Overall Average Rate Change 30M

To calculate the ***indicated overall average rate*** or ***rate change***, let's go back to the fundamental insurance equation, i.e., $(\mathrm{S5.2.0.1})$. Recall that we want to ensure the premium charged is enough to cover future losses and expenses and to generate profit.

Let P^I be the indicated premium for the forecast period.

$$\begin{array}{ccccccc} \mathbf{Premium} & = & \mathbf{Losses} & + & \mathbf{Expenses} & + & \mathbf{Profit} \\ P^I & = & L & + & \left(V \cdot P^I + E_F\right) & + & Q_T \cdot P^I \end{array}$$

Coach's Remarks

Note that L includes both losses and loss adjustment expenses, LAE. We will hold this assumption throughout this section.

Solve this equation for the indicated premium:

$$\begin{aligned} P^I &= L + VP^I + E_F + Q_T P^I \\ P^I - VP^I - Q_T P^I &= L + E_F \\ P^I &= \frac{L + E_F}{1 - V - Q_T} \end{aligned} \qquad (\mathrm{S5.3.1.1})$$

Recall that the denominator of $(\mathrm{S5.3.1.1})$ is the permissible loss ratio from $(\mathrm{S5.2.4.1})$.

Depending on what data is available to us, there are two ways to proceed from here:

1. Loss ratio method

2. Pure premium method (also known as loss cost method)

Loss Ratio Method

Let P^C be the premium at the current rate level. If we divide (S5.3.1.1) by P^C, we will have:

$$\begin{aligned}\frac{P^I}{P^C} &= \frac{\frac{L}{P^C}+\frac{E_F}{P^C}}{1-V-Q_T} \\ \frac{P^I}{P^C}-1 &= \frac{LR+F}{1-V-Q_T}-1 \\ \text{Indicated Avg. Rate Change} &= \frac{LR+F}{1-V-Q_T}-1 \quad (\text{S5.3.1.2})\end{aligned}$$

This is the formula we will use under the ***loss ratio method***, as this method calculates the indicated average rate change from the current rate level. Notice that this formula uses LR, or the loss ratio, which is why this is called the loss ratio method.

Pure Premium Method

Let n_e be the number of exposures. If we divide (S5.3.1.1) by n_e, we will have:

$$\begin{aligned}\frac{P^I}{n_e} &= \frac{\frac{L}{n_e}+\frac{E_F}{n_e}}{1-V-Q_T} \\ \text{Indicated Avg. Rate} &= \frac{\bar{L}+\bar{E}_F}{1-V-Q_T} \quad (\text{S5.3.1.3})\end{aligned}$$

This is the formula we will use for the ***pure premium method***, also known as the ***loss cost method***. This method calculates the indicated average premium per exposure. Notice this method is called the pure premium method because $\bar{L}$ is the pure premium, or loss per exposure.

Coach's Remarks

To calculate the indicated average rate under the **loss ratio method**, we can multiply the current average rate by 1 + the indicated average rate change.

The indicated average rate and indicated average rate change are not associated with a specific rating variable. They are the **overall average rate** or **overall average rate change** across all rating variables. Note that this is implied, so the word "overall" is dropped from the terms.

Let's practice using these two methods in the following example.

You are given the following information:

- Premium at current rate level = 120,000
- Projected loss and loss adjustment expenses = 74,000
- Projected number of exposures = 1,000
- Variable expense ratio = 20%
- Fixed expenses per exposure unit = 25
- Target profit and contingencies = 5%
- Current average rate = 120

Calculate the indicated average rate change under the loss ratio method and the indicated average rate under the pure premium method.

Apply $(\mathbf{S5.3.1.2})$ under the loss ratio method. The indicated average rate change is:

$$\begin{aligned}\text{Indicated Avg. Rate Change} &= \frac{\frac{74{,}000}{120{,}000} + \frac{25(1{,}000)}{120{,}000}}{1 - 0.20 - 0.05} - 1 \\ &= \mathbf{0.10}\end{aligned}$$

Apply $(\text{S}5.3.1.3)$ under the pure premium method. The indicated average rate is:

$$\begin{aligned}\text{Indicated Avg. Rate} &= \frac{\frac{74{,}000}{1{,}000} + 25}{1 - 0.20 - 0.05} \\ &= \mathbf{132}\end{aligned}$$

If we were to use the indicated average rate change of 10% to calculate the indicated average rate under the loss ratio method, we would get:

$$120\,(1 + 0.10) = 132$$

Note that in the example above, both methods resulted in the same indicated average rate. This will not be the case in general. For example, if the premium at the current rate level is calculated using the parallelogram method, there will be discrepancies between the two approaches. The equivalence of these methods depends on using consistent data and assumptions for both approaches.

