Multi Period Optimal Power Flow using Matpower and Pyomo

Taeho Nam

Changwon National University

rapitransit@gmail.com

July 5, 2025

Table of Contents I

Background

Multi Period Optimal Power Flow

- **MPOPF** (Multi-Period Optimal Power Flow) is a method for optimizing the operation of a power system over multiple time periods.
- It extends the conventional single-period Optimal Power Flow (OPF) to consider multiple time intervals.

bus	p_mw	q_mvar
1	0.000	0.000
2	0.100	0.060
3	0.090	0.040
4	0.120	0.080
5	0.060	0.030
6	0.060	0.020
7	0.200	0.100
8	0.200	0.100
9	0.060	0.020
10	0.060	0.020
11	0.045	0.030
12	0.060	0.035
13	0.060	0.035

Single-period Data
(specific time)

bus	p_mw	q_mvar	p_mw_1	q_mvar_1	p_mw_2	q_mvar_2	p_mw_3	q_mvar_3	...
									...
1	0.000	0.000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	...
2	0.100	0.060	0.061538	0.036923	0.057692	0.034615	0.053846	0.032308	...
3	0.090	0.040	0.055385	0.024615	0.051923	0.023077	0.048462	0.021538	...
4	0.120	0.080	0.073846	0.049231	0.069231	0.046154	0.064615	0.043077	...
5	0.060	0.030	0.036923	0.018462	0.034615	0.017308	0.032308	0.016154	...
6	0.060	0.020	0.036923	0.012308	0.034615	0.011538	0.032308	0.010769	...
7	0.200	0.100	0.123077	0.061538	0.115385	0.057692	0.107692	0.053846	...
8	0.200	0.100	0.123077	0.061538	0.115385	0.057692	0.107692	0.053846	...
9	0.060	0.020	0.036923	0.012308	0.034615	0.011538	0.032308	0.010769	...
10	0.060	0.020	0.036923	0.012308	0.034615	0.011538	0.032308	0.010769	...
11	0.045	0.030	0.027692	0.018462	0.025962	0.017308	0.024231	0.016154	...
12	0.060	0.035	0.036923	0.021538	0.034615	0.020192	0.032308	0.018846	...
13	0.060	0.035	0.036923	0.021538	0.034615	0.020192	0.032308	0.018846	...

Multi-period Data
(1-24h)

Multi Period Optimal Power Flow Formulation

Overview

- MPOPF Nomenclature: Slide 7
- MPOPF At glance...: Slide 10
- MPOPF Objective function: Slide 15
- MPOPF Constraints and expressions: Slide 17
  - MPOPF Load balance
  - MPOPF Power and voltage
  - MPOPF Current

Optimization problem is formulated as:

1. MPOPF Objective function:

$$\text{minimize (or maximize) } f(\mathbf{x})$$

2. MPOPF Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

3. MPOPF Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

Nomenclature

Sets, indices, parameters

- Indices

i, j	Index of bus
l	Index of line
t	Index of time

- Sets

Ω_l	Set of lines
Ω_b	Set of buses
Ω_{b_i}	Set of connected buses in the bus i
Ω_{b_g}	Set of generation buses ($\Omega_{b_g} \subset \Omega_b$)
T	The total time period (e.g 1,2...,24 for 24 hours)

Nomenclature

Sets, indices, parameters

- Parameters or constants

Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j)
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j)
$\overline{V}$, $\underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line ij
$P_{D_{i,t}}$, $Q_{D_{i,t}}$	Active and reactive power demand at bus i
$\overline{P}_{G_i}$, $\underline{P}_{G_i}$	Maximum and minimum active power from generator at bus i
$\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus i
$baseMVA$	Value of base MVA

Nomenclature

Sets, indices, parameters

- Functions

$P_{ij,t}$, $Q_{ij,t}$	Active and reactive power flow of line ij at time t
$I_{r_{ij,t}}$, $I_{Im_{ij,t}}$	Real and Imaginary current flow of line ij at time t
$P^{lineloss}_{l,t}$	Active line loss of line $l(ij)$ at time t

- Variables

$\left\|\dot{V}_{i,t}\right\|$	Voltage magnitude in bus i at time t
$\theta_{i,t}$	Voltage phase angle in bus i at time t
$P_{G_{i,t}}$, $Q_{G_{i,t}}$	Active and reactive power from generator at bus i at time t

MPOPF At glance...

