

# Optimal Power Flow using Matpower and Pyomo

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MPOPF Objective function  
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# Optimal Power Flow?

The **optimal power flow(OPF)** can integrate the computation of power flow and operational problems (such as economic dispatch and unit commitment) subject to the power system's physical and electrical constraints.

*The optimal power flow (OPF) integrates the computation of power flow and economic dispatch subject to the system's physical and electrical constraints ... (Ali et al. 2024)*

# Optimal Power Flow?

- Optimal power flow = Optimization problem + Power flow

## Optimization problem

- Find the optimal variables satisfying the objective function
- Example:

$$\begin{aligned} \min_{x \in D} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \quad i = 1, \dots, m \\ & h_j(x) = 0, \quad j = 1, \dots, r \end{aligned}$$

## Power flow

- Find the variables satisfying the power flow equation
- Example:

$$P_i = |V_i| \sum_{\forall j} |V_j| [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)]$$

# Optimal Power Flow?

## Optimization problem and power flow problem

- Optimization problem
  - Solve the problem based on the optimization techniques (convex, branch and bound, etc)
    - 조건에 따라 해가 존재하지 않을 수 있음
    - Variables: Generator status, reactive/active power, voltage magnitude/phase, etc
- Power flow
  - Solve the problem based on finding the solution on multiple equations (Newton Raphson, Gauss-Seidel)
    - 초기값 혹은 계통에 문제가 없지 않은 이상 해는 존재
    - Variables: Reactive/active power, voltage magnitude/phase
- Optimization problem + power flow
  - 다양한 목적함수를 가지며 제약조건으로 power flow를 갖는 최적화 문제
    - 손실 최소화, 비용 최소화, 이익 최대화 등

# Optimal Power Flow?

Problems defined by variable type

- Linear programming (LP)
  - 모든 목적함수와 제약조건이 선형화 되어있는 상황이며, 발전량과 전압의 크기/위상 등 continuous variable 을 결정하는 문제
  - Cplex, Gurobi 등 solver 이용 가능
  - Global optimum 을 찾을 수 있으나 비선형방정식을 선형방정식으로 바꿔야 함
- Mixed integer linear programming (MILP)
  - 모든 목적함수와 제약조건이 선형화 되어있고, 이진변수(0 or 1)이 존재하는 최적화 문제
  - 이진변수는 발전기의 On/Off, ESS의 충방전 Mode 선택 등 다양한 변수가 존재
  - Cplex, Gurobi 등 solver 이용 가능
  - Global optimum 을 찾을 수 있으나 비선형방정식을 선형방정식으로 바꿔야 함
  - Solver가 이진변수를 relaxation (이진 변수를 연속 변수로 완화) 하는 것으로 알려짐

# Optimal Power Flow?

Problems defined by variable type

- Mixed integer nonlinear programming (MINLP)
  - 목적함수와 제약조건에 비선형 방정식 등이 포함된 최적화 문제
  - 이진변수, 연속변수 등 다양한 변수와 함께 전력방정식이 그대로 포함될 수 있음
  - 문제를 선형함수로 변환하지 않아도 되기 때문에 해가 구해진다면 정확한 해일 가능성이 높음
  - 하지만, solver 성능에 따라 문제가 풀리지 않는 경우들이 대부분
  - Local optimum 과 global optimum 이 같지 않을 수 있음을 항상 고민해야 함
  - Knitro 등의 solve를 사용
- Meta heuristic
  - 비선형 방정식 등이 포함된 최적화 문제이나 문제 해결을 위해 다양한 방법을 활용하는 기법
  - 기본적으로 변수에 여러 값들을 대입하면서 구하는 문제 해결 방법이지만, 다양한 알고리즘이 사용됨
  - Genetic algorithm, particle swarm optimization, harmony search 등의 방법이 사용됨
  - Pygmo 등으로 이용할 수 있으며, matlab 을 이용하는 것이 일반적임



# Optimal Power Flow?

## How to solve?

- 문제 해결의 기본 방향
  - 최적화 문제를 처음 해결하는 상황이라면, 목적함수와 제약조건의 방정식을 그대로 이용할 수 있는 MINLP와 meta heuristic 방법을 이용하는 것도 방법임
  - 위 방법을 이용한 이후에는 relaxation 등을 통하여 MILP, LP 문제로 바꾸어 해결하는 방안을 고민할 수 있음
- 문제 유형별 사용 Solver
  - LP, MILP: Pyomo 에서 cplex, gurobi 등을 연계
  - MINLP: Pyomo 에서 knitro 등을 연계
  - Meta heuristic: Pyomo 에서는 해결하지 못하며, Pygmo 등을 사용

# Previous studies

## Typical optimal power flow problem

Minimize the loss in the overall system(Lavorato et al. 2012).

