Optimal Power Flow using Matpower and Pyomo

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Optimal Power Flow?

The optimal power flow(OPF) can integrate the compuation of power flow and operational problems (such as economic dispatch and unit commitment) subject to the power system's pysical and electrical constraints.

> *The optimal power flow (OPF) integrates the computation of power flow and economic dispatch subject to the system's physical and electrical constraints . . . (Ali et al. 2024)*

Optimal Power Flow?

- Optimal power flow = Optimization problem + Power flow

Optimization problem

- Find the optimal variables satisfying the objective function
- Example:

$$
\begin{aligned}
\min_{x \in D} \quad & f(x) \\
\text{subject to} \quad & g_i(x) \leq 0, \quad i = 1, \ldots, m \\
& h_j(x) = 0, \quad j = 1, \ldots, r
\end{aligned}
$$

Power flow

- Find the variables satisfying the power flow equation
- Example:

$$
\begin{aligned}
P_i = |V_i| \sum_{\forall j} |V_j| [&G_{ij} \cos(\theta_i - \theta_j) \\
&+ B_{ij} \sin(\theta_i - \theta_j)]
\end{aligned}
$$

Optimal Power Flow?

Optimization problem and power flow problem

- Optimization problem
  - Solve the problem based on the optimization techniques (convex, branch and bound, etc)
    - 조건에 따라 해가 존재하지 않을 수 있음
    - Variables: Generator status, reactive/active power, voltage magnitude/phase, etc
- Power flow
  - Solve the problem based on finding the solution on multiple equations (Newton Raphson, Gauss-Seidel)
    - 초기값 혹은 계통에 문제가 없지 않은 이상 해는 존재
    - Variables: Reactive/active power, voltage magnitude/phase
- Optimization problem + power flow
  - 다양한 목적함수를 가지며 제약조건으로 power flow를 갖는 최적화 문제
    - 손실 최소화, 비용 최소화, 이익 최대화 등

Optimal Power Flow?

Problems defined by variable type

- Linear programming (LP)
  - 모든 목적함수와 제약조건이 선형화 되어있는 상황이며, 발전량과 전압의 크기/위상 등 continuous variable 을 결정하는 문제
  - Cplex, Gurobi 등 solver 이용 가능
  - Global optimum 을 찾을 수 있으나 비선형방정식을 선형방정식으로 바꿔야 함
- Mixed integer linear programming (MILP)
  - 모든 목적함수와 제약조건이 선형화 되어있고, 이진변수(0 or 1)이 존재하는 최적화 문제
  - 이진변수는 발전기의 On/Off, ESS의 충방전 Mode 선택 등 다양한 변수가 존재
  - Cplex, Gurobi 등 solver 이용 가능
  - Global optimum 을 찾을 수 있으나 비선형방정식을 선형방정식으로 바꿔야 함
  - Solver가 이진변수를 relaxation (이진 변수를 연속 변수로 완화) 하는 것으로 알려짐

Optimal Power Flow?

Problems defined by variable type

- Mixed integer nonlinear programming (MINLP)
  - 목적함수와 제약조건에 비선형 방정식 등이 포함된 최적화 문제
  - 이진변수, 연속변수 등 다양한 변수와 함께 전력방정식이 그대로 포함될 수 있음
  - 문제를 선형함수로 변환하지 않아도 되기 때문에 해가 구해진다면 정확한 해일 가능성이 높음
  - 하지만, solver 성능에 따라 문제가 풀리지 않는 경우들이 대부분
  - Local optimum 과 global optimum 이 같지 않을 수 있음을 항상 고민해야 함
  - Knitro 등의 solve를 사용
- Meta heuristic
  - 비선형 방정식 등이 포함된 최적화 문제이나 문제 해결을 위해 다양한 방법을 활용하는 기법
  - 기본적으로 변수에 여러 값들을 대입하면서 구하는 문제 해결 방법이지만, 다양한 알고리즘이 사용됨
  - Genetic algorithm, particle swarm optimization, harmony search 등의 방법이 사용됨
  - Pygmo 등으로 이용할 수 있으며, matlab 을 이용하는 것이 일반적임

Optimal Power Flow?

How to solve?

- 문제 해결의 기본 방향
  - 최적화 문제를 처음 해결하는 상황이라면, 목적함수와 제약조건의 방정식을 그대로 이용할 수 있는 MINLP와 meta heuristic 방법을 이용하는 것도 방법임
  - 위 방법을 이용한 이후에는 relaxation 등을 통하여 MILP, LP 문제로 바꾸어 해결하는 방안을 고민할 수 있음
- 문제 유형별 사용 Solver
  - LP, MILP: Pyomo 에서 cplex, gurobi 등을 연계
  - MINLP: Pyomo 에서 knitro 등을 연계
  - Meta heuristic: Pyomo 에서는 해결하지 못하며, Pygmo 등을 사용

Previous studies

Typical optimal power flow problem

Minimize the loss in the overall system(Lavorato et al. 2012).

