Optimal Power Flow using Matpower and Pyomo

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Table of Contents

Optimal Power Flow?

The optimal power flow(OPF) can integrate the compuation of power flow and operational problems (such as economic dispatch and unit commitment) subject to the power system's pysical and electrical constraints.

> *The optimal power flow (OPF) integrates the computation of power flow and economic dispatch subject to the system's physical and electrical constraints ...(Ali et al. 2024)*

Optimal Power Flow?

- Optimal power flow = Optimization problem + Power flow

Optimization problem

- Find the optimal variables satisfying the objective function
- Example:

$$\begin{aligned} \min_{x \in D} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \quad i = 1, \ldots, m \\ & h_j(x) = 0, \quad j = 1, \ldots, r \end{aligned}$$

Power flow

- Find the variables satisfying the power flow equation
- Example:

$$P_i = |V_i| \sum_{\forall j} |V_j| \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right]$$

Optimal Power Flow?

Optimization problem and power flow problem

- Optimization problem
  - Solve the problem based on the optimization techniques (convex, branch and bound, etc)
    - 조건에 따라 해가 존재하지 않을 수 있음
    - Variables: Generator status, reactive/active power, voltage magnitude/phase, etc
- Power flow
  - Solve the problem based on finding the solution on multiple equations (Newton Raphson, Gauss-Seidel)
    - 초기값 혹은 계통에 문제가 없지 않은 이상 해는 존재
    - Variables: Reactive/active power, voltage magnitude/phase
- Optimization problem + power flow
  - 다양한 목적함수를 가지며 제약조건으로 power flow를 갖는 최적화 문제
    - 손실 최소화, 비용 최소화, 이익 최대화 등

Optimal Power Flow?

Problems defined by variable type

- Linear programming (LP)
  - 모든 목적함수와 제약조건이 선형화 되어있는 상황이며, 발전량과 전압의 크기/위상 등 continuous variable 을 결정하는 문제
  - Cplex, Gurobi 등 solver 이용 가능
  - Global optimum 을 찾을 수 있으나 비선형방정식을 선형방정식으로 바꿔야 함
- Mixed integer linear programming (MILP)
  - 모든 목적함수와 제약조건이 선형화 되어있고, 이진변수(0 or 1)이 존재하는 최적화 문제
  - 이진변수는 발전기의 On/Off, ESS의 충방전 Mode 선택 등 다양한 변수가 존재
  - Cplex, Gurobi 등 solver 이용 가능
  - Global optimum 을 찾을 수 있으나 비선형방정식을 선형방정식으로 바꿔야 함
  - Solver가 이진변수를 relaxation (이진 변수를 연속 변수로 완화) 하는 것으로 알려짐

Optimal Power Flow?

Problems defined by variable type

- Mixed integer nonlinear programming (MINLP)
  - 목적함수와 제약조건에 비선형 방정식 등이 포함된 최적화 문제
  - 이진변수, 연속변수 등 다양한 변수와 함께 전력방정식이 그대로 포함될 수 있음
  - 문제를 선형함수로 변환하지 않아도 되기 때문에 해가 구해진다면 정확한 해일 가능성이 높음
  - 하지만, solver 성능에 따라 문제가 풀리지 않는 경우들이 대부분
  - Local optimum 과 global optimum 이 같지 않을 수 있음을 항상 고민해야 함
  - Knitro 등의 solve를 사용
- Meta heuristic
  - 비선형 방정식 등이 포함된 최적화 문제이나 문제 해결을 위해 다양한 방법을 활용하는 기법
  - 기본적으로 변수에 여러 값들을 대입하면서 구하는 문제 해결 방법이지만, 다양한 알고리즘이 사용됨
  - Genetic algorithm, particle swarm optimization, harmony search 등의 방법이 사용됨
  - Pygmo 등으로 이용할 수 있으며, matlab 을 이용하는 것이 일반적임

Optimal Power Flow?