Objective function: Eq. (1)

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[-G_{ij} \left(\left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2 G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left(\theta_{i,t} - \theta_{j,t} \right) \right]$$

Constraints: Eqs. (5),(6),(9),(10),(11),(12),(13)

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} \left(P_{ij,t} \right) \quad \forall i \in \Omega_b, \, \forall t \in T$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} \left(Q_{ij,t} \right) \quad \forall i \in \Omega_b, \, \forall t \in T$$

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \, \forall t \in T$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \, \forall t \in T$$

$$\underline{V} \leq \left| \dot{V}_{i,t} \right| \leq \overline{V} \quad \forall i \in \Omega_b, \, \forall t \in T$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$I_{r_{ij,t}}^2 + I_{Im_{ij,t}}^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l, \, \forall t \in T$$

MPOPF At glance...

Functions or expressions: Eq. (2), (3), (4)

$$[-G_{ij}(\left|\dot{V}_{i,t}\right|^2+\left|\dot{V}_{j,t}\right|^2)+2G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos(\theta_{i,t}-\theta_{j,t})]$$
$$=P_{l,t}^{lineloss}=P_{ij,t}+P_{ji,t}$$

$$\forall l(ij)\in\Omega_l,\ \forall t\in T$$

MPOPF At glance...

$$P_{ij,t} = -G_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) + B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right)$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$P_{ji,t} = -G_{ij}\left|\dot{V}_{j,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right)$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF At Glance

Functions or expressions (Continued): Eq. (7),(8), (14), (15)

$$Q_{ij,t} = B_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right)$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$Q_{ji,t} = B_{ij}\left|\dot{V}_{j,t}\right|^2 - G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right)$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF At Glance

$$I_{r_{ij,t}} = - G_{ij} \left|\dot{V}_{i,t}\right| \cos\theta_{i,t} + B_{ij} \left|\dot{V}_{i,t}\right| \sin\theta_{i,t}$$
$$+ G_{ij} \left|\dot{V}_{j,t}\right| \cos\theta_{j,t} - B_{ij} \left|\dot{V}_{j,t}\right| \sin\theta_{j,t}$$

$$\forall l(ij) \in \Omega_l, \ \forall t \in T$$

$$I_{Im_{ij,t}} = - B_{ij} \left|\dot{V}_{i,t}\right| \cos\theta_{i,t} - G_{ij} \left|\dot{V}_{i,t}\right| \sin\theta_{i,t}$$
$$+ B_{ij} \left|\dot{V}_{j,t}\right| \cos\theta_{j,t} + G_{ij} \left|\dot{V}_{j,t}\right| \sin\theta_{j,t}$$

$$\forall l(ij) \in \Omega_l, \ \forall t \in T$$

Objective function

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[-G_{ij} \left(\left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos \left(\theta_{i,t} - \theta_{j,t} \right) \right] \tag{1}$$

$$\begin{gathered} \left[-G_{ij} \left(\left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos \left(\theta_{i,t} - \theta_{j,t} \right) \right] = P_{l,t}^{lineloss} = P_{ij,t} + P_{ji,t} \\ \forall l(ij) \in \Omega_l, \ \forall t \in T \end{gathered} \tag{2}$$

MPOPF Objective function

$$P_{ij,t} = -G_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) + B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) \tag{3}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$P_{ji,t} = -G_{ij}\left|\dot{V}_{j,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) \tag{4}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF Constraints

Load balance

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} P_{ij,t} \quad \forall i \in \Omega_b, \forall t \in T \tag{5}$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} Q_{ij,t} \quad \forall i \in \Omega_b, \forall t \in T \tag{6}$$

MPOPF Constraints

Load balance

$$Q_{ij,t} = B_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right) \tag{7}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$Q_{ji,t} = B_{ij}\left|\dot{V}_{j,t}\right|^2 - G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right) \tag{8}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF Constraints

Power and voltage

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{9}$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{10}$$

$$\underline{V} \leq \left| \dot{V}_{i,t} \right| \leq \overline{V} \quad \forall i \in \Omega_b, \ \forall t \in T \tag{11}$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{ Bus } i \text{ is slack,} \\ \text{free} & : \text{ Otherwise.} \end{cases} \quad \forall t \in T \tag{12}$$

$$I_{r_{ij,t}}^2 + I_{Im_{ij,t}}^2 \leq \overline{I}_{ij}^2 \quad \forall l(ij) \in \Omega_l, \ \forall t \in T \tag{13}$$

