the chain-ladder method, we have

$$\hat{L}^{\text{ult.}} = L^P \cdot f^{\text{ult.}}$$

which implies that the estimated proportion of paid claims is

$$\frac{L^P}{\hat{L}^{\text{ult.}}} = \frac{1}{f^{\text{ult.}}}$$

This makes

$$1 - \frac{1}{f^{\text{ult.}}}$$

the estimated proportion of **unpaid** claims.

Therefore, the unpaid claims estimate, or reserve, can be rewritten as the ultimate losses multiplied by the estimated proportion of unpaid claims.

$$R = \hat{L}^{\text{ult.}} \left(1 - \frac{1}{f^{\text{ult.}}}\right)$$

Coach's Remarks

If the age-to-ultimate factor is calculated using the **incurred** losses, then

$$\hat{L}^{\text{ult.}}\left(1-\frac{1}{f^{\text{ult.}}}\right)$$

will be the **IBNR reserve**.

Then, the Bornhuetter-Ferguson method combines the expected loss ratio method with the version of the reserve calculation shown above.

This is done by multiplying the expected loss ratio estimate of the ultimate losses by the chain-ladder estimate of the proportion of unpaid claims.

$$R_{BF}=\hat{L}_{LR}^{\text{ult.}}\left(1-\frac{1}{f_{CL}^{\text{ult.}}}\right) \tag{S5.1.3.1}$$

where

- based on the chain-ladder method, the **paid** loss age-to-ultimate factor is

$$f_{CL}^{\text{ult.}}=\prod_{k=t+1}^{\infty} f_k$$

  Note that t is the age of claims at valuation.

- based on the expected loss ratio method, the ultimate losses are

$$\hat{L}_{LR}^{\text{ult.}}=P^E\times ELR$$

Coach's Remarks

Note that $P^E \times ELR$ is **not** the Bornhuetter-Ferguson estimate of the ultimate losses.

To estimate the ultimate losses using the Bornhuetter-Ferguson method, simply sum the losses paid-to-date and the losses to be paid (i.e. reserve).

$$\hat{L}_{BF}^{\text{ult.}} = L^P + R_{BF}$$

Consider the following example.

You're given the following:

Accident Year	Earned Premium	Expected Loss Ratio
AY4	6,000	0.64
AY5	5,400	0.65
AY6	7,300	0.55
AY7	7,700	0.59

Cumulative Loss Payments				
Accident Year	Development Year			
	0	1	2	3
AY4	1,400	2,550	3,650	3,800
AY5	1,550	2,750	3,350	
AY6	1,650	2,900		
AY7	1,850			

Assume no loss development past Development Year 3.

Using the Bornhuetter-Ferguson method and volume-weighted average factor model, estimate the reserve as of 12/31/AY7.

Step 1: Estimate the ultimate losses using the expected loss ratio method.

To do this, multiply the earned premium by the expected loss ratio for each accident year.

Accident Year	Estimated Ultimate Losses
AY4	3,840
AY5	3,510
AY6	4,015
AY7	4,543

Step 2: Estimate the age-to-ultimate loss development factors using the chain-ladder method.

Note that the question tells us to use the volume-weighted average model to find the age-to-age factors.

- $f_1 = \dfrac{2{,}550 + 2{,}750 + 2{,}900}{1{,}400 + 1{,}550 + 1{,}650} = 1.783$
- $f_2 = \dfrac{3{,}650 + 3{,}350}{2{,}550 + 2{,}750} = 1.321$
- $f_3 = \dfrac{3{,}800}{3{,}650} = 1.041$

Then, we can calculate the age-to-ultimate factors.

Accident Year	Age as of 12/31/AY7	Age-to-Ultimate Factor
AY4	3	1.000
AY5	2	1.041
AY6	1	1.375
AY7	0	2.451

Step 3: Estimate the reserves using (S5.1.3.1).

