Εθνικό Μετσόβιο Πολυτεχνείο
Σχολή Ηλεκτρολόγων Μηχανικών
και Μηχανικών Υπολογιστών
Τομέας Τεχνολογίας Πληροφορικής και
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Αποδοτική εξισορρόπηση αδειών εκπομπής αερίων θερμοκηπίου στον μηχανισμό EU-ETS

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ΚΩΝΣΤΑΝΤΙΝΟΣ ΠΑΠΑΔΟΠΟΥΛΟΣ

Επιβλέπων : Δημήτριος Φωτάκης
Καθηγητής Ε.Μ.Π.

Αθήνα, Νοέμβριος 2024

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Εγκρίθηκε από την τριμελή εξεταστική επιτροπή την 5η Νοεμβρίου 2024.

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Κωνσταντίνος Παπαδόπουλος

Διπλωματούχος Ηλεκτρολόγος Μηχανικός και Μηχανικός Υπολογιστών Ε.Μ.Π.

Abstract

The allocation of emission permits in cap-and-trade systems like the European Union Emissions Trading System (EU ETS) plays a critical role in the system's fairness and efficiency. This thesis investigates the fairness of the EU ETS's free allocation mechanism, utilizing the principles of fair division and distributive justice, specifically focusing on Moulin's framework. We begin by providing a comprehensive background on cap-and-trade systems and the EU ETS, highlighting the importance of equitable permit distribution.

Through data analysis and experiments, we explore the presence of horizontal equity in the current allocation practices among EU member states. Our findings indicate significant disparities that suggest the need for a more nuanced allocation approach. To address this, we employ cluster analysis to identify groups of countries with similar economic and energy profiles, aiming to achieve a relaxed version of horizontal equity within these clusters. Despite this approach, unequal explanatory ability persists, indicating limitations in the existing allocation mechanisms.

In response, we propose a linear optimization model that incorporates the fitness principle of Moulin into its objective function, while allowing other fairness principles to be implemented through constraints. This model offers a flexible and straightforward method for achieving a balance between fairness and efficiency at both the country and sector levels. We compare our model with the Uniform Linear Mechanism from the literature, using synthetic data to assess performance. The results demonstrate that while our model may result in lower consumer surplus, it ensures higher profits for the firms involved, indicating a potential trade-off between efficiency and equity.

This research contributes to the ongoing discourse on fair allocation in emissions trading systems, providing practical insights and methodologies for enhancing fairness in the EU ETS. Future work includes further refinement of the model and exploration of its implications on a broader scale.

Key words

Fair Distribution, EU ETS, Uniform Linear Mechanism, Linear Problem, Moulin's Fairness Principles, resource allocation

Περιεχόμενα

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Κατάλογος πινάκων

List of Algorithms

Chapter 1

Introduction

1.1 Background

Climate change is one of the most urgent and complex challenges facing the global community, primarily driven by the rapid increase in greenhouse gas (GHG) emissions from industrial activities and fossil fuel consumption [33]. To address this crisis, various international agreements and mechanisms have been implemented, emphasizing the necessity of coordinated global action. Among these initiatives, the European Union Emissions Trading System (EU ETS) stands out as a significant market-based approach to reducing emissions [20]. Established in 2005, the EU ETS operates on a cap-and-trade principle, setting a maximum limit on emissions and allowing companies to buy and sell emission allowances within this cap [19]. This system incentivizes firms to reduce emissions cost-effectively and has become a cornerstone of the EU's climate policy.

Fair distribution principles are crucial in allocating emission allowances within systems like the EU ETS. The fair division is the problem of dividing resources among agents in a way that satisfies specific fairness criteria [9]. In the context of emission trading, this involves determining how to allocate limited emission permits to different countries or firms equitably. Various fairness criteria, such as proportionality, envy-freeness, and equity, are considered to ensure that the allocation is just and acceptable to all parties involved [29].

Integrating fair division theories into the allocation mechanisms of the EU ETS is essential to balance efficiency and equity in addressing climate change. By applying principles of distributive justice, policymakers aim to design allocation methods that not only reduce overall emissions but also distribute the economic burden fairly among participants [22]. This thesis explores the intersection of fair division principles and the EU ETS, focusing on developing fair and transparent allocation methods based on Moulin's framework and assessing their impact on the effectiveness of emission trading systems.

1.2 Motivation

Climate change poses an unprecedented challenge to humanity, necessitating immediate and coordinated action to reduce greenhouse gas emissions globally. The European Union Emissions Trading System (EU ETS) represents a cornerstone policy in the EU's strategy to combat climate change by setting a cap on emissions and allowing for the trading of emission permits. However, allocating these permits has significant implications for the fairness and efficiency of the system. The current allocation mechanisms have been criticized for potential inequities among member states and sectors, potentially undermining the effectiveness and acceptance of the EU ETS.

This thesis is motivated by the pressing need to enhance the fairness of emission permit allocation within the EU ETS. By leveraging the principles of fair division, mainly Moulin's distributive justice framework, we aim to develop allocation mechanisms that balance equity and efficiency. Through rigorous analysis and modeling, this research seeks to contribute to designing a more equitable and effective emissions trading system, thereby supporting global efforts to mitigate climate change.

Additionally, part of the motivation for this research stems from the need to address perspectives highlighted in previous studies. Notably, Panagiotis Koromilas's thesis demonstrated the existence of leader-follower dynamics within the EU ETS, revealing underlying discrepancies in how emission permits are managed and allocated. While Koromilas's work provided valuable insights into the operational dynamics of the EU ETS, it also underscored the importance of examining the system through a lens that prioritizes fairness and distributive justice. This thesis aims to complement and extend these findings by focusing on equitable allocation mechanisms, thereby offering a balanced approach that integrates both ecological imperatives and principles of fairness.

1.3 Literature Review

Fair division began with Steinhaus (1948) [41]. Since then, it has attracted interest from various disciplines, including mathematics, economics, and computer science [6, 43, 30, 44, 31]. The literature on fair allocation can be divided into two main categories based on the type of resources—whether they are divisible or indivisible. For the divisible, heterogeneous category, known as cake cutting, multiple publications exist [35, 6, 43].

In environmental economics, fair division principles have been applied to carbon emissions and the allocation of emission permits. Great effort has been made to construct theoretical models that describe the functions of emissions trading systems [19, 20]. Although banking and borrowing of allowances have been established, theoretical analyses remain scarce [38, 8]. Schennach [39] includes output market uncertainty in firms' abatement decisions across time. Zhang et al. [53] explore the effect of uncertainty in an intertemporal emission trading system.

Considering market power and strategic interactions, Hahn [21] introduced the idea of market power in emission trading systems. Liski and Montero [27] study the effect of market power on the equilibrium of a permit market. Chevallier [10] develops a differential Stackelberg game with non-cooperative agents. Phaneuf and Requate [34] include multiple leaders in their model.

In the context of fair allocation of emission permits, several studies have proposed methods. Ju et al. [22] suggest a simple allocation method using an interesting fairness framework. Moretti and Trabelsi [28, 45] model the allocation of permits as a bankruptcy situation.

Böhringer and Helm [7] propose allocating the efficiency gains from the initial allocation rather than the allocation itself.

The allocation method should be explainable to participants [36]. Karpf et al. [23] study the EU ETS as a transaction network, showing that a hierarchical structure has emerged. Yaveroğlu et al. [52] develop a framework for analyzing and comparing networks.

Schüller et al. [40] propose a noncooperative differential game where countries influence their neighbors. Abebe and Goldner [2] discuss the need for different mechanisms to serve social interests in challenged social groups. Abebe et al. [3] analyze the importance of subsidies in alleviating income shocks.

Abadie presents methodological advancements like the synthetic control method [1]. Müller and Teixidó [32] examine Poland's case of free allocation through Article 10c using synthetic controls.

1.4 Outline

- **Chapter 1: Introduction** This chapter introduces the context of climate change and the significance of the EU Emissions Trading System (EU ETS). It outlines the motivation behind the research and sets the stage for the subsequent chapters.
- **Chapter 2: Emission Trading - EU ETS**

  This chapter provides a detailed introduction to the EU ETS, explaining its operation, history, and the principles of cap-and-trade systems. It highlights the importance of the EU ETS in addressing global climate change and sets the foundation for analyzing the allocation mechanisms within the system.
- **Chapter 3: Fair Distribution - Fair Division**

  This chapter introduces the key concepts of fair division and distributive justice, focusing on homogeneous divisible resources like emission permits. It emphasizes Moulin's principles of fairness, particularly horizontal equity, which serves as a central theme throughout the thesis.
- **Chapter 4: Exploring Horizontal Equity and Data Analysis in the EU ETS**

  In this chapter, we explore horizontal equity within the EU ETS from the perspective of the member countries. We conduct three experiments using data analysis to assess the fairness of free allowance allocations. The findings reveal limitations in achieving horizontal equity, prompting further investigation.
- **Chapter 5: Assessing the Limitations of Relaxed Horizontal Equity through Cluster Analysis**

  This chapter employs cluster analysis to attempt a more relaxed version of horizontal equity by grouping countries with similar profiles. The analysis uncovers unequal explanatory ability even within clusters, highlighting the complexities of fair allocation in the EU ETS.

- **Chapter 6: Allowance Allocation as an Optimization Problem**

  We propose a linear optimization model that incorporates the fitness principle of Moulin into its objective function. The model allows for the inclusion of other fairness principles through constraints, offering a flexible approach to balancing fairness and efficiency in permit allocation at both the country and sector levels. Differences between our model and the actual EU ETS allocation are presented.

- **Chapter 7: Uniform Linear Mechanism for Allocation**

  This chapter introduces a mechanism from the literature, the Uniform Linear Mechanism (ULM), which is optimal under certain conditions but requires infeasible information in real-world applications. We use this model to perform comparative experiments and to prepare the grounds for testing our proposed model.

- **Chapter 8: Comparison of Allocation Models**

  In this chapter, we compare the ULM with our proposed optimization model using synthetic data. The comparison reveals that while our model may result in lower consumer surplus, it ensures higher profits for the firms involved, suggesting a trade-off between different allocation objectives.

- **Chapter 9: Future Work**

  The final chapter discusses potential directions for future research, including further refinement of the optimization model, exploration of alternative fairness principles, and application of the model to real-world data.

Chapter 2

Emission Trading - EU ETS

2.1 Introduction

Climate change remains one of the most urgent and complex challenges encountered by the global community. The rapid rise in greenhouse gas (GHG) emissions, driven primarily by industrial activities and fossil fuel consumption, has led to a warming climate with severe consequences for ecosystems, economies, and public health worldwide [33]. Recognizing the severity of this threat, governments, intergovernmental organizations, and a multitude of stakeholders have rallied to implement agreements, frameworks, and mechanisms aimed at reducing GHG emissions on a global scale. Together, these initiatives underscore the necessity of coordinated action to mitigate climate change.

Among the earliest and most impactful frameworks was the United Nations Framework Convention on Climate Change (UNFCCC) [49], adopted in 1992, which set a foundation for international cooperation on climate action. Since then, landmark agreements have built upon this foundation, each with unique mechanisms and commitments. The Kyoto Protocol [47] of 1997 marked a pivotal step by establishing binding emission reduction targets for developed nations and introducing flexible mechanisms, such as Emissions Trading and the Clean Development Mechanism (CDM) (article 12 of [47], to incentivize reductions. Later, the Paris Agreement of 2015 [48] transformed climate governance by including both developed and developing nations under a common goal to limit global temperature increases to well below 2℃, with each country setting and updating voluntary Nationally Determined Contributions (NDCs) to meet this target.

Furthermore, specialized protocols like the Montreal Protocol [46], and its Kigali Amendment have played a significant role by phasing out hydrofluorocarbons (HFCs), which contribute significantly to both ozone depletion and global warming. Sector-specific initiatives, such as the Carbon Offsetting and Reduction Scheme for International Aviation (CORSIA), underscore the commitment of industry-specific organizations, such as the International Civil Aviation Organization (ICAO), to reducing emissions in high-impact sectors.

In parallel with these global efforts, regional initiatives have demonstrated the efficacy of targeted policies. One of the most notable examples is the European Union Emission Trading System (EU ETS), established in 2005 as the world's largest and most ambitious carbon market. Designed as a cap-and-trade system, the EU ETS limits emissions from high-emitting sectors

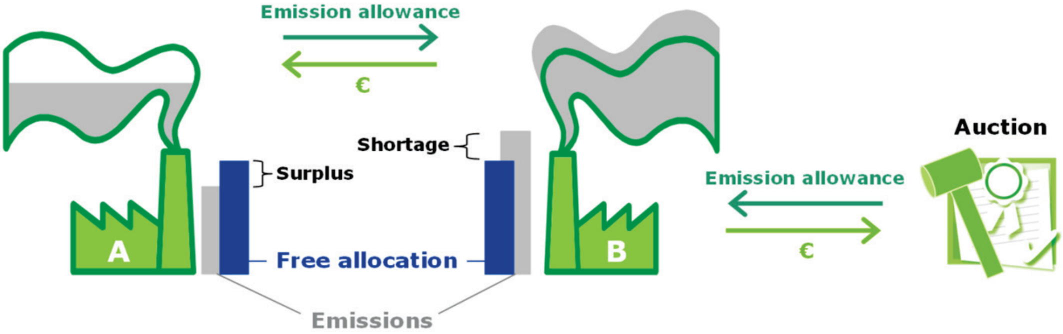


Figure 2.1: Diagram of CAT System from EU ETS Handbook [20]

across the EU and enables companies to trade allowances within this cap. This market-driven approach has made the EU ETS a cornerstone of Europe's climate policy and serves as a model for other regions aiming to reduce emissions while maintaining economic stability.

This thesis will explore the cap and Trade Systems (like the EU ETS) and the allocation of free allowances, which we will discuss shortly.

2.2 Cap And Trade Systems

A central approach in addressing emissions has been the establishment of Emissions Trading Systems (ETS) or cap-and-trade systems, which differ fundamentally from carbon taxes. While taxes set a fixed cost per unit of emissions, they do not directly ensure reductions in total emissions, as firms may continue emitting if they accept the cost. Cap-and-trade systems, on the other hand, establish a maximum (cap) on emissions and allow the market to determine the price, thereby incentivizing companies to innovate and reduce emissions while aligning with specific reduction targets.

