Chain-Ladder Method

60M

The ***chain-ladder method***, a.k.a. ***loss development method*** or ***loss triangle method***, is another popular loss reserving method. This method observes patterns in historical loss development and assumes that future losses will continue to develop in a similar manner.

Consider the following example.

Suppose we have the following incremental paid loss data (in thousands) for a line of business.

Accident Year	Development Year						
	0	**1**	**2**	**3**	**4**	**5**	**6**
2011	590	490	380	200	70	70	30
2012	590	370	260	90	40	20	
2013	690	750	430	270	170		
2014	790	610	440	170			
2015	750	660	450				
2016	770	800					
2017	870						

Assume no loss development after Development Year 6.

Estimate the loss reserve as of December 31, 2017, using the chain-ladder method and the average factor model.

This method is called the loss triangle method because of how the data is arranged. As you can see in the table above, the historical data forms a triangle at the top. This triangle is also called a run-off triangle. For this method, the goal is to project values for the lower triangle, i.e. the empty cells. These values will represent the reserve.

To start, let's denote the incremental losses as

$$l_{i,k}$$

where i represents the accident year and k represents the development year. For example, $l_{2013,2} = 430$ is the loss for accident year 2013 and Development Year 2.

Then, to read the table, it is important to understand its three dimensions:

- Rows
- Columns
- Diagonals

ROW

Each row represents one **accident year (AY)**. For example, row 2012 records the payments made on claims from AY2012, i.e. claims that occurred in 2012. Note that the payments were not all made in 2012.

Thus, the total payment that has been made on claims from AY2012 is

$$\begin{aligned}\sum_{\text{all } k} l_{2012,k} &= 590 + 370 + 260 + 90 + 40 + 20 \\ &= 1{,}370\end{aligned}$$

COLUMN

Each column represents one **development year (DY)**, which indicates the **age** of the claim. For example, column 0 records payments that were made in the calendar year in which the claims occurred. Column 1 records payments that were made in the next calendar year.

For example, $l_{2014,3} = 170$ is the payment made in DY3 on claims that occurred in AY2014.

DIAGONAL

Each upward-sloping diagonal represents one **calendar year (CY)**. For example, $l_{2011,\,2}$ is the payment made in DY2 on claims from AY2011, so the payment took place in calendar year $2011+2=2013$. Additionally, $l_{2013,\,0}$ and $l_{2012,\,1}$ also took place in 2013.

Therefore, the total payment made in CY2013 is

$$\begin{aligned}\sum_{i+k=2013} l_{i,\,k} &= l_{2013,\,0}+l_{2012,\,1}+l_{2011,\,2} \\ &= 690+370+380 \\ &= 1{,}440\end{aligned}$$

Accident Year	Development Year						
	0	1	2	3	4	5	6
2011	590	490	380	200	70	70	30
2012	590	370	260	90	40	20	
2013	690	750	430	270	170		
2014	790	610	440	170			
2015	750	660	450				
2016	770	800					
2017	870						

Now that we understand how the table is set up, we can analyze the data. Recall that our goal is to calculate the reserve by projecting ultimate losses and then subtracting losses paid-to-date.

The chain-ladder method estimates these future losses by observing loss development patterns, i.e. the change in cumulative losses. Therefore, before we can apply this method, we first need to transform the incremental losses to cumulative losses. To do this, sum the payments for claims in a given accident year up to each development year.

Accident Year	Development Year						
	0	1	2	3	4	5	6
2011	590	1,080	1,460	1,660	1,730	1,800	1,830
2012	590	960	1,220	1,310	1,350	1,370	
2013	690	1,440	1,870	2,140	2,310		
2014	790	1,400	1,840	2,010			
2015	750	1,410	1,860				
2016	770	1,570					
2017	870						

To distinguish between incremental losses and cumulative losses, we denote cumulative losses as $L_{i,\,k}$. For example, $L_{2011,\,3} = 1{,}660$ is the cumulative payment made by the end of DY3, or CY2014, on claims occurring in AY2011. It is calculated as

$$\begin{aligned} L_{2011,\,3} &= \sum_{k=0}^{3} l_{2011,\,k} \\ &= 590 + 490 + 380 + 200 \\ &= 1{,}660 \end{aligned}$$

Now that we have cumulative losses, how do we project ultimate losses? To start, note that the example assumes no loss development after DY6. In other words, all claims must be settled by DY6. Therefore, for each accident year, the cumulative losses by the end of DY6 are the ultimate losses.

$$L_i^{\text{ult.}} = L_{i,\,6}$$

This means we need to estimate the last column of the table.

