Overview

◯ 5M

Insurance operates by pooling risk together across policies. We talked about the loss count (i.e., frequency) and the individual loss sizes (i.e., severity) in the previous sections. Now, let's tie the two concepts together.

Assume we have more than one loss.

$$X_1, X_2, \ldots, X_N$$

Then, the aggregate loss is the sum of all of the individual losses:

$$S=\sum_{i=1}^N X_i, \qquad N=1,\,2,\,\ldots$$

where

- N, which describes the number of losses, is the frequency random variable.
- X_i , which describes the size of the i^{th} loss, is the **severity** random variable.

Note that S=0 when N=0.

There are two types of aggregate loss models.

Collective Risk Model

Under the collective risk model, we assume that

- All existing $X_1, X_2, ..., X_n$ are independent and identically distributed (i.i.d.) random variables.
- N and any existing X_i are independent.

In this case, the distribution of N and X_i are called the *primary distribution* and secondary distribution, respectively. And the distribution of S is called the compound distribution.

Coach's Remarks

On the exam, most questions from this section use the collective risk model. When solving problems, the independence requirement is as simple as verifying that frequency and severity are independent.

Individual Risk Model

Under the *individual risk model*, we simplify the frequency but generalize the severity.

Instead of a random variable, the frequency is now a constant, n, that is usually used to represent the number of policyholders.

 X_i is then the aggregate loss amount of the $i^{ ext{th}}$ policyholder.

$$S = \sum_{i=1}^{n} X_i$$

 $X_1, X_2, ..., X_n$ are independent but **may not be identical** random variables in this case. In other words, the X_i 's may have different distributions.