MPOPF Constraints

Current

$$I_{r_{ij,t}} = -G_{ij}\left|\dot{V}_{i,t}\right|\cos\theta_{i,t} + B_{ij}\left|\dot{V}_{i,t}\right|\sin\theta_{i,t} + G_{ij}\left|\dot{V}_{j,t}\right|\cos\theta_{j,t} - B_{ij}\left|\dot{V}_{j,t}\right|\sin\theta_{j,t} \tag{14}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

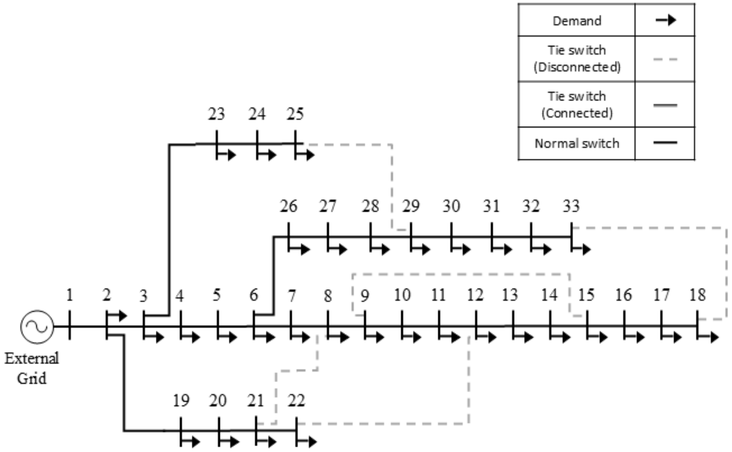
$$I_{Im_{ij,t}} = -B_{ij}\left|\dot{V}_{i,t}\right|\cos\theta_{i,t} - G_{ij}\left|\dot{V}_{i,t}\right|\sin\theta_{i,t} + B_{ij}\left|\dot{V}_{j,t}\right|\cos\theta_{j,t} + G_{ij}\left|\dot{V}_{j,t}\right|\sin\theta_{j,t} \tag{15}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

Implementation of Multi-Period Optimal Power Flow in a 33-bus distribution system

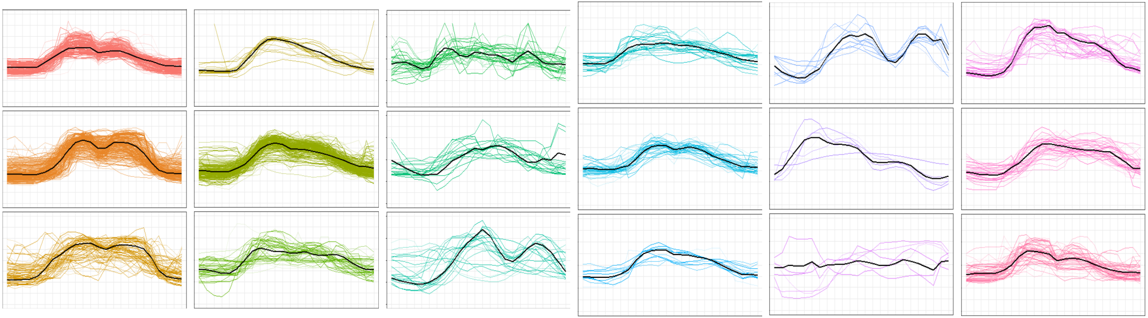
33-bus distribution system

Structure of the 33-bus distribution system



33-bus distribution system

Input Data - Load



- The 18 load patterns were randomly assigned to the existing 33-bus system.

  ELMAS: a one-year dataset of hourly electrical load profiles from 424 French industrial and tertiary sectors . . . (Bellinguer et al. 2023)

33-bus distribution system

Input Data - Load

bus	p_mw	q_mvar
1	0	0
2	0.1	0.06
3	0.09	0.04
4	0.12	0.08
5	0.06	0.03
6	0.06	0.02
7	0.2	0.1
8	0.2	0.1
9	0.06	0.02
10	0.06	0.02
11	0.045	0.03
12	0.06	0.035
13	0.06	0.035
14	0.12	0.08
15	0.06	0.01
16	0.06	0.02
17	0.06	0.02
18	0.09	0.04
19	0.09	0.04
20	0.09	0.04
21	0.09	0.04
22	0.09	0.04
23	0.09	0.05
24	0.42	0.2
25	0.42	0.2
26	0.06	0.025
27	0.06	0.025
28	0.06	0.02
29	0.12	0.07
30	0.2	0.6
31	0.15	0.07
32	0.21	0.1
33	0.06	0.04

