Multi Period Optimal Power Flow using Matpower and Pyomo

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Background

Multi Period Optimal Power Flow

- **MPOPF** (Multi-Period Optimal Power Flow) is a method for optimizing the operation of a power system over multiple time periods.
- It extends the conventional single-period Optimal Power Flow (OPF) to consider multiple time intervals.

bus	p_mw	q_mvar
1	0.000	0.000
2	0.100	0.060
3	0.090	0.040
4	0.120	0.080
5	0.060	0.030
6	0.060	0.020
7	0.200	0.100
8	0.200	0.100
9	0.060	0.020
10	0.060	0.020
11	0.045	0.030
12	0.060	0.035
13	0.060	0.035

Single-period Data
(specific time)

bus	p_mw	q_mvar	p_mw_1	q_mvar_1	p_mw_2	q_mvar_2	p_mw_3	q_mvar_3	...
1	0.000	0.000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	...
2	0.100	0.060	0.061538	0.036923	0.057692	0.034615	0.053846	0.032308	...
3	0.090	0.040	0.055385	0.024615	0.051923	0.023077	0.048462	0.021538	...
4	0.120	0.080	0.073846	0.049231	0.069231	0.046154	0.064615	0.043077	...
5	0.060	0.030	0.036923	0.018462	0.034615	0.017308	0.032308	0.016154	...
6	0.060	0.020	0.036923	0.012308	0.034615	0.011538	0.032308	0.010769	...
7	0.200	0.100	0.123077	0.061538	0.115385	0.057692	0.107692	0.053846	...
8	0.200	0.100	0.123077	0.061538	0.115385	0.057692	0.107692	0.053846	...
9	0.060	0.020	0.036923	0.012308	0.034615	0.011538	0.032308	0.010769	...
10	0.060	0.020	0.036923	0.012308	0.034615	0.011538	0.032308	0.010769	...
11	0.045	0.030	0.027692	0.018462	0.025962	0.017308	0.024231	0.016154	...
12	0.060	0.035	0.036923	0.021538	0.034615	0.020192	0.032308	0.018846	...
13	0.060	0.035	0.036923	0.021538	0.034615	0.020192	0.032308	0.018846	...

Multi-period Data
(1-24h)

Multi Period Optimal Power Flow Formulation

Overview

Optimization problem is formulated as:

1. MPOPF Objective function:

$$\text{minimize (or maximize) } f(\mathbf{x})$$

2. MPOPF Constraints:

$$g(\mathbf{x}) \leq 0, \quad h(\mathbf{x}) = 0$$

3. MPOPF Functions in the objective and constraints:

$$f(\mathbf{x}), \quad g(\mathbf{x}), \quad h(\mathbf{x})$$

Nomenclature

Sets, indices, parameters

- Indices

i, j	Index of bus
l	Index of line
t	Index of time

- Sets

Ω_l	Set of lines
Ω_b	Set of buses
Ω_{b_i}	Set of connected buses in the bus i
Ω_{b_g}	Set of generation buses ($\Omega_{b_g} \subset \Omega_b$)
T	The total time period (e.g 1,2...,24 for 24 hours)

Nomenclature

Sets, indices, parameters

- Parameters or constants

Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j)
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j)
$\overline{V}$, $\underline{V}$	Maximum and minimum voltage magnitude
$\overline{I}_{ij}$	Maximum current flow limit of line ij
$P_{D_{i,t}}$, $Q_{D_{i,t}}$	Active and reactive power demand at bus i
$\overline{P}_{G_i}$, $\underline{P}_{G_i}$	Maximum and minimum active power from generator at bus i
$\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$	Maximum and minimum reactive power from generator at bus i
$baseMVA$	Value of base MVA

Nomenclature

Sets, indices, parameters

- Functions

$P_{ij,t}$, $Q_{ij,t}$	Active and reactive power flow of line ij at time t
$I_{r_{ij,t}}$, $I_{Im_{ij,t}}$	Real and Imaginary current flow of line ij at time t
$P_{l,t}^{lineloss}$	Active line loss of line $l(ij)$ at time t

- Variables

$\left\|\dot{V}_{i,t}\right\|$	Voltage magnitude in bus i at time t
$\theta_{i,t}$	Voltage phase angle in bus i at time t
$P_{G_{i,t}}$, $Q_{G_{i,t}}$	Active and reactive power from generator at bus i at time t

MPOPF At glance...

