Network reconfiguration in distribution system with radiality constraints

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Nomenclature

Indices and sets

t / $\mathcal{T}$	Index of time / Set of time steps.
i, j / $\mathcal{N}$	Index of bus / Set of buses.
l / $\mathcal{L}$	Index of line / Set of lines.
dg / $\mathcal{G}_{dg}$	Index of distributed generator / Set of distributed generators.
$\mathcal{N}_{dg}$	Set of distributed generation buses.
$\mathcal{N}_{tf}$	Set of transfer buses.
$\mathcal{N}_{sb}$	Set of substation buses.
$\delta^{+}(i)$	Set of lines to bus i.
$\delta^{-}(i)$	Set of lines from bus i.

Parameters

n_b	Number of buses.
n_{dg}	Number of distributed generation buses.
n_{sb}	Number of substation buses.
Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j).
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j).
B^{c}_{ij}	Charging capacity of line ij (from bus i to bus j).
$Base$	Value of base MVA.

$\overline{V}$, $\underline{V}$ Maximum and minimum voltage magnitude.

$\overline{S}_{ij}$ Maximum complex power flow limit of line ij.

$P_{D_{i,t}}$, $Q_{D_{i,t}}$ Active and reactive power demand at bus i at time t.

$\overline{P}_{G_i}$, $\underline{P}_{G_i}$ Maximum and minimum active power from generator at bus i.

$\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$ Maximum and minimum reactive power from generator at bus i.

K_i Fictitious load of each distributed generator in bus i (If bus i has a distributed generator, the value is 1 [PU], otherwise 0).

Functions

$P_{ij,t}$, $Q_{ij,t}$ Active and reactive power flow of line ij at time t.

$I_{r_{ij,t}}$, $I_{Im_{ij,t}}$ Real and Imaginary current flow of line ij at time t.

$P_{l,t}^{lineloss}$ Active line loss of line $l(ij)$ at time t.

Variables

$\left|\dot{V}_{i,t}\right|$ Voltage magnitude in bus i at time t.

$\theta_{i,t}$ Voltage phase angle in bus i at time t.

$P_{G_i,t}$, $Q_{G_i,t}$ Active and reactive power from generator at bus i at time t.

x_{ij} Circuit that can be reconfigured on line ij.

y_i Status of transfer bus in bus i.

k_{ij} Fictitious flow associated with line ij.

1 Optimization problem for network reconfiguration

Based on previous researh, the optimization model for the network reconfiguration with radiality constraints can be respresented as follows [1].

Objective function:

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[-G_{ij} x_{ij} \left(\left|\dot{V}_{i,t}\right|^2 + \left|\dot{V}_{j,t}\right|^2 - 2 \left|\dot{V}_{i,t}\right| \left|\dot{V}_{j,t}\right| \cos\left(\theta_{i,t} - \theta_{j,t}\right)\right)\right] \tag{1}$$

General constraints with network, power balance, and switchable lines:

$$P_{G_i,t} - P_{D_{i,t}} = \sum_{(ij)\in\delta^-(i)} x_{ij} P_{ij,t} - \sum_{(ji)\in\delta^+(i)} x_{ji} P_{ji,t}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{2}$$

$$
\begin{aligned}
Q_{G_i,t} - Q_{D_{i,t}} = & \sum_{(ij)\in\delta^-(i)} x_{ij} Q_{ij,t} - \sum_{(ji)\in\delta^+(i)} x_{ji} Q_{ji,t} \\
& - \left|\dot{V}_{i,t}\right|^2 \Big(\sum_{(ij)\in\delta^-(i)} x_{ij} B^c_{ij}/2 + \sum_{(ji)\in\delta^+(i)} x_{ji} B^c_{ij}/2 \Big) \\
& \qquad\qquad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}.
\end{aligned} \tag{3}
$$

$$
\underline{P}_{G_i} \le P_{G_i,t} \le \overline{P}_{G_i}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{4}
$$

$$
\underline{Q}_{G_i} \le Q_{G_i,t} \le \overline{Q}_{G_i}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{5}
$$

$$
\underline{V} \le \left|\dot{V}_{i,t}\right| \le \overline{V}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{6}
$$

$$
\theta_{i,t} = 0, \quad \forall i \in \mathcal{N}_{sb}, \forall t \in \mathcal{T}. \tag{7}
$$

