Appendix 5M

Number of Exposures Needed for Full Credibility of Aggregate Claims

The goal is to derive the formula for n that satisfies the inequality below:

$$\Pr\left(\left|\,\bar{S}-\mu_S\,\right|\leq k\mu_S\right)=p$$

$$\Pr\left(-k\mu_S\leq \bar{S}-\mu_S\leq k\mu_S\right)=p$$

$$\Phi\left(\frac{k\,\mathrm{E}\left[\bar{S}\right]}{\sqrt{\mathrm{Var}\left[\bar{S}\right]}}\right)=\frac{1+p}{2}$$

$$\frac{k\,\mathrm{E}\left[\bar{S}\right]}{\sqrt{\mathrm{Var}\left[\bar{S}\right]}}=z_{(1+p)/2}$$

Note that:

- $\mathrm{E}\left[\bar{S}\right]=\mathrm{E}\left[\frac{\sum_{i=1}^{n}S_i}{n}\right]=\frac{1}{n}\cdot n\cdot\mathrm{E}[S]=\mathrm{E}[S]$

- $\mathrm{Var}\left[\bar{S}\right] = \mathrm{Var}\left[\frac{\sum_{i=1}^{n} S_i}{n}\right] = \frac{1}{n^2} \cdot n \cdot \mathrm{Var}[S] = \frac{\mathrm{Var}[S]}{n}$

Then,

$$\frac{k\,\mathrm{E}[S]}{\sqrt{\frac{\mathrm{Var}[S]}{n}}} = z_{(1+p)/2}$$

$$\sqrt{n} \cdot \frac{k\,\mathrm{E}[S]}{\sqrt{\mathrm{Var}[S]}} = z_{(1+p)/2}$$

$$\begin{aligned} \sqrt{n} &= \left[\frac{z_{(1+p)/2}}{k}\right]\left(\frac{\sqrt{\mathrm{Var}[S]}}{\mathrm{E}[S]}\right) \\ &= \left[\frac{z_{(1+p)/2}}{k}\right](CV_S) \end{aligned}$$

Therefore,

$$n = \left[\frac{z_{(1+p)/2}}{k}\right]^2 (CV_S^2)$$