Under cap-and-trade, a government or regulatory body issues a limited number of annual permits, each allowing for a specific amount of carbon dioxide (CO_2) emissions. The total number of permits sets the cap, which is typically reduced over time to ensure a progressive decrease in emissions. Companies that emit less than their allotted amount can sell (or "trade") unused permits to others, creating an economic incentive to cut emissions and drive permit prices upward as the cap tightens. This gradual increase in permit costs motivates firms to invest in cleaner technologies, ensuring that emissions reduction targets are met efficiently and at a minimal economic cost.

2.2.1 Key Examples of CAT Systems

Cap-and-trade systems have been adopted across various countries and regions as one of the most effective market-based tools for reducing emissions:

United States

Sulfur Dioxide (SO_2) One of the earliest applications of emissions trading was under the Acid Rain Program of the 1990 Clean Air Act, aimed at cutting SO_2 emissions. This program succeeded in reducing emissions by 50% from 1980 levels by 2007 and demonstrated the cost-effectiveness of cap-and-trade by reducing control costs by as much as 80% compared to traditional regulation.

Nitrogen Oxides (NOx) In 2003, the NOx Budget Trading Program began operating to lower NOx emissions, a primary cause of ground-level ozone pollution. Between 2003 and 2008, NOx emissions during ozone season dropped by 43%, even as energy demand remained steady.

Tokyo, Japan In 2010, Tokyo launched its own emissions trading scheme targeting high emitters within the city, including large industrial facilities and buildings. By its fourth year, emissions were reduced by 23% compared to baseline levels, with a goal of a 25% reduction by 2020.

European Union Emission Trading System (EU ETS) The EU ETS, launched in 2005, is the largest multinational emissions trading market and is central to the EU's climate policy. Covering over 11,000 installations in the power and industrial sectors, the EU ETS sets a progressively lower cap to reach the EU's target of a 43% reduction in emissions from 2005 levels by 2030. This system has served as a model for other emissions trading programs and continues to be a cornerstone of the EU's commitment to global climate goals.

South Korea South Korea's ETS, launched in 2015, covers 525 entities across 23 sectors, making it the second-largest carbon market globally after the EU ETS. South Korea's scheme aims to reduce emissions in alignment with its commitments to the UNFCCC, covering approximately two-thirds of the country's emissions.

New Zealand Established in 2008, the New Zealand Emissions Trading Scheme (NZ ETS) covers several sectors, including forestry, energy, and waste. The NZ ETS, though initially uncapped, has faced criticism for its high reliance on imported carbon allowances, which has diluted domestic emission reduction incentives.

China China, the world's largest GHG emitter, launched its national carbon market in 2021, initially covering the power sector, which represents over 40% of China's emissions. With a projected coverage of 3.5 billion tons of CO_2 from 1700 installations, China's ETS is anticipated to be the world's largest carbon market. China aims to reduce CO_2 emissions per unit of GDP by 65% by 2030 from 2005 levels.

2.2.2 Benefits of CAT Systems

Although counter-intuitive (to my singleton brain), CAT Systems have some strong arguments against other systems. [20]

Environmental Certainty Unlike carbon taxes, which set a price on emissions but don't guarantee a specific reduction level, CAT systems cap the total emissions allowed. This cap provides more certainty about achieving a targeted environmental outcome, making it easier to align with climate goals and international agreements.

Cost-Effectiveness By allowing companies to trade emission allowances, CAT systems enable reductions to occur where they are most economically efficient. Firms that can reduce emissions at lower costs can sell their excess allowances to those facing higher reduction costs, minimizing the overall expense of achieving emission targets.

Compatibility with International Markets CAT systems can be linked across jurisdictions, facilitating international cooperation and potentially leading to a more unified global approach to emissions reduction. For example, the European Union's Emissions Trading System (EU ETS) has explored linking with other systems to enhance market efficiency and environmental outcomes.

Minimising risk to Member State budgets The EU Emissions Trading System (EU ETS) ensures emissions reductions from installations responsible for approximately 50% of the EU's total emissions. This certainty diminishes the likelihood that Member States must acquire additional international units to fulfill their obligations under the Kyoto Protocol.

2.3 European Union Emissions Trading System (EU ETS)

2.3.1 Operation

The European Union Emissions Trading Scheme (EU ETS) operates as a cap-and-trade system to limit greenhouse gas (GHG) emissions from participating entities. It does this by issuing allowances, each representing the right to emit one tonne of CO_2 equivalent (tCOe). The total number of allowances is capped and has been decreasing annually by 1.74% since 2013, aligning with progressively ambitious emission reduction targets. Starting in 2021, the factor increased to 2.2% per year.

Annually, certain sectors—particularly those vulnerable to carbon leakage—receive a portion of these allowances for free, while the remaining allowances are sold mainly through auctions. By the end of each year, participants must surrender an allowance for every tonne of COe they have emitted. If they lack sufficient allowances, they must either reduce their emissions or purchase additional allowances through auctions or from other participants.

The value of allowances stems from their limited supply and the demand from participants for whom emission reductions are costlier. This market mechanism ensures that emissions are reduced where it is most economically efficient, benefiting both businesses and the broader economy. Compliance is strictly enforced with substantial fines—starting at €100 per tCOe and adjusted for inflation since 2013—for entities that fail to surrender enough allowances, thereby effectively maintaining the emissions cap.

Since Phase 2 began in 2008, participants with surplus allowances at the end of a trading phase can ”bank” them for future use, provided they are held in user accounts. For example, unlimited banking from Phase 2 to Phase 3 was permitted, with unused Phase 2 allowances automatically converted to Phase 3 allowances by June 2013 at no additional cost. These banked allowances are incorporated into the Phase 3 cap.

Participants can also ”borrow” allowances from their future allocations within the same trading period to meet current obligations. This is feasible because allowances are allocated in February each year, while the surrender deadline for the previous year’s allowances is at the end of April. However, borrowing across different trading periods—for instance, using Phase 3 allowances to meet Phase 2 obligations—is prohibited. [20]

2.3.2 Brief History and Goals

In this section, information from the European Commission [15] and [16] will be presented

Phase 1 (2005–2007) : Pilot Phase

The first phase served as a three-year pilot to test the system before it became a critical tool for meeting Kyoto Protocol targets in Phase 2.

Key characteristics of Phase 1 included:

- Scope: Limited to carbon dioxide (CO_2) emissions from power generators and energy-intensive industries.
- Allowance Allocation: Almost all allowances were distributed to businesses for free.
- Compliance Penalty: A penalty of €40 per tonne of CO_2 was imposed for non-compliance.

Achievements of Phase 1:

- Established a carbon pricing mechanism.
- Enabled free trade of emission allowances across EU member states.
- Developed infrastructure for monitoring, reporting, and verifying emissions.

Challenges : Due to the absence of reliable emissions data, caps were based on estimates, leading to an oversupply of allowances. This surplus caused the price of allowances to plummet to zero in 2007, as unused Phase 1 allowances could not be carried over to Phase 2.

Phase 2 (2008–2012) : Aligning with Kyoto Targets

Phase 2 aligned with the first commitment period of the Kyoto Protocol, where EU countries had specific emission reduction obligations.

Key features included:

- Reduced Cap: The overall cap was approximately 6.5% lower than 2005 emission levels.
- Expanded Membership: Iceland, Liechtenstein, and Norway joined the EU ETS.
- Broader Scope: Inclusion of nitrous oxide emissions from nitric acid production in some countries.
- Allowance Allocation: Free allocations slightly reduced to around 90%, with some allowances auctioned.
- Compliance Penalty: Increased to €100 per tonne of CO_2.
- International Credits: Businesses could purchase international credits totaling about 1.4 billion tonnes of CO_2-equivalent.
- Registry Systems: Transitioned to a Union registry and replaced the Community Independent Transaction Log (CITL) with the European Union Transaction Log (EUTL).
- Aviation Sector Inclusion: Aviation was incorporated into the EU ETS starting January 2012, although application to flights to and from non-European countries was temporarily suspended.

Despite adjusting caps based on verified emissions data, the 2008 economic crisis led to greater-than-expected emission reductions. This resulted in a surplus of allowances, exerting downward pressure on carbon prices throughout Phase 2.

Phase 3 (2013–2020) : System Overhaul

Significant reforms were introduced to enhance the effectiveness and efficiency of the EU ETS:

- Single EU-wide Cap: Replaced national caps with a unified cap across all member states.
- Auctioning of Allowances: Auctioning became the default method for allocating allowances, moving away from predominantly free allocations.
- Harmonized Allocation Rules: Standardized rules for the free allocation of allowances to address carbon leakage risks.
- Expanded Coverage: More sectors and greenhouse gases were included.
- Innovation Support: Set aside 300 million allowances in the New Entrants Reserve (NER 300) to fund innovative renewable energy technologies and carbon capture and storage projects.

Market Evolution The EU ETS has been instrumental in developing a robust carbon market:

- Trading Volumes: Increased from 321 million allowances in 2005 to over 7.9 billion in 2012.
- Global Influence: In 2010, EU allowances accounted for 84% of the global carbon market's value.
- Economic Impact: Despite challenges, the EU ETS remained a key driver in the international carbon market, with the total value of allowances traded reaching €56 billion in 2012.

Phase 4 (2021–2030) : Enhancing Ambition

Phase 4 aims to further reduce emissions in line with the EU's climate objectives:

- Allowance Allocation: Approximately 57% of total allowances are designated for auctioning, amounting to about 7.855 billion allowances for the entire phase.
- Adjustments and Funds: Portions of auctionable allowances may be redirected to support free allocation buffers, the Innovation Fund, the Modernisation Fund, and the Social Climate Fund.
- Actual Auction Volumes: Due to these adjustments, around 51.5% of the annual cap is expected to be auctioned between 2021 and 2025.
- Market Stability Reserve (MSR): Continues to regulate the supply of allowances to stabilize the market.
- Member State Derogations: Bulgaria, Hungary, and Romania opted to continue providing free allowances to their energy sectors, deducted from their auction volumes, under Article 10c of the ETS Directive.

Aviation Sector in Phase 4

The aviation sector remains a distinct component:

- Allowance Allocation: 15% of aviation allowances are auctioned, with the cap determined using a bottom-up approach.
- Linear Reduction Factor: Applied to the aviation cap to align with overall emission reduction efforts.
- Phase-Out of Free Allocation: Free allocations to aircraft operators will be phased out by 2026, increasing the auctioned share to support sustainable aviation fuels.
- Between 2024 and 2030, 20 million aviation allowances are reserved to support the adoption of alternative fuels.

Maritime Transport Inclusion Starting in 2024, maritime transport is covered by the EU ETS. This sector inclusion increased the cap by 78.4 million allowances, based on emissions data from 2018 and 2019.

Cap and its Reduction Trajectory

- Between 2024 and 2027, the annual reduction factor for the emissions cap will increase to 4.3%, and from 2028 onward, it will rise to 4.4%.
- A 2023 revision targets a 62% reduction in emissions by 2030, compared to 2005 levels.
- Scheduled reductions in allowances: 90 million allowances in 2024 and 27 million allowances in 2026.
- The EU ETS cap for 2024 is specified as 1,386,051,745 allowances.
- New Entrant Reserve: A reserve of allowances from Phase 3 (2013–2020) is retained to accommodate new entrants.

By systematically identifying vulnerable sectors and adjusting free allowance allocations, the European Commission aims to prevent carbon leakage while encouraging industries to reduce emissions.

2.4 Allocation of Permits

With this groundwork laid, we now arrive at the crux of the ETS introduction: the allocation of emission permits. In the subsequent section, the distribution of permits will take center stage, as the cap established earlier represents the limited resource agents are competing for. This process mirrors the allocation of any scarce resource, underscoring both its value and the complexities in its fair distribution. In our analysis, these agents may be individual firms or entire countries, each with unique stakes in securing their share of this finite resource.

2.4.1 Auctioning

According to: [13] (Commission, Auctioning of allowances) Since 2013, auctioning has been the primary method for distributing emission allowances in the EU ETS, upholding the 'polluter pays' principle by requiring emitters to purchase the right to emit pollutants. Member States conduct these auctions under the EU ETS Directive and Auctioning Regulation, ensuring they are open, transparent, harmonized, and non-discriminatory. These rules also apply to Iceland, Liechtenstein, Norway, and the UK's electricity generation in Northern Ireland.

For the 2021–2030 period, the European Commission has specified auction shares for general and aviation allowances. Up to 57% of general allowances are auctioned, with the remainder allocated for free to mitigate carbon leakage risk and promote low-carbon technologies. Of the auctioned allowances:

- 90% are distributed among Member States based on their historical emissions.
- 10% are allocated to 16 specific Member States to promote solidarity.

From July 2023 to August 2026, a portion of allowances is redirected to the Recovery and Resilience Facility to fund the REPowerEU Plan. In aviation, approximately 15% of allowances have been auctioned, increasing gradually so that most are auctioned from 2026 onward.

Use of Auction Revenues Auctioning generates significant revenues for national budgets and funds like the Innovation Fund and Modernisation Fund. Since 2013, the EU ETS has raised over EUR 200 billion, with EUR 43.6 billion generated in 2023 alone, of which EUR 33 billion was distributed to Member States.

Member States are required to use at least 50% of auction revenues—and all revenues from aviation allowances—for climate and energy-related purposes. Between 2013 and 2020, they reported spending an average of 75% of these revenues on projects such as renewable energy, energy efficiency, and low-emission transport. From mid-2023, all EU ETS revenues (or an equivalent amount) must support the green transition, including measures to address social impacts. Member States are also encouraged to fund the decarbonization of maritime transport and protect marine biodiversity.

Under the Regulation on the Governance of the Energy Union and Climate Action, Member States report annually on how they use EU ETS revenues, which the Commission summarizes in its annual Climate Action Progress Reports.

Auctioning Rules and Platforms The Auctioning Regulation outlines the procedures to ensure auctions are conducted fairly. In October 2023, a new regulation was adopted to reflect updates to the EU ETS and the REPowerEU plan.

Currently, 28 countries—including 25 EU Member States plus Iceland, Liechtenstein, and Norway—use the common auction platform, the European Energy Exchange (EEX) in Leipzig, Germany. Germany, Poland, and the UK (for Northern Ireland) have opted out of the common platform but continue to use EEX for their auctions.

Auction Calendars and Results Auction calendars are set by the auctioning platform in accordance with regulatory requirements, detailing the dates and volumes of allowances to be auctioned per Member State and for specific funds. These calendars and daily auction results are available on the platform's website, with detailed quarterly reports providing additional information on auction outcomes, participants, and revenues.

2.4.2 Benchmark

As described in [20] EU ETS Handbook.

