Partial Credibility

20M

In Section S4.1.1, we discussed the sample size required to assign 100% credibility to the observations, i.e. $Z = 1$. However, if the experience is inadequate for full credibility, then ***partial credibility*** is assigned to the sample, i.e. $Z < 1$.

Recall that the concept of credibility is to assign weights to different sources to produce the best estimate of the future outcome. This estimate of the future outcome is called the ***credibility premium*** and is denoted P_C.

In the case of partial credibility, P_C is calculated by assigning a weight of Z to the sample mean, and a weight of $1 - Z$ to the value from a more general source, commonly known as the ***manual premium***.

$$\begin{aligned} P_C &= Z\bar{x} + (1 - Z)M \\ &= M + Z(\bar{x} - M) \end{aligned} \tag{S4.1.2.1}$$

where

- $\bar{x}$ represents the sample mean
- M represents the manual premium

Coach's Remarks

In a classical credibility problem, $\bar{x}$ is determined by taking the average of the observations provided, but M should be explicitly specified by the problem because it is set "manually," without any consideration to the observations.

Also, since P_C is a weighted average of two different sources, the "sources" should have comparable quantities. For example, if M is for the **annual claim frequency per insured**, then $\bar{x}$ should also be the average of the observed **annual claim frequency per insured**. This is a good way to check if $\bar{x}$ is calculated correctly.

Under classical credibility theory, Z can be calculated using the ***square root rule***. The rule states that if there is not full credibility, then

$$Z = \sqrt{\frac{n}{n_e}} = \sqrt{\frac{n'}{n_c}} \qquad \text{(S4.1.2.2)}$$

where

- n_e is the number of exposures **needed** for full credibility
- n is the **actual** number of exposures
- n_c is the expected number of claims **needed** for full credibility
- n' is the **actual** number of claims

Note the two forms of the square root rule. It can be applied for partial credibility as long as the numerator and the denominator are consistent in units (exposures or claims).

Coach's Remarks

Technically, n' should equal $n \cdot \mu_N$, so that it mirrors the relationship between n_e and n_c as stated in (S4.1.1.3). This is why there are two forms for (S4.1.2.2). However, μ_N may not always be known (and in practice, it is often assumed to be unknown). Thus, the **actual** number of claims is used as an approximation for n'.

Consider the following example:

If the number of exposures needed for full credibility is 80,000, how much credibility should be assigned to the observations if 60,000 exposures are observed? What if 120,000 exposures are observed instead?

Using the square root rule, if there are only 60,000 exposures, then

$$Z = \sqrt{\frac{60{,}000}{80{,}000}} = \mathbf{0.866}$$

because less than 80,000 exposures have been observed.

On the other hand, if 120,000 exposures are observed, then

$$Z = \mathbf{1}$$

This is because the full credibility standard is achieved as long as at least 80,000 exposures are observed.

Example S4.1.2.1

For a specific type of policy, you are given:

- The standard for full credibility is set so that the claim frequency is within 3% of the true value 90% of the time.
- A group with 800 insureds has a credibility of 80%.

Then, the standard for full credibility is changed so that the claim frequency is within 3% of the true value 99% of the time.

Calculate the revised credibility factor for this same group of insureds.

Solution

In the first part of this problem, we are given enough information to calculate the number of exposures/claims needed for full credibility. Then, since the full credibility standard has changed, the number of exposures/claims needed for full credibility needs to be revised as well. We will use the revised number of exposures/claims needed for full credibility to revise the credibility factor.

$$Z \to n_e \to n_c \to \text{revised } n_c \to \text{revised } n_e \to \text{revised } Z$$

Let's start by defining N as the claim frequency.

Since a group of 800 insureds has partial credibility of $Z = 0.8$, apply the square root rule to determine the number of exposures needed for full credibility.

$$Z = 0.8 = \sqrt{\frac{800}{n_e}}$$

$$n_e = 1{,}250$$

From $(\text{S}4.1.1.3)$, we know

$$n_c = 1{,}250\mu_N$$

Next, apply $(\text{S}4.1.1.4)$:

- $n_c = 1{,}250\mu_N$
- $z_{(1+0.9)/2} = z_{0.95} = 1.64485$
- $k = 0.03$
- $CV_X^2 = 0$ because full credibility is calculated for the **claim frequency only**.