Objective function: Eq. (1)

$$\min \sum_{\forall t} \sum_{\forall i,j} [-G_{ij}(\left|\dot{V}_{i,t}\right|^2 + \left|\dot{V}_{j,t}\right|^2) + 2G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos(\theta_{i,t} - \theta_{j,t})]$$

Constraints: Eqs. (5),(6),(9),(10),(11),(12),(13)

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (P_{ij,t}) \quad \forall i \in \Omega_b,\ \forall t \in T$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} (Q_{ij,t}) \quad \forall i \in \Omega_b,\ \forall t \in T$$

$$\underline{P}_{G_i} \le P_{G_{i,t}} \le \overline{P}_{G_i} \quad \forall i \in \Omega_b,\ \forall t \in T$$

$$\underline{Q}_{G_i} \le Q_{G_{i,t}} \le \overline{Q}_{G_i} \quad \forall i \in \Omega_b,\ \forall t \in T$$

$$\underline{V} \le \left|\dot{V}_{i,t}\right| \le \overline{V} \quad \forall i \in \Omega_b,\ \forall t \in T$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases}$$

$$I^2_{r_{ij,t}} + I^2_{Im_{ij,t}} \le \overline{I}^2_{ij} \quad \forall I(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF At glance...

Functions or expressions: Eq. (2), (3), (4)

$$[-G_{ij}(\left|\dot{V}_{i,t}\right|^2 + \left|\dot{V}_{j,t}\right|^2) + 2G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos(\theta_{i,t} - \theta_{j,t})]$$
$$= P_{l,t}^{lineloss} = P_{ij,t} + P_{ji,t}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF At glance...

$$
\begin{aligned}
P_{ij,t} = & -G_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) \\
& + B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right)
\end{aligned}
$$

$$
\forall l(ij) \in \Omega_l,\ \forall t \in T
$$

$$
\begin{aligned}
P_{ji,t} = & -G_{ij}\left|\dot{V}_{j,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) \\
& - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right)
\end{aligned}
$$

$$
\forall l(ij) \in \Omega_l,\ \forall t \in T
$$

MPOPF At Glance

Functions or expressions (Continued): Eq. (7),(8), (14), (15)

$$Q_{ij,t} = B_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right)$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$Q_{ji,t} = B_{ij}\left|\dot{V}_{j,t}\right|^2 - G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right)$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF At Glance

$$
\begin{aligned}
I_{r_{ij,t}} = &- G_{ij} \left| \dot{V}_{i,t} \right| \cos\theta_{i,t} + B_{ij} \left| \dot{V}_{i,t} \right| \sin\theta_{i,t} \\
&+ G_{ij} \left| \dot{V}_{j,t} \right| \cos\theta_{j,t} - B_{ij} \left| \dot{V}_{j,t} \right| \sin\theta_{j,t}
\end{aligned}
$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$
\begin{aligned}
I_{Im_{ij,t}} = &- B_{ij} \left| \dot{V}_{i,t} \right| \cos\theta_{i,t} - G_{ij} \left| \dot{V}_{i,t} \right| \sin\theta_{i,t} \\
&+ B_{ij} \left| \dot{V}_{j,t} \right| \cos\theta_{j,t} + G_{ij} \left| \dot{V}_{j,t} \right| \sin\theta_{j,t}
\end{aligned}
$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

Objective function

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[-G_{ij} \left(\left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left(\theta_{i,t} - \theta_{j,t}\right) \right] \tag{1}$$

$$\left[-G_{ij} \left(\left| \dot{V}_{i,t} \right|^2 + \left| \dot{V}_{j,t} \right|^2 \right) + 2G_{ij} \left| \dot{V}_{i,t} \right| \left| \dot{V}_{j,t} \right| \cos\left(\theta_{i,t} - \theta_{j,t}\right) \right] = P_{l,t}^{lineloss} = P_{ij,t} + P_{ji,t} \tag{2}$$

$$\forall l(ij) \in \Omega_l, \ \forall t \in T$$

MPOPF Objective function

$$P_{ij,t} = -G_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) + B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) \tag{3}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$P_{ji,t} = -G_{ij}\left|\dot{V}_{j,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) \tag{4}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF Constraints

Load balance

$$P_{G_{i,t}} - P_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} P_{ij,t} \quad \forall i \in \Omega_b,\ \forall t \in T \tag{5}$$

$$Q_{G_{i,t}} - Q_{D_{i,t}} = \sum_{j \in \Omega_{b_i}} Q_{ij,t} \quad \forall i \in \Omega_b,\ \forall t \in T \tag{6}$$

MPOPF Constraints

Load balance

$$Q_{ij,t} = B_{ij}\left|\dot{V}_{i,t}\right|^2 + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) \tag{7}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

$$Q_{ji,t} = B_{ij}\left|\dot{V}_{j,t}\right|^2 - G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) - B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right) \tag{8}$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

