# Multi Period Optimal Power Flow using Matpower and Pyomo

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## Multi-Period Overview

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Optimization problem is formulated as:

1. MPOPF Objective function:

$$\text{minimize (or maximize) } f(\mathbf{x})$$

2. MPOPF Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

3. MPOPF Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

# Nomenclature
## Sets, indices, parameters

- Indices
- $i, j$    Index of bus
- $l$    Index of line
- $t$    Index of time

- Sets
- $\Omega_l$    Set of lines
- $\Omega_b$    Set of buses
- $\Omega_{b_i}$    Set of connected buses in the bus $i$
- $\Omega_{b_g}$    Set of generation buses ($\Omega_{b_g} \subset \Omega_b$)
- $T$    The total time period (e.g 1,2...,24 for 24 hours)

# Nomenclature
## Sets, indices, parameters

- Parameters or constants

| | | | |
|---|---|---|---|
| $Z_{ij}$, $Y_{ij}$ | Impedance and admittance of line $ij$ (from bus $i$ to bus $j$) | $P_{D_{i,t}}$, $Q_{D_{i,t}}$ | Active and reactive power demand at bus $i$ |
| $G_{ij}$, $B_{ij}$ | Conductance and susceptance of line $ij$ (from bus $i$ to bus $j$) | $\overline{P}_{G_i}$, $\underline{P}_{G_i}$ | Maximum and minimum active power from generator at bus $i$ |
| $\overline{V}$, $\underline{V}$ | Maximum and minimum voltage magnitude | $\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$ | Maximum and minimum reactive power from generator at bus $i$ |
| $\overline{I}_{ij}$ | Maximum current flow limit of line $ij$ | $baseMVA$ | Value of base MVA |

## Nomenclature
Sets, indices, parameters

- Functions

$P_{ij,t}$, $Q_{ij,t}$   Active and reactive power flow of line $ij$ at time $t$

$I_{r_{ij,t}}$, $I_{Im_{ij,t}}$   Real and Imaginary current flow of line $ij$ at time $t$

$P_{l,t}^{lineloss}$   Active line loss of line $l(ij)$ at time $t$

- Variables

$\left|\dot{V}_{i,t}\right|$   Voltage magnitude in bus $i$ at time $t$

$\theta_{i,t}$   Voltage phase angle in bus $i$ at time $t$

$P_{G_{i,t}}$, $Q_{G_{i,t}}$   Active and reactive power from generator at bus $i$ at time $t$

## MPOPF At glance...

Objective function: Eq. (1)

$$\min \sum_{\forall t} \sum_{\forall i,j} [-G_{ij}(\left|\dot{V}_{i,t}\right|^2 + \left|\dot{V}_{j,t}\right|^2) + 2G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos(\theta_{i,t} - \theta_{j,t})]$$

Constraints: Eqs. (5),(**??**),(9),(10),(11),(12),(13)

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (P_{ij,t}) \quad \forall i \in \Omega_b, \ \forall t \in T$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (Q_{ij,t}) \quad \forall i \in \Omega_b, \ \forall t \in T$$

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \ \forall t \in T$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \ \forall t \in T$$

$$\underline{V} \leq \left|\dot{V}_{i,t}\right| \leq \overline{V} \quad \forall i \in \Omega_b, \ \forall t \in T$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$I_{r_{ij,t}}^2 + I_{Im_{ij,t}}^2 \leq \overline{I}_{ij}^2 \quad \forall I(ij) \in \Omega_I, \ \forall t \in T$$

## MPOPF At glance...

Functions or expressions: Eq. (2), (3), (4)

$$[-G_{ij}(\left|\dot{V}_{i,t}\right|^2 + \left|\dot{V}_{j,t}\right|^2) + 2G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos(\theta_{i,t} - \theta_{j,t})]$$
$$= P_{l,t}^{lineloss} = P_{ij,t} + P_{ji,t}$$

$$\forall l(ij) \in \Omega_l, \ \forall t \in T$$

## MPOPF At glance...

$$P_{ij,t} = - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{i,t} \right| + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left( \theta_{i,t} - \theta_{j,t} \right)$$
$$+ B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin\left( \theta_{i,t} - \theta_{j,t} \right)$$

$$P_{ji,t} = - G_{ij} \left| \dot{V}_{j,t} \right| \left| \dot{V}_{j,t} \right| + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left( \theta_{i,t} - \theta_{j,t} \right)$$
$$- B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin\left( \theta_{i,t} - \theta_{j,t} \right)$$

$$\forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \quad \text{and} \quad \forall t \in T$$