AY2011 claims have fully matured, so start with AY2012 claims. The losses up to DY5 are known, so we only need to project $L_{2012,\,6}$.

The chain-ladder method assumes that historical loss developments are indicative of the future. We will quantify the loss development between DY5 and DY6 by calculating the ratio of the cumulative losses recorded at DY6 to those recorded at DY5 for AY2011.

$$\frac{L_{2011,6}}{L_{2011,5}} = \frac{1{,}830}{1{,}800} = 1.017$$

This value is called a *loss development factor*. A loss development factor of 1.017 means we expect the cumulative losses to grow roughly 1.7% from DY5 to DY6. So, we can multiply $L_{2012,5}$ by this factor to estimate $L_{2012,6}$.

$$\hat{L}_{2012,6} = 1{,}370\,(1.017) = 1{,}393$$

Let's move on to AY2013 claims. This is a little different because we only have data up to DY4, so we'll need to estimate $L_{2013,5}$ before we can find $L_{2013,6}$.

To do this, we'll need to calculate the loss development factors from DY4 to DY5.

Accident Year	Loss Development Factor
2011	$\frac{1{,}800}{1{,}730} = 1.040$
2012	$\frac{1{,}370}{1{,}350} = 1.015$

Note that we have two possible factors. Which should we use to project $L_{2013,5}$? This is where actuarial judgment is needed. In the real world, actuaries need to study the data and select a factor that reflects their best estimate of the future.

To simplify this demonstration, let's use the average of the factors.

$$\frac{1.040 + 1.015}{2} = 1.028$$

A factor of 1.028 means that we expect cumulative losses to grow roughly 2.8% from DY4 to DY5.

So, multiply $L_{2013,4}$ by the factor of 1.028 to estimate $L_{2013,5}$.

$$\hat{L}_{2013,\,5} = 2{,}310\,(1.028) = 2{,}374$$

Then, multiply $\hat{L}_{2013,\,5}$ by the loss development factor from DY5 to DY6 to estimate the ultimate losses.

$$\hat{L}_{2013,\,6} = 2{,}374\,(1.017) = 2{,}413$$

What we just described are the basic mechanics of the chain-ladder method. Now, we will generalize the steps and apply them to the entire data set. First, calculate the **loss development factors**, which are also known as ***age-to-age factors*** or ***link ratios***.

For accident year i, the $(k-1)$-to-k factor is calculated as

$$f_{i,\,k} = \frac{L_{i,\,k}}{L_{i,\,k-1}} \tag{S5.1.2.1}$$

For our example, we get the following values:

Accident Year, i	$f_{i,\,1}$	$f_{i,\,2}$	$f_{i,\,3}$	$f_{i,\,4}$	$f_{i,\,5}$	$f_{i,\,6}$
2011	1.831	1.352	1.137	1.042	1.040	1.017
2012	1.627	1.271	1.074	1.031	1.015	
2013	2.087	1.299	1.144	1.079		
2014	1.772	1.314	1.092			
2015	1.880	1.319				
2016	2.039					

The next step is to select the age-to-age factors. There are many ways to select the factors, but we'll cover three common models:

- **Average** — Finds the arithmetic average of all loss development factors for a given age. Under the average factor model, the selected age-to-age factors are

f_1	f_2	f_3	f_4	f_5	f_6
1.873	1.311	1.112	1.051	1.028	1.017

- **n-year average** — Finds the arithmetic average of the n **most recent** loss development factors. Under a 3-year average model, we get

f_1	f_2	f_3	f_4	f_5	f_6
1.897	1.311	1.104	1.051	1.028	1.017

- **Mean**, or **volume-weighted average** — Finds the sum of the cumulative losses in a given development year divided by the sum of the corresponding cumulative losses from the previous development year. Note that the numerator and denominator must have the same number of entries. For example,

$$f_4 = \frac{1{,}730 + 1{,}350 + 2{,}310}{1{,}660 + 1{,}310 + 2{,}140} = 1.055$$

  This model gives us selected age-to-age factors of

f_1	f_2	f_3	f_4	f_5	f_6
1.880	1.312	1.114	1.055	1.029	1.017

For our example, we are asked to use the average factor model. Therefore, the selected age-to-age factors are

f_1	f_2	f_3	f_4	f_5	f_6
1.873	1.311	1.112	1.051	1.028	1.017

With these factors, we can estimate the cumulative losses at any age T as the cumulative losses at age t multiplied by the product of the factors from age $t+1$ to age T, i.e.

$$\hat{L}_{i,T} = L_{i,t} \cdot \prod_{k=t+1}^{T} f_k \qquad \text{(S5.1.2.2)}$$

where t is the age of the claims at valuation, which in this case is Dec 31, 2017.

