Network reconfiguration in distribution system with radiality constraints

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Nomenclature

Indices and sets

t / $\mathcal{T}$	Index of time / Set of time steps.
t_a / $\mathcal{T}_a$	Index of time interval / Set of time intervals.
i, j / $\mathcal{N}$	Index of bus / Set of buses.
l / $\mathcal{L}$	Index of line / Set of lines.
dg / $\mathcal{G}_{dg}$	Index of distributed generator / Set of distributed generators.
$\mathcal{N}_{dg}$	Set of distributed generation buses.
$\mathcal{N}_{tf}$	Set of transfer buses.
$\mathcal{N}_{sb}$	Set of substation buses.
$\delta^{+}(i)$	Set of lines to bus i.
$\delta^{-}(i)$	Set of lines from bus i.

Parameters

n_b	Number of buses.
n_{dg}	Number of distributed generation buses.
n_{sb}	Number of substation buses.
Z_{ij}, Y_{ij}	Impedance and admittance of line ij (from bus i to bus j).
G_{ij}, B_{ij}	Conductance and susceptance of line ij (from bus i to bus j).
B^{c}_{ij}	Charging capacity of line ij (from bus i to bus j).

$Base$ Value of base MVA.

$\overline{V}$, $\underline{V}$ Maximum and minimum voltage magnitude.

$\overline{S}_{ij}$ Maximum complex power flow limit of line ij.

$P_{D_{i,t}}$, $Q_{D_{i,t}}$ Active and reactive power demand at bus i at time t.

$\overline{P}_{G_i}$, $\underline{P}_{G_i}$ Maximum and minimum active power from generator at bus i.

$\overline{Q}_{G_i}$, $\underline{Q}_{G_i}$ Maximum and minimum reactive power from generator at bus i.

K_i Fictitious load of each distributed generator in bus i (If bus i has a distributed generator, the value is 1 [PU], otherwise 0).

Functions

$P_{ij,t}$, $Q_{ij,t}$ Active and reactive power flow of line ij at time t.

$I_{r_{ij,t}}$, $I_{Im_{ij,t}}$ Real and Imaginary current flow of line ij at time t.

$P_{l,t}^{lineloss}$ Active line loss of line $l(ij)$ at time t.

Variables

$\left|\dot{V}_{i,t}\right|$ Voltage magnitude in bus i at time t.

$\theta_{i,t}$ Voltage phase angle in bus i at time t.

$P_{G_i,t}$, $Q_{G_i,t}$ Active and reactive power from generator at bus i at time t.

$x_{ij,t}$ Circuit that can be reconfigured on line ij at time t.

y_{i,t_a} Status of transfer bus in bus i at time interval t_a.

k_{ij,t_a} Fictitious flow associated with line ij at time interval t_a.

1 Optimization problem for network reconfiguration

Based on previous researh, the optimization model for the network reconfiguration with radiality constraints can be respresented as follows [1].

Objective function:

$$\min \sum_{\forall t} \sum_{\forall i,j} \left[-G_{ij} x_{ij} \left(\left|\dot{V}_{i,t}\right|^2 + \left|\dot{V}_{j,t}\right|^2 - 2 \left|\dot{V}_{i,t}\right| \left|\dot{V}_{j,t}\right| \cos\left(\theta_{i,t} - \theta_{j,t}\right)\right)\right] \quad (1)$$

General constraints with network, power balance, and switchable lines:

$$P_{G_i,t} - P_{D_{i,t}} = \sum_{(ij)\in\delta^-(i)} x_{ij,t}P_{ij,t} - \sum_{(ji)\in\delta^+(i)} x_{ji,t}P_{ji,t}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{2}$$

$$\begin{aligned} Q_{G_i,t} - Q_{D_{i,t}} = &\sum_{(ij)\in\delta^-(i)} x_{ij,t}Q_{ij,t} - \sum_{(ji)\in\delta^+(i)} x_{ji,t}Q_{ji,t} \\ &- \left|\dot{V}_{i,t}\right|^2 \Big(\sum_{(ij)\in\delta^-(i)} x_{ij,t}B^c_{ij}/2 + \sum_{(ji)\in\delta^+(i)} x_{ji,t}B^c_{ij}/2 \Big) \\ &\forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \end{aligned} \tag{3}$$

$$\underline{P}_{G_i} \le P_{G_i,t} \le \overline{P}_{G_i}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{4}$$

$$\underline{Q}_{G_i} \le Q_{G_i,t} \le \overline{Q}_{G_i}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{5}$$

$$\underline{V} \le \left|\dot{V}_{i,t}\right| \le \overline{V}, \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}. \tag{6}$$

$$\theta_{i,t} = 0, \quad \forall i \in \mathcal{N}_{sb}, \forall t \in \mathcal{T}. \tag{7}$$

$$-\pi \le \theta_{i,t} \le \pi, \quad \forall i \in \mathcal{N} \setminus \mathcal{N}_{sb}, \forall t \in \mathcal{T}. \tag{8}$$

$$P^2_{ij,t} + Q^2_{ij,t} \le x_{ij,t}\overline{S}^2_{ij}, \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}. \tag{9}$$

$$x_{ij,t} \in \{0,1\}, \quad \forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}. \tag{10}$$