A. Distribution System Reconfiguration Problem

The DSR problem is modeled as follows:

$$\min \quad v = \sum_{(ij)\in\Omega_l} (g_{ij} x_{ij} (V_i^2 + V_j^2 - 2V_i V_j \cos\theta_{ij})) \quad (18)$$

s.t.

$$P_{S_i} - P_{D_i} - \sum_{j\in\Omega_{b_i}} (x_{ij} P_{ij}) = 0 \quad \forall i \in \Omega_b \quad (19)$$

$$Q_{S_i} - Q_{D_i} - \sum_{j\in\Omega_{b_i}} (x_{ij} Q_{ij}) = 0 \quad \forall i \in \Omega_b \quad (20)$$

$$\underline{V} \le V_i \le \overline{V} \quad \forall i \in \Omega_b \quad (21)$$

$$x_{ij}(I_{r_{ij}}^2 + I_{m_{ij}}^2) \le \overline{I}_{ij}^2 \quad \forall (ij) \in \Omega_l \quad (22)$$

$$x_{ij} \in \{0, 1\} \quad \forall (ij) \in \Omega_l \quad (23)$$

Eqs. (14) – (17)

$$\sum_{(ij)\in\Omega_l} x_{ij} = n_b - 1 - \sum_{j\in\Omega_{b_p}} (1 - y_j). \quad (24)$$

Previous studies

Typical optimal power flow problem

- Objective function: Minimize the loss

$$\min \quad v = \sum_{(ij)\in\Omega_l} (g_{ij}(|V_i|^2 + |V_j|^2 - 2|V_i||V_j|\cos\theta_{ij}))$$

Previous studies

Typical optimal power flow problem

Equality constraints:

- Balance
  - Active power

$$P_{S_i} - P_{D_i} - \sum_{j \in \Omega_{b_i}} P_{ij} = 0$$

$$\left(P_{ij} = -\left|V_i\right|^2 G_{ij} + \left|V_i\right|\left|V_j\right|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right)\right)$$

  - Reactive power

$$Q_{S_i} - Q_{D_i} - \sum_{j \in \Omega_{b_i}} Q_{ij} = 0$$

$$Q_{ij} = \left|V_i\right|^2 B_{ij} + \left|V_i\right|\left|V_j\right|\left(G_{ij}\sin\theta_{ij} - B_{ij}\sin\theta_{ij}\right)$$

Typical optimal power flow problem

Constraints

Inequality constraints:

- Operation condition
  - Voltage

$$\underline{|V_i|} \leq |V_i| \leq \overline{|V_i|}$$

  - Line rating

$$I_{real,ij}^2 + I_{imag,ij}^2 \leq \overline{I_{ij}^2}$$

Our main objective is to...

- Formulate the objective function and constraints of the Optimal Power Flow (OPF) problem.
- Implement the formulated objective function and constraints using Matpower and Pyomo.
- Derive simulation results of the implemented model on a 33-bus test system.

Formulation

Overview

- Nomenclature: Slide 35
- At glance...: Slide 38
- Objective function: Slide 41
- Constraints and expressions: Slide 46
  - Load balance
  - Power and voltage
  - Current

Optimization problem is formulated as:

1. Objective function:

$$\text{minimize (or maximize) } f(\mathbf{x})$$

2. Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

3. Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

Nomenclature

Sets, indices, parameters

- Indices

i, j	Index of bus
l	Index of line

- Sets

Ω_l	Set of lines
Ω_b	Set of buses
Ω_{b_i}	Set of connected buses in the bus i
Ω_{b_g}	Set of generation buses ($\Omega_{b_g} \subset \Omega_b$)

- Parameters or constants

Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j)
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j)
$baseMVA$	Value of base MVA
$\overline{V}$, $\underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line ij

Nomenclature

Sets, indices, parameters

- Parameters or constants (Continued)

P_{D_i}, Q_{D_i}	Active and reactive power demand at bus i
$\overline{P}_{G_i}$, $\underline{P}_{G_i}$	Maximum and minimum active power from generator at bus i
$\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus i

- Functions

P_{ij}, Q_{ij}	Active and reactive power flow of line ij
$I_{r_{ij}}$, $I_{Im_{ij}}$	Real and Imaginary current flow of line ij
$P_l^{lineloss}$	Active line loss of line $l(ij)$

Nomenclature

Variables

- Varaibles

$\left\lvert \dot{V}_i \right\rvert$	Voltage magnitude in bus i
θ_i	Voltage phase angle in bus i
P_{G_i}, Q_{G_i}	Active and reactive power from generator at bus i

At glance...