Coach's Remarks

When given **multiple years** of experience data, use the following procedure to calculate LR for $(\text{S}5.3.1.2)$:

- For each year, determine L and P^C using methods presented in Section S5.2.
- For each year, calculate the loss ratio as $\frac{L}{P^C}$.
- Calculate the weighted average of the loss ratios from each year; weights will be specified in the problem.

Use the same procedure to calculate $\bar{L}$ for $(\mathbf{S5.3.1.3})$, just replacing P^C with n_e. Note that while P^C can be calculated for each year, n_e will be given.

Let's apply the concepts we have learned so far to the following examples.

Example S5.3.1.1

Paige has been asked to calculate a rate change for annual policies effective October 1, 2018. New rates are assumed to be effective for one year. Policies are assumed to be written uniformly throughout the year.

She uses the following ultimate losses to project future loss experience. An annual effective rate of 3% is selected as the loss trend.

Accident Year	Estimated Ultimate Losses (in thousands)
2015	970
2016	967
2017	956

To match the loss experience, she uses the corresponding calendar year earned premiums.

Calendar Year	Earned Premium (in thousands)
2015	1,122
2016	1,155
2017	1,281

In the last three years, there have only been two rate changes:

- +3% on July 1, 2015
- +5% on January 1, 2017

Paige calculates the earned premiums at the current rate level using the parallelogram method. Paige then calculates the loss ratio for each year and selects the average as the projected loss ratio for the forecast period. All expenses are variable, and she decides on a permissible loss ratio of 80%.

Calculate the indicated average rate change using the loss ratio method.

Solution

Using the loss ratio method, the indicated average rate change is:

$$\text{Indicated Avg. Rate Change} = \frac{LR + F}{1 - V - Q_T} - 1$$

We are given that all expenses are variable, thus $F = 0$, and that the permissible loss ratio is 80%, hence $1 - V - Q_T = 0.80$. Our equation reduces to:

$$\text{Indicated Avg. Rate Change} = \frac{LR}{0.80} - 1$$

To calculate the projected loss ratio, we need to trend losses and bring premiums to the current rate level.

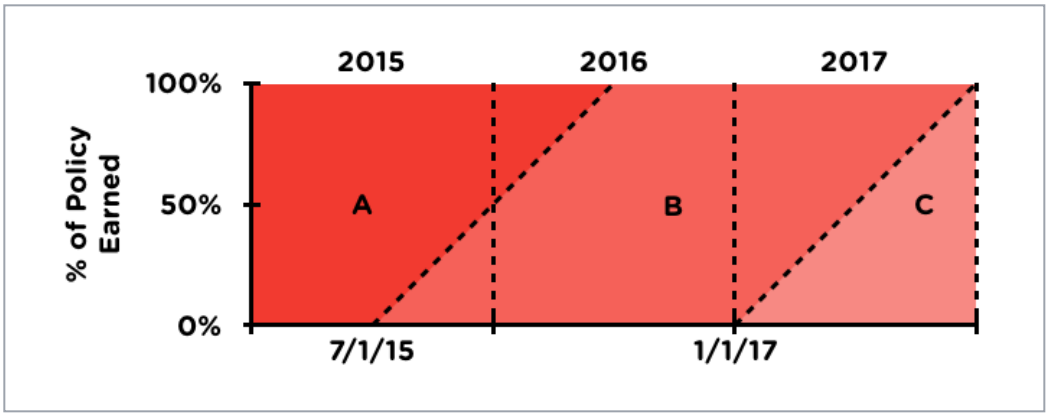
Step 1: Calculate the trended ultimate losses for each accident year. Recall that we learned how to trend losses in Section S5.2.3.

- First, calculate the average loss occurrence date for every experience period and the forecast period.
- Then, calculate the trend period and trend the ultimate losses for each accident year. Note that the annual trend factor is 1.03.

Accident Year	Trend From Date	Trend To Date	Trend Period	Trended Ultin
2015	7/1/2015	10/1/2019	4.25	1,1(
2016	7/1/2016	10/1/2019	3.25	1,0(
2017	7/1/2017	10/1/2019	2.25	1,02

Step 2: Calculate the earned premiums at the current rate level using the parallelogram method, as shown in Section S5.2.7.

- Draw this diagram and calculate the cumulative rate level index for each rate group.