### A. Distribution System Reconfiguration Problem

The DSR problem is modeled as follows:

$$\min \quad v = \sum_{(ij) \in \Omega_l} (g_{ij} x_{ij} (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij})) \quad (18)$$

s.t.

$$P_{S_i} - P_{D_i} - \sum_{j \in \Omega_{b_i}} (x_{ij} P_{ij}) = 0 \quad \forall i \in \Omega_b \quad (19)$$

$$Q_{S_i} - Q_{D_i} - \sum_{j \in \Omega_{b_i}} (x_{ij} Q_{ij}) = 0 \quad \forall i \in \Omega_b \quad (20)$$

$$\underline{V} \leq V_i \leq \overline{V} \quad \forall i \in \Omega_b \quad (21)$$

$$x_{ij} (I_{r_{ij}}^2 + I_{m_{ij}}^2) \leq \overline{I}_{ij}^2 \quad \forall (ij) \in \Omega_l \quad (22)$$

$$x_{ij} \in \{0, 1\} \quad \forall (ij) \in \Omega_l \quad (23)$$

Eqs. (14) – (17)

$$\sum_{(ij) \in \Omega_l} x_{ij} = n_b - 1 - \sum_{j \in \Omega_{bp}} (1 - y_j). \quad (24)$$

# Previous studies

## Typical optimal power flow problem

- Objective function: Minimize the loss

$$\min \quad v = \sum_{(ij) \in \Omega_l} (g_{ij}(|V_i|^2 + |V_j|^2 - 2|V_i||V_j|\cos\theta_{ij}))$$

# Previous studies

## Typical optimal power flow problem

Equality constraints:

- Balance
  - Active power

$$P_{S_i} - P_{D_i} - \sum_{j \in \Omega_{b_i}} P_{ij} = 0$$

$$(P_{ij} = -|V_i|^2 G_{ij} + |V_i| |V_j| (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}))$$

- Reactive power

$$Q_{S_i} - Q_{D_i} - \sum_{j \in \Omega_{b_i}} Q_{ij} = 0$$

$$Q_{ij} = |V_i|^2 B_{ij} + |V_i| |V_j| (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$

# Typical optimal power flow problem

## Constraints

Inequality constraints:

- Operation condition
  - Voltage

$$\underline{|V_i|} \leq |V_i| \leq \overline{|V_i|}$$

- Line rating

$$I_{real,ij}^2 + I_{imag,ij}^2 \leq \overline{I_{ij}^2}$$

## Our main objective is to...

- Formulate the objective function and constraints of the Optimal Power Flow (OPF) problem.
- Implement the formulated objective function and constraints using Matpower and Pyomo.
- Derive simulation results of the implemented model on a 33-bus test system.

# Formulation

# Overview

- Nomenclature: Slide 17
- At glance...: Slide 20
- Objective function: Slide 24
- Constraints and expressions: Slide 29
  - Load balance
  - Power and voltage
  - Current

Optimization problem is formulated as:

① Objective function:

minimize (or maximize)  $f(\mathbf{x})$

② Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

③ Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$



# Nomenclature

## Sets, indices, parameters

- Indices

$i, j$  Index of bus

$l$  Index of line

- Sets

$\Omega_l$  Set of lines

$\Omega_b$  Set of buses

$\Omega_{b_i}$  Set of connected buses  
in the bus  $i$

$\Omega_{b_g}$  Set of generation buses  
( $\Omega_{b_g} \subset \Omega_b$ )

- Parameters or constants

$Z_{ij}, Y_{ij}$  Impedance and  
admittance of line  $ij$   
(from bus  $i$  to bus  $j$ )

$G_{ij}, B_{ij}$  Conductance and  
susceptance of line  $ij$   
(from bus  $i$  to bus  $j$ )

$baseMVA$  Value of base MVA

$\bar{V}, \underline{V}$  Maximum and minimum  
voltage magnitude

$\bar{I}_{ij}$  Maximum current  
flow limit of line  $ij$

# Nomenclature

## Sets, indices, parameters

- Parameters or constants (Continued)

$P_{D_i}, Q_{D_i}$	Active and reactive power demand at bus $i$
$\overline{P}_{G_i}, \underline{P}_{G_i}$	Maximum and minimum active power from generator at bus $i$
$\overline{Q}_{G_i}, \underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus $i$

- Functions

$P_{ij}, Q_{ij}$	Active and reactive power flow of line $ij$
$I_{rij}, I_{lmij}$	Real and Imaginary current flow of line $ij$
$P_I^{lineloss}$	Active line loss of line $I(ij)$

# Nomenclature

## Variables

- Variables

$ \dot{V}_i $	Voltage magnitude in bus $i$
$\theta_i$	Voltage phase angle in bus $i$
$P_{G_i}, Q_{G_i}$	Active and reactive power from generator at bus $i$

# At glance...

Objective function: Eq. (1)

$$\min \sum_{\forall i,j} [-G_{ij}(|\dot{V}_i|^2 + |\dot{V}_j|^2) + 2G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j)]$$