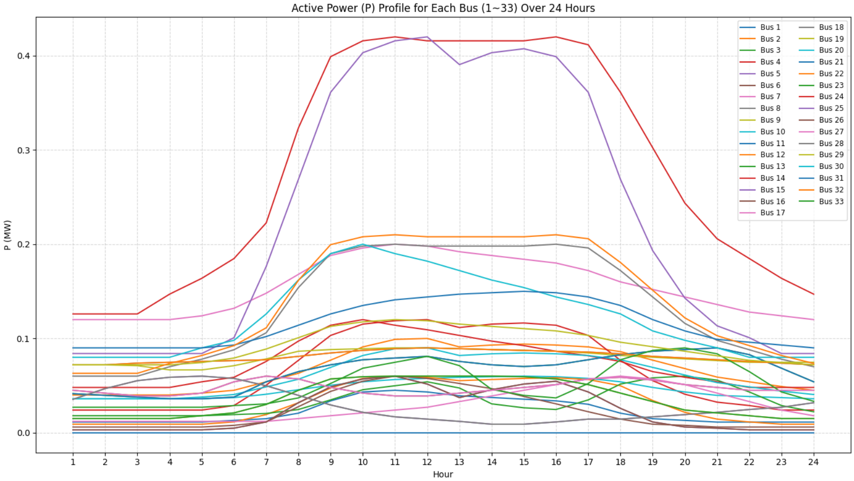
×

bus	cluster	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	17	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.25	0.3	0.35	0.4	0.45	0.55	0.65	0.75	0.85	0.95	1	0.95	0.85	0.7	0.55	0.4	0.3
2	3	0.4	0.4	0.4	0.4	0.42	0.45	0.54	0.63	0.77	0.91	0.99	1	0.91	0.93	0.94	0.93	0.91	0.86	0.77	0.68	0.59	0.54	0.49	0.45
3	13	0.46	0.44	0.42	0.4	0.4	0.42	0.6	0.72	0.8	0.86	0.88	0.9	0.84	0.8	0.78	0.8	0.86	0.92	0.96	0.98	1	0.92	0.76	0.6
4	5	0.2	0.2	0.2	0.2	0.2	0.24	0.42	0.64	0.86	0.96	0.99	1	0.93	0.96	0.97	0.95	0.86	0.64	0.46	0.34	0.27	0.24	0.2	0.2
5	14	0.59	0.78	0.92	0.98	1	0.96	0.83	0.66	0.49	0.36	0.28	0.24	0.2	0.15	0.15	0.19	0.24	0.24	0.28	0.32	0.36	0.41	0.45	0.53
6	7	0.75	0.7	0.65	0.65	0.7	0.9	1	0.95	0.8	0.7	0.65	0.65	0.7	0.75	0.8	0.85	0.95	0.98	0.93	0.85	0.8	0.75	0.75	0.75
7	16	0.6	0.6	0.6	0.6	0.62	0.66	0.74	0.84	0.94	0.98	1	0.99	0.96	0.94	0.92	0.9	0.86	0.8	0.76	0.72	0.68	0.64	0.62	0.6
8	1	0.3	0.3	0.3	0.35	0.39	0.44	0.53	0.77	0.95	0.99	1	0.99	0.99	0.99	0.99	1	0.98	0.86	0.72	0.58	0.49	0.44	0.39	0.35
9	6	0.25	0.25	0.25	0.25	0.3	0.35	0.5	0.75	0.95	0.99	1	1	1	1	0.99	0.95	0.85	0.7	0.55	0.4	0.35	0.3	0.25	0.25
10	8	0.6	0.6	0.6	0.6	0.6	0.62	0.68	0.76	0.84	0.9	0.94	0.96	0.98	0.99	1	0.99	0.96	0.9	0.8	0.72	0.66	0.64	0.62	0.6
11	11	0.25	0.25	0.25	0.25	0.25	0.29	0.33	0.46	0.75	0.96	1	0.96	0.87	0.83	0.79	0.75	0.67	0.46	0.33	0.29	0.25	0.25	0.25	0.25
12	2	0.15	0.15	0.15	0.15	0.15	0.19	0.32	0.53	0.79	0.96	1	0.98	0.92	0.94	0.96	0.97	0.94	0.83	0.57	0.36	0.24	0.19	0.15	0.15
13	9	0.3	0.3	0.3	0.3	0.3	0.32	0.34	0.41	0.58	0.76	0.83	0.9	0.79	0.51	0.44	0.41	0.58	0.86	0.97	1	0.93	0.72	0.48	0.37
14	12	0.4	0.4	0.4	0.4	0.45	0.49	0.63	0.81	0.95	1	0.95	0.91	0.86	0.81	0.77	0.72	0.68	0.63	0.54	0.49	0.45	0.4	0.4	0.4
15	18	0.05	0.05	0.05	0.05	0.05	0.08	0.19	0.53	0.81	0.95	1	0.86	0.62	0.76	0.86	0.91	0.72	0.43	0.19	0.1	0.08	0.05	0.05	0.05
16	4	0.1	0.1	0.1	0.1	0.1	0.13	0.19	0.46	0.73	0.91	1	0.96	0.87	0.78	0.64	0.51	0.37	0.24	0.15	0.13	0.1	0.1	0.1	0.1
