Summary 5M

Aggregate Loss Models

Sum of all individual losses.

Collective Risk Model

$$S = \sum_{i=1}^{N} X_i, \qquad N = 1,\ 2,\ \ldots$$

- All existing severities are i.i.d.
- Frequency and severity are independent.

$$\mathrm{E}\left[S\right] = \mathrm{E}\left[N\right]\mathrm{E}\left[X\right]$$

$$\mathrm{Var}\left[S\right] = \mathrm{E}\left[N\right]\mathrm{Var}\left[X\right] + \mathrm{Var}\left[N\right]\mathrm{E}[X]^2$$

Individual Risk Model

$$S = \sum_{i=1}^{n} X_i$$

- Severities are independent but may not be identical.
- Frequency is a constant.

$$\mathrm{E}\left[S\right] = \sum_{i=1}^{n} \mathrm{E}\left[X_i\right]$$

$$\mathrm{Var}\left[S\right] = \sum_{i=1}^{n} \mathrm{Var}\left[X_i\right]$$

Normal Approximation

Approximate S with a normal distribution.

$$S \sim \mathrm{Normal}\left(\mu = \mathrm{E}\left[S\right],\, \sigma^2 = \mathrm{Var}\left[S\right]\right)$$

Lognormal Approximation

Approximate S with a lognormal distribution.

1. Calculate the aggregate mean and variance.
2. Equate lognormal mean and variance to aggregate mean and variance to solve for the parameters μ and σ^2.
3. Calculate the desired quantity based on the approximating distribution, i.e.,

$$S \sim \mathrm{Lognormal}\left(\mu,\, \sigma^2\right)$$

Continuity Correction

Needed when both frequency and severity are discrete.

Stop-Loss Insurance

A stop-loss insurance refers to an insurance that imposes an aggregate deductible. An aggregate deductible is applied to the aggregate loss. In contrast, an ordinary deductible is applied to individual claims.

The net stop-loss premium

$$\mathrm{E}\left[(S-d)_+\right] = \mathrm{E}[S] - \mathrm{E}[S \wedge d]$$

Aggregate Payments

Sum of all individual payments. The key is to make sure both frequency and severity are in terms of either losses or payments.

N is the number of losses; Y^L is the payment per loss.

$$S = \sum_{i=1}^{N} Y_i^L, \qquad N = 1,\, 2,\, \ldots$$

N' is the number of payments; Y^P is the payment per payment.

$$S = \sum_{i=1}^{N'} Y_i^P, \qquad N' = 1,\, 2,\, \ldots$$