Constraints: Eqs. (5),(6),(9),(10),(11),(12),(13)

$$s.t. \quad P_{G_i} - P_{D_i} = \sum_{j \in \Omega_{b_i}} (P_{ij}) \quad \forall i \in \Omega_b$$

$$Q_{G_i} - Q_{D_i} = \sum_{j \in \Omega_{b_i}} (Q_{ij}) \quad \forall i \in \Omega_b$$

$$\underline{P}_{G_i} \leq P_{G_i} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b$$

$$\underline{Q}_{G_i} \leq Q_{G_i} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b$$

$$\underline{V} \leq |\dot{V}_i| \leq \overline{V} \quad \forall i \in \Omega_b$$

$$\theta_i = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$I_{r_{ij}}^2 + I_{l_{m_{ij}}}^2 \leq \overline{I}_{ij}^2 \quad \forall (ij) \in \Omega_l$$

# At glance...

Functions or expressions: Eq. (2), (3), (4)

$$[-G_{ij}(|\dot{V}_i|^2 + |\dot{V}_j|^2) + 2G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j)] = P_i^{lineloss} = P_{ij} + P_{ji} \quad \forall (ij) \in \Omega_l$$

$$\begin{aligned} P_{ij} = & -G_{ij}|\dot{V}_i||\dot{V}_i| + G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j) \\ & + B_{ij}|\dot{V}_i||\dot{V}_j|\sin(\theta_i - \theta_j) \quad \forall i \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned}$$

$$\begin{aligned} P_{ji} = & -G_{ij}|\dot{V}_j||\dot{V}_j| + G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j) \\ & - B_{ij}|\dot{V}_i||\dot{V}_j|\sin(\theta_i - \theta_j) \quad \forall i \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned}$$

# At glance...

Functions or expressions(Continued): Eq. (7), (8), (14), (15)

$$Q_{ij} = B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) \\ - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l$$

$$Q_{ji} = B_{ij} \left| \dot{V}_j \right|^2 - G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) \\ - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l$$

# At glance...

$$I_{rij} = -G_{ij} \left| \dot{V}_i \right| \cos \theta_i + B_{ij} \left| \dot{V}_i \right| \sin \theta_i \\ + G_{ij} \left| \dot{V}_j \right| \cos \theta_j - B_{ij} \left| \dot{V}_j \right| \sin \theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l$$

$$I_{lmij} = -B_{ij} \left| \dot{V}_i \right| \cos \theta_i - G_{ij} \left| \dot{V}_i \right| \sin \theta_i \\ + B_{ij} \left| \dot{V}_j \right| \cos \theta_j + G_{ij} \left| \dot{V}_j \right| \sin \theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l$$

# Objective function

$$\min \sum_{\forall i,j} [-G_{ij}(|\dot{V}_i|^2 + |\dot{V}_j|^2) + 2G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j)] \quad (1)$$

$$[-G_{ij}(|\dot{V}_i|^2 + |\dot{V}_j|^2) + 2G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j)] = P_i^{lineloss} = P_{ij} + P_{ji} \quad \forall l(ij) \in \Omega_l \quad (2)$$

$$\begin{aligned} P_{ij} = & -G_{ij}|\dot{V}_i||\dot{V}_i| + G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j) \\ & + B_{ij}|\dot{V}_i||\dot{V}_j|\sin(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \quad (3)$$

$$\begin{aligned} P_{ji} = & -G_{ij}|\dot{V}_j||\dot{V}_j| + G_{ij}|\dot{V}_i||\dot{V}_j|\cos(\theta_i - \theta_j) \\ & - B_{ij}|\dot{V}_i||\dot{V}_j|\sin(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \quad (4)$$



# Objective function (Derivation of the formula)

Sending flow

$$\begin{aligned}
 \dot{S}_{ij} &= \dot{V}_i \dot{I}_{ij}^* = \dot{V}_i \left( \frac{\dot{V}_i - \dot{V}_j}{\dot{Z}_{ij}} \right)^* \\
 &= \dot{V}_i \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_i \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \quad \left( Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
 &= \left| \dot{V}_i \right|^2 (-G_{ij} + jB_{ij}) - \left| \dot{V}_i \right| \left| \dot{V}_j \right| e^{j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) \\
 &= -G_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos \theta_{ij} + B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin \theta_{ij} \\
 &\quad + j \left( B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin \theta_{ij} - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos \theta_{ij} \right)
 \end{aligned}$$