Definition : A benchmark is a reference value for greenhouse gas (GHG) emissions, measured in tonnes of CO_2 (tCO_2) per unit of production activity. It is used to determine the level of free emission allowances each installation within a sector receives under the EU Emissions Trading System (EU ETS).

Purpose : A benchmark is not an emission limit or a reduction target. All installations within a sector receive the same allocation of allowances per unit of activity. Installations with GHG emissions lower than the benchmark may receive more free allowances than they need, potentially allowing them to sell the surplus.

Basis : Benchmarks are set on an output basis whenever possible, considering all GHG emissions from the entire production process for a specific product. In the EU ETS, product benchmarks are based on the average GHG performance of the top 10% most efficient installations producing that product in the EU.

To establish these benchmarks, industry sectors collected GHG emissions data from ETS installations during 2007 and 2008, following Article 10a(2) of the EU ETS Directive. By plotting the specific emissions (emissions per unit of output) of all installations in ascending order, a "benchmarking curve" was created for each sector. The average efficiency of the top 10% best-performing installations was then determined from this curve and set as the benchmark for allocation rules. If insufficient data was available, the best available techniques were used as a starting point to develop the benchmarks.

This algorithm provides a systematic method for establishing benchmarks in the EU ETS, ensuring that free allocation of allowances is based on actual performance data and encourages efficiency within sectors. It rewards installations that are more efficient than the benchmark while incentivizing others to reduce their emissions.

2.4.3 Free Allocagtion & Carbon Leakage

Carbon leakage occurs when businesses relocate production to countries with less stringent emission constraints due to increased costs from climate policies, rendering the climate policies both useless and harmful to the economy at the regulator. This risk is particularly significant in energy-intensive industries. [14]

European Commission's Algorithm to Address Carbon Leakage

To combat carbon leakage, the European Commission implements a systematic approach under the EU Emissions Trading System (EU ETS) [14] [20]

Financial Compensation for Indirect Emissions:

- Member States can offer compensation to electro-intensive sectors for higher electricity costs due to the EU ETS.

Algorithm 1: Algorithm for Establishing Product Benchmarks in the EU ETS

Result: Benchmark value BM for each product
Input: Set of Installations I producing a specific product;
For each installation $i \in I$:
GHG emissions data $E(i)$ over the period 2007–2008;
Production output data $P(i)$ over the same period.

```
for each installation i in I do
    Calculate specific emissions SE(i):
```

$$SE(i) = \frac{E(i)}{P(i)}$$

```
end for
Sort installations I in ascending order of SE(i).
Determine the number of installations representing the top 10% most efficient:
```

$$n_{10\%} = \lceil 0.10 \times |I| \rceil$$

Select the set I_{top} of the $n_{10\%}$ installations with the lowest $SE(i)$.
Calculate the benchmark BM as the average specific emissions of I_{top}:

$$\text{BM} = \frac{1}{n_{10\%}} \sum_{i \in I_{\text{top}}} SE(i)$$

```
if Insufficient data is available for the product then
    Use Best Available Techniques (BAT) to estimate BM.
end if
```

Output: Benchmark value BM for the product, to be used in allocation rules.

- This is governed by EU state aid rules and is limited to 25% of auction revenues, unless justified otherwise.

- Transparency measures require regular publication of compensation amounts.

With the allocation $A(i)$, we proceed to distribute free emission permits. The algorithm provided above is simplified, as a full exploration of the ETS free allocation mechanism lies outside the scope of this thesis. However, it is worth noting that Article 10c of the EU ETS Directive enables certain lower-income Member States to allocate free permits specifically for modernizing their energy sectors. This mechanism supports investment in energy diversification, infrastructure upgrades, and clean technologies, contributing to fairer and more sustainable emissions reductions. Further details can be found in Article 10c and the allocation for Phase IV and how countries responded to their claim for free Allocation is described on Table 2.1. [12]

2.5 Epilogue

This chapter has outlined the fundamental principles and frameworks of cap-and-trade systems, highlighting their importance in addressing global climate change. The next chapters will investigate the fairness of free allocation in the EU Emissions Trading System, analyzing its impact and effectiveness in ensuring equitable emissions reductions.

Algorithm 2: Identifying and Addressing Carbon Leakage under the EU ETS

Result: Allocation Plan of Free Emission Allowances to Sectors
Input: Set of Sectors S;
For each sector $s \in S$: Direct Costs $C_d(s)$, Indirect Costs $C_i(s)$, Gross Value Added GVA(s), Trade Intensity TI(s), Historical Activity Level HAL(s), Benchmark BM(s)

for *each sector* s *in* S **do**
  Calculate cost impact $CI(s)$:

$$CI(s) = \frac{C_d(s) + C_i(s)}{\text{GVA}(s)}$$

  if *(*$CI(s) \geq 5\%$ ***and*** TI$(s) \geq 10\%$*)* ***or*** *(*$CI(s) \geq 30\%$ ***or*** TI$(s) \geq 30\%$*)* **then**
    Mark sector s as **At Significant Risk of Carbon Leakage**
end for
Compile the **Carbon Leakage List** $\mathcal{L}$ with all sectors marked **At Significant Risk**.
for *each installation* i *in sector* s **do**
  Determine the applicable benchmark BM(i) based on the product produced. algorithm 1;
  Calculate the Historical Activity Level HAL(i) as the median production in a baseline period (e.g., 2005–2008 or 2009–2010).
  Calculate allocation $A(i)$ using:

$$A(i) = \text{BM}(i) \times \text{HAL}(i) \times \text{CLEF}(s) \times \text{CF}$$

  where:
    CLEF(s) is the Carbon Leakage Exposure Factor:
      if s *is* ***At Significant Risk*** **then**
    CLEF$(s) = 100\%$
  else
    CLEF(s) decreases from 80% in 2013 to 30% in 2020.
  end if
    CF is the Correction Factor:
      if i *is a non-electricity generator* **then**
    CF = Cross-Sectoral Correction Factor (CSCF), ensuring total allocation stays within limits.
  else if i *is an electricity generator* **then**
    CF = Linear Reduction Factor (LRF), in line with emission reduction targets.
  end if
end for
for *Phase 4 (2021–2030)* **do**
  Update the Carbon Leakage List using refined criteria based on trade and emissions intensity.
  for *each sector* s *in* S **do**
    if s *is* ***Highly Exposed*** **then**
      Continue allocating allowances at 100% of the benchmark.
    else if s *is* ***Less Exposed*** **then**
      Allocate allowances at 30% until 2026, then phase out by 2030.
    end if
  end for
end for
Output: Detailed allocation plan $A(s)$ for all sectors to mitigate the risk of carbon leakage.

Eligible Member States	Maximum Article 10c derogation (40% of regular allowances)	Amount to be used under Article 10c	Amount transferred from Article 10c to the Modernisation Fund	Amount to be auctioned
Bulgaria	51,599,838	51,599,838	0	0
Czechia	111,462,281	0	111,462,281	0
Estonia	17,583,702	0	0	17,583,702
Croatia	11,957,703	0	5,978,852	5,978,851
Latvia	3,794,677	0	0	3,794,677
Lithuania	8,696,818	0	8,696,818	0
Hungary	34,610,750	20,748,000	0	13,862,750
Poland	273,211,665	0	0	273,211,665
Romania	91,673,704	5,600,000	86,073,704	0
Slovakia	33,228,414	0	33,228,414	0
Total	637,819,552	77,947,838	245,440,068	314,431,646

Table 2.1: Allocation of allowances under Article 10c for the modernization of the energy sector in eligible Member States in Phase IV.

Chapter 3

Fair Distribution - Fair Division

3.1 Introduction

Fair division is the problem of dividing one or several goods among two or more agents in a way that satisfies a suitable fairness criterion. It is a fundamental issue in economics, mathematics, and computer science, and is part of the larger research area of multiagent resource allocation [9]. The goal is to find an allocation that is considered fair by all participants, despite their differing preferences.

Fair division studies the allocation of scarce resources among interested agents, with the objective of finding an allocation that is fair to all participants involved [4]. However, fairness is a complex and multifaceted concept, making it challenging to define precisely. Different fairness criteria may be appropriate in different contexts, and what is considered fair by one agent may not be viewed the same way by another.

In this chapter, we will explore the fundamental concepts of fair division, including definitions, types of goods, preference representations, and fairness criteria. We aim to provide a structured understanding of how resources can be allocated fairly among agents with varying preferences.

3.2 Definition

To understand fair division comprehensively, we need to break down its components:

> *Fair division is the act of dividing or distributing something among agents who desire it, using an algorithm that adheres to a specified fairness criterion.*

We need to define the following elements to make this statement meaningful:

1. **Who is dividing or distributing?**
2. **Who are the recipients?**
3. **What is being distributed?**
4. **How can we define the agents' preferences?**

5. **What constitutes a fairness criterion?**

3.2.1 Who is Dividing or Distributing?

The division can be performed by a central authority, such as a benevolent dictator, or through a collaborative process involving all agents. The method of division influences the fairness perception of the allocation. In some cases, an impartial mediator may oversee the process to ensure adherence to fairness principles.

3.2.2 Who are the Recipients?

Let $N = \{1, \dots, n\}$ be a finite set of agents (or players). These agents are participants in the fair division problem and must find a way to divide or distribute resources among themselves. Agents can represent individuals, organizations, or any entities with preferences over the resources.

3.2.3 What is Being Distributed? (Types of Goods)

The resources to be divided can take various forms, from tangible items like land and commodities to intangible assets like intellectual property or broadcast rights. According to [9], the main types of resources (or goods, items, objects, commodities) are:

- **Continuous vs. Discrete**: A resource may be continuous (e.g., energy) or discrete (e.g., individual fruits). Continuous resources are typically divisible into any fraction, while discrete resources are indivisible units.

- **Divisible vs. Indivisible**: Divisibility depends on the allocation mechanism. Divisible goods can be shared among agents in any proportion, whereas indivisible goods cannot be split without losing their value.

- **Sharable vs. Non-Sharable**: Sharable resources can be allocated to multiple agents simultaneously without diminishing in value (e.g., digital goods like software licenses). Non-sharable resources cannot be used by more than one agent at the same time.

- **Static vs. Dynamic**: Static resources maintain the same properties during the allocation process, while dynamic resources can lose value, spoil, or change properties over time (e.g., perishable goods).

- **Single-Unit vs. Multi-Unit**: In multi-unit settings, identical resources (e.g., bottles of champagne) are grouped together, whereas in single-unit settings, each resource is unique and distinguishable. Multi-unit settings allow for more compact representations of allocations and agent preferences.

- **Resources vs. Tasks**: Task allocation is a form of resource allocation where tasks are treated as resources with associated costs or negative utilities. Tasks may involve additional constraints such as dependencies or sequencing, adding complexity to the allocation problem.

To this list we must add another categorization.

- **Homogenous vs. Heterogeneous**: Resources may be identical and interchangeable (homogeneous) or differ in characteristics, desirability, or utility to the agents (heterogeneous). For example, a cake with varying toppings represents a heterogeneous resource, as different parts may hold different values for different agents.

3.2.4 Preference Representation

Understanding agents' preferences is crucial in fair division. A preference structure models an agent's preferences over alternatives X. The main types are:

- **Cardinal Preferences**: Represented by a utility function $u : X \to \mathbb{R}$, assigning a numerical value to each alternative, indicating the level of satisfaction.
- **Ordinal Preferences**: Represented by a binary relation $\succsim$, where $x \succsim y$ means the agent prefers x at least as much as y. This defines strict preference ($x \succ y$) and indifference ($x \sim y$).
- **Binary Preferences**: A partition of X into "acceptable" and "unacceptable" alternatives, indicating whether an option meets the agent's minimal requirements.
- **Fuzzy Preferences**: A function $\mu : X \times X \to [0, 1]$, indicating the degree of preference of one alternative over another. Fuzzy preferences generalize both ordinal and cardinal structures, allowing for more nuanced preference representation.

3.2.5 Fairness Criteria

Fairness criteria establish the standards by which allocations are deemed equitable among agents. These criteria not only ensure that allocations meet certain ethical standards but also interact with efficiency concepts like Pareto optimality and social welfare orderings. Common fairness criteria include:

- **Envy-Freeness**: An allocation is envy-free if no agent prefers another agent's allocation over their own. Formally, for all agents i and j,

  $$u_i(A_i) \geq u_i(A_j),$$

  where A_i is the allocation to agent i. This criterion aligns with the notion of individual rationality and complements Pareto efficiency by ensuring that no agent has a justified complaint against another's allocation, thereby promoting a sense of fairness and satisfaction among all participants.

- **Proportionality**: Proportionality requires that each agent receives at least $\frac{1}{n}$ of the total value according to their own valuation. Formally,

$$u_i(A_i) \geq \frac{1}{n} u_i(T),$$

where T is the total set of resources and n is the number of agents. This criterion ensures a basic level of fairness by guaranteeing that every agent receives a fair share relative to the total available resources, preventing scenarios where some agents receive disproportionately less.

- **Equitability**: An allocation is equitable if all agents derive equal utility from their respective allocations. Formally, for all agents i and j,

$$u_i(A_i) = u_j(A_j).$$

Equitability emphasizes fairness by striving for uniform satisfaction levels across all agents. While it promotes equality, it may sometimes conflict with efficiency, as ensuring equal utilities might prevent reaching Pareto optimal allocations.

- **Maximin Share Guarantee**: This criterion ensures that each agent receives a share that is at least as good as the minimum value they could guarantee for themselves by partitioning the goods into n shares and receiving the least valuable share. Formally, for each agent i,

$$u_i(A_i) \geq \text{MMS}_i,$$

where MMS_i is the maximin share for agent i. The maximin share guarantee balances individual guarantees with overall fairness, ensuring that no agent is left with an allocation worse than a certain threshold, thereby enhancing collective welfare without necessarily compromising efficiency.

- **Pareto Efficiency**: An allocation is Pareto efficient if there is no other feasible allocation that can make at least one agent better off without making any other agent worse off. Formally, an allocation A is Pareto efficient if there does not exist another allocation A' such that,

$$u_i(A') \geq u_i(A) \quad \forall i \in N,$$

with at least one strict inequality. Pareto efficiency ensures that resources are allocated in a way that no further mutual gains are possible, complementing fairness by maximizing overall welfare without disadvantaging any agent.

Selecting an appropriate fairness criterion depends on the specific context and objectives of the allocation process.

In subsequent sections, we will explore additional fairness criteria and examine their interactions with various social welfare orderings and collective utility functions, providing a comprehensive framework for evaluating and designing fair allocations.

