Summary 10M

Payment Per Loss

Y^L is the payment per loss variable.

Policy Limits

The policy limit is the maximum amount the **insurer** will pay for a single loss. For an insurance policy with a policy limit u,

$$Y^L = X \wedge u = \begin{cases} X, & X < u \\ u, & X \geq u \end{cases}$$

$$\begin{aligned} \mathrm{E}\left[\left(Y^L\right)^k\right] &= \mathrm{E}\left[(X \wedge u)^k\right] \\ &= \int_0^u x^k f(x)\,\mathrm{d}x + u^k \cdot S(u) \\ &= \int_0^u k\,x^{k-1} S(x)\,\mathrm{d}x \end{aligned}$$

$(X \wedge u)$ is the limited loss variable, and its mean, $\mathrm{E}[X \wedge u]$, is the limited expected value.

INCREASED LIMIT FACTOR

For b is the original limit and u is the increased limit,

$$ILF = \frac{\mathrm{E}[X \wedge u]}{\mathrm{E}[X \wedge b]}$$

Deductibles

The deductible is the amount the **policyholder** has to pay before receiving any reimbursement.

ORDINARY DEDUCTIBLE

$$Y^L = (X-d)_+ = \begin{cases} 0, & X < d \\ X-d, & X \geq d \end{cases} = X - (X \wedge d)$$

$$\mathrm{E}\left[Y^L\right] = \mathrm{E}\left[(X-d)_+\right] = \mathrm{E}[X] - \mathrm{E}[X \wedge d]$$

$$\begin{aligned} \mathrm{E}\left[\left(Y^L\right)^k\right] &= \mathrm{E}\left[(X-d)_+^k\right] \\ &= \int_d^\infty (x-d)^k f(x)\,\mathrm{d}x \\ &= \int_d^\infty k\,(x-d)^{k-1} S(x)\,\mathrm{d}x \end{aligned}$$

LOSS ELIMINATION RATIO

For a policy with an ordinary deductible d,

$$LER = \frac{\mathrm{E}[X \wedge d]}{\mathrm{E}[X]}$$

FRANCHISE DEDUCTIBLE

$$Y^L = \begin{cases} 0, & X \le d \\ X, & X > d \end{cases}$$

$$\mathrm{E}\left[Y^L\right] = \mathrm{E}\left[(X-d)_+\right] + d \cdot S(d)$$

Payment Per Payment

Y^P is the payment per payment variable, i.e., $Y^P = Y^L \mid Y^L > 0$.

$$\mathrm{E}\left[Y^P\right] = \frac{\mathrm{E}\left[Y^L\right]}{S(d)} \qquad \Leftrightarrow \qquad \mathrm{E}\left[Y^L\right] = \mathrm{E}\left[Y^P\right] \cdot S(d)$$

$(X - d \mid X > d)$ is the excess loss variable, and its mean, $e(d)$, is the mean excess loss.

$$e(d) = \mathrm{E}[X - d \mid X > d] = \frac{\mathrm{E}\left[(X-d)_+\right]}{S(d)}$$

HELPFUL PROPERTIES

Loss, X	**Mean,** [illegible]	**Excess Loss,** [illegible]	**Mean Excess Loss,** [illegible]
Exponential (θ)	θ	Exponential (θ)	θ
Uniform $(a,\ b)$	$\frac{a+b}{2}$	Uniform $(0,\ b-d)$	$\frac{b-d}{2}$

Loss, $\boldsymbol{X}$	**Mean,** $\mathbf{E}[\boldsymbol{X}]$	**Excess Loss,** $\boldsymbol{X-d \mid X>d}$	**Mean Excess Loss,** $\mathbf{E}[\boldsymbol{X-d \mid X>d}]$
Pareto $(\alpha,\, \theta)$	$\dfrac{\theta}{\alpha-1}$	Pareto $(\alpha,\, \theta+d)$	$\dfrac{\theta+d}{\alpha-1}$
S-P Pareto $(\alpha,\, \theta)$	$\dfrac{\alpha\theta}{\alpha-1}$	Pareto $(\alpha,\, d)$	$\dfrac{d}{\alpha-1}$

The Ultimate Formula

$$\mathrm{E}\left[Y^L\right] = \alpha\,(1+r)\left(\mathrm{E}\left[X \wedge \frac{m}{1+r}\right] - \mathrm{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

where

- X is the loss variable,
- d is the deductible (set to 0 if not applicable),
- u is the policy limit (set to ∞ if not applicable),
- α is the coinsurance (set to 1 if not applicable),