Summary 5M

Value-at-Risk (VaR)

For a continuous random variable X, its VaR at the 100p% level is:

$$\mathrm{VaR}_p(X) = \pi_p = F_X^{-1}(p)$$

The VaR formula for most distributions can be found on the exam table.

Tail-Value-at-Risk (TVaR)

For a continuous random variable X, its TVaR at the 100p% level is:

$$\begin{aligned}\mathrm{TVaR}_p(X) &= \mathrm{E}[X \mid X > \mathrm{VaR}_p(X)] \\ &= \frac{\displaystyle\int_{\mathrm{VaR}_p(X)}^{\infty} x \cdot f(x)\,\mathrm{d}x}{1-p}\end{aligned}$$

The TVaR formulas for normal and lognormal distributions are:

	$\mathbf{TVaR_p(X)}$
Normal	$\mu + \sigma\left[\frac{\phi(z_p)}{1-p}\right]$
Lognormal	$\mathrm{E}[X] \cdot \left[\frac{\Phi(\sigma - z_p)}{1-p}\right]$

Coherence

A risk measure is coherent if it satisfies **all** properties below:

- Translation invariance

$$\rho(X+c) = \rho(X) + c$$

- Positive homogeneity

$$\rho(cX) = c \cdot \rho(X)$$

- Subadditivity

$$\rho(X+Y) \leq \rho(X) + \rho(Y)$$

- Monotonicity

$$\rho(X) \leq \rho(Y), \quad \text{if } \Pr(X \leq Y) = 1$$

Also, note that:

- VaR is not coherent because it fails to satisfy the subadditivity property.
- TVaR is coherent.

Tail Weight

The **fewer** positive raw moments that exist, the **greater** the tail weight.