📖 Summary

🕐 5M

(a, b, 0) Class

Poisson

The sum of independent Poisson random variables follows a Poisson distribution with mean equal to the sum of all means.

Bernoulli and Binomial

The Bernoulli distribution is a special case of a binomial distribution with $m = 1$.

The sum of n **i.i.d.** Bernoulli random variables follows a binomial distribution with parameters $m = n$ and q.

Geometric and Negative Binomial

The geometric distribution is a special case of a negative binomial distribution with $r = 1$.

The sum of n **i.i.d.** geometric random variables follows a negative binomial distribution with parameters $r = n$ and β.

The geometric distribution is memoryless.

Important Property

$$\frac{p_n}{p_{n-1}} = a + \frac{b}{n}, \qquad n = 1, 2, \ldots$$

Poisson-Gamma Mixture

If X follows a Poisson distribution where its mean follows a gamma distribution, then the unconditional distribution of X is a negative binomial distribution:

$$(X \mid \Lambda = \lambda) \sim \text{Poisson}(\lambda)$$
$$\Lambda \sim \text{Gamma}(\alpha, \theta) \qquad \Rightarrow \qquad X \sim \text{Negative Binomial}(r = \alpha, \, $$

Choosing from the (a, b, 0) Class

There are two methods to determine the most appropriate $(a, b, 0)$ class distribution for a set of data:

- Compare the sample mean and unbiased sample variance.
- Observe the slope of $\frac{k n_k}{n_{k-1}}$.

Distribution	Mean vs. Variance	Slope, a
Poisson	$\bar{x} = s^2$	0
Binomial	$\bar{x} > s^2$	-
Negative binomial	$\bar{x} < s^2$	+

(a, b, 1) Class

Zero-Truncated

$$p_n^T = \frac{1}{1 - p_0} \cdot p_n, \qquad n = 1, 2, \ldots$$

$$\mathrm{E}\left[\left(N^T\right)^k\right] = \frac{1}{1 - p_0} \cdot \mathrm{E}\left[N^k\right]$$

Zero-Modified

$$p_n^M = \frac{1 - p_0^M}{1 - p_0} \cdot p_n, \qquad n = 1, 2, \ldots$$

$$\mathrm{E}\left[\left(N^M\right)^k\right] = \frac{1 - p_0^M}{1 - p_0} \cdot \mathrm{E}\left[N^k\right]$$

Important Property

$$\frac{p_n^T}{p_{n-1}^T} = a + \frac{b}{n}, \qquad n = 2, 3, \ldots$$

$$\frac{p_n^M}{p_{n-1}^M} = a + \frac{b}{n}, \qquad n = 2, 3, \ldots$$