Inflation

25M

We briefly touched on inflation when we discussed scaling random variables in Section S2.2.7. Exam questions often pair inflation with coverage modifications. Consider the following scenario:

Example S2.3.4.1

This year, losses follow a single-parameter Pareto distribution with the following PDF:

$$f(x) = \frac{20{,}000}{x^3}, \qquad x > 100$$

Individual losses increase 10% each year due to inflation.

An insurance policy has a policy limit of 220.

Calculate the expected insurance payment next year.

Solution

Let X represent this year's losses.

$$X \sim \text{S-P Pareto}\ (2,\ 100)$$

$1.1X$ represents **next** year's losses. The goal is to calculate

$$\mathrm{E}[1.1X \wedge 220]$$

Let's start with first principles.

$$
\begin{aligned}
\mathrm{E}[1.1X \wedge 220] &= \int_0^\infty \min(1.1x,\, 220)\, f(x)\,\mathrm{d}x \\
&= \int_0^\infty 1.1 \min\left(x,\, \frac{220}{1.1}\right) f(x)\,\mathrm{d}x \\
&= 1.1 \int_0^\infty \min(x,\, 200)\, f(x)\,\mathrm{d}x \\
&= 1.1\mathrm{E}[X \wedge 200]
\end{aligned}
$$

Look up the limited expected value formula from the exam table.

$$
\begin{aligned}
\mathrm{E}[1.1X \wedge 220] &= 1.1\mathrm{E}[X \wedge 200] \\
&= 1.1 \left[\frac{2 \cdot 100}{2-1} - \frac{(1)100^2}{(2-1)200^{2-1}}\right] \\
&= \mathbf{165}
\end{aligned}
$$

■

Alternative Solution

We can apply the concept of scaling discussed in Section S2.2.7. The single-parameter Pareto distribution is a scale distribution with θ as its scale parameter. Thus, next year's losses follow a single-parameter Pareto distribution, where α remains as 2 and $\theta = 1.1(100) = 110$.

$$1.1X \sim \text{S-P Pareto}\ (2, 110)$$

The limited expected value is

$$\begin{aligned}\mathrm{E}[1.1X \wedge 220] &= \frac{2 \cdot 110}{2-1} - \frac{(1)110^2}{(2-1)220^{2-1}} \\ &= \mathbf{165}\end{aligned}$$

From the example above, we arrive at the conclusion that

$$\mathrm{E}[(1+r)X \wedge d] = (1+r)\mathrm{E}\left[X \wedge \frac{d}{1+r}\right]$$

where r is the inflation rate.

Example S2.3.4.2

In 2016, losses were uniformly distributed on the interval $[0,\ 1{,}000]$.

Losses in 2017 are 20% higher than in 2016.

An insurance policy covers each loss subject to an ordinary deductible of 300.

Calculate the difference between the loss elimination ratios in 2016 and 2017, $LER_{2016} - LER_{2017}$.

Solution

Recall the loss elimination ratio formula for an ordinary deductible.

$$LER = \frac{\mathrm{E}[X \wedge d]}{\mathrm{E}[X]}$$

Let X represent the loss in 2016.

$$X \sim \textbf{Uniform}\ (0,\ 1{,}000)$$

Calculate the mean and limited expected value of losses in 2016.

$$\begin{aligned} \mathrm{E}[X] &= \frac{0 + 1{,}000}{2} \\ &= 500 \end{aligned}$$

$$\begin{aligned} \mathrm{E}[X \wedge 300] &= \int_0^{300} S(x)\,\mathrm{d}x \\ &= \int_0^{300} \left(1 - \frac{x}{1{,}000}\right)\mathrm{d}x \\ &= \left[x - \frac{x^2}{2{,}000}\right]_0^{300} \\ &= 255 \end{aligned}$$

Thus the loss elimination ratio in 2016 is

$$\begin{aligned} LER_{2016} &= \frac{255}{500} \\ &= 0.51 \end{aligned}$$

Losses in 2017 are 20% higher. Thus, losses in 2017 are represented by $1.2X$.

$$\begin{aligned} \mathrm{E}[1.2X] &= 1.2\,\mathrm{E}[X] \\ &= 1.2\,(500) \\ &= 600 \end{aligned}$$

$$\begin{aligned}
\mathrm{E}[1.2X \wedge 300] &= 1.2\,\mathrm{E}[X \wedge 250] \\
&= 1.2 \int_0^{250} S(x)\,\mathrm{d}x \\
&= 1.2 \int_0^{250} \left(1 - \frac{x}{1{,}000}\right) \mathrm{d}x \\
&= 1.2 \left[x - \frac{x^2}{2{,}000}\right]_0^{250} \\
&= 262.5
\end{aligned}$$