Note that $L_{i,t}$ is the most recent cumulative paid losses recorded for claims of accident year i, which is essentially the losses paid-to-date for those claims.

$$L_i^P = L_{i,t}$$

Now, let's apply this to estimate the ultimate losses in our example:

- The AY2011 claims are age 6, so we assume they are fully matured. Thus, the ultimate losses equal 1,830 based on the cumulative losses table.
- The AY2012 claims are age 5 at valuation. So, the estimated ultimate losses are

$$\begin{aligned} \hat{L}_{2012,6} &= L_{2012,5} \cdot f_6 \\ &= 1{,}370\,(1.017) \\ &= 1{,}393 \end{aligned}$$

- The AY2013 claims are age 4 at valuation. So, the estimated ultimate losses are

$$\begin{aligned} \hat{L}_{2013,6} &= L_{2013,4} \cdot f_5 \cdot f_6 \\ &= 2{,}310\,(1.028)\,(1.017) \\ &= 2{,}413 \end{aligned}$$

- And so on:

Accident Year	Estimated Ultimate Losses
2012	1,393

Accident Year	Estimated Ultimate Losses
2013	2,413
2014	2,206
2015	2,270
2016	2,512
2017	2,607

Coach's Remarks

If you are interested in estimating the cumulative losses for all development years, here are the values for all accident years and development years.

Accident Year	Development Year						
	0	1	2	3	4	5	6
2011	590	1,080	1,460	1,660	1,730	1,800	1,830
2012	590	960	1,220	1,310	1,350	1,370	1,393
2013	690	1,440	1,870	2,140	2,310	2,374	2,413
2014	790	1,400	1,840	2,010	2,112	2,170	2,206
2015	750	1,410	1,860	2,068	2,173	2,233	2,270
2016	770	1,570	2,058	2,288	2,405	2,471	2,512
2017	870	1,629	2,136	2,375	2,495	2,564	2,607

Now that we have the estimated ultimate losses and losses paid-to-date for all accident years, we can calculate the loss reserve for each accident year. Note that we don't need to hold a reserve for AY2011 because those claims have fully matured.

Accident Year	Estimated Ultimate Losses	Losses Paid-to-Date	Loss Reserve
2012	1,393	1,370	23
2013	2,413	2,310	103
2014	2,206	2,010	196
2015	2,270	1,860	410
2016	2,512	1,570	942

Accident Year	Estimated Ultimate Losses	Losses Paid-to-Date	Loss Reserve
2017	2,607	870	1,737

Finally, the total loss reserve is the sum of the Loss Reserve column.

$$23 + 103 + \cdots + 1{,}737 = \mathbf{3{,}412}$$

To conclude this example, the steps for the chain-ladder method are:

1. Calculate and select the age-to-age factors.
2. Project the ultimate losses.
3. Subtract the losses paid-to-date to estimate reserves.

It is worth noting that we can combine loss development factors from the current age to the ultimate age to create *age-to-ultimate* factors, which directly develop current losses to ultimate losses. The age-to-ultimate factor is calculated as the product of all age-to-age factors beyond current age, t.

$$f_i^{\text{ult.}} = \prod_{k=t+1}^{\infty} f_k \tag{S5.1.2.3}$$

Thus, the estimated ultimate losses can be calculated as the losses paid-to-date times the age-to-ultimate factor.

$$\hat{L}_i^{\text{ult.}} = L_{i,t} \cdot f_i^{\text{ult.}} \tag{S5.1.2.4}$$

Coach's Remarks

Tying back to the previous example, since we assume no loss development after DY6,

$$f_{i,7} = f_{i,8} = \ldots = f_{i,\infty} = 1$$

Therefore,

$$f_i^{\text{ult.}} = \prod_{k=t+1}^{\infty} f_k = \prod_{k=t+1}^{6} f_k$$

and

$$\hat{L}_i^{\text{ult.}} = L_{i,t} \cdot \prod_{k=t+1}^{6} f_k$$

Example S5.1.2.1

For claims of AY4, the losses paid-to-date as of 12/31/AY7 is $100,000.

You're given the following:

Age	Age-to-Ultimate Factor
0	2.189
1	1.600
2	1.243
3	1.108
4	1.044
5	1.016

Determine the chain-ladder estimate of the unpaid losses of AY4 claims as of 12/31/AY7.

Solution

Note that the reserve is an estimate of the outstanding payments. Thus, we are looking to calculate the reserve for AY4 claims.