3.3 Moulin's Framework of Distributive Justice

Hervé Moulin, in his work [29], builds upon Aristotle's maxim: *"Equals should be treated equally, and unequals unequally, in proportion to the relevant similarities and differences."* He uses this principle to define **Horizontal Equity**, which emphasizes treating individuals with similar relevant characteristics equally.

Moulin's framework identifies four key principles of distributive justice that guide the fair allocation of resources:

1. **Compensation**: Involuntary differences in individual characteristics, such as disabilities or socio-economic disadvantages, justify unequal shares of a resource to level the playing field. This principle holds that inequalities arising from factors beyond an individual's control should be addressed through redistribution to achieve equity.

2. **Reward**: Voluntary differences, such as effort, skill, or productivity, are rewarded, leading to unequal resource distribution. This principle asserts that fairness requires acknowledging and rewarding the contributions or choices that individuals actively make, encouraging productivity and recognizing individual responsibility.

3. **Exogenous Rights**: Resources are allocated based on rights or claims that are independent of individual characteristics. Certain rights are universal and unrelated to personal effort or circumstances, such as the right to vote or freedom of speech. This principle ensures that fundamental rights are protected and that allocations respect these inherent entitlements.

4. **Fitness**: Resources are allocated to those who can make the best use of them, maximizing utility or outcomes. This principle focuses on efficiency as a core consideration in determining fairness, promoting societal benefit by encouraging the efficient use of resources.

3.3.1 Illustrative Example: The Lifeboat Dilemma

To demonstrate the application of Moulin's distributive justice principles in real-world scenarios, consider the following lifeboat dilemma:

[Lifeboat Dilemma] Imagine a situation where a ship is sinking, and there is only one lifeboat available that cannot accommodate all passengers. The decision-maker, acting as a benevolent dictator, must determine who gets to board the lifeboat and who does not. This scenario is a quintessential example of resource allocation under severe constraints. Similar critical situations include medical triage during emergencies, allocation of scarce organs for transplantation, and setting immigration policies under limited capacity.

Different allocation strategies based on Moulin's principles would approach this dilemma as follows:

- **Exogenous Rights**: Applying strict equality, the lifeboat seats could be allocated randomly through a lottery system. This ensures that every individual has an equal probability of being selected, regardless of their personal attributes or circumstances.
- **Compensation**: This approach might allow physically stronger individuals to attempt to swim to safety, while those deemed weaker, such as women and children, remain in the lifeboat. The aim is to balance the chances of survival by compensating for inherent physical disparities.
- **Reward**: Here, the focus would be on accountability. Individuals responsible for causing the ship to sink, such as those whose actions led to the emergency, might be excluded from boarding the lifeboat as a form of punishment, thereby rewarding those who are not at fault.
- **Fitness**: Allocation based on fitness would prioritize individuals who can contribute most effectively to the group's survival or future well-being. This could mean giving priority to the crew members with essential skills for navigation or selecting women and children due to their potential to contribute to future generations.

This lifeboat scenario highlights the inherent tensions between the four principles of distributive justice. While the **Exogenous Rights** principle emphasizes equal opportunity, the **Compensation** principle seeks to address and rectify inherent inequalities. The **Reward** principle introduces considerations of accountability and merit, and the **Fitness** principle focuses on maximizing overall utility and future benefits. Balancing these principles requires careful ethical consideration, as each principle may lead to different allocation outcomes. In practice, a benevolent dictator must weigh these competing principles to arrive at a decision that strives to be both fair and ethically justifiable.

3.4 Application of Moulin's Principles in Cap and Trade Systems

3.4.1 Introduction

As we've established in the previous chapter, CAT systems rely on the free allocation of permits to combat carbon leakage [14]. The permits are another resource that needs to be distributed among agents (Firms) who desire it by a Regulator using an algorithm that adheres to a specified fairness criterion.

characterization of our problem

Permits can be characterized as follows:

- **Continuous and Divisible**. This is not accurate for obvious reasons. one permit represents emission rights to one tone of CO_2, this is the quantum of our system. On the other hand, one tone is such a small number compared to the actual numbers in question, that it can be considered continuous in our study.

- **Homogenuous**. Our permits are identical to one another.
- **Non Sharable, Static**. It cannot be allocated to multiple agents nor does it change states.
- **Multi-Unit**. Permits are grouped together as ”permits” or ”free allocation” and regarded as one thing.

The initial allocation of European Union Allowances (EUAs) plays a crucial role in the stability and efficiency of the Emissions Trading System (ETS), as it determines the overall scarcity in the market [50]. A significant challenge in allocating emission allowances is establishing an allocation principle that appropriately shares the responsibility of CO_2 emissions among different member states. Equity in the distribution process is vital because the unfair allocation of free allowances can negatively impact the economic development of countries.

Over the years, numerous studies have advocated a variety of allocation criteria for emissions allowances, which can be summarized into two main principles: (1) fairness in terms of distributive justice, and (2) economic efficiency in terms of minimizing abatement costs [54]. Focusing on the first phase of the EU ETS, [11] argues that the allocation was unfair and proposes a method for the equitable reallocation of emission permits among member states. [22] propose axioms based on population, historical emissions, and business-as-usual emissions to establish equal-per-capita allowance allocation rules that favor developing countries with large populations over developed countries with substantial historical emissions. Aiming to balance economic activity and the production of renewable energy, [28] introduce a Double-Weighted Constrained Equal Awards Rule to allocate emission allowances and investigate similarities in the resulting allocation using an unsupervised clustering approach. Examining China’s Emission Trading System, [37] propose a multi-criteria model that balances equity and efficiency in allocating carbon allowances.

3.4.2 Alignment of Existing Allocation Principles with Moulin’s Framework

Many allocation principles have been proposed in the literature to guide the distribution of emission allowances. These principles can be aligned with Moulin’s four key principles of distributive justice: *Compensation*, *Reward*, *Exogenous Rights*, and *Fitness*. In this section, we expand on existing allocation criteria and indicators by incorporating Moulin’s principles into the framework.

In their review [54], collect and present all the Allocation Criteria and Indicators used in the literature to allocate permits; we extend the tables they present to include the Fairness principle they adhere to with respect to Moulin’s framework. Table 3.1 presents the main allocation criteria, their interpretations, operational rules, and alignment with Moulin’s principles. The first 3 columns are from table 1 of [54].

Similarly, Table 3.2 lists the main indicators used for CO_2 emissions allocation, their associated allocation criteria, operational rules, and alignment with Moulin’s principles.

These tables illustrate how existing allocation principles and indicators align with Moulin’s framework. The *Exogenous Rights* principle corresponds to criteria that distribute permits equally based on inherent rights, such as population. The *Compensation* principle aligns with criteria that adjust allocations based on economic well-being, aiming to level the playing field. The *Reward* principle corresponds to criteria that allocate based on historical responsibility or efficiency, rewarding or penalizing past actions.

3.4.3 Interpretation of Moulin's Principles in Cap and Trade Systems

Moulin's principles of distributive justice can be directly interpreted and applied within Cap and Trade (CAT) systems to guide the allocation of emission allowances among participants.

Compensation: This principle suggests that entities with involuntary disadvantages, such as lower economic capacity or higher abatement costs due to structural factors, should receive more allowances to compensate for these factors beyond their control. In a CAT system, this could translate to allocating more allowances to less developed countries or industries facing higher costs due to technological limitations, thereby promoting equity and facilitating their participation in emission reduction efforts.

Reward: Under the Reward principle, entities that have made voluntary efforts to reduce emissions or have historically maintained low emission levels should be acknowledged and potentially receive fewer allowances, reflecting their lower need for emission rights. Conversely, entities with high historical emissions may receive fewer allowances, incentivizing them to reduce emissions. This aligns with the "Polluter Pays" concept, promoting accountability and encouraging proactive environmental practices.

Exogenous Rights: This principle emphasizes equal rights to resources, irrespective of individual characteristics. In a CAT system, this could manifest as allocating emission allowances equally on a per capita basis, reflecting the notion that all individuals have an equal right to the atmosphere's absorptive capacity. Such an approach promotes fairness by ensuring that each participant starts with the same allocation, regardless of their economic status or historical emissions.

Fitness: The Fitness principle focuses on allocating resources to those who can make the best use of them, maximizing overall utility or outcomes. In the context of a CAT system, this could involve allocating more allowances to sectors or entities that can achieve greater emission reductions at lower costs, thereby enhancing the system's overall efficiency. This approach promotes cost-effectiveness and ensures that emission reductions are achieved where they are most economical.

By interpreting Moulin's principles within CAT systems, policymakers can design allocation mechanisms that balance equity and efficiency. For instance, a hybrid allocation method could combine per capita allocations (*Exogenous Rights*) with adjustments based on economic capacity (*Compensation*) and historical emissions (*Reward*). Additionally, incorporating efficiency considerations (*Fitness*) can ensure that the system not only distributes allowances fairly but also achieves emission reductions in a cost-effective manner. In practice, implementing these principles requires careful consideration of the specific context and the potential trade-offs between fairness and efficiency.

Table 3.1: Main Allocation Criteria, as per table 1 of [54], Aligned with Moulin's Principles

Criterion	Interpretation	Operational Rule	Moulin's Principle
Sovereignty / Grandfathering	All nations (firms) have equal rights to pollute and to be protected from pollution	Distribute permits in proportion to historical emissions or energy consumption	Exogenous Rights
Egalitarianism	All people have equal rights to pollute and to be protected from pollution	Distribute permits in proportion to population	Exogenous Rights
Ability to Pay	Mitigation costs vary directly with national economic well-being	Distribute reductions inversely to GDP or per capita GDP	Compensation
Economic Activity	All nations should maintain their standard of living	Distribute permits in proportion to GDP	Compensation
Horizontal Equity	All countries should face equal welfare changes as a share of GDP	Distribute permits to equalize net welfare change (net loss as a proportion of GDP equal for each nation)	Compensation
Vertical Equity	Welfare gains vary inversely with economic well-being, and welfare losses vary directly with GDP	Progressively distribute permits inversely/directly correlated with per capita GDP	Compensation
Polluter Pays / Historical Responsibility	Nations with more historical emissions need to take greater abatement burdens	Distribute reductions in proportion to cumulative emissions	Reward
Merit (Efficiency)	Nations should be compensated for prior emission reduction efforts	Distribute reductions inversely to emissions per unit of GDP or production intensity	Reward

Table 3.2: Main Indicators and Allocation Rules, as per table 2 [54], Aligned with Moulin's Principles

Indicator	Allocation Criterion	Allocation Rule	Moulin's Principle
Population	Egalitarianism	Equal per capita permits	Exogenous Rights
		Equal adult per capita permits	Exogenous Rights
		Equal per capita permits with discounted historical responsibility	Exogenous Rights and Reward
		Equal past and future per capita permits	Exogenous Rights
		Equal per capita permits by Contraction and Convergence (C&C)	Exogenous Rights and Fitness
		Equal per capita permits by Common but Differentiated Convergence (CDC)	Exogenous Rights and Fitness
Emission	Sovereignty / Grandfathering;	Proportional permits to historical emissions (country/firm)	Reward
	Polluter Pays	Proportional reductions to historical emissions	Reward
	Historical Responsibility	Proportional reductions to cumulative emissions	Reward
Energy	Sovereignty / Grandfathering	Proportional permits to energy consumption	Compensation or Exogenous Rights
		Proportional permits to energy production	Reward
GDP	Economic Activity	Proportional permits to GDP	Fitness
	Ability to Pay	Proportional reductions to GDP	Fitness
	Horizontal Equity	Equal net abatement cost to GDP	Compensation
Per Capita GDP	Ability to Pay	Proportional reductions to per capita GDP	Reward (punishment) or Fitness
	Vertical Equity	Equal net abatement cost to per capita GDP	Compensation
Emission Intensity	Merit (Efficiency)	Proportional reductions to emissions per unit of GDP	Reward or Fitness
		Proportional reductions to emissions per unit of GDP under C&C	Reward or Fitness
		Proportional permits to emissions per unit of production outputs (Benchmarking)	Reward or Fitness

Chapter 4

Exploring Horizontal Equity and the data available on the EU ETS

4.1 Introduction

Fairness in the allocation of free allowances under the European Union Emissions Trading System (EU ETS) is critical to its effectiveness and acceptance among member countries. This chapter presents three experiments that explore different dimensions of fairness using the same dataset of economic and energy attributes of EU member countries.

1. **Experiment 1**: Investigates the relationship between the similarity of countries' profiles and the similarity of their free allowance allocations by analyzing all pairwise combinations of countries.

2. **Experiment 2**: Examines whether using the median country as a reference point improves the explanation of free allowance allocations.

3. **Experiment 3**: Explores the extent to which optimal linear combinations of features can explain the allocations, identifying countries that cannot explain the rest even when overfitting is allowed.

4.1.1 Data Collection

The dataset of the following experiments is common. The dataset includes economic and energy attributes for EU member countries from 2005 to 2020. The indicators used are listed in Table 5.1, sourced from the World Bank and Eurostat. The data used are presented in this table, but can also be found in the Appendix more descriptively.

Here we can also see the correlation of the data (this is from 2010, but the grand picture remains the same).

4.1.2 Correlation Analysis

The following experiments, investigate the relationship between 2 values. To accomplish this, tools measuring the existence of a correlation between two values are needed. In the upcoming

Table 4.1: List of Indicators along with the Allocation Principles of [54] (Zhou & Wang 2016), aligned with Mulin's principles as presented in Table 3.2

Indicators	**Principle**	**Data Source**	**Moulin's Principle**
Population	Fairness	`https://data.worldbank.org/indicator/SP.POP.TOTL`	Exogenous Rights and possibly Reward
GDP per capita	Fairness	`https://data.worldbank.org/indicator/NY.GDP.PCAP.CD`	Fitness and Compensation
Inflation	Fairness	`https://data.worldbank.org/indicator/FP.CPI.TOTL.ZG`	Compensation or Fitness
Agriculture	Fairness	`http://wdi.worldbank.org/table/4.2#`	Fitness or Compensation
Industry	Fairness	`http://wdi.worldbank.org/table/4.2#`	Reward or Fitness or Compensation
Manufacturing	Fairness	`http://wdi.worldbank.org/table/4.2#`	Reward or Fitness or Compensation
Total Energy Supply	Fairness	`https://ec.europa.eu/eurostat/databrowser/view/nrg_bal_s/`	Reward or Fitness
Energy Intensity	Economic Efficiency	`https://ec.europa.eu/eurostat/databrowser/view/NRG_IND_EI`	Reward or Fitness
Verified Emissions	Fairness	`https://www.eea.europa.eu/data-and-maps/dashboards/emissions-trading-viewer-1`	Reward

experiments, we will primarily present the R^2 value as the key indicator of linearity. Additional metrics—including the Pearson correlation coefficient, p-values from regression analysis, and error metrics such as Mean Squared Error (MSE)—that further support or challenge the linearity assumption will be provided in the Appendix for comprehensive reference.