How to solve?

- 문제 해결의 기본 방향
  - 최적화 문제를 처음 해결하는 상황이라면, 목적함수와 제약조건의 방정식을 그대로 이용할 수 있는 MINLP와 meta heuristic 방법을 이용하는 것도 방법임
  - 위 방법을 이용한 이후에는 relaxation 등을 통하여 MILP, LP 문제로 바꾸어 해결하는 방안을 고민할 수 있음
- 문제 유형별 사용 Solver
  - LP, MILP: Pyomo 에서 cplex, gurobi 등을 연계
  - MINLP: Pyomo 에서 knitro 등을 연계
  - Meta heuristic: Pyomo 에서는 해결하지 못하며, Pygmo 등을 사용

Typical optimal power flow problem

Minimize the loss in the overall system(Lavorato et al. 2012).

A. Distribution System Reconfiguration Problem

The DSR problem is modeled as follows:

$$
\begin{aligned}
\min \quad & v = \sum_{(ij)\in\Omega_l} \left(g_{ij} x_{ij} \left(V_i^2 + V_j^2 - 2V_iV_j\cos\theta_{ij}\right)\right) && (18)\\
\text{s.t.} \quad & \\
& P_{S_i} - P_{D_i} - \sum_{j\in\Omega_{b_i}} (x_{ij}P_{ij}) = 0 \quad \forall i \in \Omega_b && (19)\\
& Q_{S_i} - Q_{D_i} - \sum_{j\in\Omega_{b_i}} (x_{ij}Q_{ij}) = 0 \quad \forall i \in \Omega_b && (20)\\
& \underline{V} \le V_i \le \overline{V} \quad \forall i \in \Omega_b && (21)\\
& x_{ij}\left(I_{r_{ij}}^2 + I_{m_{ij}}^2\right) \le \overline{I}_{ij}^2 \quad \forall (ij) \in \Omega_l && (22)\\
& x_{ij} \in \{0,1\} \quad \forall (ij) \in \Omega_l && (23)\\
& \text{Eqs. } (14) - (17) \\
& \sum_{(ij)\in\Omega_l} x_{ij} = n_b - 1 - \sum_{j\in\Omega_{b_p}} (1 - y_j). && (24)
\end{aligned}
$$

Typical optimal power flow problem

Objective function

- Objective function: Minimize the loss

$$\min \quad v = \sum_{(ij)\in\Omega_l} (g_{ij}(|V_i|^2 + |V_j|^2 - 2|V_i||V_j|\cos\theta_{ij}))$$

Typical optimal power flow problem

Constraints

- Balance
  - Active power

$$P_{S_i} - P_{D_i} - \sum_{j \in \Omega_{b_i}} P_{ij} = 0$$

$$\left(P_{ij} = -\left|V_i\right|^2 G_{ij} + \left|V_i\right| \left|V_j\right| \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} \right) \right)$$

  - Reactive power

$$Q_{S_i} - Q_{D_i} - \sum_{j \in \Omega_{b_i}} Q_{ij} = 0$$

$$Q_{ij} = \left|V_i\right|^2 B_{ij} + \left|V_i\right| \left|V_j\right| \left(G_{ij} \sin\theta_{ij} - B_{ij} \sin\theta_{ij} \right)$$

Typical optimal power flow problem

Constraints

- Inequality constraints
  - Voltage

$$\underline{|V_i|} \leq |V_i| \leq \overline{|V_i|}$$

  - Line rating

$$I_{real,ij}^2 + I_{imag,ij}^2 \leq \overline{I_{ij}^2}$$

Our main objective is to...

- Formulate the objective function and constraints of the Optimal Power Flow (OPF) problem.
- Implement the formulated objective function and constraints using Matpower and Pyomo.
- Derive simulation results of the implemented model on a 33-bus test system.

Overview

Depicted by latex..

