

# Multi Period Optimal Power Flow using Matpower and Pyomo

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# Multi-Period Overview

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  - MPOPF Load balance
  - MPOPF Power and voltage
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Optimization problem is formulated as:

- 1 MPOPF Objective function:

minimize (or maximize)  $f(\mathbf{x})$

- 2 MPOPF Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

- 3 MPOPF Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

# Nomenclature

## Sets, indices, parameters

- Indices

$i, j$  Index of bus  
 $l$  Index of line  
 $t$  Index of time

- Sets

$\Omega_l$  Set of lines  
 $\Omega_b$  Set of buses  
 $\Omega_{b_i}$  Set of connected buses  
 in the bus  $i$   
 $\Omega_{b_g}$  Set of generation buses  
 $(\Omega_{b_g} \subset \Omega_b)$   
 $T$  The total time period  
 (e.g 1,2...,24 for 24 hours)

# Nomenclature

## Sets, indices, parameters

- Parameters or constants

$Z_{ij}, Y_{ij}$	Impedance and admittance of line $ij$ (from bus $i$ to bus $j$ )
$G_{ij}, B_{ij}$	Conductance and susceptance of line $ij$ (from bus $i$ to bus $j$ )
$\overline{V}, \underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line $ij$

$P_{D_{i,t}}, Q_{D_{i,t}}$	Active and reactive power demand at bus $i$
$\overline{P}_{G_i}, \underline{P}_{G_i}$	Maximum and minimum active power from generator at bus $i$
$\overline{Q}_{G_i}, \underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus $i$
$baseMVA$	Value of base MVA

# Nomenclature

Sets, indices, parameters

- Functions

$P_{ij,t}, Q_{ij,t}$	Active and reactive power flow of line $ij$ at time $t$
$I_{r_{ij,t}}, I_{Im_{ij,t}}$	Real and Imaginary current flow of line $ij$ at time $t$
$P_{l,t}^{lineloss}$	Active line loss of line $l(ij)$ at time $t$

- Variables

$ \dot{V}_{i,t} $	Voltage magnitude in bus $i$ at time $t$
$\theta_{i,t}$	Voltage phase angle in bus $i$ at time $t$
$P_{G_{i,t}}, Q_{G_{i,t}}$	Active and reactive power from generator at bus $i$ at time $t$

# MPOPF At glance...

Objective function: Eq. (1)

$$\min \sum_{\forall t} \sum_{\forall i,j} [-G_{ij}(|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij}|\dot{V}_{i,t}||\dot{V}_{j,t}|\cos(\theta_{i,t} - \theta_{j,t})]$$

Constraints: Eqs. (5),(6),(9),(10),(11),(12),(13)

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (P_{ij,t}) \quad \forall i \in \Omega_b, \forall t \in T$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (Q_{ij,t}) \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{V} \leq |\dot{V}_{i,t}| \leq \overline{V} \quad \forall i \in \Omega_b, \forall t \in T$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$I_{r_{ij,t}}^2 + I_{lm_{ij,t}}^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l, \forall t \in T$$

# MPOPF At glance...

Functions or expressions: Eq. (2), (3), (4)

$$\begin{aligned} & [-G_{ij}(|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij}|\dot{V}_{i,t}||\dot{V}_{j,t}|\cos(\theta_{i,t} - \theta_{j,t})] \\ & = P_{l,t}^{line loss} = P_{ij,t} + P_{ji,t} \end{aligned}$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$



# MPOPF At glance...

$$P_{ij,t} = -G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{i,t} \right| + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ + B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t})$$

$$P_{ji,t} = -G_{ij} \left| \dot{V}_{j,t} \right| \left| \dot{V}_{j,t} \right| + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t})$$

$$\forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \quad \text{and} \quad \forall t \in \mathcal{T}$$

# MPOPF At Glance

Functions or expressions (Continued): Eq. (7),(8), (14), (15)

$$\begin{aligned}
 Q_{ij,t} = & B_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \\
 & - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\
 & \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}
 \end{aligned}$$

$$\begin{aligned}
 Q_{ji,t} = & B_{ij} \left| \dot{V}_{j,t} \right|^2 - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \\
 & - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\
 & \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}
 \end{aligned}$$

# MPOPF At Glance

$$\begin{aligned}
 I_{r_{ij},t} = & -G_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\
 & + G_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \\
 & \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T
 \end{aligned}$$

$$\begin{aligned}
 I_{lm_{ij},t} = & -B_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\
 & + B_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \\
 & \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T
 \end{aligned}$$

# Objective function

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[ -G_{ij} (|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij} |\dot{V}_{i,t}| |\dot{V}_{j,t}| \cos(\theta_{i,t} - \theta_{j,t}) \right] \quad (1)$$

$$\begin{aligned} & \left[ -G_{ij} (|\dot{V}_{i,t}|^2 + |\dot{V}_{j,t}|^2) + 2G_{ij} |\dot{V}_{i,t}| |\dot{V}_{j,t}| \cos(\theta_{i,t} - \theta_{j,t}) \right] \\ & = P_{l,t}^{line loss} = P_{ij,t} + P_{ji,t} \quad \forall l(ij) \in \Omega_l, \forall t \in T \end{aligned} \quad (2)$$

# MPOPF Objective function

$$P_{ij,t} = -G_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ + B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \quad (3)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T$$

$$P_{ji,t} = -G_{ij} \left| \dot{V}_{j,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \\ - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) \quad (4)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T$$

# MPOPF Constraints

## Load balance

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} P_{ij,t} \quad \forall i \in \Omega_b, \forall t \in T \quad (5)$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} Q_{ij,t} \quad \forall i \in \Omega_b, \forall t \in T \quad (6)$$

# MPOPF Constraints

## Load balance

$$Q_{ij,t} = B_{ij} \left| \dot{V}_{i,t} \right|^2 + G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \quad (7)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

$$Q_{ji,t} = B_{ij} \left| \dot{V}_{j,t} \right|^2 - G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \sin(\theta_{i,t} - \theta_{j,t}) - B_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos(\theta_{i,t} - \theta_{j,t}) \quad (8)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

# MPOPF Constraints

## Power and voltage

$$\underline{P}_{G_i} \leq P_{G_i,t} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad (9)$$

$$\underline{Q}_{G_i} \leq Q_{G_i,t} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad (10)$$

$$\underline{V} \leq |\dot{V}_{i,t}| \leq \overline{V} \quad \forall i \in \Omega_b, \forall t \in T \quad (11)$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \quad \forall t \in T \quad (12)$$



# MPOPF Constraints

## Current

$$I_{r_{ij,t}}^2 + I_{lm_{ij,t}}^2 \leq \bar{I}_{ij}^2 \quad \forall (ij) \in \Omega_l, \forall t \in \mathcal{T} \quad (13)$$

$$\begin{aligned} I_{r_{ij,t}} = & -G_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\ & + G_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \end{aligned} \quad (14)$$

$$\forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

$$\begin{aligned} I_{lm_{ij,t}} = & -B_{ij} \left| \dot{V}_{i,t} \right| \cos \theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin \theta_{i,t} \\ & + B_{ij} \left| \dot{V}_{j,t} \right| \cos \theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin \theta_{j,t} \end{aligned} \quad (15)$$

$$\forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

# References

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# The End

Questions? Comments?