## MPOPF At Glance

Functions or expressions (Continued): Eq. (7),(8), (14), (15)

$$Q_{ij,t} = B_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin \left( \theta_{i,t} - \theta_{j,t} \right)$$
$$- B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos \left( \theta_{i,t} - \theta_{j,t} \right)$$
$$\forall I \in \Omega_I \text{ or } \forall (ij) \in \Omega_I, \ \forall t \in T$$

$$Q_{ji,t} = B_{ij} \left| \dot{V}_{j,t} \right|^2 - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin \left( \theta_{i,t} - \theta_{j,t} \right)$$
$$- B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos \left( \theta_{i,t} - \theta_{j,t} \right)$$
$$\forall I \in \Omega_I \text{ or } \forall (ij) \in \Omega_I, \ \forall t \in T$$

# MPOPF At Glance

$$I_{r_{ij,t}} = - G_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t}$$
$$+ G_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t}$$
$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \ \forall t \in T$$

$$I_{Im_{ij,t}} = - B_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t}$$
$$+ B_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t}$$
$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \ \forall t \in T$$

# Objective function

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[ -G_{ij} \left( \left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2 G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos \left( \theta_{i,t} - \theta_{j,t} \right) \right] \tag{1}$$

$$\left[ -G_{ij} \left( \left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2 G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos \left( \theta_{i,t} - \theta_{j,t} \right) \right]$$
$$= P_{l,t}^{lineloss} = P_{ij,t} + P_{ji,t} \qquad \forall l(ij) \in \Omega_l, \ \forall t \in T \tag{2}$$

# MPOPF Objective function

$$P_{ij,t} = -G_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right)$$
$$+ B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) \tag{3}$$

$$\forall l \in \Omega_l \text{ or } \forall(ij) \in \Omega_l, \ \forall t \in T$$

$$P_{ji,t} = -G_{ij}\left|\dot{V}_{j,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right)$$
$$- B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) \tag{4}$$

$$\forall l \in \Omega_l \text{ or } \forall(ij) \in \Omega_l, \ \forall t \in T$$

# MPOPF Constraints
## Load balance

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} P_{ij,t} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{5}$$

If a line has a charging susceptance, the model must consider the impact of susceptance($-\sum_{\forall j} B_{ij,t} \left| \dot{V}_{i,t} \right|^2$).

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} \left( Q_{ij,t} \right) - \sum_{\forall j} B_{ij,t} \left| \dot{V}_{i,t} \right|^2 \quad \forall i \in \Omega_b, \ \forall t \in T \tag{6}$$

# MPOPF Constraints
Load balance

$$Q_{ij,t} = B_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin\left(\theta_{i,t} - \theta_{j,t}\right)$$
$$- B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left(\theta_{i,t} - \theta_{j,t}\right) \tag{7}$$

$$\forall I \in \Omega_I \text{ or } \forall(ij) \in \Omega_I, \ \forall t \in T$$

$$Q_{ji,t} = B_{ij} \left| \dot{V}_{j,t} \right|^2 - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin\left(\theta_{i,t} - \theta_{j,t}\right)$$
$$- B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left(\theta_{i,t} - \theta_{j,t}\right) \tag{8}$$

$$\forall I \in \Omega_I \text{ or } \forall(ij) \in \Omega_I, \ \forall t \in T$$

# MPOPF Constraints
Power and voltage

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{9}$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{10}$$

$$\underline{V} \leq \left| \dot{V}_{i,t} \right| \leq \overline{V} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{11}$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \quad \forall t \in T \tag{12}$$

# MPOPF Constraints
Current

$$I_{r_{ij,t}}^2 + I_{Im_{ij,t}}^2 \leq \bar{I}_{ij}^2 \quad \forall I(ij) \in \Omega_I, \ \forall t \in T \tag{13}$$

$$\begin{aligned}
I_{r_{ij,t}} = &- G_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\
&+ G_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \\
&\forall I(ij) \in \Omega_I, \ \forall t \in T
\end{aligned} \tag{14}$$

$$\begin{aligned}
I_{Im_{ij,t}} = &- B_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\
&+ B_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \\
&\forall I(ij) \in \Omega_I, \ \forall t \in T
\end{aligned} \tag{15}$$

# References

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# The End

Questions? Comments?