17	17	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.25	0.3	0.35	0.4	0.45	0.55	0.65	0.75	0.85	0.95	1	0.95	0.85	0.7	0.55	0.4	0.3
18	15	0.8	0.8	0.82	0.83	0.84	0.85	0.86	0.9	0.94	0.97	0.99	1	0.99	0.98	0.97	0.96	0.95	0.93	0.9	0.88	0.86	0.84	0.82	0.8
19	10	0.8	0.8	0.79	0.74	0.74	0.79	0.87	0.96	0.98	0.99	1	1	0.99	0.98	0.97	0.96	0.94	0.91	0.89	0.87	0.85	0.83	0.83	0.83
20	3	0.4	0.4	0.4	0.4	0.42	0.45	0.54	0.63	0.77	0.91	0.99	1	0.91	0.93	0.94	0.93	0.91	0.86	0.77	0.68	0.59	0.54	0.49	0.45
21	13	0.46	0.44	0.42	0.4	0.4	0.42	0.6	0.72	0.8	0.86	0.88	0.9	0.84	0.8	0.78	0.8	0.86	0.92	0.96	0.98	1	0.92	0.76	0.6
22	15	0.8	0.8	0.82	0.83	0.84	0.85	0.86	0.9	0.94	0.97	0.99	1	0.99	0.98	0.97	0.96	0.95	0.93	0.9	0.88	0.86	0.84	0.82	0.8
23	9	0.3	0.3	0.3	0.3	0.3	0.32	0.34	0.41	0.58	0.76	0.83	0.9	0.79	0.51	0.44	0.41	0.58	0.86	0.97	1	0.93	0.72	0.48	0.37
24	1	0.3	0.3	0.3	0.35	0.39	0.44	0.53	0.77	0.95	0.99	1	0.99	0.99	0.99	0.99	1	0.98	0.86	0.72	0.58	0.49	0.44	0.39	0.35
25	5	0.2	0.2	0.2	0.2	0.2	0.24	0.42	0.64	0.86	0.96	0.99	1	0.93	0.96	0.97	0.95	0.86	0.64	0.46	0.34	0.27	0.24	0.2	0.2
26	18	0.05	0.05	0.05	0.05	0.05	0.08	0.19	0.53	0.81	0.95	1	0.86	0.62	0.76	0.86	0.91	0.72	0.43	0.19	0.1	0.08	0.05	0.05	0.05
27	7	0.75	0.7	0.65	0.65	0.7	0.9	1	0.95	0.8	0.7	0.65	0.65	0.7	0.75	0.8	0.85	0.95	0.98	0.93	0.85	0.8	0.75	0.75	0.75
28	14	0.59	0.78	0.92	0.98	1	0.96	0.83	0.66	0.49	0.36	0.28	0.24	0.2	0.15	0.15	0.19	0.24	0.24	0.28	0.32	0.36	0.41	0.45	0.53
29	16	0.6	0.6	0.6	0.6	0.62	0.66	0.74	0.84	0.94	0.98	1	0.99	0.96	0.94	0.92	0.9	0.86	0.8	0.76	0.72	0.68	0.64	0.62	0.6
30	12	0.4	0.4	0.4	0.4	0.45	0.49	0.63	0.81	0.95	1	0.95	0.91	0.86	0.81	0.77	0.72	0.68	0.63	0.54	0.49	0.45	0.4	0.4	0.4
31	8	0.6	0.6	0.6	0.6	0.6	0.62	0.68	0.76	0.84	0.9	0.94	0.96	0.98	0.99	1	0.99	0.96	0.9	0.8	0.72	0.66	0.64	0.62	0.6
32	1	0.3	0.3	0.3	0.35	0.39	0.44	0.53	0.77	0.95	0.99	1	0.99	0.99	0.99	0.99	1	0.98	0.86	0.72	0.58	0.49	0.44	0.39	0.35
33	6	0.25	0.25	0.25	0.25	0.3	0.35	0.5	0.75	0.95	0.99	1	1	1	1	0.99	0.95	0.85	0.7	0.55	0.4	0.35	0.3	0.25	0.25

- Assume the given load as the maximum load.
- Apply load patterns to the 33-bus system.