Objective function: Eq. (16)

$$\min \sum_{\forall i,j} [-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right| \cos(\theta_i - \theta_j)]$$

Constraints: Eqs. (20),(21),(23),(24),(25),(26),(28)

$$
\begin{aligned}
s.t. \quad P_{G_i} - P_{D_i} &= \sum_{j \in \Omega_{b_i}} (P_{ij}) \quad \forall i \in \Omega_b \\
Q_{G_i} - Q_{D_i} &= \sum_{j \in \Omega_{b_i}} (Q_{ij}) \quad \forall i \in \Omega_b \\
\underline{P}_{G_i} \leq P_{G_i} &\leq \overline{P}_{G_i} \quad \forall i \in \Omega_b \\
\underline{Q}_{G_i} \leq Q_{G_i} &\leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b
\end{aligned}
$$

$$\underline{V} \leq \left|\dot{V}_i\right| \leq \overline{V} \quad \forall i \in \Omega_b$$

$$\theta_i = \begin{cases} 0 & : \text{ Bus } i \text{ is slack,} \\ \text{free} & : \text{ Otherwise.} \end{cases}$$

$$I_{r_{ij}}^2 + I_{Im_{ij}}^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l$$

At glance...

Functions or expressions: Eq. (17), (18), (19)

$$[-G_{ij}(\left|\dot{V}_i\right|^2+\left|\dot{V}_j\right|^2)+2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i-\theta_j)] = P_l^{lineloss} = P_{ij}+P_{ji} \qquad \forall l(ij)\in\Omega_l$$

$$\begin{aligned} P_{ij} = & -G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_i\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i-\theta_j) \\ & + B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i-\theta_j) \qquad \forall l\in\Omega_l \quad \text{or} \quad \forall(ij)\in\Omega_l \end{aligned}$$

$$\begin{aligned} P_{ji} = & -G_{ij}\left|\dot{V}_j\right|\left|\dot{V}_j\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i-\theta_j) \\ & - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i-\theta_j) \qquad \forall l\in\Omega_l \quad \text{or} \quad \forall(ij)\in\Omega_l \end{aligned}$$

At glance...

Functions or expressions(Continued): Eq. (22), (29), (30)

$$\begin{aligned} Q_{ij} =& B_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i-\theta_j\right) \\ &- B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i-\theta_j\right) \quad \forall l\in\Omega_l \quad \text{or} \quad \forall (ij)\in\Omega_l \end{aligned}$$

$$\begin{aligned} I_{r_{ij}} =& -G_{ij}\left|\dot{V}_i\right|\cos\theta_i + B_{ij}\left|\dot{V}_i\right|\sin\theta_i \\ &+ G_{ij}\left|\dot{V}_j\right|\cos\theta_j - B_{ij}\left|\dot{V}_j\right|\sin\theta_j \quad \forall l\in\Omega_l \quad \text{or} \quad \forall (ij)\in\Omega_l \end{aligned}$$

$$\begin{aligned} I_{Im_{ij}} =& -B_{ij}\left|\dot{V}_i\right|\cos\theta_i - G_{ij}\left|\dot{V}_i\right|\sin\theta_i \\ &+ B_{ij}\left|\dot{V}_j\right|\cos\theta_j + G_{ij}\left|\dot{V}_j\right|\sin\theta_j \quad \forall l\in\Omega_l \quad \text{or} \quad \forall (ij)\in\Omega_l \end{aligned}$$

Objective function

$$\min \sum_{\forall i,j} [-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j)] \tag{1}$$

$$[-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j)] = P_l^{lineloss} = P_{ij} + P_{ji} \qquad \forall l(ij) \in \Omega_l \tag{2}$$

$$\begin{aligned} P_{ij} = &- G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_i\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j) \\ &+ B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i - \theta_j) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{3}$$

$$\begin{aligned} P_{ji} = &- G_{ij}\left|\dot{V}_j\right|\left|\dot{V}_j\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j) \\ &- B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i - \theta_j) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{4}$$

Objective function (Derivation of the formula)

Sending flow

$$
\begin{aligned}
\dot{S}_{ij} &= \dot{V}_i \dot{I}_{ij}^* = \dot{V}_i \left(\frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \right)^* \\
&= \dot{V}_i \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_i \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \qquad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= \left|\dot{V}_i\right|^2 (-G_{ij} + jB_{ij}) - \left|\dot{V}_i\right| \left|\dot{V}_j\right| e^{j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) \\
&= -G_{ij} \left|\dot{V}_i\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} + B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} \\
&\quad + j \left(B_{ij} \left|\dot{V}_i\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} \right)
\end{aligned}
$$