4.2 Experiment 1: Analyzing Pairwise Similarities Among Countries

Objective To determine whether countries with similar economic and energy attributes receive comparable levels of free allowances by analyzing all possible pairwise combinations of countries, and to assess whether this relationship varies across different phases of the EU ETS. This experiment aligns with the "Horizontal Equity" discussed in section 3.3

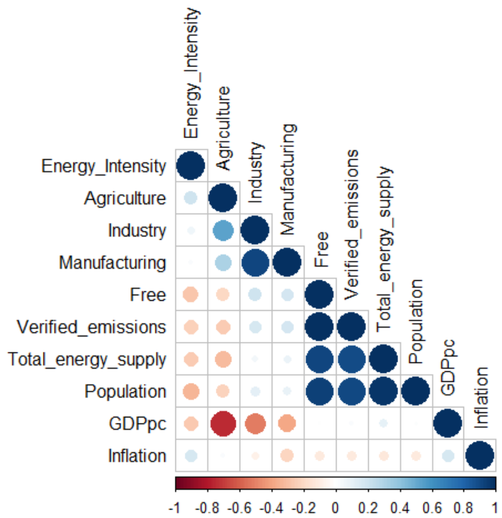


Figure 4.1: Correlation Matrix of the Data for 2010

Hypothesis There is a positive correlation between the Euclidean distances of countries' attribute profiles and the differences in their free allowance allocations across all pairs of countries. Specifically, countries with similar attributes should have similar allocations. We expect that the strength and nature of this correlation may differ depending on the phase of the EU ETS being examined, due to changes in allocation methodologies and regulatory adjustments over time.

4.2.1 Methodology

Data Segmentation by ETS Phases The EU ETS has been implemented in distinct phases, each characterized by different allocation rules and market conditions:

- **Phase I (2005-2007)**
- **Phase II (2008-2012)**
- **Phase III (2013-2020)**

For this experiment, we divide the dataset according to these phases and perform the analysis separately for each phase to investigate potential differences in the relationship between countries' attribute similarities and allocation differences.

Data Normalization Normalize all attributes to ensure comparability across different scales. This is achieved by dividing each attribute by its maximum value within each phase to account for temporal changes in the data.

Calculate Pairwise Distances

- **Attribute Distance ($D_{x_{ij}}$)**: For every pair of countries i and j, calculate the Euclidean distance between their attribute vectors $\vec{x_i}$ and $\vec{x_j}$.

$$D_{x_{ij}} = \sqrt{\sum_{k=1}^{n} (x_{ik} - x_{jk})^2}$$

- **Allocation Difference ($D_{Y_{ij}}$)**: Calculate the absolute difference between their free allowance allocation Y_i and Y_j.

$$D_{Y_{ij}} = |Y_i - Y_j|$$

Correlation Analysis

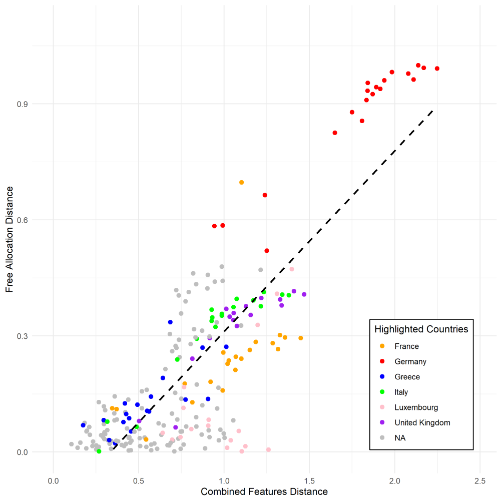
- **Scatter Plot**: Plot $D_{x_{ij}}$ against $D_{Y_{ij}}$ $\forall$ country pairs.
- **Statistical Analysis**: Compute r^2 value of the linear regression to assess the strength of the relationship.

4.2.2 Results and Analysis

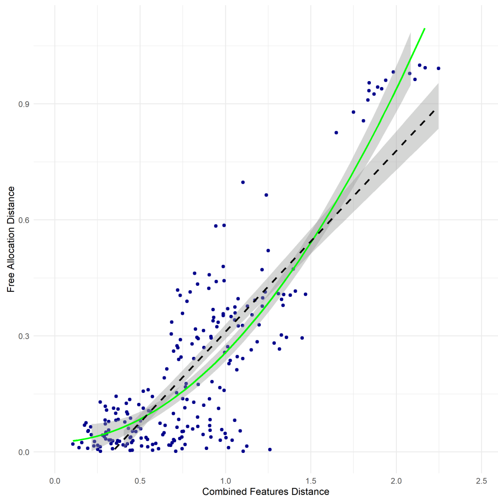
Table 4.2: Analytic data for the linear regression of experiment section 4.2

year	pearson cor.n	spearman cor.	kendall tau	linear p value	linear r^2	mse	rmse	mae	quad p value	quad coeff
2005	0.84	0.71	0.53	0	0.71	0.02	0.13	0.11	0	0.22
2006	0.84	0.69	0.52	0	0.70	0.02	0.13	0.11	0	0.23
2007	0.79	0.65	0.48	0	0.62	0.02	0.15	0.12	0	0.25
2008	0.80	0.69	0.51	0	0.64	0.02	0.15	0.12	0	0.19
2009	0.76	0.62	0.46	0	0.58	0.03	0.16	0.13	0	0.22
2010	0.74	0.59	0.43	0	0.55	0.03	0.16	0.13	0	0.22
2011	0.79	0.64	0.47	0	0.62	0.02	0.15	0.13	0	0.25
2012	0.79	0.64	0.46	0	0.63	0.02	0.15	0.13	0	0.27
2013	0.77	0.60	0.44	0	0.59	0.02	0.15	0.12	0	0.30
2014	0.77	0.59	0.44	0	0.59	0.02	0.15	0.12	0	0.30
2015	0.80	0.62	0.47	0	0.64	0.02	0.14	0.11	0	0.28
2016	0.77	0.59	0.44	0	0.60	0.02	0.15	0.11	0	0.27
2017	0.79	0.60	0.44	0	0.62	0.02	0.14	0.11	0	0.30
2018	0.81	0.63	0.47	0	0.65	0.02	0.14	0.11	0	0.28
2019	0.81	0.63	0.47	0	0.65	0.02	0.14	0.11	0	0.28
2020	0.77	0.57	0.42	0	0.59	0.02	0.15	0.12	0	0.32

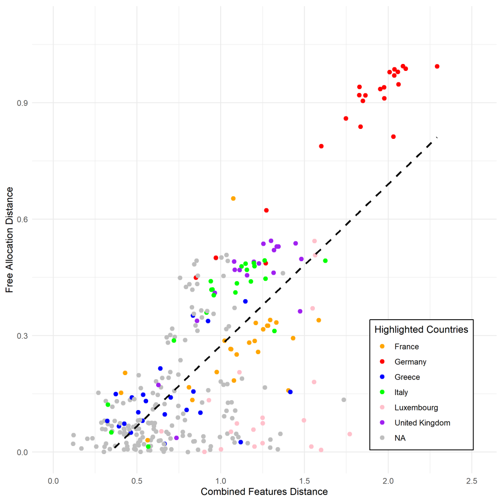
Figure 4.2: Experiment 1 Distances from all the countries to all the others



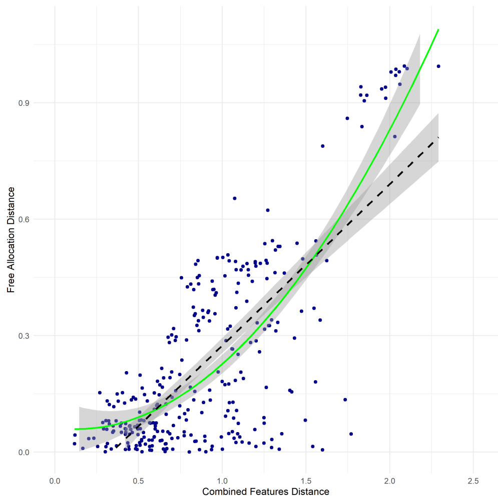
(a) Phase I: 2006



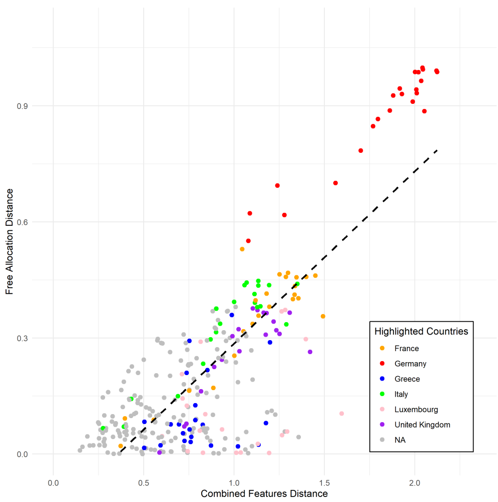
(b) Phase I: 2006



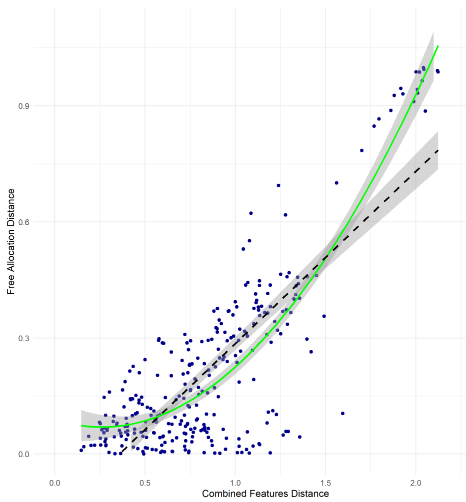
(c) Phase II: 2010



(d) Phase II: 2010



(e) Phase III: 2018



(f) Phase III: 2018

The findings from Experiment 1 provide compelling insights into the relationship between economic and energy attribute similarities and free allowance allocations across EU member countries. The results confirm our initial hypothesis, but also reveal intriguing patterns and variations across different phases of the EU ETS, especially as the allocation methodology evolved.

Key Observations

1. **Positive Correlation between Similarity and Allocation**: The analysis shows a notable correlation between countries' profile similarities (measured by Euclidean distance) and the similarity in their free allowance allocations. This finding supports our hypothesis that countries with comparable economic and energy profiles tend to receive similar levels of allowances, aligning with fairness in allocation.

2. **Impact of ETS Phase on Correlation**: As expected, the strength of this correlation diminishes slightly in Phase III (2013-2020), reflecting changes in allocation methods, including the gradual phase-out of the grandfathering principle. The decline is mostly apparent through the values of Pearson Correlation coefficient, Spearman correlation coefficient, r^2, and the prize of the Quadratic coefficient, which can be found on Table 4.2 This decline suggests that adjustments in the regulatory framework influenced the predictability of allocation based on attribute similarity.

3. **Country-Specific Variations**: Analysis of individual countries reveals parallel trends with differing y-intercepts, suggesting that country-specific factors influence baseline allocations, even among countries with similar profiles. Some of the most influential countries are displayed in different colors.

4. **Quadratic Fit and Country-Based Lines**: A quadratic model appears better to capture the data patterns than a simple linear approximation. However, closer inspection reveals that this improvement may be due to distinct clusters of country-specific lines rather than a single, overarching quadratic relationship. This insight motivates the design of our next experiment, which investigates whether using a single "median" country as a reference point could refine our understanding of allocation consistency across the dataset.

4.3 Experiment 2: Using the Median Country as a Reference Point

Objective To investigate whether using the median country as a reference improves the explanation of free allowance allocations, thereby assessing the fairness of allocations relative to a central benchmark. This experiment still aligns mainly with the broad idea of "Horizontal Equity" discussed in section 3.3

Hypothesis The Euclidean distance of each country's attribute profile from the median country's profile is positively correlated with the difference in their free allowance allocations from that of the median country.

4.3.1 Methodology

Algorithm 3: FIND MEDIAN COUNTRY (Ρεαλιστικά αυτό δε λέει τίποτα, αλλά πρόσφατα κάποιο PAPER εγραφε τόσο και πιο αυτιστικά έναν αλγόειθμο, οπότε αποφάσισα να το κάνω και εγώ)

Result: The median Country
Input: The Attributes $\vec{X}$

```
Initialize rank accumulation vector R with R(c) = 0 for each country c.
for each attribute A_i in X do
    Sort countries c by A_i, assigning ranks r(c, A_i) from 1 to |C|.
end for
for each country c do
    Update cumulative rank: R(c) ← R(c) + r(c, A_i)
end for
Sort countries by R(c) in ascending order. Define the median country c_median as:
```

$$c_{\text{median}} = \text{sorted}(R)\left[\frac{|C|}{2}\right]$$

Output c_{median} to find its $\overrightarrow{X_{mid}}$.

Determine the median country Use algorithm 3 to identify the median country based on cumulative attribute rankings.