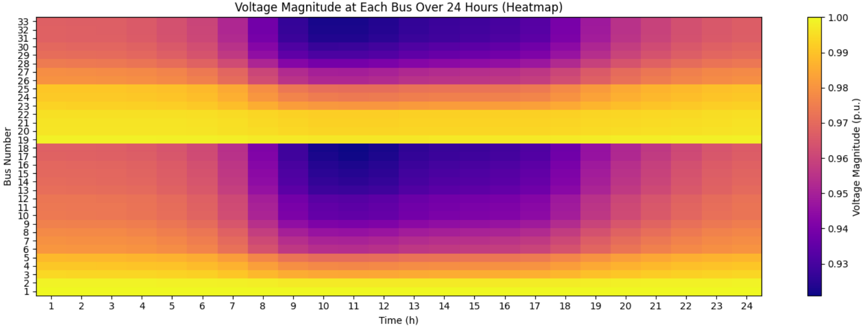
33-bus distribution system

Input Data - Load



33-bus distribution system

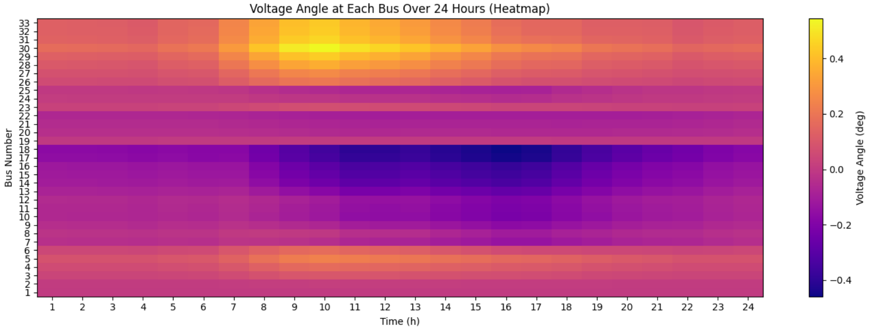
Output Data - Voltage magnitude



- Increased demand leads to a larger voltage drop
- The voltage drop increases with distance from the generator(BUS 1).

33-bus distribution system

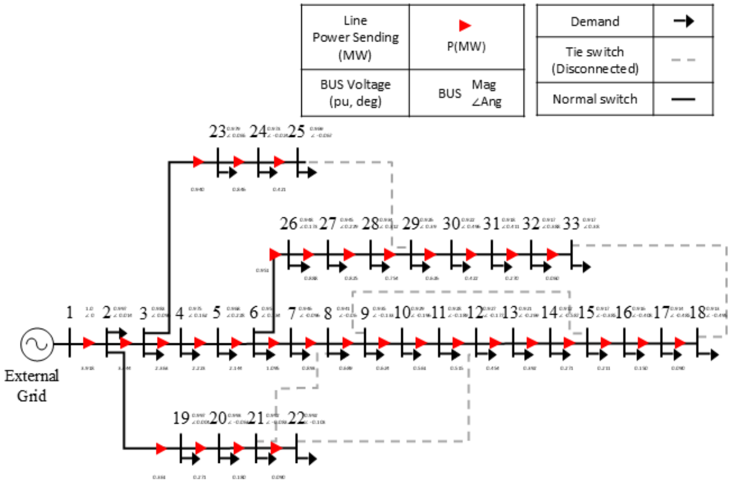
Output Data - Voltage angle



- Increased demand leads to a greater phase angle.
- The phase angle tends to decrease with increasing distance from the generator.
- Bus 30 has a substantial reactive power demand.

33-bus distribution system

Output Data - P line flow sending(at 13:00)



References

Bellinguer, Kevin et al. (2023). "ELMAS: a one-year dataset of hourly electrical load profiles from 424 French industrial and tertiary sectors". In: *Scientific Data* 10.1, p. 686. DOI: 10.1038/s41597-023-02542-z. URL: https://doi.org/10.1038/s41597-023-02542-z.

Acknowledgements

CWNU Power System Economis Lab

- Prof. Woong Ko
- Taeho Nam

The End

Questions? Comments?