Objective function (Derivation of the formula)

Sending flow (continued)

$$\therefore P_{ij} = -G_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) + B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right)$$

$$Q_{ij} = B_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right)$$

Objective function (Derivation of the formula)

Receving flow

$$
\begin{aligned}
\dot{S}_{ji} &= \dot{V}_j \dot{I}_{ji}^* = -\dot{V}_j \dot{I}_{ij}^* \\
&= \dot{V}_j \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_j \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \quad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= -\left|\dot{V}_i\right| \left|\dot{V}_j\right| e^{-j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) + \left|\dot{V}_j\right|^2 (-G_{ij} + jB_{ij}) \\
&= -G_{ij} \left|\dot{V}_j\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} \\
&\quad + j \left(B_{ij} \left|\dot{V}_j\right|^2 - G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} \right)
\end{aligned}
$$

Objective function (Derivation of the formula)

Receving flow (continued)

$$\therefore P_{ji} = -G_{ij}\left|\dot{V}_j\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right)$$

$$Q_{ij} = B_{ij}\left|\dot{V}_j\right|^2 - G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right)$$

Constraints

Load balance

$$P_{G_i} - P_{D_i} = \sum_{j \in \Omega_{b_i}} (P_{ij}) \quad \forall i \in \Omega_b \tag{5}$$

$$Q_{G_i} - Q_{D_i} = \sum_{j \in \Omega_{b_i}} (Q_{ij}) \quad \forall i \in \Omega_b \tag{6}$$

$$\begin{aligned} Q_{ij} = & B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) \\ & - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{7}$$

Constraints

Power and voltage

$$\underline{P}_{G_i} \leq P_{G_i} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b \tag{8}$$

$$\underline{Q}_{G_i} \leq Q_{G_i} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b \tag{9}$$

$$\underline{V} \leq \left|\dot{V}_i\right| \leq \overline{V} \quad \forall i \in \Omega_b \tag{10}$$

$$\theta_i = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \tag{11}$$

(12)

Constraints

Current

$$I_{r_{ij}}^2 + I_{Im_{ij}}^2 \leq \bar{I}_{ij}^2 \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \tag{13}$$

$$\begin{aligned} I_{r_{ij}} = & - G_{ij} \left| \dot{V}_i \right| \cos\theta_i + B_{ij} \left| \dot{V}_i \right| \sin\theta_i \\ & + G_{ij} \left| \dot{V}_j \right| \cos\theta_j - B_{ij} \left| \dot{V}_j \right| \sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{14}$$

$$\begin{aligned} I_{Im_{ij}} = & - B_{ij} \left| \dot{V}_i \right| \cos\theta_i - G_{ij} \left| \dot{V}_i \right| \sin\theta_i \\ & + B_{ij} \left| \dot{V}_j \right| \cos\theta_j + G_{ij} \left| \dot{V}_j \right| \sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{15}$$

Constraints(Derivation of the formula)

Current

$$
\begin{aligned}
I_{ij} &= \frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \qquad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= \left(-G_{ij} - jB_{ij}\right)\left(\dot{V}_i - \dot{V}_j\right) \\
&= \left(-G_{ij} - jB_{ij}\right)\left(\left|\dot{V}_i\right| \cos\theta_i + j\left|\dot{V}_i\right| \sin\theta_i - \left|\dot{V}_j\right| \cos\theta_j - j\left|\dot{V}_j\right| \sin\theta_j\right) \\
&= -G_{ij}\left|\dot{V}_i\right| \cos\theta_i + B_{ij}\left|\dot{V}_i\right| \sin\theta_i + G_{ij}\left|\dot{V}_j\right| \cos\theta_j - B_{ij}\left|\dot{V}_j\right| \sin\theta_j \\
&\quad + j\left(-B_{ij}\left|\dot{V}_i\right| \cos\theta_i - G_{ij}\left|\dot{V}_i\right| \sin\theta_i + B_{ij}\left|\dot{V}_j\right| \cos\theta_j + G_{ij}\left|\dot{V}_j\right| \sin\theta_j\right)
\end{aligned}
$$

Constraints(Derivation of the formula)