Accident Year	Reserve
AY5	139
AY6	1,095
AY7	2,690

The total reserve is the sum of the Reserve column.

$$139 + 1{,}095 + 2{,}690 = \mathbf{3{,}923}$$

The Bornhuetter-Ferguson reserve can also be calculated as the weighted average of the chain-ladder and expected loss ratio reserves.

$$R_{BF} = w \cdot R_{CL} + (1 - w) \cdot R_{LR} \tag{S5.1.3.2}$$

where

- R_{BF} denotes the Bornhuetter-Ferguson reserve;
- R_{CL} denotes the chain-ladder reserve;
- R_{LR} denotes the expected loss ratio reserve;
- $w = \dfrac{1}{f_{CL}^{\text{ult.}}}$.

Let's try this out with the example above. We get the following expected loss ratio reserves:

Accident Year	Estimated Ultimate Losses	Losses Paid-to-Date	Reserve, R_{LR}
AY4	3,840	3,800	40
AY5	3,510	3,350	160
AY6	4,015	2,900	1,115
AY7	4,543	1,850	2,693

Then, we can calculate the chain-ladder reserves as:

Accident Year	Estimated Ultimate Losses	Losses Paid-to-Date	Reserve, R_{CL}
AY4	3,800	3,800	0
AY5	3,488	3,350	138
AY6	3,988	2,900	1,088
AY7	4,535	1,850	2,685

We also get the following weights:

Accident Year	Age-to-Ultimate Factor	Weight, w
AY4	1.000	1.000
AY5	1.041	0.961
AY6	1.375	0.727
AY7	2.451	0.408

Finally, calculate the Bornhuetter-Ferguson reserves using (S5.1.3.2). Notice the reserves match the ones calculated in Step 3 above.

Accident Year	w	R_{CL}	$1-w$	R_{LR}	R_{BF}
AY4	1	0	0	40	0
AY5	0.961	138	0.039	160	139
AY6	0.727	1,088	0.273	1,115	1,095
AY7	0.408	2,685	0.592	2,693	2,690

One observation we can make is that as claims approach maturity (i.e. as DY increases), the age-to-ultimate factors decrease and the weights assigned to the chain-ladder reserves increase. This should make logical sense. The closer the claims are to maturity, the more data is available on those claims. Therefore, higher weight is given to reserves calculated using actual claims experience, i.e. R_{CL}.

Let's practice with a couple examples.

Example S5.1.3.1

You're given the following:

Age	Age-to-Ultimate Factor
0	2.422
1	1.450
2	1.209
3	1.000

Accident Year	Earned Premium	Expected Loss Ratio
AY1	1,150	0.74
AY2	1,250	0.71
AY3	1,200	0.72
AY4	1,500	0.69

Estimate the loss reserve at the end of AY4 using the Bornhuetter-Ferguson method.

Solution

- For AY1, the reserve is

$$1{,}150\,(0.74)\left(1-\frac{1}{1.000}\right)=0$$

- For AY2, the reserve is

$$1{,}250\,(0.71)\left(1-\frac{1}{1.209}\right)=153$$

- For AY3, the reserve is

$$1{,}200\,(0.72)\left(1-\frac{1}{1.450}\right)=268$$

- For AY4, the reserve is

$$1{,}500\,(0.69)\left(1-\frac{1}{2.422}\right)=608$$

Sum to get the final answer.

$$0+153+268+608=\mathbf{1{,}029}$$

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Example S5.1.3.2

For a block of business, you're given:

- Bornhuetter-Ferguson reserve = 77,900
- Expected loss ratio reserve = 78,600
- Earned premium = 250,000
- Expected loss ratio = 0.65

Determine the chain-ladder reserve.

Solution

To calculate the chain-ladder reserve, we need the losses paid-to-date and the age-to-ultimate factor.

Using (S5.1.3.1), we can determine the age-to-ultimate factor.

$$77{,}900 = 250{,}000\,(0.65)\left(1 - \frac{1}{f^{\text{ult.}}}\right) \Downarrow f^{\text{ult.}} = 1.9208$$

Recall that we can find the losses paid-to-date using the expected loss ratio method.

$$R = \left(P^E \times ELR\right) - L^P$$

$$78{,}600 = 250{,}000\,(0.65) - L^P$$

$$\Downarrow$$

$$L^P = 83{,}900$$

Then, the chain-ladder estimate of the ultimate losses is

$$83{,}900\,(1.9208) = 161{,}155$$

Finally, the chain-ladder reserve is

$$161{,}155 - 83{,}900 = \mathbf{77{,}255}$$

Alternative Solution