MPOPF Constraints

Power and voltage

$$\underline{P}_{G_i} \leq P_{G_{i,t}} \leq \overline{P}_{G_i} \quad \forall i \in \Omega_b,\ \forall t \in T \tag{9}$$

$$\underline{Q}_{G_i} \leq Q_{G_{i,t}} \leq \overline{Q}_{G_i} \quad \forall i \in \Omega_b,\ \forall t \in T \tag{10}$$

$$\underline{V} \leq \left| \dot{V}_{i,t} \right| \leq \overline{V} \quad \forall i \in \Omega_b,\ \forall t \in T \tag{11}$$

$$\theta_{i,t} = \begin{cases} 0 & : \text{Bus } i \text{ is slack,} \\ \text{free} & : \text{Otherwise.} \end{cases} \quad \forall t \in T \tag{12}$$

$$I^2_{r_{ij,t}} + I^2_{Im_{ij,t}} \leq \overline{I}^2_{ij} \quad \forall I(ij) \in \Omega_l,\ \forall t \in T \tag{13}$$

MPOPF Constraints

Current

$$
\begin{aligned}
I_{r_{ij,t}} = & -G_{ij}\left|\dot{V}_{i,t}\right|\cos\theta_{i,t} + B_{ij}\left|\dot{V}_{i,t}\right|\sin\theta_{i,t} \\
& + G_{ij}\left|\dot{V}_{j,t}\right|\cos\theta_{j,t} - B_{ij}\left|\dot{V}_{j,t}\right|\sin\theta_{j,t}
\end{aligned} \tag{14}
$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

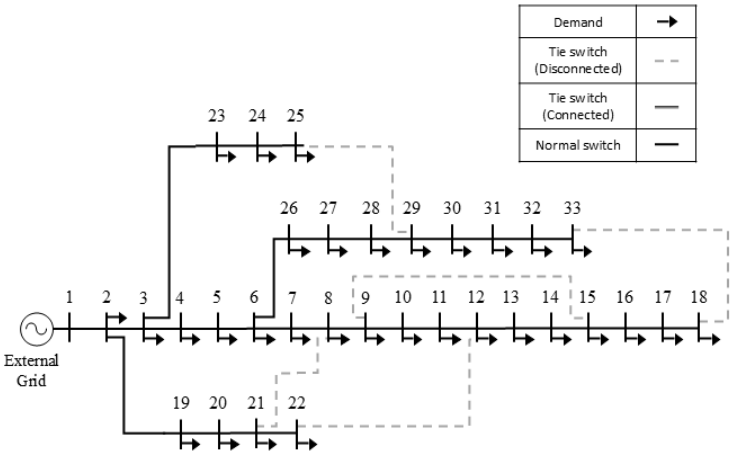
$$
\begin{aligned}
I_{Im_{ij,t}} = & -B_{ij}\left|\dot{V}_{i,t}\right|\cos\theta_{i,t} - G_{ij}\left|\dot{V}_{i,t}\right|\sin\theta_{i,t} \\
& + B_{ij}\left|\dot{V}_{j,t}\right|\cos\theta_{j,t} + G_{ij}\left|\dot{V}_{j,t}\right|\sin\theta_{j,t}
\end{aligned} \tag{15}
$$

$$\forall l(ij) \in \Omega_l,\ \forall t \in T$$

Implementation of Multi-Period Optimal Power Flow in a 33-bus distribution system

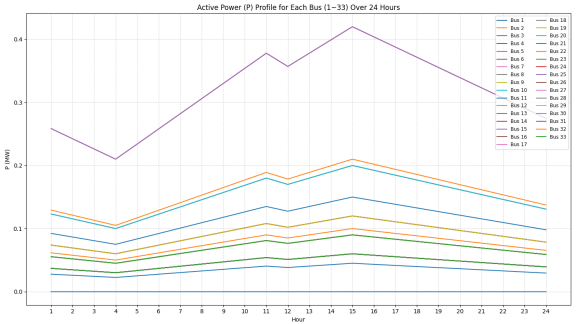
Disconnected 33-bus distribution system

Structure of the Disconnected 33-bus distribution system



Disconnected 33-bus distribution system

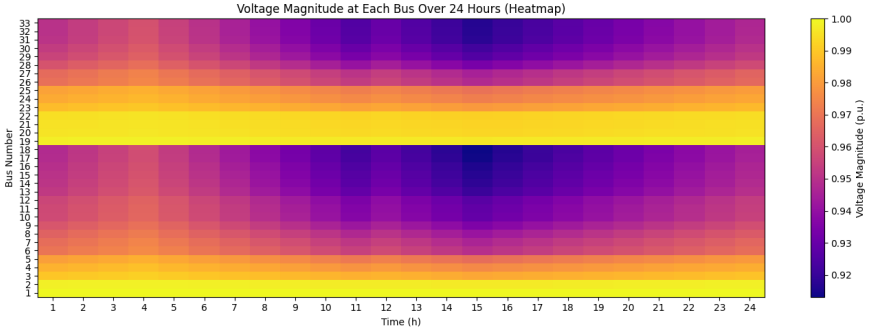
Input Data - Load



- The load data is based on the 24-hour load profile of a typical distribution system.
- The load varies throughout the day, with the highest demand at 13:00 and the lowest at 4:00.