Calculating the Distances from the median Country

- **Attribute Distance** (D_{X_i}): Calculate the Euclidean distance between each country's attribute vector $\vec{x_i}$ and the median country's attribute vector $\overrightarrow{x_{median}}$.

$$D_{x_i} = \sqrt{\sum_{k=1}^{n} x_{ik} - x_{median,k}}$$

- **Allocation Difference** (D_{Y_i}): Calculate the absolute difference between each country's free allocation Y_i and that of the median country Y_{median}.

$$D_{Y_i} = |Y_i - Y_{median}|$$

4.3.2 Results and Analysis

Table 4.4: Analytic data for the linear regression of experiment section 4.3

Mid country	year	pearson cor.n	spearman cor.	kendall tau	linear p value	linear r^2	mse	rmse	mae	quad p value	quad coeff
Sweden	2005	0.76	0.42	0.28	0	0.58	0.02	0.15	0.12	0.00	0.39
Estonia	2006	0.87	0.61	0.45	0	0.76	0.01	0.11	0.09	0.01	0.24
Estonia	2007	0.81	0.52	0.36	0	0.66	0.02	0.13	0.10	0.01	0.30
Lithuania	2008	0.85	0.67	0.50	0	0.72	0.01	0.12	0.09	0.05	0.20
Sweden	2009	0.73	0.50	0.36	0	0.53	0.02	0.15	0.11	0.04	0.30
Bulgaria	2010	0.71	0.53	0.35	0	0.51	0.02	0.14	0.10	0.03	0.31
Austria	2011	0.78	0.61	0.45	0	0.61	0.02	0.13	0.11	0.00	0.35
Sweden	2012	0.75	0.45	0.34	0	0.56	0.02	0.14	0.11	0.00	0.46
Estonia	2013	0.71	0.46	0.34	0	0.51	0.03	0.16	0.12	0.01	0.42
Bulgaria	2014	0.76	0.52	0.39	0	0.58	0.02	0.14	0.11	0.01	0.33
Ireland	2015	0.79	0.31	0.23	0	0.62	0.02	0.13	0.11	0.00	0.76
Hungary	2016	0.76	0.47	0.36	0	0.57	0.02	0.14	0.10	0.02	0.29
Austria	2017	0.69	0.37	0.30	0	0.48	0.02	0.13	0.09	0.00	0.43
United Kingdom	2018	0.76	0.67	0.49	0	0.57	0.01	0.09	0.06	0.33	-0.23
Austria	2019	0.75	0.43	0.33	0	0.56	0.01	0.11	0.08	0.00	0.39
Bulgaria	2020	0.75	0.47	0.36	0	0.56	0.02	0.14	0.11	0.01	0.34

Key Observations

1. **Limited Explanatory Power of the Median Country**: The median country does not serve as an effective benchmark for explaining free allowance allocations across other countries, indicating that central tendencies alone may not capture the nuances in allocations.

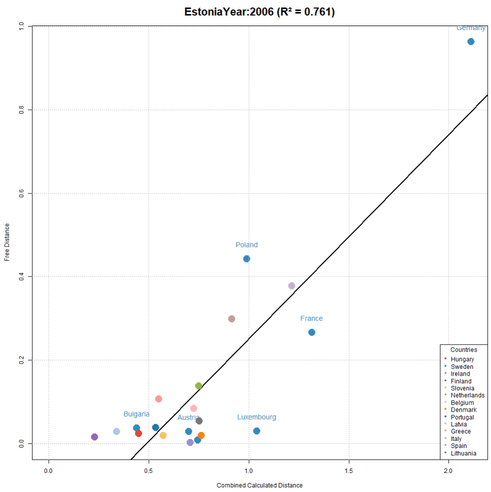
2. **Potential Explanatory Role of Specific Countries**: Certain countries, like Germany, display inherent differences that make them potentially better reference points for explaining allocations in other countries. This suggests that specific national profiles might be more representative than a simple median.

3. **Phase-Independence of Results**: The weak correlation observed in this experiment appears largely unaffected by the phase of the EU ETS, suggesting that these poor results are consistent across different regulatory periods.

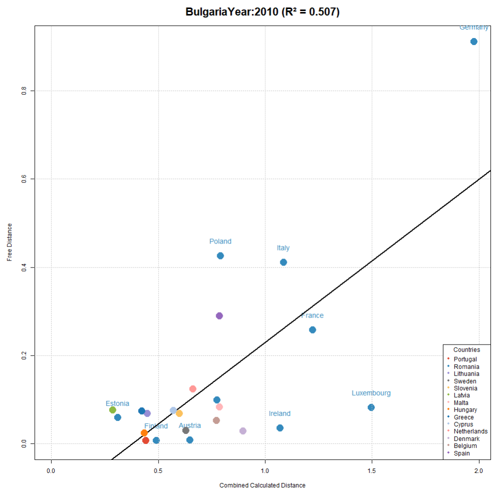
4.4 Experiment 3: Optimal Feature Weights

Objective To find the best linear combination of attributes (allowing for potential overfitting) for each country to assess whether some countries cannot explain the allocations of others, regardless of the model used. This experiment questions essentially the ”relevant” in Aristotle’s ”Horizontal Equity” as discuss in section 3.3

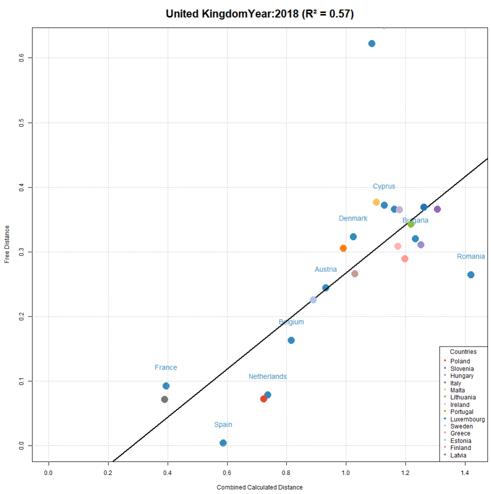
Figure 4.3: Experiment 2 Distances from from the median country



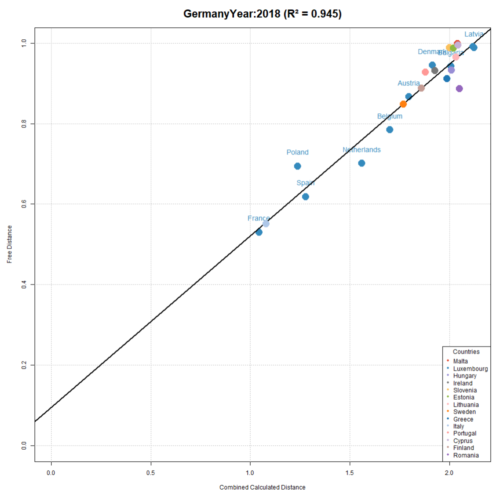
(a) Phase I: 2006



(b) Phase II: 2010



(c) Phase III: 2018



(d) Phase III: 2018 NOT THE MEDIAN

Hypothesis Even with optimized models that allow overfitting, certain countries cannot adequately explain the free allocations of others, highlighting disparities in the allocation mechanism.

4.4.1 Methodology

Feature Selection and Model Building:

- For each country i, build a linear regression model to predict free allocations Y of other countries using their attributes Y and the country's own attributes $\vec{x_i}$. The following attributes were included:

1. Total energy supply
2. GDP per capita
3. Population
4. Inflation
5. Agriculture
6. Industry
7. Manufacturing
8. Energy Intensity

In this experiment, we avoided using verified emissions as this would be too correlated on its own.

- Allow the model to find the optimal weights for each attribute, using algorithm R's built in optim package that utilizes "L-BFGS-B" (a general purpose optimizer), potentially leading to overfitting.

Evaluation of Model Performance: Record all the R^2 values using another country every time.

4.4.2 Results and Analysis

Table 4.6: R^2 values for all the countries throughout the years of the ETS section 4.4

Country	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Austria	0.82	0.81	0.80	0.84	0.83	0.83	0.82	0.82	0.89	NA	0.89	0.90	0.89	0.87	0.85	0.87
Belgium	0.77	0.75	0.73	0.77	0.75	0.77	0.75	0.75	0.81	0.80	0.79	0.81	0.78	0.75	0.74	0.75
Bulgaria	0.83	0.82	NA	0.83	0.81	0.83	0.80	0.80	0.92	NA	0.92	0.93	0.92	0.91	0.91	0.92
Denmark	0.82	0.82	0.80	0.85	0.84	0.84	0.83	0.83	0.92	0.92	0.92	0.93	0.92	0.92	0.91	0.92
Estonia	0.84	0.83	0.81	0.86	0.85	0.86	0.84	NA	0.93	0.93	NA	0.93	0.93	0.92	0.92	0.93
Finland	0.80	0.79	0.77	0.82	0.81	0.82	0.81	0.81	0.89	0.89	0.90	0.91	NA	0.89	0.89	0.90
France	0.31	0.35	0.23	NA	0.09	0.20	0.18	NA	0.73	0.71	0.56	0.58	0.55	0.53	0.52	0.50
Germany	0.83	0.82	0.80	0.86	0.85	0.85	0.84	0.84	0.93	0.93	0.93	0.94	0.94	0.93	0.92	0.93
Greece	0.74	0.72	0.67	0.71	0.71	0.70	0.68	0.72	0.92	0.92	0.91	0.92	0.92	0.90	NA	0.91
Hungary	0.82	0.81	0.80	0.84	0.82	0.83	0.82	0.82	0.91	NA	0.91	0.92	0.91	0.90	0.90	0.91
Ireland	0.84	0.82	0.81	NA	0.85	0.85	0.84	0.83	0.92	0.92	0.92	0.93	0.92	NA	0.90	0.91
Italy	0.72	NA	0.64	0.86	0.85	0.85	0.83	0.77	0.87	0.84	0.85	0.86	0.85	0.81	0.81	0.84
Latvia	0.84	0.83	0.82	0.87	0.86	0.86	0.85	0.85	0.93	0.93	0.93	0.94	0.93	0.92	0.92	0.93
Lithuania	0.84	0.83	0.82	0.86	0.86	0.86	0.85	0.84	0.93	0.93	0.93	0.93	0.93	NA	0.92	0.93

(continued)

Country	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Luxembourg	0.84	0.83	0.82	0.87	0.86	0.86	0.85	0.85	0.93	0.93	0.93	0.94	0.93	0.92	0.92	0.93
Netherlands	0.69	0.68	0.64	0.67	0.60	0.62	0.57	0.59	0.69	0.68	0.62	0.65	0.60	0.55	0.53	0.59
Poland	0.72	0.72	0.69	0.68	NA	0.62	0.64	0.68	0.79	0.79	0.78	0.81	0.80	0.78	0.78	0.80
Portugal	NA	0.80	0.79	0.83	0.82	0.83	0.81	0.82	0.92	0.92	0.91	0.92	0.92	NA	0.91	0.92
Slovenia	0.84	0.83	0.82	0.87	0.86	0.86	0.85	0.84	0.93	0.93	0.93	0.93	0.93	NA	0.92	0.93
Spain	0.67	0.68	0.57	0.72	0.66	0.66	0.62	0.66	0.83	0.84	0.82	0.84	0.83	0.80	0.79	0.82
Sweden	0.82	0.81	0.80	0.84	0.83	0.83	0.81	0.81	0.86	0.87	0.87	0.88	0.87	0.86	0.86	0.88
United Kingdom	0.64	0.66	0.63	0.82	0.83	0.83	0.86	0.90	0.75	0.79	0.71	0.76	0.67	0.64	0.58	0.65

Table 4.8: The weights for all the countries throughout the years of the ETS section 4.4

Country	Period	Energy Supply	GDPpc	Population	Inflation	Agriculture	Industry	Manufacturing	Energy Intensity	Max Std
Austria	Period 1	7.51	0.00	100.00	0.02	0.70	1.71	1.18	0.84	12.01
	Period 2	50.53	0.17	78.39	0.00	0.21	2.27	0.28	0.00	43.53
	Period 3	100.00	1.12	0.00	0.15	0.00	1.91	0.00	0.37	2.02
Belgium	Period 1	30.30	6.51	99.99	0.61	1.31	6.29	0.00	1.59	27.68
	Period 2	78.00	5.17	56.29	0.54	0.00	0.00	2.11	2.15	51.91
	Period 3	99.99	9.20	0.00	0.37	0.28	0.00	1.07	0.83	1.31
Bulgaria	Period 1	4.22	0.00	99.98	3.87	0.15	0.50	0.53	0.29	8.43
	Period 2	54.38	0.01	82.49	0.54	0.00	2.28	2.83	0.04	48.80
	Period 3	100.00	0.00	3.98	0.00	0.00	0.00	0.00	0.00	3.63
Denmark	Period 1	8.56	0.42	99.91	0.76	0.96	5.42	0.63	0.67	13.04
	Period 2	61.90	0.00	67.08	0.18	0.21	4.55	0.00	0.31	45.30
	Period 3	100.00	0.00	3.32	0.00	0.00	0.00	0.00	0.00	2.93
Estonia	Period 1	6.22	0.00	100.00	0.51	0.37	1.55	5.63	0.00	9.92
	Period 2	57.65	0.00	61.58	0.03	0.00	1.39	0.02	0.00	48.90
	Period 3	100.00	0.00	5.09	0.00	0.00	0.00	0.00	0.00	4.02
Finland	Period 1	31.42	6.35	99.98	0.41	2.04	0.00	0.74	1.34	35.50
	Period 2	82.92	0.45	53.95	0.13	0.00	2.17	0.24	0.00	50.78
	Period 3	100.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.16
France	Period 1	0.00	9.43	8.84	10.34	3.07	0.00	99.94	13.15	19.52
	Period 2	0.00	4.29	42.90	11.46	0.00	0.00	99.00	15.03	23.34
	Period 3	0.00	6.09	80.11	7.68	22.89	54.66	55.70	0.00	50.64
Germany	Period 1	4.28	0.00	85.35	1.45	16.15	67.19	58.46	7.30	43.12
	Period 2	53.63	0.00	73.23	0.00	13.90	0.00	77.29	12.62	41.06
	Period 3	84.75	0.00	3.11	8.31	9.57	16.67	75.05	11.87	40.82
Greece	Period 1	5.36	4.26	99.99	6.08	8.63	12.72	3.97	0.94	17.42

(continued)

