Appendix

(L) 5M

Number of Exposures Needed for Full Credibility of Aggregate Claims

The goal is to derive the formula for n that satisfies the inequality below:

$$ext{Pr}\Big(ig|ar{S}-\mu_Sig|\leq k\mu_S\Big)=p$$

$$\Pr\Bigl(-k\mu_S \leq ar{S} - \mu_S \leq k\mu_S\Bigr) = p$$

$$\Phi\left(rac{k\,\mathrm{E}igl[ar{S}igr]}{\sqrt{\mathrm{Var}igl[ar{S}igr]}}
ight) = rac{1+p}{2}$$

$$rac{k\,\mathrm{E}igl[ar{S}igr]}{\sqrt{\mathrm{Var}igl[ar{S}igr]}}=z_{(1+p)/2}$$

Note that:

•
$$\mathrm{E}igl[ar{S}igr] = \mathrm{E}iggl[rac{\sum_{i=1}^{n} S_i}{n}iggr] = rac{1}{n} \cdot n \cdot \mathrm{E}[S] = \mathrm{E}[S]$$

$$\bullet \ \, \mathrm{Var}\Big[\bar{S}\Big] = \mathrm{Var}\Bigg[\frac{\sum_{i=1}^n S_i}{n}\Bigg] = \frac{1}{n^2} \cdot n \cdot \mathrm{Var}[S] = \frac{\mathrm{Var}[S]}{n}$$

Then,

$$rac{k\,\mathrm{E}[S]}{\sqrt{rac{\mathrm{Var}[S]}{n}}} = z_{(1+p)/2}$$

$$\sqrt{n} \cdot rac{k \operatorname{E}[S]}{\sqrt{\operatorname{Var}[S]}} = z_{(1+p)/2}$$

$$egin{aligned} \sqrt{n} &= \left[rac{z_{(1+p)/2}}{k}
ight] \left(rac{\sqrt{ ext{Var}[S]}}{ ext{E}[S]}
ight) \ &= \left[rac{z_{(1+p)/2}}{k}
ight] (CV_S) \end{aligned}$$

Therefore,

$$n=\left[rac{z_{(1+p)/2}}{k}
ight]^2\!\left(CV_S^2
ight)$$