# Objective function (Derivation of the formula)

Sending flow (continued)

$$\therefore P_{ij} = -G_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) + B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j)$$

$$Q_{ij} = B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j)$$

# Objective function (Derivation of the formula)

Receiving flow

$$\begin{aligned}
 \dot{S}_{ji} &= \dot{V}_j \dot{I}_{ji}^* = -\dot{V}_j \dot{I}_{ij}^* \\
 &= \dot{V}_j \frac{\dot{V}_i^*}{\dot{Z}_{ij}^*} - \dot{V}_j \frac{\dot{V}_j^*}{\dot{Z}_{ij}^*} \quad \left( Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
 &= -\left| \dot{V}_i \right| \left| \dot{V}_j \right| e^{-j(\theta_i - \theta_j)} (-G_{ij} + jB_{ij}) + \left| \dot{V}_j \right|^2 (-G_{ij} + jB_{ij}) \\
 &= -G_{ij} \left| \dot{V}_j \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos \theta_{ij} - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin \theta_{ij} \\
 &\quad + j \left( B_{ij} \left| \dot{V}_j \right|^2 - G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin \theta_{ij} - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos \theta_{ij} \right)
 \end{aligned}$$

# Objective function (Derivation of the formula)

Receiving flow (continued)

$$\therefore P_{ji} = -G_{ij} \left| \dot{V}_j \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j)$$

$$Q_{ij} = B_{ij} \left| \dot{V}_j \right|^2 - G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j)$$

# Constraints

## Load balance

$$P_{G_i} - P_{D_i} = \sum_{j \in \Omega_{b_i}} (P_{ij}) \quad \forall i \in \Omega_b \quad (5)$$

$$Q_{G_i} - Q_{D_i} = \sum_{j \in \Omega_{b_i}} (Q_{ij}) \quad \forall i \in \Omega_b \quad (6)$$

# Constraints

## Load balance

$$Q_{ij} = B_{ij} \left| \dot{V}_i \right|^2 + G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \quad (7)$$

$$Q_{ji} = B_{ij} \left| \dot{V}_j \right|^2 - G_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \sin(\theta_i - \theta_j) - B_{ij} \left| \dot{V}_i \right| \left| \dot{V}_j \right| \cos(\theta_i - \theta_j) \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \quad (8)$$

# Constraints

## Power and voltage

$$\underline{P}_{G_i} \leq P_{G_i} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b \quad (9)$$

$$\underline{Q}_{G_i} \leq Q_{G_i} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b \quad (10)$$

$$\underline{V} \leq |\dot{V}_i| \leq \overline{V} \quad \forall i \in \Omega_b \quad (11)$$

$$\theta_i = \begin{cases} 0 & \text{if bus } i \text{ is slack,} \\ \text{free} & \text{otherwise.} \end{cases} \quad (12)$$

# Constraints

## Current

$$I_{rij}^2 + I_{lmij}^2 \leq \bar{I}_{ij}^2 \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \quad (13)$$

$$\begin{aligned} I_{rij} = & -G_{ij} \left| \dot{V}_i \right| \cos \theta_i + B_{ij} \left| \dot{V}_i \right| \sin \theta_i \\ & + G_{ij} \left| \dot{V}_j \right| \cos \theta_j - B_{ij} \left| \dot{V}_j \right| \sin \theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \quad (14)$$

$$\begin{aligned} I_{lmij} = & -B_{ij} \left| \dot{V}_i \right| \cos \theta_i - G_{ij} \left| \dot{V}_i \right| \sin \theta_i \\ & + B_{ij} \left| \dot{V}_j \right| \cos \theta_j + G_{ij} \left| \dot{V}_j \right| \sin \theta_j \quad \forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \end{aligned} \quad (15)$$



# Constraints(Derivation of the formula)

Current

$$\begin{aligned}
 I_{ij} &= \frac{\dot{V}_i - \dot{V}_j}{Z_{ij}} \quad \left( Y_{ij} = -\frac{1}{Z_{ij}} = G_{ij} + jB_{ij} \right) \\
 &= (-G_{ij} - jB_{ij}) (\dot{V}_i - \dot{V}_j) \\
 &= (-G_{ij} - jB_{ij}) \left( |\dot{V}_i| \cos \theta_i + j |\dot{V}_i| \sin \theta_i - |\dot{V}_j| \cos \theta_j - j |\dot{V}_j| \sin \theta_j \right) \\
 &= -G_{ij} |\dot{V}_i| \cos \theta_i + B_{ij} |\dot{V}_i| \sin \theta_i + G_{ij} |\dot{V}_j| \cos \theta_j - B_{ij} |\dot{V}_j| \sin \theta_j \\
 &\quad + j \left( -B_{ij} |\dot{V}_i| \cos \theta_i - G_{ij} |\dot{V}_i| \sin \theta_i + B_{ij} |\dot{V}_j| \cos \theta_j + G_{ij} |\dot{V}_j| \sin \theta_j \right)
 \end{aligned}$$

# Constraints(Derivation of the formula)

Current (Continued)

$$\begin{aligned}\therefore I_{rij} &= -G_{ij} \left| \dot{V}_i \right| \cos \theta_i + B_{ij} \left| \dot{V}_i \right| \sin \theta_i + G_{ij} \left| \dot{V}_j \right| \cos \theta_j - B_{ij} \left| \dot{V}_j \right| \sin \theta_j \\ I_{lmij} &= -B_{ij} \left| \dot{V}_i \right| \cos \theta_i - G_{ij} \left| \dot{V}_i \right| \sin \theta_i + B_{ij} \left| \dot{V}_j \right| \cos \theta_j + G_{ij} \left| \dot{V}_j \right| \sin \theta_j\end{aligned}$$