Country	Period	Energy Supply	GDPpc	Population	Inflation	Agriculture	Industry	Manufacturing	Energy Intensity	Max Std
	Period 2	59.33	2.63	68.22	0.47	0.00	0.00	2.26	2.88	52.30
	Period 3	100.00	0.00	3.64	0.00	0.09	0.00	0.00	0.00	2.12
Hungary	Period 1	8.17	0.00	99.99	0.01	1.57	2.58	1.60	0.44	14.68
	Period 2	60.50	0.00	82.76	0.37	0.01	3.79	0.00	0.00	35.12
	Period 3	100.00	0.00	7.53	0.01	0.23	0.00	0.00	0.00	2.53
Ireland	Period 1	7.67	0.00	99.99	0.00	0.08	0.59	1.79	1.01	11.33
	Period 2	62.09	0.20	67.49	0.08	0.35	4.49	0.00	0.48	44.79
	Period 3	100.00	0.00	1.37	0.00	0.00	0.04	0.12	0.09	1.87
Italy	Period 1	99.87	0.00	15.25	0.00	2.47	0.00	0.01	0.00	17.86
	Period 2	99.96	0.05	17.30	3.15	1.89	0.73	2.73	0.00	11.61
	Period 3	99.98	0.35	2.06	1.08	4.04	0.91	0.00	0.24	3.69
Latvia	Period 1	4.30	0.00	100.00	0.58	0.00	0.00	0.18	1.00	7.92
	Period 2	59.54	0.00	69.29	0.00	0.00	0.00	0.00	0.04	43.22
	Period 3	100.00	0.00	4.87	0.00	0.00	0.00	0.00	0.00	4.00
Lithuania	Period 1	6.44	0.00	100.00	0.94	0.00	0.00	1.20	0.32	9.52
	Period 2	60.19	0.00	68.12	0.00	0.03	0.00	0.00	0.00	43.86
	Period 3	100.00	0.00	3.60	0.00	0.07	0.00	0.03	0.01	3.31
Luxembourg	Period 1	6.72	0.00	100.00	0.43	0.00	0.39	0.18	0.71	11.14
	Period 2	59.65	0.00	69.55	0.00	0.00	0.02	0.00	0.05	44.08
	Period 3	100.00	0.00	5.64	0.00	0.00	0.00	0.00	0.00	4.13
Netherlands	Period 1	44.27	14.34	99.77	1.09	2.26	0.99	15.57	1.65	33.97
	Period 2	98.51	9.55	41.52	2.43	0.00	0.00	8.15	16.93	45.54
	Period 3	100.00	12.09	0.00	3.77	4.25	0.00	0.84	29.53	9.96
Poland	Period 1	0.00	0.12	99.87	8.05	18.01	0.00	3.21	0.00	17.34
	Period 2	23.00	0.00	99.99	6.55	2.53	0.00	3.12	0.00	32.88
	Period 3	96.36	0.00	34.96	2.69	2.77	0.00	0.00	4.44	39.15
Portugal	Period 1	18.11	0.00	99.99	0.00	2.49	2.01	0.45	0.17	11.91
	Period 2	73.47	0.00	67.41	0.00	0.30	6.47	0.00	0.00	44.94
	Period 3	100.00	0.00	1.70	0.00	0.00	0.00	0.00	0.00	2.34
Slovenia	Period 1	8.49	0.00	100.00	0.00	0.03	0.00	0.19	1.01	13.46
	Period 2	59.30	0.00	69.99	0.00	0.00	0.00	0.00	0.08	43.79
	Period 3	100.00	0.00	7.29	0.00	0.00	0.00	0.00	0.00	4.31
Spain	Period 1	100.00	0.45	40.90	7.94	18.04	0.00	38.87	0.11	31.84
	Period 2	100.00	1.49	62.89	2.20	0.00	0.00	34.49	0.00	22.45
	Period 3	100.00	0.00	29.18	3.71	0.00	14.26	0.00	0.00	7.11
Sweden	Period 1	0.00	0.00	99.91	0.00	0.90	2.50	2.26	0.83	3.26
	Period 2	40.52	1.04	63.11	0.10	0.59	3.28	0.35	0.00	54.31
	Period 3	100.00	0.42	0.96	0.02	0.00	0.02	0.00	0.00	1.47

(continued)

Country	Period	Energy Supply	GDPpc	Population	Inflation	Agriculture	Industry	Manufacturing	Energy Intensity	Max Std
United Kingdom	Period 1	91.27	0.00	10.68	16.05	0.00	0.00	42.88	0.09	43.90
	Period 2	99.98	0.00	11.57	0.00	0.00	0.00	15.02	0.46	21.23
	Period 3	100.00	0.00	0.00	0.38	0.00	2.66	15.15	0.00	8.92

The outcomes of Experiment 3 offer profound insights into the complexities of modeling free allowance allocations among EU member countries. By seeking the optimal linear combination of eight key economic and energy attributes for each country, we aimed to determine the extent to which individual countries can explain the allocation patterns of others, even when overfitting is permitted.

Variability in Predictive Accuracy: Table Table 4.6 presents the highest R^2 values achieved for each country across different years using any linear combination of the selected attributes. The results reveal significant variability in predictive accuracy among countries:

- **High Predictive Power**: Countries like Austria, Denmark, Sweden, and Germany consistently achieved high R^2 values, often exceeding 0.85 across all EU ETS phases. This suggests that these countries' economic and energy profiles are sufficiently representative to model the allocations of other member states effectively.

- **Low Predictive Power**: Conversely, countries such as Poland and France exhibited lower R^2 values, indicating a weaker ability to explain others' allocations. Poland's R^2 values, for instance, hovered around 0.72 in Phase I and only marginally improved in later phases, highlighting inherent differences in its profile that the linear models do not capture.

Diversity in Optimal Attribute Weights: Table Table 4.8 illustrates the optimal weights assigned to each attribute that yielded the best R^2 values for each country during the three EU ETS phases. Key observations include:

- **Attribute Dominance**: Many countries heavily weighted the Total Energy Supply attribute. For example, Italy, Spain, and Germany assigned nearly 100% weight to this attribute in certain periods, underscoring its significance in their allocation models.

- **Varied Attribute Importance**: Other countries displayed a more diversified weighting. France, for instance, placed substantial weight on attributes like Manufacturing and Agriculture, reflecting the unique aspects of its economic structure.

- **Shifts Across Phases**: The optimal weights for some countries changed notably across different ETS phases, indicating adjustments in their economic profiles or in the allocation mechanism itself.

Systematic Patterns and Anomalies:

1. **Consistent Underperformance**: Despite overfitting allowances, some countries like Poland and France struggled to achieve high R^2 values consistently. This suggests that the selected attributes may not fully capture the factors influencing their allocation patterns or that unique, unmodeled factors influence their allocations.

2. **Robust Predictors**: Countries such as Sweden and Germany maintained high predictive power, indicating that their economic and energy profiles are more aligned with the overall allocation mechanisms of the EU ETS.

4.4.3 Discussion

The findings of Experiment 3, combined with insights from Experiments 1 and 2, reveal the complexities and nuances in attempting to model the fairness of EU ETS free allowance allocations. The observed variability in predictive accuracy among member states highlights the limitations of linear models when applied to the unique economic and energy profiles of each country. Given these findings, cluster analysis may provide a more effective approach to account for the diversity among member states, grouping countries with similar economic and energy profiles to understand underlying patterns in allocation outcomes better.

The limitations observed, such as Polands's distinct reliance on coal or France's diverse economic activities, indicate that these countries may belong to clusters that differ significantly from the rest of the EU. Instead of striving for a one-size-fits-all model, cluster analysis would allow for identifying groups of countries that share similar structural characteristics, thereby enabling a more granular approach to model their allocation patterns.

4.5 Conclusion

The analysis across the three experiments underscores the need for a more nuanced approach to modeling free allowance allocations within the EU ETS. Notably, the significant differences in predictive accuracy suggest that a cluster analysis could uncover groups of countries with similar economic and energy profiles, offering insights beyond those provided by linear models. This approach would allow for a better understanding of shared allocation patterns and reduce the risk of oversimplification inherent in standardized linear models.

Chapter 5

Assessing the Limitations of Relaxed Horizontal Equity in EU ETS Allocation

5.1 Introduction

In this chapter, we aim to deepen our understanding of the free allowance allocation under the European Union Emissions Trading System (EU ETS) by employing a clustering analysis of EU Member States based on selected economic and energy indicators. The allocation of emission allowances is a critical component of the EU ETS, impacting the fairness and efficiency of the system. By examining how different countries are grouped according to relevant attributes, we can gain insights into the underlying principles governing the allocation process and assess whether the allocation aligns with notions of equity and economic efficiency [17] (Dimos et al., 2023).

Following the literature on allowance allocation, we select multiple complementary criteria as features that describe EU Member States in terms of size, economic health, and energy intensity. Our aim is to observe whether equity and efficiency are achieved in the EU ETS allocation procedure. Using a clustering approach, we first categorize the Member States based on the selected features and then compare each cluster regarding free allocation. Previous literature has used cluster analysis primarily to group EU countries based on greenhouse gas (GHG) emissions [24, 42] or allowance transfer patterns in the EU ETS [5].

5.2 Data Collection and Indicator Selection

Following the indicator selection methodology outlined by [54] (Zhou and Wang, 2016), we select a set of economic and energy indicators that represent the principles of fairness and economic efficiency in the context of emission allowance allocation. The indicators are chosen to capture various aspects of each country's profile, including population size, economic health, energy consumption, and intensity.

The selected indicators, along with their associated allocation principles and data sources, are presented in Table 3.2.

We consider data from 2005 to 2020, covering the first three phases of the EU ETS:

- **Phase I** (2005-2007): The pilot phase, primarily utilizes grandfathering allocation meth-

Table 5.1: List of Indicators along with the Allocation Principles of [54] (Zhou & Wang 2016), aligned with Mulin's principles as presented in Table 3.2

Indicators	**Principle**	**Data Source**	**Moulin's Principle**
Population	Fairness	`https://data.worldbank.org/indicator/SP.POP.TOTL`	Exogenous Rights and possibly Reward
GDP per capita	Fairness	`https://data.worldbank.org/indicator/NY.GDP.PCAP.CD`	Fitness and Compensation
Inflation	Fairness	`https://data.worldbank.org/indicator/FP.CPI.TOTL.ZG`	Compensation or Fitness
Agriculture	Fairness	`http://wdi.worldbank.org/table/4.2#`	Fitness or Compensation
Industry	Fairness	`http://wdi.worldbank.org/table/4.2#`	Reward or Fitness or Compensation
Manufacturing	Fairness	`http://wdi.worldbank.org/table/4.2#`	Reward or Fitness or Compensation
Total Energy Supply	Fairness	`https://ec.europa.eu/eurostat/databrowser/view/nrg_bal_s/`	Reward or Fitness
Energy Intensity	Economic Efficiency	`https://ec.europa.eu/eurostat/databrowser/view/NRG_IND_EI`	Reward or Fitness
Verified Emissions	Fairness	`https://www.eea.europa.eu/data-and-maps/dashboards/emissions-trading-viewer-1`	Reward

ods.

- **Phase II** (2008-2012): The first commitment period under the Kyoto Protocol, with continued use of grandfathering.

- **Phase III** (2013-2020): A shift towards benchmarking and auctioning, with more centralized allocation rules.

The selection of these indicators aims to capture the principles of fairness, such as the ability to pay and vertical equity, and economic efficiency, which relates to the effective use of resources in reducing emissions.

5.3 Methodology

5.3.1 Data Normalization

Before conducting the cluster analysis, it is essential to normalize the data to ensure that each indicator contributes equally to the clustering process. **In this chapter, we focus on normalizing each indicator by dividing the value for each country by the corresponding average across all countries for each year**. This approach provides a straightforward and balanced comparison across countries, ensuring that each indicator's influence on clustering is consistent.

Nonetheless, other normalization methods could be employed, each offering unique advantages:

- **Division by Maximum**: Dividing each value by the maximum of the dataset ensures that all values lie between zero and one, emphasizing the relative size of each country's indicator.
- **Linear Normalization or "Max-Min" Method**: Using the $(x - x_{\text{min}})/(x_{\text{max}} - x_{\text{min}})$ formula allows for a balanced scaling between minimum and maximum values, helping to mitigate the impact of outliers.
- **Principal Component Analysis (PCA) Normalization**: PCA employs a normalization that focuses on preserving the variance structure, highlighting the most impactful indicators across dimensions.

Each alternative method provides a distinct perspective and could influence how clusters are perceived and interpreted. However, for the purposes of this chapter, average-based normalization remains the primary method used in the analysis.

5.3.2 Clustering Method

We employ the k-means clustering algorithm to categorize the EU Member States based on the selected indicators. K-means clustering is a partitioning method that aims to divide a set of observations into k clusters in which each observation belongs to the cluster with the nearest mean, serving as a cluster prototype.

5.3.3 Determination of Optimal Number of Clusters

Determining the optimal number of clusters, k, is a crucial step in the clustering process. We utilize the NbClust package in R to identify the best number of clusters. The NbClust function provides 30 indices for determining the number of clusters and proposes the best clustering scheme based on the majority rule.

Given the relatively small number of countries (25 EU Member States considered), we restrict the possible number of clusters to between 3 and 5 to ensure meaningful and interpretable groupings.

For every year, those indicators are summarized in a table like that:

```
    *******************************************************************
* Among all indices:
* 9 proposed 3 as the best number of clusters
* 2 proposed 4 as the best number of clusters
* 8 proposed 5 as the best number of clusters
* 5 proposed 7 as the best number of clusters

                   ***** Conclusion *****

* According to the majority rule, the best number of clusters is  3
```

Table 5.2 presents the results from every year with the votes on every Number of clusters. The algorithm used normalized data -using the mean of each attribute - and the range in which the indices could search was 3 to 7.

It is clear that the clustering is not unique, but we need to standardize one clustering to perform our analysis. We chose the clustering that results from the data of 2018 by using the mean for normalization. Here we can observe its difference from other clusterings produced on data from other years. In the first plot we let each year produce the best clustering, even with a different number of clusters. On the second, we forced it to equal the 3 clusters of 2018.

5.3.4 Results of Clustering

The NbClust analysis suggests that the optimal number of clusters is 3. The resulting clusters of EU Member States are illustrated in Figure 5.2

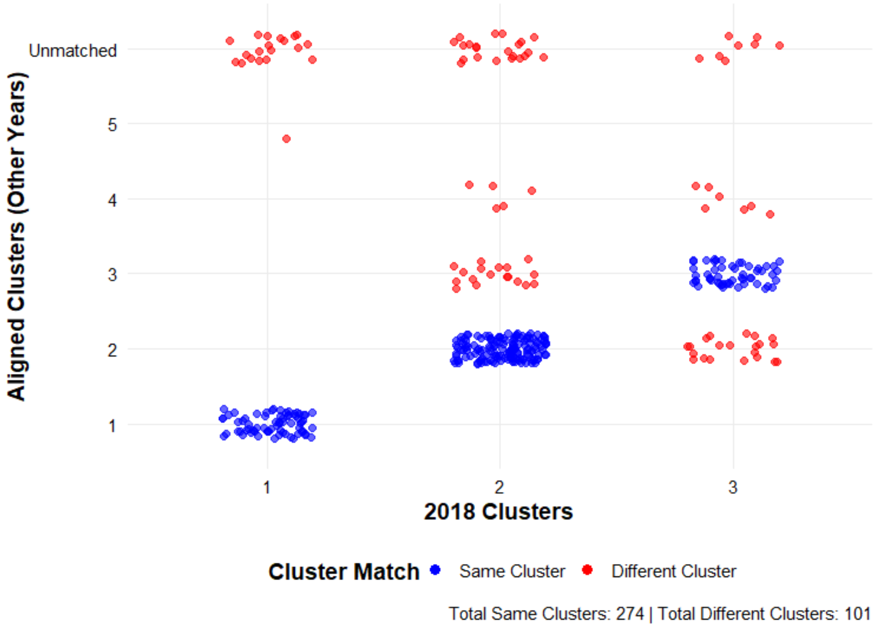
Temporal Consistency of Clustering

To assess the stability of these clusters over time, we compared the clustering results of 2018 with those from other years. When allowing each year to determine its optimal number of clusters (Figure 5.2a), we observed variations in cluster composition and number. However, when we fixed the number of clusters to three for all years (Figure 5.2 b), the clusters remained relatively consistent over time. This consistency suggests that the clustering based on the 2018 data provides a reasonable representation of the Member States' grouping throughout the EU ETS phases.