Thus, the loss elimination ratio in 2017 is

$$\begin{aligned}
LER_{2017} &= \frac{262.5}{600} \\
&= 0.4375
\end{aligned}$$

Calculate the final answer.

$$\begin{aligned}
LER_{2016} - LER_{2017} &= 0.51 - 0.4375 \\
&= \mathbf{0.0725}
\end{aligned}$$

Coach's Remarks

Alternatively, use the fact that

$$1.2X \sim \text{Uniform}\ (0,\ 1{,}200)$$

to calculate the mean and limited expected value of losses in 2017.

$$
\begin{aligned}
\mathrm{E}\left[1.2X \wedge 300\right] &= \int_0^{300} \left(1 - \frac{x}{1{,}200}\right) \mathrm{d}x \\
&= \left[x - \frac{x^2}{2{,}400}\right]_0^{300} \\
&= 262.5
\end{aligned}
$$

We can add the effects of inflation to (S2.3.3.1) as follows:

$$\mathrm{E}\left[Y^L\right] = \alpha\left(1+r\right)\left(\mathrm{E}\left[X \wedge \frac{m}{1+r}\right] - \mathrm{E}\left[X \wedge \frac{d}{1+r}\right]\right) \quad (\text{S2.$$

where

- X is the loss variable,
- u is the policy limit (set $u = \infty$ if policy limit doesn't apply),
- d is the deductible (set $d = 0$ if deductible doesn't apply),
- α is the coinsurance (set $\alpha = 1$ if coinsurance doesn't apply),
- r is the inflation rate (set $r = 0$ if inflation doesn't apply), and
- m is the maximum covered loss and equals $\dfrac{u}{\alpha} + d$.

Likewise, (S2.3.2.4) becomes

$$\mathrm{E}\left[\left(Y^P\right)^k\right] = \frac{\mathrm{E}\left[\left(Y^L\right)^k\right]}{S_X\left(\dfrac{d}{1+r}\right)}$$

$$\mathrm{E}\left[\left(Y^L\right)^k\right] = \mathrm{E}\left[\left(Y^P\right)^k\right] \cdot S_X\left(\frac{d}{1+r}\right) \tag{S2.3.4.2}$$

Alternatively, you can choose to stick with $(\mathrm{S}2.3.3.1)$ and replace X in the equation with $X^* = (1+r)X$. For example,

$$X \sim \text{Pareto}\ (\alpha,\ \theta) \quad \Rightarrow \quad X^* \sim \text{Pareto}\ (\alpha,\ \theta^*)$$

where $X^* = (1+r)X$ and $\theta^* = (1+r)\theta$.

The expected payment per loss would then be

$$\mathrm{E}\left[Y^L\right] = \alpha\left(\mathrm{E}[X^* \wedge m] - \mathrm{E}[X^* \wedge d]\right)$$

The alternative method only works when the distribution of X^* has a scale parameter.

Coach's Remarks

$(\mathrm{S}2.3.4.1)$ is applicable to almost all situations we have discussed, except for the following two cases that are not commonly seen.

For a **franchise deductible**, since the full amount is paid when the loss is greater than the deductible, two changes need to be made to the ultimate formula. First, the maximum covered loss will be d less than it would be for an ordinary deductible.

$$m = \frac{u}{\alpha}$$

Second, add back the effect of the deductible for the portion of loss above the deductible, i.e.

$$+ \alpha \cdot d \cdot S_X\left(\frac{d}{1+r}\right)$$

For **coinsurance that is applied before the deductible**, replace d with the deductible divided by the coinsurance factor.

$$d = \frac{\text{Deductible}}{\alpha}$$

Example S2.3.4.3

This year's losses follow a distribution that has the following CDF values and limited expected values:

x	$F(x)$	$\mathrm{E}[X \wedge x]$
500	0.44	375
600	0.49	429
1,800	0.79	818
2,415	0.85	925
3,000	0.89	1,000
4,350	0.93	1,115
∞	1.00	1,500

A policy covering the losses has an ordinary deductible of 600.

Assume losses are 20% higher next year.

The insurer is considering adding one of the following to the policy starting next year:

- A policy limit of 3,000.
- A coinsurance of α.

Determine α such that the expected payments will be equal regardless of the insurer's decision.

Solution

Calculate the expected payments under each option by modifying $(S2.3.4.1)$.