Therefore,

$$1{,}250\mu_N = \left[\frac{1.64485}{0.03}\right]^2 \left(\frac{\sigma_N^2}{\mu_N}\right)$$

$$\sigma_N^2 = 0.4158\mu_N^2$$

Then, to calculate the number of claims needed to meet the revised standard for full credibility, reapply (S4.1.1.4) with an adjusted z-value:

- $z_{(1+0.99)/2} = 2.57583$

$$\begin{aligned}\text{revised } n_c &= \left[\frac{2.57583}{0.03}\right]^2\left(\frac{0.4158\mu_N^2}{\mu_N}\right)\\ &= 3{,}065.43\mu_N\end{aligned}$$

Then,

$$\text{revised } n_e = \frac{3{,}065.43\mu_N}{\mu_N} = 3{,}065.43$$

Under the new standard, 3,065.43 exposures are needed to achieve full credibility. Since there are only 800 insureds, the revised credibility factor is calculated as:

$$\text{revised } Z = \sqrt{\frac{800}{3{,}065.43}} = \mathbf{0.5109}$$

Coach's Remarks

Alternatively, instead of solving for n_c and its revised version, we can skip the middle part and do:

$$Z \rightarrow n_e \rightarrow \textbf{revised } n_e \rightarrow \textbf{revised } Z$$

However, this "shorter" approach requires the formula of n_e for the full credibility for claim frequency, which can easily be derived from n_c:

$$\begin{aligned} n_e &= \left[\frac{z_{(1+p)/2}}{k}\right]^2 \left(\frac{\sigma_N^2}{\mu_N} + 0\right) \cdot \frac{1}{\mu_N} \\ &= \left[\frac{z_{(1+p)/2}}{k}\right]^2 (CV_N^2) \end{aligned}$$

Example S4.1.2.2

You are given:

- The annual claim frequency follows a Poisson distribution.
- The individual claim sizes follow an inverse Gaussian distribution with parameters $\mu = 4$ and $\theta = 8$.
- The claim frequency and severity are independent.
- A full credibility standard is established so that the aggregate claim is within 10% of its mean 90% of the time.
- The manual premium for the annual aggregate claim is 10,000.

In the previous year, 200 claims were observed with a total claim size of 12,000.

Using limited fluctuation credibility, calculate the credibility premium for the current year.

Solution

Notice the actual observations are given in the form of **number of claims**. This means the full credibility standard should be calculated in terms of n_c so that the units are consistent.

Let N and X be the annual claim frequency and severity, respectively.

$$N \sim \textbf{Poisson}\ (\lambda)$$

$$X \sim \textbf{Inverse Gaussian}\ (4,\ 8)$$

Next, apply (S4.1.1.4) to calculate n_c:

- $z_{(1+0.90)/2} = z_{0.95} = 1.64485$
- $k = 0.1$
- For the frequency distribution,

$$\text{E}[N] = \text{Var}[N] = \lambda$$

- For the severity distribution,

$$\text{E}[X] = 4$$

$$\text{Var}[X] = \frac{4^3}{8} = 8$$

Therefore,

$$\begin{aligned} n_c &= \left[\frac{1.64485}{0.1}\right]^2 \left(\frac{\lambda}{\lambda} + \frac{8}{4^2}\right) \\ &= \left[\frac{1.64485}{0.1}\right]^2 (1 + 0.5) \\ &= 405.83 \end{aligned}$$

Since only 200 claims were observed in the previous year, apply the square root rule to calculate the partial credibility factor:

$$Z = \sqrt{\frac{200}{405.83}} = 0.702$$

To determine the credibility premium for this year, apply (S4.1.2.1). From the question,

$$\bar{x} = 12{,}000$$

$$M = 10{,}000$$

Therefore,

$$\begin{aligned} P_C &= 0.702(12{,}000) + (1 - 0.702)(10{,}000) \\ &= \mathbf{11{,}404.02} \end{aligned}$$

Coach's Remarks

In this question, notice that the $\bar{x}$ used in the calculation of P_C is:

$$\bar{x} = 12{,}000 \neq \frac{12{,}000}{200}$$

Recall that the sample mean and the manual premium should have comparable quantities. For this problem, the manual premium, M, is for the annual aggregate claim. Therefore, $\bar{x}$ should be the average of the observed annual aggregate claims.

As alluded to by the phrase "In the previous year," the observed number of exposures (years) is 1, i.e. $n = 1$. The sample mean is the observed total amount across all exposures divided by n. Thus,

$$\bar{x} = \frac{12{,}000}{1} = 12{,}000$$