Current (Continued)

$$\therefore I_{r_{ij}} = -G_{ij}\left|\dot{V}_i\right|\cos\theta_i + B_{ij}\left|\dot{V}_i\right|\sin\theta_i + G_{ij}\left|\dot{V}_j\right|\cos\theta_j - B_{ij}\left|\dot{V}_j\right|\sin\theta_j$$

$$I_{Im_{ij}} = -B_{ij}\left|\dot{V}_i\right|\cos\theta_i - G_{ij}\left|\dot{V}_i\right|\sin\theta_i + B_{ij}\left|\dot{V}_j\right|\cos\theta_j + G_{ij}\left|\dot{V}_j\right|\sin\theta_j$$

Formulation(Periodic)

Overview

- Nomenclature: Slide 35
- At glance...: Slide 38
- Objective function: Slide 41
- Constraints and expressions: Slide 46
  - Load balance
  - Power and voltage
  - Current

Optimization problem is formulated as:

1. Objective function:

$$\text{minimize (or maximize) } f(\mathbf{x})$$

2. Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

3. Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

Nomenclature

Sets, indices, parameters

- Indices

i, j	Index of bus
l	Index of line

- Sets

Ω_l	Set of lines
Ω_b	Set of buses
Ω_{b_i}	Set of connected buses in the bus i
Ω_{b_g}	Set of generation buses ($\Omega_{b_g} \subset \Omega_b$)

- Parameters or constants

Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j)
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j)
$baseMVA$	Value of base MVA
$\overline{V}$, $\underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line ij

Nomenclature

Sets, indices, parameters

- Parameters or constants (Continued)

P_{D_i}, Q_{D_i}	Active and reactive power demand at bus i
$\overline{P}_{G_i}, \underline{P}_{G_i}$	Maximum and minimum active power from generator at bus i
$\overline{Q}_{G_i}, \underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus i

- Functions

P_{ij}, Q_{ij}	Active and reactive power flow of line ij
$I_{r_{ij}}, I_{Im_{ij}}$	Real and Imaginary current flow of line ij
$P_l^{lineloss}$	Active line loss of line $l(ij)$

Nomenclature

Variables

- Varaibles

$\left\lvert \dot{V}_i \right\rvert$	Voltage magnitude in bus i
θ_i	Voltage phase angle in bus i
P_{G_i}, Q_{G_i}	Active and reactive power from generator at bus i

At glance...

Objective function: Eq. (16)

$$\min \sum_{\forall i,j} [-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j)]$$

Constraints: Eqs. (20),(21),(23),(24),(25),(26),(28)

$$s.t. \quad P_{G_i} - P_{D_i} = \sum_{j \in \Omega_{b_i}} (P_{ij}) \quad \forall i \in \Omega_b$$

$$Q_{G_i} - Q_{D_i} = \sum_{j \in \Omega_{b_i}} (Q_{ij}) \quad \forall i \in \Omega_b$$

$$\underline{P}_{G_i} \le P_{G_i} \le \overline{P}_{G_i} \quad \forall i \in \Omega_b$$

$$\underline{Q}_{G_i} \le Q_{G_i} \le \overline{Q}_{G_i} \quad \forall i \in \Omega_b$$

$$\underline{V} \le \left|\dot{V}_i\right| \le \overline{V} \quad \forall i \in \Omega_b$$

$$\theta_i = \begin{cases} 0 & : \text{ Bus } i \text{ is slack,} \\ \text{free} & : \text{ Otherwise.} \end{cases}$$

$$I_{r_{ij}}^2 + I_{Im_{ij}}^2 \le \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l$$

At glance...

Functions or expressions: Eq. (17), (18), (19)

$$[-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j)] = P_l^{lineloss} = P_{ij} + P_{ji} \qquad \forall l(ij) \in \Omega_l$$

$$\begin{aligned} P_{ij} = & - G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_i\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j) \\ & + B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i - \theta_j) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned}$$

$$\begin{aligned} P_{ji} = & - G_{ij}\left|\dot{V}_j\right|\left|\dot{V}_j\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j) \\ & - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i - \theta_j) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned}$$

At glance...

Functions or expressions(Continued): Eq. (22), (29), (30)

$$
\begin{aligned}
Q_{ij} =& B_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) \\
& - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l
\end{aligned}
$$

$$
\begin{aligned}
I_{r_{ij}} =& - G_{ij}\left|\dot{V}_i\right|\cos\theta_i + B_{ij}\left|\dot{V}_i\right|\sin\theta_i \\
& + G_{ij}\left|\dot{V}_j\right|\cos\theta_j - B_{ij}\left|\dot{V}_j\right|\sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l
\end{aligned}
$$

$$
\begin{aligned}
I_{Im_{ij}} =& - B_{ij}\left|\dot{V}_i\right|\cos\theta_i - G_{ij}\left|\dot{V}_i\right|\sin\theta_i \\
& + B_{ij}\left|\dot{V}_j\right|\cos\theta_j + G_{ij}\left|\dot{V}_j\right|\sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l
\end{aligned}
$$