The AY4 claims are age 3 at the end of AY7. Thus, the estimated ultimate losses are

$$100{,}000(1.108) = 110{,}800$$

This means the estimated unpaid losses are

$$110{,}800 - 100{,}000 = \mathbf{10{,}800}$$

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Example S5.1.2.2

You are given the following cumulative paid losses:

Accident Year	Development Year			
	0	1	2	3
AY1	340	570	670	810
AY2	360	580	710	
AY3	360	620		
AY4	410			

Assume no loss development beyond Development Year 3.

Using the chain-ladder method and average factor model, estimate the loss reserve as of December 31, AY4.

Solution

We are given cumulative losses, so we can calculate the age-to-age factors.

Accident Year, i	$f_{i,1}$	$f_{i,2}$	$f_{i,3}$
AY1	1.676	1.175	1.209
AY2	1.611	1.224	
AY3	1.722		

To select the factors using the average factor model, take the average of each column.

f_1	f_2	f_3
1.670	1.200	1.209

Next, estimate the ultimate losses.

- The claims of AY1 are fully developed. There is no need to hold a reserve for these claims.
- $\hat{L}_2^{\text{ult.}} = 710\,(1.209) = 858$
- $\hat{L}_3^{\text{ult.}} = 620\,(1.200)\,(1.209) = 899$
- $\hat{L}_4^{\text{ult.}} = 410\,(1.670)\,(1.200)\,(1.209) = 993$

Then, estimate the reserves.

Accident Year	Estimated Ultimate Losses	Losses Paid-to-Date	Reserve
AY2	858	710	148
AY3	899	620	279
AY4	993	410	583

Finally, the total reserve is the sum of the Reserve column.

$$148 + 279 + 583 = \mathbf{1{,}011}$$

Although we estimated the ultimate losses by developing **paid loss** data in the examples above, we could have done the same with **incurred loss** data. This is because, in theory, incurred losses should equal paid losses at the end of loss development. Recall that this was demonstrated at the beginning of Section S5.1, where incurred and paid losses were ultimately both equal to 285,000.

However, in practice, developing paid losses will likely yield a different estimate of the ultimate losses than developing incurred losses. Therefore, do not make the assumption that they will be equal unless the problem explicitly indicates it. That said, we should only develop paid losses with paid loss development factors and develop incurred losses with incurred loss development factors.

After estimating the ultimate losses, whether by developing the paid or incurred losses, subtract losses paid-to-date to calculate loss reserve, or subtract losses incurred-to-date to calculate IBNR reserve.

Example S5.1.2.3

You are given the following cumulative incurred losses experience:

Accident Half-Year	Age in Months				Paid-to-Date as of 12/31/2015
	6	12	18	24	
2014-1	2,893	3,782	4,144	4,227	4,227
2014-2	2,937	3,869	4,255		3,627
2015-1	3,068	3,876			3,379
2015-2	3,285				2,601

Using the chain-ladder method, estimate the loss reserve at the end of 2015. Use the volume-weighted average factor model with no development past 24 months.

Solution

The selected 6-to-12-months factor is

$$\frac{3{,}782 + 3{,}869 + 3{,}876}{2{,}893 + 2{,}937 + 3{,}068} = 1.295$$

The selected 12-to-18-months factor is

$$\frac{4{,}144 + 4{,}255}{3{,}782 + 3{,}869} = 1.098$$

The selected 18-to-24-months factor is

$$\frac{4{,}227}{4{,}144} = 1.020$$

The claims of 2014-1 are fully developed and paid.

The claims of 2014-2 are age 18 months at valuation. Thus, the estimated ultimate losses are

$$4{,}255\,(1.020) = 4{,}340$$

The claims of 2015-1 are age 12 months at valuation. Thus, the estimated ultimate losses are

$$3{,}876\,(1.098)\,(1.020) = 4{,}340$$

The claims of 2015-2 are age 6 months at valuation. Thus, the estimated ultimate losses are

$$3{,}285\,(1.295)\,(1.098)\,(1.020) = 4{,}765$$

Finally, subtract the losses paid-to-date from the ultimate losses to estimate reserves.

Estimated Ultimate Losses	Losses Paid-to-Date	Reserve
4,340	3,627	713
4,340	3,379	961
4,765	2,601	2,164

The total reserve is the sum of the Reserve column.

$$713 + 961 + 2{,}164 = \mathbf{3{,}839}$$

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The chain-ladder method looks promising, but it has its disadvantages. First, since the method fully relies on claims experience, instability is a concern. A few outliers