Disconnected 33-bus distribution system

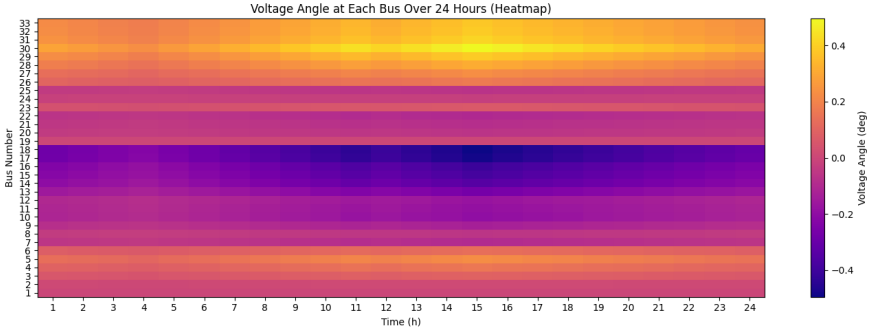
Output Data - Voltage magnitude



- The smallest voltage drop occurs at 4:00 under the lightest demand, and the largest drop occurs at 13:00 under the heaviest demand.
- The voltage drop increases with distance from the generator(BUS 1).

Disconnected 33-bus distribution system

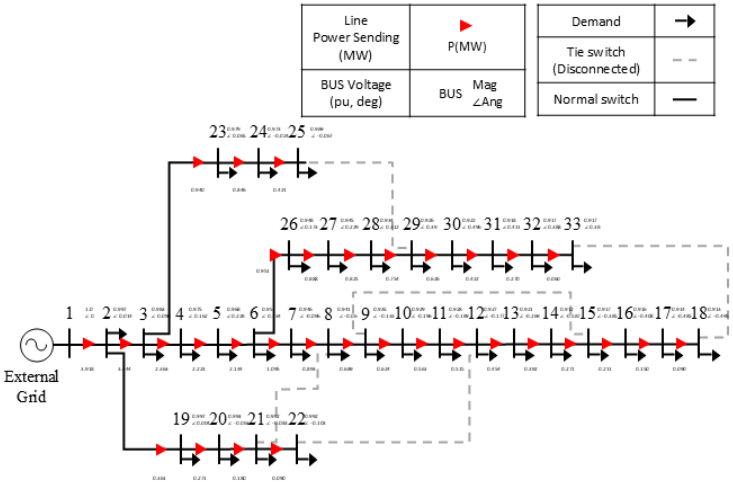
Output Data - Voltage angle



- The phase difference is smallest at 4:00 with minimum demand, and largest at 13:00 with maximum demand.
- The phase difference increases with distance from the generator(BUS 1).

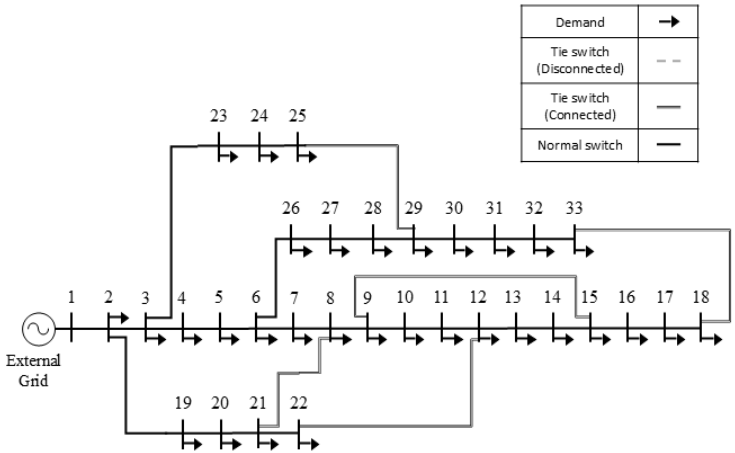
Disconnected 33-bus distribution system

Output Data - P line flow sending(at 13:00)



Connected 33-bus distribution system

Structure of the Connected 33-bus distribution system



References

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The End

Questions? Comments?