Objective function

$$\min \sum_{\forall i,j} [-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j)] \tag{16}$$

$$[-G_{ij}(\left|\dot{V}_i\right|^2 + \left|\dot{V}_j\right|^2) + 2G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j)] = P_l^{lineloss} = P_{ij} + P_{ji} \qquad \forall l(ij) \in \Omega_l \tag{17}$$

$$\begin{aligned} P_{ij} = & -G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_i\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j) \\ & + B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i - \theta_j) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{18}$$

$$\begin{aligned} P_{ji} = & -G_{ij}\left|\dot{V}_j\right|\left|\dot{V}_j\right| + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos(\theta_i - \theta_j) \\ & - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin(\theta_i - \theta_j) \qquad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{19}$$

Objective function (Derivation of the formula)

Sending flow

$$
\begin{aligned}
\dot{S}_{ij} &= \dot{V}_i \dot{I}_{ij}^* = \dot{V}_i \left(\frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \right)^* \\
&= \dot{V}_i \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_i \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \qquad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= \left|\dot{V}_i\right|^2 (-G_{ij} + jB_{ij}) - \left|\dot{V}_i\right| \left|\dot{V}_j\right| e^{j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) \\
&= -G_{ij} \left|\dot{V}_i\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} + B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} \\
&\quad + j \left(B_{ij} \left|\dot{V}_i\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} \right)
\end{aligned}
$$

Objective function (Derivation of the formula)

Sending flow (continued)

$$\therefore P_{ij} = -G_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) + B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right)$$

$$Q_{ij} = B_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right)$$

Objective function (Derivation of the formula)

Receving flow

$$
\begin{aligned}
\dot{S}_{ji} &= \dot{V}_j \dot{I}_{ji}^* = -\dot{V}_j \dot{I}_{ij}^* \\
&= \dot{V}_j \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_j \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \quad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= -\left|\dot{V}_i\right| \left|\dot{V}_j\right| e^{-j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) + \left|\dot{V}_j\right|^2 (-G_{ij} + jB_{ij}) \\
&= -G_{ij} \left|\dot{V}_j\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} \\
&\quad + j \left(B_{ij} \left|\dot{V}_j\right|^2 - G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\theta_{ij} - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\theta_{ij} \right)
\end{aligned}
$$

Objective function (Derivation of the formula)

Receving flow (continued)

$$\therefore P_{ji} = -G_{ij}\left|\dot{V}_j\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right)$$

$$Q_{ij} = B_{ij}\left|\dot{V}_j\right|^2 - G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_i - \theta_j\right) - B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_i - \theta_j\right)$$

Constraints

Load balance

$$P_{G_i} - P_{D_i} = \sum_{j \in \Omega_{b_i}} \left(P_{ij} \right) \quad \forall i \in \Omega_b \tag{20}$$

$$Q_{G_i} - Q_{D_i} = \sum_{j \in \Omega_{b_i}} \left(Q_{ij} \right) \quad \forall i \in \Omega_b \tag{21}$$

$$\begin{aligned} Q_{ij} = & B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin\left(\theta_i - \theta_j\right) \\ & - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos\left(\theta_i - \theta_j\right) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{22}$$

Constraints

Power and voltage

$$\underline{P}_{G_i} \le P_{G_i} \le \overline{P}_{G_i} \quad \forall i \in \Omega_b \tag{23}$$

$$\underline{Q}_{G_i} \le Q_{G_i} \le \overline{Q}_{G_i} \quad \forall i \in \Omega_b \tag{24}$$

$$\underline{V} \le \left| \dot{V}_i \right| \le \overline{V} \quad \forall i \in \Omega_b \tag{25}$$

$$\theta_i = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \tag{26}$$

(27)

Constraints

Current

$$I_{r_{ij}}^2 + I_{Im_{ij}}^2 \leq \bar{I}_{ij}^2 \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \tag{28}$$

$$\begin{aligned} I_{r_{ij}} =& - G_{ij} \left|\dot{V}_i\right| \cos\theta_i + B_{ij} \left|\dot{V}_i\right| \sin\theta_i \\ &+ G_{ij} \left|\dot{V}_j\right| \cos\theta_j - B_{ij} \left|\dot{V}_j\right| \sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{29}$$

$$\begin{aligned} I_{Im_{ij}} =& - B_{ij} \left|\dot{V}_i\right| \cos\theta_i - G_{ij} \left|\dot{V}_i\right| \sin\theta_i \\ &+ B_{ij} \left|\dot{V}_j\right| \cos\theta_j + G_{ij} \left|\dot{V}_j\right| \sin\theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \tag{30}$$

