

Multi Period Optimal Power Flow using Matpower and Pyomo

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Multi Period Optimal Power Flow

Multi-Period Overview

- MPOPF Nomenclature: Slide 5
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- MPOPF Objective function: Slide 13
- MPOPF Constraints and expressions: Slide 15
 - MPOPF Load balance
 - MPOPF Power and voltage
 - MPOPF Current

Optimization problem is formulated as:

- 1 MPOPF Objective function:

minimize (or maximize) $f(\mathbf{x})$

- 2 MPOPF Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

- 3 MPOPF Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

Nomenclature

Sets, indices, parameters

- Indices

i, j Index of bus
 l Index of line
 t Index of time

- Sets

Ω_l Set of lines
 Ω_b Set of buses
 Ω_{b_i} Set of connected buses
in the bus i
 Ω_{b_g} Set of generation buses
($\Omega_{b_g} \subset \Omega_b$)
 T The total time period
(e.g 1,2,...,24 for 24 hours)

Nomenclature

Sets, indices, parameters

- Parameters or constants

Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j)
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j)
$\overline{V}, \underline{V}$	Maximum and minimum voltage magnitude
\overline{I}_{ij}	Maximum current flow limit of line ij

$P_{D_{i,t}}, Q_{D_{i,t}}$	Active and reactive power demand at bus i
$\overline{P}_{G_i}, \underline{P}_{G_i}$	Maximum and minimum active power from generator at bus i
$\overline{Q}_{G_i}, \underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus i
$baseMVA$	Value of base MVA

Nomenclature

Sets, indices, parameters

- Functions

$P_{ij,t}, Q_{ij,t}$	Active and reactive power flow of line ij at time t
$I_{r_{ij,t}}, I_{Im_{ij,t}}$	Real and Imaginary current flow of line ij at time t
$P_{l,t}^{lineloss}$	Active line loss of line $l(ij)$ at time t

- Variables

$ \dot{V}_{i,t} $	Voltage magnitude in bus i at time t
$\theta_{i,t}$	Voltage phase angle in bus i at time t
$P_{G_{i,t}}, Q_{G_{i,t}}$	Active and reactive power from generator at bus i at time t

MPOPF At glance...

Objective function: Eq. (1)

$$\min \sum_{\forall t} \sum_{\forall i,j} [-G_{ij}(|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij}|\dot{V}_{i,t}||\dot{V}_{j,t}|\cos(\theta_{i,t} - \theta_{j,t})]$$

Constraints: Eqs. (5),(6),(9),(10),(11),(12),(13)

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (P_{ij,t}) \quad \forall i \in \Omega_b, \forall t \in T$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (Q_{ij,t}) \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{V} \leq |\dot{V}_{i,t}| \leq \overline{V} \quad \forall i \in \Omega_b, \forall t \in T$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$I_{r_{ij,t}}^2 + I_{lm_{ij,t}}^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l, \forall t \in T$$

MPOPF At glance...

Functions or expressions: Eq. (2), (3), (4)

$$[-G_{ij}(|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij}|\dot{V}_{i,t}||\dot{V}_{j,t}|\cos(\theta_{i,t} - \theta_{j,t})]$$

$$= P_{l,t}^{line loss} = P_{ij,t} + P_{ji,t}$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

MPOPF At glance...

$$P_{ij,t} = -G_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ + B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t})$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

$$P_{ji,t} = -G_{ij} \left| \dot{V}_{j,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t})$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

MPOPF At Glance

Functions or expressions (Continued): Eq. (7),(8), (14), (15)

$$Q_{ij,t} = B_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \\ - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t})$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

$$Q_{ji,t} = B_{ij} \left| \dot{V}_{j,t} \right|^2 - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \\ - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t})$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

MPOPF At Glance

$$I_{rij,t} = -G_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\ + G_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t}$$

$$\forall (ij) \in \Omega_l, \forall t \in T$$

$$I_{lmij,t} = -B_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\ + B_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t}$$

$$\forall (ij) \in \Omega_l, \forall t \in T$$

Objective function

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[-G_{ij} (|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij} |\dot{V}_{i,t}| |\dot{V}_{j,t}| \cos(\theta_{i,t} - \theta_{j,t}) \right] \quad (1)$$

$$\left[-G_{ij} (|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij} |\dot{V}_{i,t}| |\dot{V}_{j,t}| \cos(\theta_{i,t} - \theta_{j,t}) \right] = P_{l,t}^{lineloss} = P_{ij,t} + P_{ji,t} \quad (2)$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

MPOPF Objective function

$$P_{ij,t} = -G_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ + B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \quad (3)$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

$$P_{ji,t} = -G_{ij} \left| \dot{V}_{j,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \quad (4)$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