# Multi-Period Optimal Power Flow (MPOPF) Formulation

# Multi-Period Overview

- MPOPF Nomenclature: Slide 37
- MPOPF At glance...: Slide 40
- MPOPF Objective function: Slide 45
- MPOPF Constraints and expressions: Slide 47
  - MPOPF Load balance
  - MPOPF Power and voltage
  - MPOPF Current

Optimization problem is formulated as:

- 1 MPOPF Objective function:

minimize (or maximize)  $f(\mathbf{x})$

- 2 MPOPF Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

- 3 MPOPF Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

# MPOFP Nomenclature

## Sets, indices, parameters

- Indices

$i, j$  Index of bus  
 $l$  Index of line  
 $t$  Index of time

- Sets

$\Omega_l$  Set of lines  
 $\Omega_b$  Set of buses  
 $\Omega_{b_i}$  Set of connected buses in the bus  $i$   
 $\Omega_{b_g}$  Set of generation buses ( $\Omega_{b_g} \subset \Omega_b$ )  
 $T$  The total time period (e.g 1,2,...,24 for 24 hours)

# MPOFP Nomenclature

## Sets, indices, parameters

- Parameters or constants

$Z_{ij}, Y_{ij}$	Impedance and admittance of line $ij$ (from bus $i$ to bus $j$ )
$G_{ij}, B_{ij}$	Conductance and susceptance of line $ij$ (from bus $i$ to bus $j$ )
$\overline{V}, \underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line $ij$

$P_{D_i}, Q_{D_i}$	Active and reactive power demand at bus $i$
$\overline{P}_{G_i}, \underline{P}_{G_i}$	Maximum and minimum active power from generator at bus $i$
$\overline{Q}_{G_i}, \underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus $i$
$baseMVA$	Value of base MVA

# MPOFP Nomenclature

## Sets, indices, parameters

### • Functions

$P_{ij}(t), Q_{ij}(t)$	Active and reactive power flow of line $ij$ at time $t$
$I_{rij}(t), I_{lmij}(t)$	Real and Imaginary current flow of line $ij$ at time $t$
$P_I^{lineloss}(t)$	Active line loss of line $I(ij)$ at time $t$

### • Variables

$ \dot{V}_i(t) $	Voltage magnitude in bus $i$ at time $t$
$\theta_i(t)$	Voltage phase angle in bus $i$ at time $t$
$P_{G_i}, Q_{G_i}$	Active and reactive power from generator at bus $i$ at time $t$

# MPOPF At glance...

Objective function: Eq. (16)

$$\min \sum_t \sum_{i,j} [-G_{ij} (|\dot{V}_i(t)|^2 + |\dot{V}_j(t)|^2) + 2G_{ij} |\dot{V}_i(t)| |\dot{V}_j(t)| \cos(\theta_i(t) - \theta_j(t))]$$

Constraints: Eqs. (20),(21),(24),(25),(26),(27),(28)

*s.t.*

$$P_{G_i}(t) - P_{D_i}(t) = \sum_{j \in \Omega_{b_i}} (P_{ij}(t)) \quad \forall i \in \Omega_b, \forall t \in T$$

$$\underline{V} \leq |\dot{V}_i(t)| \leq \overline{V} \quad \forall i \in \Omega_b, \forall t \in T$$

$$Q_{G_i}(t) - Q_{D_i}(t) = \sum_{j \in \Omega_{b_i}} (Q_{ij}(t)) \quad \forall i \in \Omega_b, \forall t \in T$$

$$\theta_i(t) = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$\underline{P}_{G_i} \leq P_{G_i}(t) \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad I_{rij}(t)^2 + I_{lmij}(t)^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l, \forall t \in T$$

$$\underline{Q}_{G_i} \leq Q_{G_i}(t) \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T$$



# MPOPF At glance...

Functions or expressions: Eq. (17), (18), (19)

$$\begin{aligned} & [-G_{ij}(|\dot{V}_i(t)|^2 + |\dot{V}_j(t)|^2) + 2G_{ij}|\dot{V}_i(t)||\dot{V}_j(t)|\cos(\theta_i(t) - \theta_j(t))] \\ & = P_l^{line loss} = P_{ij} + P_{ji} \end{aligned}$$

$$\forall l(ij) \in \Omega_l, \forall t \in T$$

# MPOPF At glance...

$$P_{ij}(t) = -G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_i(t) \right| + G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \\ + B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t))$$

$$P_{ji}(t) = -G_{ij} \left| \dot{V}_j(t) \right| \left| \dot{V}_j(t) \right| + G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \\ - B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t))$$

$$\forall l \in \Omega_l \quad \text{or} \quad \forall (ij) \in \Omega_l \quad \text{and} \quad \forall t \in \mathcal{T}$$