Constraints(Derivation of the formula)

Current

$$
\begin{aligned}
I_{ij} &= \frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \qquad \left(Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
&= (-G_{ij} - jB_{ij}) \left(\dot{V}_i - \dot{V}_j \right) \\
&= (-G_{ij} - jB_{ij}) \left(\left| \dot{V}_i \right| \cos\theta_i + j \left| \dot{V}_i \right| \sin\theta_i - \left| \dot{V}_j \right| \cos\theta_j - j \left| \dot{V}_j \right| \sin\theta_j \right) \\
&= -G_{ij} \left| \dot{V}_i \right| \cos\theta_i + B_{ij} \left| \dot{V}_i \right| \sin\theta_i + G_{ij} \left| \dot{V}_j \right| \cos\theta_j - B_{ij} \left| \dot{V}_j \right| \sin\theta_j \\
&\quad + j \left(-B_{ij} \left| \dot{V}_i \right| \cos\theta_i - G_{ij} \left| \dot{V}_i \right| \sin\theta_i + B_{ij} \left| \dot{V}_j \right| \cos\theta_j + G_{ij} \left| \dot{V}_j \right| \sin\theta_j \right)
\end{aligned}
$$

Constraints(Derivation of the formula)

Current (Continued)

$$
\begin{aligned}
\therefore I_{r_{ij}} &= -G_{ij}\left|\dot{V}_i\right|\cos\theta_i + B_{ij}\left|\dot{V}_i\right|\sin\theta_i + G_{ij}\left|\dot{V}_j\right|\cos\theta_j - B_{ij}\left|\dot{V}_j\right|\sin\theta_j \\
I_{Im_{ij}} &= -B_{ij}\left|\dot{V}_i\right|\cos\theta_i - G_{ij}\left|\dot{V}_i\right|\sin\theta_i + B_{ij}\left|\dot{V}_j\right|\cos\theta_j + G_{ij}\left|\dot{V}_j\right|\sin\theta_j
\end{aligned}
$$

Implementation in Matpower and Pyomo

Matpower and Pyomo

Matpower:
MATPOWER is a package of free, open-source Matlab-language M-files for solving steady-state power system simulation and optimization problems…(Zimmerman, Murillo-Sánchez, and Thomas 2011)
Site: `https://matpower.org/`

Pyomo:
Pyomo is a Python-based open-source software package that supports a diverse set of optimization capabilities for formulating, solving, and analyzing optimization models…(Bynum et al. 2021)
Site: `https://www.pyomo.org/`

Matpower and Pyomo

Matpower can load the test system easily. In addition, it can extract the system data utilized in the Pyomo...

Optimal Power Flow: IEEE 33 Bus Case with Matpower

- Matpower
  - Matpower python 버전 설치: https://pypi.org/project/matpower/
  - Octave 설치: https://github.com/yasirroni/mypower/blob/master/README_INSTALL_OCTAVE_AND_OCT2PY.md
    - 환경변수에 octave-cli.exe 반드시 추가 (참고경로: C:\Program Files\GNU Octave\Octave-10.1.0\mingw64\bin, 홈페이지의 경로와 실제 경로가 다름 주의!)
- Matpower OPF 결과 도출
- Matpower 참고: https://pypi.org/project/matpower/
- IEEE 33 bus 데이터 구조
  - https://github.com/MATPOWER/matpower/blob/master/data/case33bw.m

1. 계통 불러오기

```python
from matpower import start_instance
from oct2py import octave
import pandas as pd
import numpy as np

# Set and load Matpower case
m = start_instance()
mpc = m.loadcase('case33bw')

#Base MVA
base_MVA = mpc['baseMVA']
```

Python

Matpower and Pyomo

Pyomo can design and solve the optimization problem in python...

Pyomo Tutorial v1 (2025.03.19)

test_nonlinear.py

예제 파일

- 비선형 최적화 문제 예제

$$
\begin{aligned}
Minimize \quad & x + sin(y) \\
s.t. \quad & 10 \le x \le 20 \\
& \pi \le y \le \frac{5}{2}\pi
\end{aligned}
$$

- 예제에서의 답은 x가 10이고, $sin(y)$가 -1이 되는 지점일 것이다.
  - Optimal Point: $x = 10, y = \frac{3}{2}\pi$
- Code step by step
  - Module 불러오기
    - ```
      import pyomo.environ as pyo
      ```
  - Model 만들기
    - ```
      model = pyo.ConcreteModel()
      ```

Implementation procedures

Algorithm 1: Optimal Power Flow Implementation in Matpower and Pyomo

Input: IEEE test system (e.g., case33)
Output: Optimization results (Bus voltage, active/reactive power)