MPOPF Constraints

Load balance

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} P_{ij,t} \quad \forall i \in \Omega_b, \forall t \in T \quad (5)$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} Q_{ij,t} \quad \forall i \in \Omega_b, \forall t \in T \quad (6)$$

MPOPF Constraints

Load balance

$$Q_{ij,t} = B_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \quad (7)$$

$$\forall l(ij) \in \Omega_l, \forall t \in \mathcal{T}$$

$$Q_{ji,t} = B_{ij} \left| \dot{V}_{j,t} \right|^2 - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \quad (8)$$

$$\forall l(ij) \in \Omega_l, \forall t \in \mathcal{T}$$

MPOPF Constraints

Power and voltage

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad (9)$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad (10)$$

$$\underline{V} \leq |\dot{V}_{i,t}| \leq \overline{V} \quad \forall i \in \Omega_b, \forall t \in T \quad (11)$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \quad \forall t \in T \quad (12)$$

$$I_{r_{ij,t}}^2 + I_{lm_{ij,t}}^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l, \forall t \in T \quad (13)$$

MPOPF Constraints

Current

$$I_{rij,t} = -G_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\ + G_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \quad (14)$$

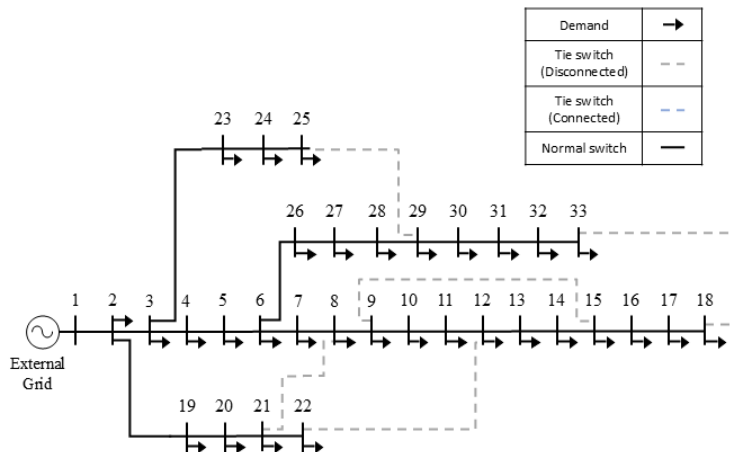
$$\forall l(ij) \in \Omega_l, \forall t \in T$$

$$I_{lmij,t} = -B_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\ + B_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \quad (15)$$

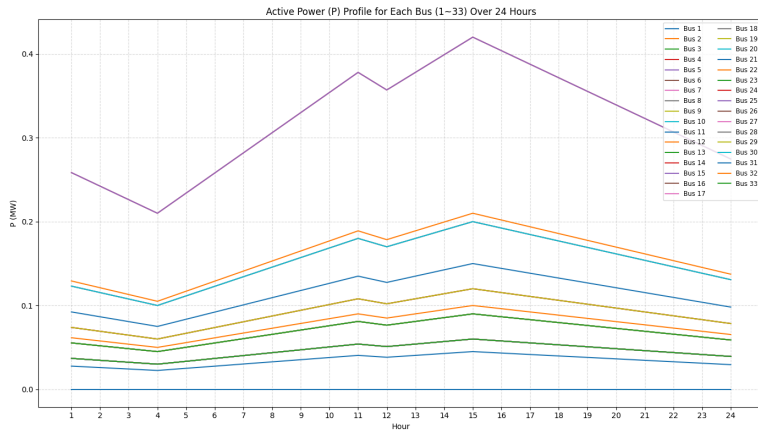
$$\forall l(ij) \in \Omega_l, \forall t \in T$$

Multi-Period Optimal Power Flow in a 33-Bus Distribution System

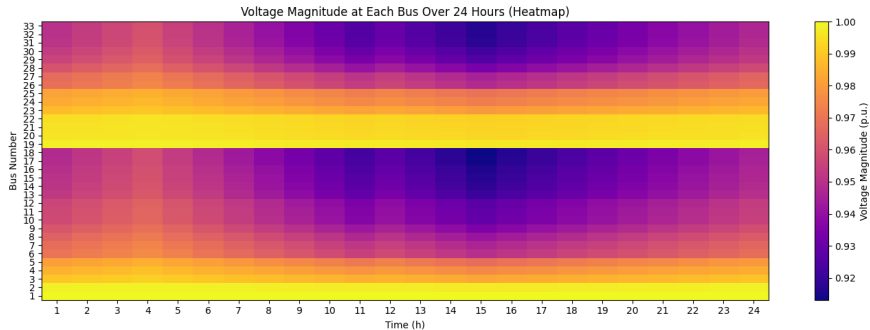
33-Bus Distribution System



Load



V_mag



References

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The End

Questions? Comments?