# MPOPF At Glance

Functions or expressions (Continued): Eq. (22),(23), (29), (30)

$$\begin{aligned}
 Q_{ij}(t) = & B_{ij} \left| \dot{V}_i(t) \right|^2 + G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t)) \\
 & - B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \\
 & \forall I \in \Omega_I \text{ or } \forall (ij) \in \Omega_I, \forall t \in T
 \end{aligned}$$

$$\begin{aligned}
 Q_{ji}(t) = & B_{ij} \left| \dot{V}_j(t) \right|^2 - G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t)) \\
 & - B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \\
 & \forall I \in \Omega_I \text{ or } \forall (ij) \in \Omega_I, \forall t \in T
 \end{aligned}$$

# MPOPF At Glance

$$\begin{aligned}
 I_{rij}(t) = & -G_{ij} \left| \dot{V}_i(t) \right| \cos \theta_i(t) + B_{ij} \left| \dot{V}_i(t) \right| \sin \theta_i(t) \\
 & + G_{ij} \left| \dot{V}_j(t) \right| \cos \theta_j(t) - B_{ij} \left| \dot{V}_j(t) \right| \sin \theta_j(t) \\
 & \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T
 \end{aligned}$$

$$\begin{aligned}
 I_{lmij}(t) = & -B_{ij} \left| \dot{V}_i(t) \right| \cos \theta_i(t) - G_{ij} \left| \dot{V}_i(t) \right| \sin \theta_i(t) \\
 & + B_{ij} \left| \dot{V}_j(t) \right| \cos \theta_j(t) + G_{ij} \left| \dot{V}_j(t) \right| \sin \theta_j(t) \\
 & \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T
 \end{aligned}$$

# MPOPF Objective function

$$\min \sum_{t \in T} \sum_{\forall i,j} \left[ -G_{ij} (|\dot{V}_i(t)|^2 + |\dot{V}_j(t)|^2) + 2G_{ij} |\dot{V}_i(t)| |\dot{V}_j(t)| \cos(\theta_i(t) - \theta_j(t)) \right] \quad (16)$$

$$\begin{aligned} & \left[ -G_{ij} (|\dot{V}_i(t)|^2 + |\dot{V}_j(t)|^2) + 2G_{ij} |\dot{V}_i(t)| |\dot{V}_j(t)| \cos(\theta_i(t) - \theta_j(t)) \right] \\ & = P_i^{line loss}(t) = P_{ij}(t) + P_{ji}(t) \quad \forall l(ij) \in \Omega_l, \forall t \in T \end{aligned} \quad (17)$$

# MPOPF Objective function

$$\begin{aligned}
 P_{ij}(t) = & -G_{ij} \left| \dot{V}_i(t) \right|^2 + G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \\
 & + B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t))
 \end{aligned} \tag{18}$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T$$

$$\begin{aligned}
 P_{ji}(t) = & -G_{ij} \left| \dot{V}_j(t) \right|^2 + G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \\
 & - B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t))
 \end{aligned} \tag{19}$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in T$$

# MPOPF Constraints

## Load balance

$$P_{G_i}(t) - P_{D_i}(t) = \sum_{j \in \Omega_{b_i}} P_{ij}(t) \quad \forall i \in \Omega_b, \forall t \in T \quad (20)$$

$$Q_{G_i}(t) - Q_{D_i}(t) = \sum_{j \in \Omega_{b_i}} Q_{ij}(t) \quad \forall i \in \Omega_b, \forall t \in T \quad (21)$$

# MPOPF Constraints

## Load balance

$$\begin{aligned} Q_{ij}(t) = & B_{ij} \left| \dot{V}_i(t) \right|^2 + G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t)) \\ & - B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \end{aligned} \quad (22)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

$$\begin{aligned} Q_{ji}(t) = & B_{ij} \left| \dot{V}_j(t) \right|^2 - G_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \sin(\theta_i(t) - \theta_j(t)) \\ & - B_{ij} \left| \dot{V}_i(t) \right| \left| \dot{V}_j(t) \right| \cos(\theta_i(t) - \theta_j(t)) \end{aligned} \quad (23)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$



# MPOPF Constraints

## Power and voltage

$$\underline{P}_{G_i} \leq P_{G_i}(t) \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad (24)$$

$$\underline{Q}_{G_i} \leq Q_{G_i}(t) \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \forall t \in T \quad (25)$$

$$\underline{V} \leq |\dot{V}_i(t)| \leq \overline{V} \quad \forall i \in \Omega_b, \forall t \in T \quad (26)$$

$$\theta_i(t) = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \quad \forall t \in T \quad (27)$$