If the insurer chooses to add the policy limit, the expected payment will be

$$u = 3{,}000 \quad d = 600 \quad \alpha = 1 \quad r = 0.2 \quad m = 3{,}600$$

$$\begin{aligned} \mathrm{E}\left[Y^L\right] &= 1.2\left(\mathrm{E}\left[X \wedge \frac{3{,}600}{1.2}\right] - \mathrm{E}\left[X \wedge \frac{600}{1.2}\right]\right) \\ &= 1.2\left(\mathrm{E}[X \wedge 3{,}000] - \mathrm{E}[X \wedge 500]\right) \\ &= 1.2\left(1{,}000 - 375\right) \\ &= 750 \end{aligned}$$

If the insurer chooses to add the coinsurance, the expected payment will be

$$u = \infty \quad d = 600 \quad \alpha = \alpha \quad r = 0.2 \quad m = \infty$$

$$\begin{aligned} \mathrm{E}\left[Y^L\right] &= \alpha\,(1.2)\left(\mathrm{E}\left[X \wedge \frac{\infty}{1.2}\right] - \mathrm{E}\left[X \wedge \frac{600}{1.2}\right]\right) \\ &= \alpha\,(1.2)\left(\mathrm{E}[X] - \mathrm{E}[X \wedge 500]\right) \\ &= \alpha\,(1.2)\left(1{,}500 - 375\right) \\ &= 1{,}350\alpha \end{aligned}$$

Equate the two expected payments to solve for α:

$$\begin{aligned} 750 &= 1{,}350\alpha \\ \alpha &= \mathbf{0.5556} \end{aligned}$$

It is uncommon for a question to ask for the variance of the payment. If it does, it usually

- involves a distribution that has a special property, such as memoryless, so that the variance can be calculated easily (as illustrated in Example S2.3.2.4), or
- has a simple payment structure so that you can solve it using first principles (see Example S2.3.4.4 below).

Example S2.3.4.4

Losses follow a probability distribution that has the following survival function:

$$S(x) = 1 - \frac{x}{200}, \qquad 0 \le x \le 200$$

An insurance policy has a deductible of 20 and a maximum covered loss of 100.

Calculate the variance of payment per payment, $\mathbf{Var}\left[Y^P\right]$.

Solution

Based on the survival function, losses follow a uniform distribution on the interval 0 to 200.

Also, a maximum covered loss of 100 indicates a policy limit of 80.

$$u = m - d = 100 - 20 = 80$$

The variance is the second moment minus the first moment squared.

$$\mathrm{Var}\left[Y^P\right] = \mathrm{E}\left[\left(Y^P\right)^2\right] - \mathrm{E}\left[Y^P\right]^2$$

A simpler way to calculate Y^P's moments is by calculating Y^L's moments, and then converting them.

Start by determining the payment per loss variable. The insurer pays

- nothing for losses below the deductible;
- the amount in excess of the deductible for losses between the deductible and the maximum covered loss;
- the policy limit for losses greater than the maximum covered loss.

The payment per loss variable is

$$Y^L = \begin{cases} 0, & X \le 20 \\ X - 20, & 20 < X < 100 \\ 80, & X \ge 100 \end{cases}$$

Calculate the first and second moments using first principles.

$$\begin{aligned} \mathrm{E}\left[Y^L\right] &= \int_{20}^{100} (x - 20) \cdot \frac{1}{200}\,\mathrm{d}x + 80\,S(100) \\ &= \left[\frac{(x-20)^2}{400}\right]_{20}^{100} + 80\left(1 - \frac{100}{200}\right) \\ &= 56 \end{aligned}$$

$$
\begin{aligned}
\mathrm{E}\left[\left(Y^L\right)^2\right] &= \int_{20}^{100} (x-20)^2 \cdot \frac{1}{200}\,\mathrm{d}x + 80^2 S(100) \\
&= \left[\frac{(x-20)^3}{600}\right]_{20}^{100} + 80^2(0.5) \\
&= 4{,}053.3333
\end{aligned}
$$

Using $(\mathrm{S}2.3.2.5)$, convert Y^L's moments to Y^P's moments:

$$\mathrm{E}\left[Y^P\right] = \frac{\mathrm{E}\left[Y^L\right]}{S(d)} = \frac{56}{0.9} = 62.2222$$

$$\mathrm{E}\left[\left(Y^P\right)^2\right] = \frac{\mathrm{E}\left[\left(Y^L\right)^2\right]}{S(d)} = \frac{4{,}053.3333}{0.9} = 4{,}503.7037$$

where $S(d) = S(20) = 1 - \dfrac{20}{200} = 0.9$.

Finally, calculate the variance.