Appendix

(L) 5M

Compound Mean and Variance

For a collective risk model,

$$S=\sum_{i=1}^N X_i, \qquad N=1,\,2,\,\ldots$$

The compound mean and variance can be calculated using Law of Total Expectation and Law of Total Variance.

$$egin{aligned} \mathbf{E}\left[S
ight] &= \mathbf{E}\left[\mathbf{E}\left[S\mid N
ight]
ight] \ &= \mathbf{E}\left[\sum_{i=1}^{N}X_{i}\mid N
ight]
ight] \ &= \mathbf{E}\left[\sum_{i=1}^{N}\mathbf{E}\left[X_{i}\mid N
ight]
ight] \ &= \mathbf{E}\left[N\cdot\mathbf{E}\left[X
ight]
ight] \ &= \mathbf{E}\left[N
ight]\mathbf{E}\left[X
ight] \end{aligned}$$

$$egin{aligned} \operatorname{Var}\left[S
ight] &= \operatorname{E}\left[\operatorname{Var}\left[S\mid N
ight]
ight] + \operatorname{Var}\left[\operatorname{E}\left[S\mid N
ight]
ight] \\ &= \operatorname{E}\left[\operatorname{Var}\left[\sum_{i=1}^{N}X_{i}\mid N
ight]
ight] + \operatorname{Var}\left[\operatorname{E}\left[\sum_{i=1}^{N}X_{i}\mid N
ight]
ight] \\ &= \operatorname{E}\left[\sum_{i=1}^{N}\operatorname{Var}\left[X_{i}\mid N
ight]
ight] + \operatorname{Var}\left[\sum_{i=1}^{N}\operatorname{E}\left[X_{i}\mid N
ight]
ight] \\ &= \operatorname{E}\left[N\cdot\operatorname{Var}\left[X
ight]
ight] + \operatorname{Var}\left[N\cdot\operatorname{E}\left[X
ight]
ight] \\ &= \operatorname{E}\left[N
ight]\operatorname{Var}\left[X
ight] + \operatorname{Var}\left[N
ight]\operatorname{E}\left[X
ight]^{2} \end{aligned}$$