- Nomenclature
- Objective function
- Constraints
    - Balance

Nomenclature

Sets, indices, parameters

- Indices

i, j	Index of bus
l	Index of line

- Sets

Ω_l	Set of lines
Ω_b	Set of buses
Ω_{b_i}	Set of connected buses in the bus i
Ω_{b_g}	Set of generation buses ($\Omega_{b_g} \subset \Omega_b$)

- Parameters or constants

Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j)
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j)
$baseMVA$	Value of base MVA
$\overline{V}$, $\underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line ij

Nomenclature

Sets, indices, parameters

- Parameters or constants (Continued)

P_{D_i}, Q_{D_i}	Active and reactive power demand at bus i
$\overline{P}_{G_i}$, $\underline{P}_{G_i}$	Maximum and minimum active power from generator at bus i
$\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus i

- Functions

P_i, Q_i	Active and reactive power calculated at bus i
P_{ij}, Q_{ij}	Active and reactive power flow of line ij
$I_{r_{ij}}$, $I_{Im_{ij}}$	Real and Imaginary current flow of line ij
P_{ij}^{loss}, Q_{ij}^{loss}	Active and reactive line loss of line ij

Nomenclature

Variables

- Varaibles

$\left\lvert \dot{V}_i \right\rvert$	Voltage magnitude in bus i
θ_i	Voltage phase angle in bus i
P_{G_i}, Q_{G_i}	Active and reactive power from generator at bus i

Objective function

$$\min \sum_{\forall l} P_l^{lineloss} \tag{1}$$

$$P_l^{lineloss} = P_{ij} + P_{ji} \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \tag{2}$$

$$\begin{aligned} P_{ij} = & - G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_i\right| + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\left(\theta_i - \theta_j\right) \\ & + B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\left(\theta_i - \theta_j\right) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{3}$$

$$\begin{aligned} P_{ji} = & - G_{ij} \left|\dot{V}_j\right| \left|\dot{V}_j\right| + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\left(\theta_i - \theta_j\right) \\ & - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\left(\theta_i - \theta_j\right) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{4}$$

Constraints

Load balance

$$P_{G_i} - P_{D_i} = \sum_{j \in \Omega_{b_i}} (P_{ij}) \quad \forall i \in \Omega_b \tag{5}$$

$$Q_{G_i} - Q_{D_i} = \sum_{j \in \Omega_{b_i}} (Q_{ij}) \quad \forall i \in \Omega_b \tag{6}$$

$$\begin{aligned} Q_{ij} = & B_{ij} \left|\dot{V}_i\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin(\theta_i - \theta_j) \\ & - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{7}$$

Constraints

Power and voltage

$$\underline{P}_{G_i} \le P_{G_i} \le \overline{P}_{G_i} \quad \forall i \in \Omega_b \tag{8}$$

$$\underline{Q}_{G_i} \le Q_{G_i} \le \overline{Q}_{G_i} \quad \forall i \in \Omega_b \tag{9}$$

$$\underline{V} \le \left|\dot{V}_i\right| \le \overline{V} \quad \forall i \in \Omega_b \tag{10}$$

$$\theta_i = 0 \quad \text{if bus } i \quad \text{is slack bus.} \tag{11}$$

Constraints

Current

$$I_{r_{ij}}^2 + I_{Im_{ij}}^2 \leq \bar{I}_{ij}^2 \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \tag{12}$$

$$\begin{aligned} I_{r_{ij}} = &- G_{ij} \left| \dot{V}_i \right| \cos\theta_i + B_{ij} \left| \dot{V}_i \right| \sin\theta_i \\ &+ G_{ij} \left| \dot{V}_j \right| \cos\theta_j - B_{ij} \left| \dot{V}_j \right| \sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{13}$$

$$\begin{aligned} I_{Im_{ij}} = &- B_{ij} \left| \dot{V}_i \right| \cos\theta_i - G_{ij} \left| \dot{V}_i \right| \sin\theta_i \\ &+ B_{ij} \left| \dot{V}_j \right| \cos\theta_j + G_{ij} \left| \dot{V}_j \right| \sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{14}$$