Rate Group	Effective Date	Cumulative Rate Level Index
A	Initial	1.0000
B	7/1/2015	1.0300
C	1/1/2017	1.0815

- Compute the portion of the premium for each calendar year that corresponds to each rate group. Then, calculate the average cumulative rate level indexes.

Calendar Year	Portion			Average Cumulative Rate Level Index
	A	B	C	
2015	0.875	0.125	-	1.00375
2016	0.125	0.875	-	1.02625
2017	-	0.500	0.500	1.05575

- Calculate the on-level factors and use them to calculate the earned premiums at the current rate level. The values are tabulated below.

Calendar Year	On-Level Factor	Premium at Current Rate Level
2015	1.0775	1,209
2016	1.0538	1,217
2017	1.0244	1,312

Step 3: Compute the loss ratio for each year.

Year	Trended Ultimate Losses	Premium at Current Rate Level	Loss Ratio
2015	1,100	1,209	0.9098
2016	1,065	1,217	0.8746
2017	1,022	1,312	0.7786

Step 4: Then, compute the projected loss ratio for the forecast period as the average of the loss ratios.

$$LR = \frac{0.9098 + 0.8746 + 0.7786}{3} = 0.8543$$

Step 5: Finally, we can calculate the indicated average rate change as:

$$\text{Indicated Avg. Rate Change} = \frac{0.8543}{0.80} - 1 = \mathbf{0.0679}$$

Example S5.3.1.2

Cheap Insurance Corporation plans to expand its business by breaking into an existing market with a new insurance product. The product is expected to be launched on April 1, 2019. Annual policies are expected to be written uniformly during the year.

Caeden, an actuary in the company, is instructed to estimate the premium rate for the product. The rates are assumed to be effective for one year. He plans to use industry-wide loss data to predict future losses. A loss trend of 1.5% per year compounded continuously is selected.

Accident Year	Ultimate Losses	Earned Exposure	Weight
2016	5,160,000	12,760	40%
2017	4,110,000	12,800	60%

Caeden is told to include a variable expense provision of 33% and a target profit provision of 5%. A fixed underwriting cost of 25 is expected for each exposure.

Calculate the average rate charged for the new product using the pure premium method.

Solution

Using the pure premium method, the indicated average rate is:

$$\text{Indicated Avg. Rate} = \frac{\bar{L} + \bar{E}_F}{1 - V - Q_T}$$

We are given that $V = 0.33$, $Q_T = 0.05$, and $\bar{E}_F = 25$. Therefore, our equation reduces to:

$$\text{Indicated Avg. Rate} = \frac{\bar{L} + 25}{1 - 0.33 - 0.05}$$

To calculate the projected pure premium, we need to trend the ultimate losses.

Step 1: First, calculate the average loss occurrence date for every experience period and the forecast period.

- For AY2016, the average loss occurrence date is July 1, 2016.
- For AY2017, the average loss occurrence date is July 1, 2017.

Since annual policies are written beginning April 1, 2019, the average loss occurrence date for the forecast period is April 1, 2020.

Step 2: Calculate the trend period and the trended ultimate losses.

- For AY2016, $t = 3.75$; thus, the trended ultimate losses are:

$$5{,}160{,}000e^{0.015(3.75)} = 5{,}458{,}569$$

- For AY2017, $t = 2.75$; thus, the trended ultimate losses are:

$$4{,}110{,}000e^{0.015(2.75)} = 4{,}283{,}083$$

Step 3: Next, calculate the pure premium for each year as the trended ultimate losses divided by the earned exposures.

- For AY2016, the trended pure premium is:

$$\frac{5{,}458{,}569}{12{,}760} = 427.79$$

- For AY2017, the trended pure premium is:

$$\frac{4{,}283{,}083}{12{,}800} = 334.62$$

Step 4: Then, the projected pure premium is the weighted average of the trended pure premiums:

$$\begin{aligned}\bar{L} &= 0.4\,(427.79) + 0.6\,(334.62) \\ &= 371.88\end{aligned}$$

Step 5: Finally, the indicated premium per exposure is:

$$\begin{aligned}\text{Indicated Avg. Rate} &= \frac{371.88 + 25}{1 - 0.33 - 0.05} \\ &= \mathbf{640.14}\end{aligned}$$

Coach's Remarks

Instead of trending the ultimate losses and then calculating the pure premiums, we can also calculate and then trend the pure premiums. Divide the ultimate losses by the earned exposures to calculate the pure premium. Then, calculate the trended pure premium by applying the trend factor. The results will be identical.

Accident Year	Pure Premium	Trended Pure Premium
2016	404.39	427.79