$$
-\pi \le \theta_{i,t} \le \pi, \quad \forall i \in \mathcal{N} \setminus \mathcal{N}_{sb}, \forall t \in \mathcal{T}. \tag{8}
$$

$$
P^2_{ij,t} + Q^2_{ij,t} \le x_{ij,t} \overline{S}^2_{ij}, \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}. \tag{9}
$$

$$
x_{ij} \in \{0,1\}, \quad \forall l(ij) \in \mathcal{L}. \tag{10}
$$

Functions (Appendix):

$$
\begin{aligned}
P_{ij,t} = & - G_{ij} \left|\dot{V}_{i,t}\right| \left|\dot{V}_{i,t}\right| + G_{ij} \left|\dot{V}_{i,t}\right| \left|\dot{V}_{j,t}\right| \cos\left(\theta_{i,t} - \theta_{j,t}\right) \\
& + B_{ij} \left|\dot{V}_{i,t}\right| \left|\dot{V}_{j,t}\right| \sin\left(\theta_{i,t} - \theta_{j,t}\right), \\
& \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}.
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
P_{ji,t} = & - G_{ij} \left|\dot{V}_{j,t}\right| \left|\dot{V}_{j,t}\right| + G_{ij} \left|\dot{V}_{i,t}\right| \left|\dot{V}_{j,t}\right| \cos\left(\theta_{i,t} - \theta_{j,t}\right) \\
& - B_{ij} \left|\dot{V}_{i,t}\right| \left|\dot{V}_{j,t}\right| \sin\left(\theta_{i,t} - \theta_{j,t}\right), \\
& \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}.
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
Q_{ij,t} = & B_{ij} \left|\dot{V}_i\right|^2 + G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\left(\theta_{i,t} - \theta_{j,t}\right) \\
& - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\left(\theta_{i,t} - \theta_{j,t}\right), \\
& \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}.
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
Q_{ji,t} = & B_{ij} \left|\dot{V}_j\right|^2 - G_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \sin\left(\theta_{i,t} - \theta_{j,t}\right) \\
& - B_{ij} \left|\dot{V}_i\right| \left|\dot{V}_j\right| \cos\left(\theta_{i,t} - \theta_{j,t}\right), \\
& \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}.
\end{aligned} \tag{14}
$$

- Radiality constraints using transfer buses which don't have distributed generators and demands from [1]:

$$x_{ij} \leq y_i, \quad \forall ij \in \delta^-(i), i \in \mathcal{N}_{tf}. \tag{15}$$
$$x_{ji} \leq y_i, \quad \forall ji \in \delta^+(i), i \in \mathcal{N}_{tf}. \tag{16}$$

$$\sum_{(ij)\in\delta^-(i)} x_{ij} + \sum_{(ji)\in\delta^+(i)} x_{ji} \geq 2y_i, \quad i \in \mathcal{N}_{tf}. \tag{17}$$

$$y_i \in 0, 1, \quad i \in \mathcal{N}_{tf}. \tag{18}$$
$$\sum_{(ij)\in\mathcal{L}} x_{ij} = n_b - n_{sb} - \sum_{j\in\mathcal{N}_{tf}} (1 - y_j). \tag{19}$$

- Radiality constraints considering distributed generators with fictious load(K) from [1]:

$$-K_i = \sum_{(ij)\in\delta^-(i)} k_{ij} - \sum_{(ji)\in\delta^+(i)} k_{ji}, \quad \forall i \in \mathcal{N}. \tag{20}$$

$$K_i = -n_{dg}, \quad \forall i \in \mathcal{N}_{sb}. \tag{21}$$
$$K_i = 1, \quad \forall i \in \mathcal{N}_{dg}. \tag{22}$$
$$K_i = 0, \quad \forall i \notin \mathcal{N}_{sb} \cup \mathcal{N}_{dg}. \tag{23}$$
$$|k_{ij}| \leq n_{dg} x_{ij}, \quad \forall (ij) \in \mathcal{L}. \tag{24}$$

References

[1] Marina Lavorato, John F. Franco, Marcos J. Rider, and Rubén Romero. Imposing radiality constraints in distribution system optimization problems. *IEEE Transactions on Power Systems*, 27(1):172–180, 2012.