# MPOPF Constraints

## Current

$$I_{r_{ij}}(t)^2 + I_{l_{m_{ij}}}(t)^2 \leq \bar{I}_{ij}^2 \quad \forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T} \quad (28)$$

$$\begin{aligned} I_{r_{ij}}(t) = & -G_{ij} \left| \dot{V}_i(t) \right| \cos \theta_i(t) + B_{ij} \left| \dot{V}_i(t) \right| \sin \theta_i(t) \\ & + G_{ij} \left| \dot{V}_j(t) \right| \cos \theta_j(t) - B_{ij} \left| \dot{V}_j(t) \right| \sin \theta_j(t) \end{aligned} \quad (29)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

$$\begin{aligned} I_{l_{m_{ij}}}(t) = & -B_{ij} \left| \dot{V}_i(t) \right| \cos \theta_i(t) - G_{ij} \left| \dot{V}_i(t) \right| \sin \theta_i(t) \\ & + B_{ij} \left| \dot{V}_j(t) \right| \cos \theta_j(t) + G_{ij} \left| \dot{V}_j(t) \right| \sin \theta_j(t) \end{aligned} \quad (30)$$

$$\forall l \in \Omega_l \text{ or } \forall (ij) \in \Omega_l, \forall t \in \mathcal{T}$$

# Implementation in Matpower and Pyomo

# Matpower and Pyomo

## Matpower:

MATPOWER is a package of free, open-source Matlab-language M-files for solving steady-state power system simulation and optimization problems...(Zimmerman, Murillo-Sánchez, and Thomas 2011)

Site: <https://matpower.org/>

## Pyomo:

Pyomo is a Python-based open-source software package that supports a diverse set of optimization capabilities for formulating, solving, and analyzing optimization models...(Bynum et al. 2021)

Site: <https://www.pyomo.org/>

# Matpower and Pyomo

Matpower can load the test system easily. In addition, it can extract the system data utilized in the Pyomo...

## Optimal Power Flow: IEEE 33 Bus Case with Matpower

- Matpower
  - Matpower python 버전 설치: <https://pypi.org/project/matpower/>
  - Octave 설치: [https://github.com/yasinoni/mypower/blob/master/README\\_INSTALL\\_OCTAVE\\_AND\\_OCT2PY.md](https://github.com/yasinoni/mypower/blob/master/README_INSTALL_OCTAVE_AND_OCT2PY.md)
    - 환경변수에 octave-cli.exe 반드시 추가 (참고경로: C:\Program Files\GNU Octave\Octave-10.1.0\mingw64\bin, 홈페이지의 경로와 실제 경로가 다름 주의)
- Matpower OPF 결과 도움
- Matpower 참고: <https://pypi.org/project/matpower/>
- IEEE 33 bus 데이터 구조
  - <https://github.com/MATPOWER/matpower/blob/master/data/case33bw.m>

### 1. 계통 불러오기

```
from matpower import start_instance
from oct2py import octave
import pandas as pd
import numpy as np

# Set and load Matpower case
m = start_instance()
mpc = m.loadcase("case33bw")

#Base MVA
base_MVA = mpc['baseMVA']
```

Python

# Matpower and Pyomo

Pyomo can design and solve the optimization problem in python...

## Pyomo Tutorial v1 (2025.03.19)

[test\\_nonlinear.py](#)

### 예제 파일

- 비선형 최적화 문제 예제

$$\begin{aligned} \text{Minimize} \quad & x + \sin(y) \\ \text{s.t.} \quad & 10 \leq x \leq 20 \\ & \pi \leq y \leq \frac{5}{2}\pi \end{aligned}$$

- 예제에서의 답은  $x$ 가 10이고,  $\sin(y)$ 가 -1이 되는 지점일 것이다.
  - Optimal Point:  $x = 10, y = \frac{3}{2}\pi$
- Code step by step
  - Module 불러오기

```
import pyomo.environ as pyo
```

- Model 만들기

```
model = pyo.ConcreteModel()
```

# Implementation procedures

---

## Algorithm 1: Optimal Power Flow Implementation in Matpower and Pyomo

---

**Input:** IEEE test system (e.g., case33)

**Output:** Optimization results (Bus voltage, active/reactive power)

- 1 Load IEEE test system in Matpower:
  - Use the IEEE test system (4,33,...)
- 2 Extract parameters from system...:
  - Base MVA, slack bus
  - Bus:  $V_{min}/V_{max}$
  - Line:  $R/X$ , tap ratio
  - Generator:  $P_{min}/P_{max}$ , cost
  - Load data
  - Build Y-bus
- 3 Formulate OPF model:
  - Sets, parameters, variables...
  - Objective: Minimize loss
- 4 Solve with Pyomo:
  - Select solver (Ipopt,neos,...)
  - Execute
- 5 Save results:
  - Voltage profiles
  - Generation dispatch

# Including Code - Matpower

See [▶ file](#) for the detailed code.