Constraints

Sending flow

$$
\begin{aligned}
\dot{S}_{ij} &= \dot{V}_i \dot{I}_{ij}^* = \dot{V}_i \left(\frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \right)^* \\
&= \dot{V}_i \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_i \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \qquad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= \left| \dot{V}_i \right|^2 \left(-G_{ij} + jB_{ij} \right) - \left| \dot{V}_i \right| \left| \dot{V}_j \right| e^{j\left(\theta_i - \theta_j\right)} \left(-G_{ij} + jB_{ij} \right) \\
&= -G_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos\theta_{ij} + B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin\theta_{ij} \\
&\quad + j \left(B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin\theta_{ij} - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos\theta_{ij} \right)
\end{aligned}
$$

Constraints

Sending flow (continued)

$$\therefore P_{ij} = -G_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) + B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right)$$

$$Q_{ij} = B_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right)$$

Constraints

Receiving flow

$$
\begin{aligned}
\dot{S}_{ji} &= \dot{V}_j \dot{I}_{ji}^* = -\dot{V}_j \dot{I}_{ij}^* \\
&= \dot{V}_j \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_j \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \quad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= -\left|\dot{V}_i\right| \left|\dot{V}_j\right| e^{-j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) + \left|\dot{V}_j\right|^2 (-G_{ij} + jB_{ij}) \\
&= -G_{ij} \left|\dot{V}_j\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} \\
&\quad + j\left(B_{ij} \left|\dot{V}_j\right|^2 - G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} \right)
\end{aligned}
$$

Constraints

Receiving flow (continued)

$$\therefore P_{ji} = -G_{ij}\left|\dot{V}_j\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right)$$

$$Q_{ij} = B_{ij}\left|\dot{V}_j\right|^2 - G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right)$$

Constraints

Current

$$
\begin{aligned}
I_{ij} &= \frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \qquad\qquad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= \left(-G_{ij} - jB_{ij}\right) \left(\dot{V}_i - \dot{V}_j \right) \\
&= \left(-G_{ij} - jB_{ij}\right) \left(\left|\dot{V}_i\right| \cos\theta_i + j \left|\dot{V}_i\right| \sin\theta_i - \left|\dot{V}_j\right| \cos\theta_j - j \left|\dot{V}_j\right| \sin\theta_j \right) \\
&= -G_{ij} \left|\dot{V}_i\right| \cos\theta_i + B_{ij} \left|\dot{V}_i\right| \sin\theta_i + G_{ij} \left|\dot{V}_j\right| \cos\theta_j - B_{ij} \left|\dot{V}_j\right| \sin\theta_j \\
&\quad + j \left(-B_{ij} \left|\dot{V}_i\right| \cos\theta_i - G_{ij} \left|\dot{V}_i\right| \sin\theta_i + B_{ij} \left|\dot{V}_j\right| \cos\theta_j + G_{ij} \left|\dot{V}_j\right| \sin\theta_j \right)
\end{aligned}
$$

Constraints

Current (Continued)

$$\therefore I_{ij,real} = -G_{ij}\left|\dot{V}_i\right|\cos\theta_i + B_{ij}\left|\dot{V}_i\right|\sin\theta_i + G_{ij}\left|\dot{V}_j\right|\cos\theta_j - B_{ij}\left|\dot{V}_j\right|\sin\theta_j$$

$$I_{ij,imaginary} = -B_{ij}\left|\dot{V}_i\right|\cos\theta_i - G_{ij}\left|\dot{V}_i\right|\sin\theta_i + B_{ij}\left|\dot{V}_j\right|\cos\theta_j + G_{ij}\left|\dot{V}_j\right|\sin\theta_j$$

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The End

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