1. Load IEEE test system in Matpower:
   - Use the IEEE test system (4,33,...)
2. Extract parameters from system...:
   - Base MVA, slack bus
   - Bus: `Vmin/Vmax`
   - Line: `R/X`, tap ratio
   - Generator: `Pmin/Pmax`, cost
   - Load data
   - Build Y-bus
3. Formulate OPF model:
   - Sets, parameters, variables...
   - Objective: Minimize loss
4. Solve with Pyomo:
   - Select solver (Ipopt,neos,...)
   - Execute
5. Save results:
   - Voltage profiles
   - Generation dispatch

Including Code - Matpower

See ▸ file for the detailed code.
Load the test case and execute optimal power flow:

1. 계통 불러오기

```python
from matpower import start_instance
from oct2py import octave
import pandas as pd
import numpy as np

# Set and load Matpower case
m = start_instance()
mpc = m.loadcase('case33bw')

#Base MVA
base_MVA = mpc['baseMVA']
```

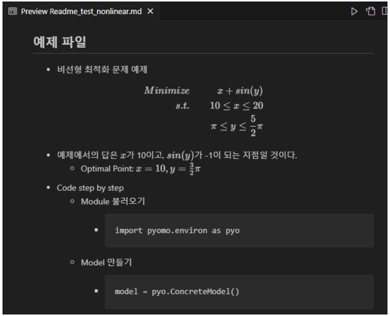
Matpower OPF 결과

```python
# Run OPF
mpopt = m.mpoption('verbose', 2)
[baseMVA, bus, gen, gencost, branch, f, success, et] = m.runopf(mpc, mpopt, nout='ma

mat_gen_index = range(1,len(gen)+1)
mat_gen_info_columns = ['bus','Pg', 'Qg','Qmax','Qmin','Vg','mBase','status','Pmax',
mat_gen_info = pd.DataFrame(gen,index = mat_gen_index, columns = mat_gen_info_colum

matpower_gen_mw_total = mat_gen_info['Pg'].sum()

print('total gen MW:', matpower_gen_mw_total)
```

```
MATPOWER Version 8.0, 17-May-2024
Optimal Power Flow -- AC-polar-power formulation
MATPOWER Interior Point Solver -- MIPS, Version 1.5.1, 10-May-2024
 (using built-in linear solver)
 it    objective   step size   feascond     gradcond     compcond     costcond
----  ------------ --------- ------------ ------------ ------------ ------------
```

Including Code - Pyomo

See file for the detailed code.
Construct model and solve the optimization problem (Concrete model case):



Preview Readme_test_nonlinear.md

예제 파일

- 비선형 최적화 문제 예제

$$\begin{aligned} Minimize \quad & x + sin(y) \\ s.t. \quad & 10 \le x \le 20 \\ & \pi \le y \le \frac{5}{2}\pi \end{aligned}$$

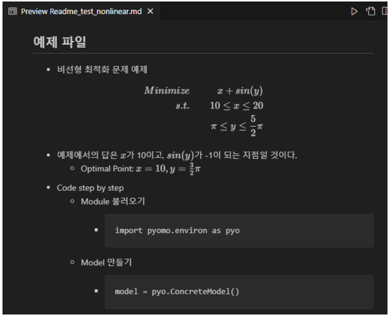
- 예제에서의 답은 x가 10이고, $sin(y)$가 -1이 되는 지점일 것이다.
  - Optimal Point: $x = 10, y = \frac{3}{2}\pi$
- Code step by step
  - Module 불러오기
    -
```
import pyomo.environ as pyo
```
  - Model 만들기
    -
```
model = pyo.ConcreteModel()
```

Optimal power flow problem using Matpower and Pyomo

See ▸ file for the detailed code.

Construct OPF model and solve the OPF problem (Abstract model case):

(Abstract model example: ▸ site1, ▸ site2)



Preview Readme_test_nonlinear.md ×

예제 파일

- 비선형 최적화 문제 예제

$$\begin{array}{rl} Minimize & x + sin(y) \\ s.t. & 10 \le x \le 20 \\ & \pi \le y \le \frac{5}{2}\pi \end{array}$$

- 예제에서의 답은 x가 10이고, $sin(y)$가 -1이 되는 지점일 것이다.
  - Optimal Point: $x = 10, y = \frac{3}{2}\pi$
- Code step by step
  - Module 불러오기
    - ```
      import pyomo.environ as pyo
      ```
  - Model 만들기
    - ```
      model = pyo.ConcreteModel()
      ```

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- Woong Ko

The End

Questions? Comments?