Functions (Appendix):

$$\begin{aligned} P_{ij,t} = &- G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{i,t}\right| + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right) \\ &+ B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right), \\ &\forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}. \end{aligned} \tag{11}$$

$$\begin{aligned} P_{ji,t} = &- G_{ij}\left|\dot{V}_{j,t}\right|\left|\dot{V}_{j,t}\right| + G_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right) \\ &- B_{ij}\left|\dot{V}_{i,t}\right|\left|\dot{V}_{j,t}\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right), \\ &\forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}. \end{aligned} \tag{12}$$

$$\begin{aligned} Q_{ij,t} = &B_{ij}\left|\dot{V}_i\right|^2 + G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_{i,t} - \theta_{j,t}\right) \\ &- B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_{i,t} - \theta_{j,t}\right), \\ &\forall l(ij) \in \mathcal{L}, \forall t \in \mathcal{T}. \end{aligned} \tag{13}$$

$$
\begin{aligned}
Q_{ji,t} =& B_{ij}\left|\dot{V}_j\right|^2 - G_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\sin\left(\theta_{i,t}-\theta_{j,t}\right) \\
&- B_{ij}\left|\dot{V}_i\right|\left|\dot{V}_j\right|\cos\left(\theta_{i,t}-\theta_{j,t}\right), \\
&\forall l(ij)\in\mathcal{L}, \forall t\in\mathcal{T}.
\end{aligned} \tag{14}
$$

- Radiality constraints using transfer buses which don't have distributed generators and demands from [1]:

$$x_{ij,1+(t_a-1)T_a} \leq y_{i,t_a}, \quad \forall ij\in\delta^-(i), i\in\mathcal{N}_{tf}, \forall t_a\in\mathcal{T}_a. \tag{15}$$

$$x_{ji,1+(t_a-1)T_a} \leq y_{i,t_a}, \quad \forall ji\in\delta^+(i), i\in\mathcal{N}_{tf}, \forall t_a\in\mathcal{T}_a. \tag{16}$$

$$\sum_{(ij)\in\delta^-(i)} x_{ij,1+(t_a-1)T_a} + \sum_{(ji)\in\delta^+(i)} x_{ji,1+(t_a-1)T_a} \geq 2y_{i,t_a}, \quad i\in\mathcal{N}_{tf}, \forall_a\in\mathcal{T}_a. \tag{17}$$

$$y_{i,t_a} \in 0,1, \quad i\in\mathcal{N}_{tf}, \forall t_a\in\mathcal{T}_a. \tag{18}$$

$$\sum_{(ij)\in\mathcal{L}} x_{ij,1+(t_a-1)T_a} = n_b - n_{sb} - \sum_{j\in\mathcal{N}_{tf}}(1-y_{j,t_a}), \quad \forall t_a\in\mathcal{T}_a. \tag{19}$$

- Radiality constraints considering distributed generators with fictious load(K) from [1]:

$$-K_i = \sum_{(ij)\in\delta^-(i)} k_{ij,t_a} - \sum_{(ji)\in\delta^+(i)} k_{ji,t_a}, \quad \forall i\in\mathcal{N}, \quad \forall t_a\in\mathcal{T}_a. \tag{20}$$

$$K_i = -n_{dg}, \quad \forall i\in\mathcal{N}_{sb}. \tag{21}$$

$$K_i = 1, \quad \forall i\in\mathcal{N}_{dg}. \tag{22}$$

$$K_i = 0, \quad \forall i\notin\mathcal{N}_{sb}\cup\mathcal{N}_{dg}. \tag{23}$$

$$|k_{ij,t_a}| \leq n_{dg}x_{ij,1+(t_a-1)T_a}, \quad \forall(ij)\in\mathcal{L}, \quad \forall t_a\in\mathcal{T}_a. \tag{24}$$

- Radiality constraints for maintaining line status during time interval:

$$\sum_{h=1+(t_a-1)T_a}^{t_aT_a} x_{ij,h} = T_a x_{ij,1+(t_a-1)T_a}, \quad \forall l(ij)\in\mathcal{L}, \forall t_a\in\mathcal{T}_a. \tag{25}$$

$$x_{ij,t+1} = x_{ij,t}, (\text{if} \quad 1+(t_a-1)T_a \leq t \leq t_aT_a-1), \quad \forall l(ij)\in\mathcal{L}, \forall t_a\in\mathcal{T}_a. \tag{26}$$

References

[1] Marina Lavorato, John F. Franco, Marcos J. Rider, and Rubén Romero. Imposing radiality constraints in distribution system optimization problems. *IEEE Transactions on Power Systems*, 27(1):172–180, 2012.