Load the test case and execute optimal power flow:

## 1. 계통 불러오기

```
from matpower import start_instance
from oct2py import octave
import pandas as pd
import numpy as np

# Set and load Matpower case
m = start_instance()
mpc = m.loadcase('case33bw')

#Base MVA
base_MVA = mpc['baseMVA']
```

## Matpower OPF 결과

```
# Run OPF
mpopt = m.mpopion('verbose', 2)
[baseMVA, bus, gen, gencost, branch, f, success, et] = m.runopf(mpc, mpop, nout='ma

mat_gen_index = range(1,len(gen)+1)
mat_gen_info_columns = ['bus','Pg', 'Qg','Qmax','Qmin','Vg','mBase','status','Pmax',
mat_gen_info = pd.DataFrame(gen,index = mat_gen_index, columns = mat_gen_info_colum

matpower_gen_mw_total = mat_gen_info['Pg'].sum()

print('total gen MW:', matpower_gen_mw_total)
```

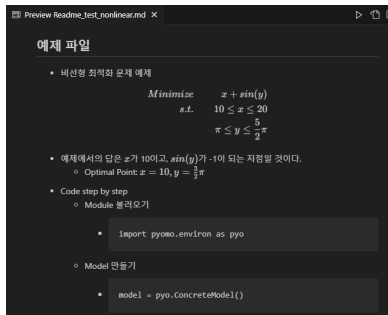
```
MATPOWER Version 8.0, 17-May-2024
Optimal Power Flow -- AC-polar-power formulation
MATPOWER Interior Point Solver -- MIPS, Version 1.5.1, 10-May-2024
(using built-in linear solver)
it   objective   step size   feascond   gradcond   compond   costcond
```



# Including Code - Pyomo

See [▶ file](#) for the detailed code.

Construct model and solve the optimization problem (Concrete model case):



Preview Readme\_test\_nonlinear.md

### 예제 파일

- 비선형 최적화 문제 예제

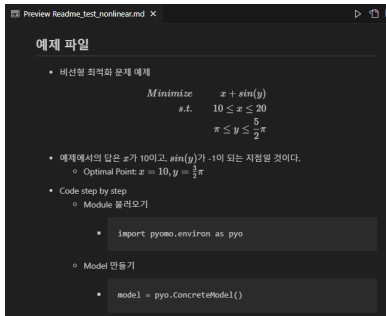
$$\begin{array}{ll} \text{Minimize} & x + \sin(y) \\ \text{s.t.} & 10 \leq x \leq 20 \\ & \pi \leq y \leq \frac{5}{2}\pi \end{array}$$

- 예제에서의 답은  $x$ 가 10이고,  $\sin(y)$ 가 -1이 되는 지점일 것이다.
  - Optimal Point:  $x = 10, y = \frac{3}{2}\pi$
- Code step by step
  - Module 불러오기
    - `import pyomo.environ as pyo`
  - Model 만들기
    - `model = pyo.ConcreteModel()`

# Optimal power flow problem using Matpower and Pyomo

See [▶ file](#) for the detailed code.

Construct OPF model and solve the OPF problem (Abstract model case):  
(Abstract model example: [▶ site1](#), [▶ site2](#))







Preview Readme\_test\_nonlinear.md X

### 예제 파일

- 비선형 최적화 문제 예제
$$\begin{aligned} & \text{Minimize} && x + \sin(y) \\ & \text{s.t.} && 10 \leq x \leq 20 \\ & && \pi \leq y \leq \frac{5}{2}\pi \end{aligned}$$
- 예제에서의 답은  $x$ 가 10이고,  $\sin(y)$ 가 -1이 되는 지점일 것이다.
  - Optimal Point:  $x = 10, y = \frac{3}{2}\pi$
- Code step by step
  - Module 불러오기
    - ```
import pyomo.environ as pyo
```
  - Model 만들기
    - ```
model = pyo.ConcreteModel()
```

# References

-  Ali, Aamir et al. (2024). "A novel solution to optimal power flow problems using composite differential evolution integrating effective constrained handling techniques". In: *Scientific Reports* 14.1. DOI: [10.1038/s41598-024-56590-5](https://doi.org/10.1038/s41598-024-56590-5).
-  Bynum, Michael L. et al. (2021). *Pyomo-optimization modeling in python*. Third. Vol. 67. Springer Science & Business Media.
-  Lavorato, Marina et al. (2012). "Imposing Radiality Constraints in Distribution System Optimization Problems". In: *IEEE Transactions on Power Systems* 27.1, pp. 172–180. DOI: [10.1109/TPWRS.2011.2161349](https://doi.org/10.1109/TPWRS.2011.2161349).
-  Zimmerman, Ray Daniel, Carlos Edmundo Murillo-Sánchez, and Robert John Thomas (2011). "MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education". In: *IEEE Transactions on Power Systems* 26.1, pp. 12–19. DOI: [10.1109/TPWRS.2010.2051168](https://doi.org/10.1109/TPWRS.2010.2051168).

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## **CWNU Power System Economis Lab**

- Woong Ko

# The End

Questions? Comments?