Policy Limits

10M

The policy limit is the maximum amount the **insurer** will pay for a single loss.

Let X represent the loss variable. Note that X can never go below zero, as negative values do not make sense in the context of loss amounts. Then, for insurance with a policy limit u, the insurance payment is defined as

$$X \wedge u = \begin{cases} X, & X < u \\ u, & X \geq u \end{cases}$$

which implies the payment amount is the loss amount capped at u. In other words, the insurer pays whichever is lower: the loss or the policy limit. $(X \wedge u)$ is also called the ***limited loss variable***.

The average payment amount in this case is called the ***limited expected value***.

$$\mathrm{E}[X \wedge u]$$

For losses **below** the policy limit, i.e. $X < u$, the payment amount is the loss amount, and the average payment amount for this portion is:

$$\int_0^u x\, f(x)\, \mathrm{d}x$$

For losses **above** the policy limit, i.e. $X \geq u$, the payment amount is the policy limit, and the average payment amount for this portion is:

$$\int_u^\infty u\, f(x)\, \mathrm{d}x = u \cdot S(u)$$

Thus, the average payment amount is:

$$\mathrm{E}[X \wedge u] = \int_0^u x\, f(x)\,\mathrm{d}x + u \cdot S(u)$$

This can be extended to the k^{th} moment of the limited loss variable:

$$\mathrm{E}\left[(X \wedge u)^k\right] = \int_0^u x^k\, f(x)\,\mathrm{d}x + u^k \cdot S(u) \qquad \text{(S2.3.1.1)}$$

Using the survival function method, the limited loss moments can also be expressed as

$$\mathrm{E}[X \wedge u] = \int_0^u S(x)\,\mathrm{d}x$$

$$\mathrm{E}\left[(X \wedge u)^k\right] = \int_0^u k x^{k-1} S(x)\,\mathrm{d}x \qquad \text{(S2.3.1.2)}$$

The derivation of the survival function method is provided in the appendix at the end of this section.

Coach's Remarks

Claim size usually has a continuous distribution; thus, we will only introduce the formulas in continuous form. In the rare case where losses are discretely distributed, substitute sums for the integrals. For example:

$$\mathrm{E}\left[(X \wedge u)^k\right] = \left[\sum_{x \le u} x^k p(x)\right] + u^k \cdot S(u)$$

Note then that if X follows an empirical distribution, its limited expected value for

$k = 1$ can be calculated as:

$$\mathrm{E}\left[X \wedge u\right] = \frac{\sum_{i=1}^{n} \left(x_i \wedge u\right)}{n}$$

Example S2.3.1.1

Claim size for a medical insurance policy follows a Pareto distribution with parameters $\alpha = 5$ and $\theta = 1{,}000$.

The medical insurance has a policy limit of 500.

Calculate the expected insurance payment for a claim.

Solution

Let X represent the claim size.

$$X \sim \text{Pareto}\ (5,\ 1{,}000)$$

The Pareto distribution's limited expected value formula is given in the exam table. Look up the formula and plug in the parameters and policy limit to calculate the answer.

$$\begin{aligned}\mathrm{E}[X \wedge 500] &= \frac{1{,}000}{5-1}\left[1 - \left(\frac{1{,}000}{500 + 1{,}000}\right)^{5-1}\right] \\ &\approx \mathbf{200.62}\end{aligned}$$

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Example S2.3.1.2

For an auto insurance policy, claim amounts follow a distribution with the following CDF:

$$F(x) = 1 - 0.6e^{-0.01x} - 0.4e^{-0.002x}$$

The auto insurance has a policy limit of 200.

Calculate the insurance company's expected payment for one claim.

Solution

$$\begin{aligned}
\mathrm{E}[X \wedge u] &= \int_0^u S(x)\,\mathrm{d}x \\
&= \int_0^{200} \left(0.6e^{-0.01x} + 0.4e^{-0.002x}\right) \mathrm{d}x \\
&= \left[-\frac{0.6}{0.01}e^{-0.01x} - \frac{0.4}{0.002}e^{-0.002x}\right]_0^{200} \\
&= \mathbf{117.82}
\end{aligned}$$

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Increased Limit Factor

The ***increased limit factor (ILF)*** measures how much more the insurer expects to pay by increasing the policy limit. As an example, if the ILF is 1.1, then the insurer expects to pay 10% more with the given policy limit increase.

It is calculated as:

$$ILF = \frac{\mathrm{E}[X \wedge u]}{\mathrm{E}[X \wedge b]} \tag{S2.3.1.3}$$

where b is the original limit and u is the increased limit.

Let's extend Example S2.3.1.1 to practice calculating the ILF.

Example S2.3.1.3

Claim size for a medical insurance policy follows a Pareto distribution with parameters $\alpha = 5$ and $\theta = 1{,}000$.

The insurance company considers raising that policy's limit from 500 to 800.

Determine the increased limit factor for a claim.

Solution

The insurer's expected payout per claim will increase from $\mathrm{E}[X \wedge 500]$ to $\mathrm{E}[X \wedge 800]$. Therefore,

$$ILF = \frac{\mathrm{E}[X \wedge 800]}{\mathrm{E}[X \wedge 500]}$$

From Example S2.3.1.1, we know that $\mathrm{E}[X \wedge 500] = 200.62$. Using the same formula from the exam table,

$$\begin{aligned}\mathrm{E}[X \wedge 800] &= \frac{1{,}000}{5-1}\left[1 - \left(\frac{1{,}000}{800 + 1{,}000}\right)^{5-1}\right] \\ &= 226.19\end{aligned}$$

Thus,

$$ILF = \frac{226.19}{200.62} = \mathbf{1.1274}$$

which means that the insurer expects to pay 12.74% more per claim.