Stop-Loss Insurance 15M

An insurance policy can impose an ***aggregate deductible***. In contrast to a per-claim deductible, an aggregate deductible is applied to the aggregate loss , S.

$$(S-d)_+ = \begin{cases} 0, & S \le d \\ S-d, & S > d \end{cases}$$

Such insurance is called ***stop-loss*** insurance. The expected cost of this insurance to the insurer, $\mathrm{E}\left[(S-d)_+\right]$, is called the ***net stop-loss premium***.

Similar to an individual payment, the aggregate payment variable with an aggregate deductible can be expressed as

$$(S-d)_+ = S - (S \wedge d)$$

Because the distribution of S can be difficult to determine, there is not a consistent formula to calculate $\mathrm{E}\left[(S-d)_+\right]$ or $\mathrm{E}[S \wedge d]$. The rule of thumb is to **identify patterns and apply basic probability principles**. Fortunately, on the exam, S is usually discrete for this type of questions.

Example S2.5.4.1

For a collective risk model:

- The number of claims received by the company has a geometric distribution with mean 3.
- The individual loss amounts have the following probability function:

$$\Pr(X=x) = \frac{5-x}{10}, \qquad x = 1, 2, 3, 4$$

An insurance covers the aggregate loss subject to a deductible of 2.

Calculate the net stop-loss premium.

Solution

From the question, $N \sim \text{Geometric}\ (3)$, and the loss amount can be 1, 2, 3, or 4 with probabilities 0.4, 0.3, 0.2, and 0.1, respectively.

The net stop-loss premium is

$$\mathrm{E}\left[(S-2)_+\right] = \mathrm{E}[S] - \mathrm{E}[S \wedge 2]$$

Start with the expected aggregate loss, which requires the frequency and severity means.

$$\mathrm{E}[N] = 3$$

$$\begin{aligned}\mathrm{E}[X] &= 0.4(1) + 0.3(2) + 0.2(3) + 0.1(4) \\ &= 2\end{aligned}$$

$$\mathrm{E}[S] = 3(2) = 6$$

To calculate the aggregate loss limited expected value, construct this table.

S	Probability	$S \wedge 2$
0	0.25	0
1	0.075	1
≥ 2	0.675	2

The probabilities in the middle column are calculated as follows.

$$\begin{aligned}\Pr(S=0) &= \Pr(N=0)\\ &= \frac{1}{1+3}\\ &= 0.25\end{aligned}$$

$$\begin{aligned}\Pr(S=1) &= \Pr(N=1)\Pr(X=1)\\ &= \frac{3}{(1+3)^2}\cdot(0.4)\\ &= 0.075\end{aligned}$$

$$\begin{aligned}\Pr(S\ge 2) &= 1-0.25-0.075\\ &= 0.675\end{aligned}$$

Calculate the aggregate loss limited expected value from first principles.

$$\begin{aligned}\mathrm{E}[S\wedge 2] &= \sum_{\text{all } s_i}(s_i\wedge 2)\cdot\Pr(S=s_i)\\ &= 0(0.25)+1(0.075)+2(0.675)\\ &= 1.425\end{aligned}$$

Finally, calculate the net stop-loss premium.

$$\mathrm{E}\left[(S-2)_+\right] = 6-1.425 = \mathbf{4.575}$$

Coach's Remarks

Another way to calculate $\mathrm{E}\left[(S-2)_+\right]$ is to start with $\mathrm{E}[S-2]$ and make adjustments where $\mathrm{E}\left[(S-2)_+\right]$ differs from $\mathrm{E}[S-2]$. Construct the following table:

$\boldsymbol{S}$	$\boldsymbol{(S-2)_+}$	$\boldsymbol{S-2}$
0	0	-2
1	0	-1
2	0	0
3	1	1
4	2	2
5	3	3
$\vdots$	$\vdots$	$\vdots$

Therefore,

$$\mathrm{E}\left[(S-2)_+\right] = \mathrm{E}[S-2] + 2 \cdot p_S(0) + 1 \cdot p_S(1)$$

Example S2.5.4.2

The number of claims follows a logarithmic distribution with the following probability function:

$$\Pr(N=n) = \frac{0.9^n}{n \ln(10)}, \qquad n = 1,\ 2,\ 3,\ \ldots$$

The amount of each claim is 80.

Reinsurance covers 80% of the aggregate claims in excess of 200.

Calculate the reinsurance net premium.

Solution

The reinsurance net premium is

$$0.8 \cdot \mathrm{E}\big[(S-200)_+\big] = 0.8 \cdot (\mathrm{E}[S] - \mathrm{E}[S \wedge 200])$$

Look up the logarithmic PMF in the exam tables, and compare it to the PMF provided by the question. The parameter β has a value of 9. Therefore,

$$\mathrm{E}[N] = \frac{9}{\ln(1+9)} = 3.9087$$

The claim amount is a constant of 80 per claim.

$$\mathrm{E}[X] = \mathrm{E}[80] = 80$$

Multiply them together to get the aggregate mean.

$$\mathrm{E}[S] = 3.9087(80) = 312.6920$$

Construct the following table to calculate the aggregate limited expected value.

S	Probability	$S \wedge 200$
80	0.3909	80
160	0.1759	160
≥ 200	0.4332	200

The probabilities are calculated as

$$\begin{aligned}\Pr(S=80) &= \Pr(N=1)\Pr(X=80) \\ &= \frac{0.9}{\ln(10)}(1) \\ &= 0.3909\end{aligned}$$

$$\begin{aligned}\Pr(S=160) &= \Pr(N=2)\Pr(X=80)^2 \\ &= \frac{0.9^2}{2\ln(10)}\left(1^2\right) \\ &= 0.1759\end{aligned}$$

$$\begin{aligned}\Pr(S\geq 200) &= 1-0.3909-0.1759 \\ &= 0.4332\end{aligned}$$

Thus, the aggregate limited expected value is

$$\begin{aligned}\mathrm{E}[S\wedge 200] &= 0.3909(80)+0.1759(160)+0.4332(200) \\ &= 146.0606\end{aligned}$$

Finally, calculate the reinsurance net premium.

$$\begin{aligned}0.8\cdot\mathrm{E}\left[(S-200)_+\right] &= 0.8\cdot(\mathrm{E}[S]-\mathrm{E}[S\wedge 200]) \\ &= 0.8\cdot(312.6920-146.0606) \\ &= \mathbf{133.3051}\end{aligned}$$