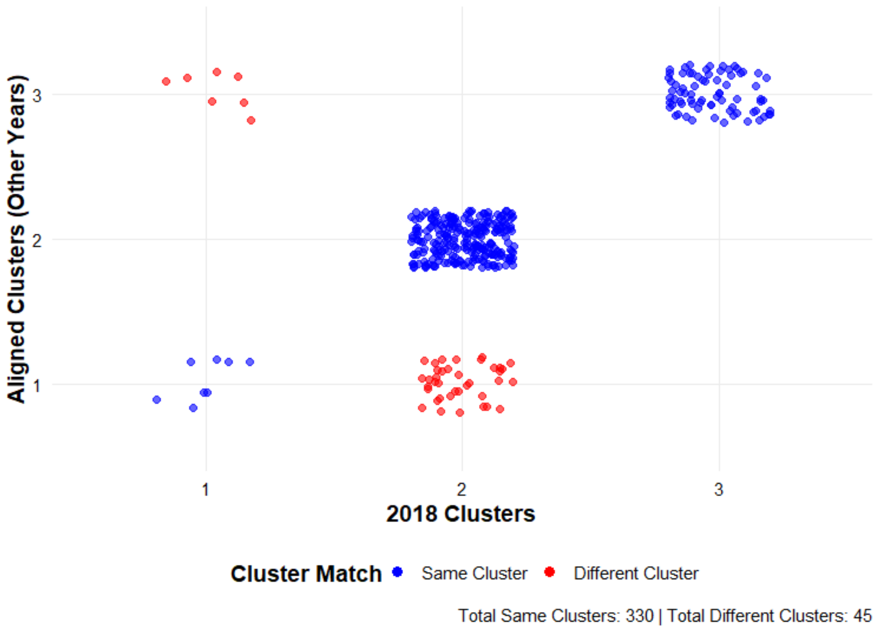
Cluster Composition

- **Cluster 1**: France, Germany, Italy, Poland, Spain, United Kingdom
- **Cluster 2**: Bulgaria, Estonia, Hungary, Latvia, Lithuania, Romania
- **Cluster 3**: Austria, Belgium, Cyprus, Denmark, Finland, Greece, Ireland, Luxembourg, Malta, Netherlands, Portugal, Slovenia, Sweden

Figure 5.1: Comparisons of clusters of 2018 vs other years



(a) Each year could yield different number of clusters



(b) All years forced to have 3 clusters

Year	Clusters = 3	Clusters = 4	Clusters = 5	Clusters = 6	Clusters = 7	Majority Cluster
Phase I (2005-2007)						
2005	9	2	8	0	5	3
2006	6	7	7	2	1	4
2007	10	2	10	2	0	3
Summary	Phase I shows a preference for clusters of 3 and 5, with the majority clustering at 3 in most years.					
Phase II (2008-2012)						
2008	5	3	13	0	2	5
2009	8	9	1	3	3	4
2010	6	1	6	2	9	7
2011	8	2	3	6	4	3
2012	6	3	2	10	3	6
Summary	Phase II exhibits more variation, with clusters of 4, 5, 6, and 7 all being favored in different years, although cluster 3 remains the most voted.					
Phase III (2013-2020)						
2013	10	6	1	3	4	3
2014	10	4	5	1	4	3
2015	6	2	4	7	5	6
2016	10	6	2	2	3	3
2017	13	3	6	0	2	3
2018	10	4	3	0	7	3
2019	10	4	3	0	7	3
2020	4	4	6	4	6	5
Summary	Phase III heavily favors cluster 3, with occasional years leaning towards clusters 5 or 6, but the majority rule for most years remains at 3.					

Table 5.2: Yearly cluster analysis with majority votes for optimal clusters in each phase.

Cluster 1 comprises some of the largest economies and most populous countries in the EU. These countries have significant industrial sectors, higher total energy consumption, and larger verified emissions. Their economies are diverse, with substantial contributions from industry and manufacturing, which are energy-intensive sectors.

Cluster 2 includes several Eastern European countries with smaller economies and lower GDP per capita. These countries often have higher energy intensity due to less efficient energy use and reliance on older technologies. Their industrial sectors may be significant relative to their economies, but in absolute terms, they are smaller than those in Cluster 1.

Cluster 3 consists of a mix of smaller and medium-sized economies, many of which have higher GDP per capita and more advanced energy efficiency measures. These countries often

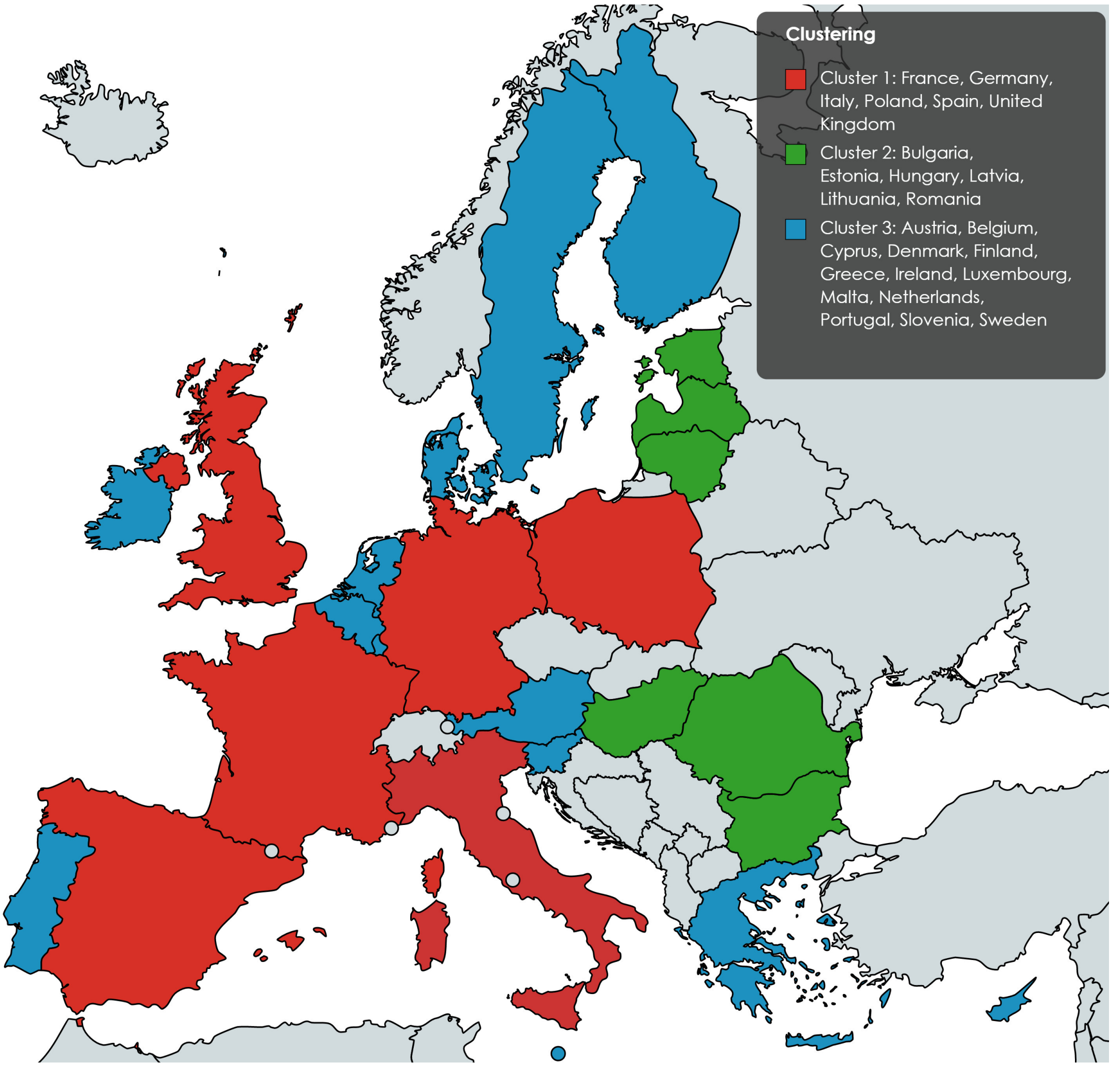


Figure 5.2: Map of Clustering

have significant service sectors and have made substantial investments in renewable energy and energy efficiency technologies.

Analysis of Clusters in Terms of Free Allowance Allocation

The Figure 5.3 show the allocation of free emission permits. The following observation can be made:

1. Average Free Allocation Across Countries and Phases. This plot shows the average free allocation for each country across three phases: Phase 1 (2005-2007), Phase 2 (2008-2012), and Phase 3 (2013-2020), grouped by clusters.

2. Average Free Allocation Across Countries and Phases (Log Scale). This plot uses a logarithmic scale to visualize average free allocation across countries and phases, allowing better data comparison with a wide range of values.
3. Average Free Allocation per Capita Across Countries and Phases. This plot displays the average free allocation per capita for each country across the three phases, facilitating comparison of allocations on a per-person basis.

Key Observations:

1. **Declining Allocation Over Phases**: There is a clear declining trend in the allocation of free allowances across all clusters from Phase I to Phase III. This reflects the EU ETS's design, which progressively reduces the total number of allowances to encourage emissions reductions.
2. **Differences in Absolute Allocation**: Cluster 1 countries receive the highest absolute amounts of free allowances, consistent with their larger economies and higher emissions. Cluster 2 receives the lowest absolute allocations, while Cluster 3 sits between the other two clusters.
3. **Per Capita Allocation Differences**: When considering free allocation per capita (Figure 5.3c), Cluster 2 and 3 countries receive significantly higher allocations compared to Cluster 1, particularly in Phases I and II.
4. **Shift Towards Uniformity**: Over time, the per capita allocations among the clusters converge, indicating a shift towards a more uniform allocation approach in Phase III.

5.4 Regression Analysis

To further investigate the relationship between the selected indicators and the free allowance allocation, we perform a regression analysis. The goal is to assess which indicators are significant predictors of the allocation and how this relationship varies across the different phases of the EU ETS.

5.4.1 Regression Model Specification

Due to multicollinearity among some indicators, we select a subset of variables to include in the regression model:

- Population
- GDP per capita
- Composite Indicator: Total Energy Supply multiplied by Energy Intensity

The composite indicator captures the overall energy consumption adjusted for efficiency, providing a meaningful variable for analysis.

5.4.2 Regression Results

We perform the regression analysis separately for each phase of the EU ETS to account for changes in allocation rules and market conditions.

Table 5.3 summarizes the regression results of the different attributes, of different clusters through the phases.

Table 5.3: R^2 Values Across Phases and Clusters

Attribute	**Phase**	**Cluster 1** R^2	**Cluster 2** R^2	**Cluster 3** R^2
Last Year's Verified emissions	Phase I	All clusters	0.9842	—
	Phase II	All clusters	0.9793	—
	Phase III	All clusters	0.9111	—
Last Year's Verified emissions	Phase I	0.9510	0.7115	0.9798
	Phase II	0.9573	0.9315	0.9697
	Phase III	0.8224	0.8245	0.7859
Population	Phase I	0.4515	0.08624	0.8275
	Phase II	0.4247	0.8879	0.8353
	Phase III	0.2807	0.7941	0.8011
GDP per Capita	Phase I	0.005165	0.127	0.006142
	Phase II	0.06907	0.4672	0.01339
	Phase III	0.07561	0.4774	6.58×10^{-7}
Total Energy Supply	Phase I	0.3634	0.1156	0.7121
	Phase II	0.3593	0.8039	0.7248
	Phase III	0.4334	0.6891	0.9706
Total Energy Supply $\times$ Energy Intensity	Phase I	0.3639	0.01803	0.6036
	Phase II	0.2556	0.6993	0.6321
	Phase III	0.4269	0.5703	0.9140

5.4.3 Key Observations

The regression analysis yields several important insights regarding the predictors of free allowance allocations across different phases and clusters:

1. **Verified Emissions**

    - Exhibits consistently high R^2 values ranging from 0.78 to 0.98 across all phases and clusters.
    - Indicates that verified emissions are a strong and reliable predictor of free allowance allocations.

2. **Population**

   - Shows variable R^2 values, from as low as 0.08 to as high as 0.89.
   - Significant predictor in Cluster 2 and Cluster 3, particularly during Phase II.

3. **GDP per Capita**

   - Generally low R^2 values below 0.5 across all phases and clusters.
   - Suggests that GDP per capita is not a significant determinant of free allowance allocations.

4. **Total Energy Supply**

   - Displays moderate to high R^2 values, increasing in later phases.
   - Strong predictor in Cluster 3 during Phase III with an R^2 of 0.9706.

5. **Composite Indicator (Total Energy Supply $\times$ Energy Intensity)**

   - Exhibits a wide range of R^2 values from 0.02 to 0.91.
   - Particularly strong predictor in Cluster 3 during Phase III ($R^2 = 0.9140$).

6. **Overall Trends**

   - Increasing relevance of energy-related indicators in later phases of the EU ETS.
   - Declining influence of GDP per capita over time, highlighting a shift towards energy efficiency and emissions-focused allocation criteria.

5.4.4 Implications of the Results

- **Grandfathering and Energy**: The near perfect R^2 values for verified emissions can be attributed to the grandfathering which, even though is declining, it was prevalent on Phase I and Phase II

- **Fairness Considerations**: The varying impact of population metrics suggests that fairness principles, such as per capita allocations, are significant, especially in certain clusters and phases.

- **Limited Role of Economic Wealth**: The minimal influence of GDP per capita indicates that economic prosperity per individual is not a primary factor in the allocation process, potentially avoiding biases towards wealthier nations.

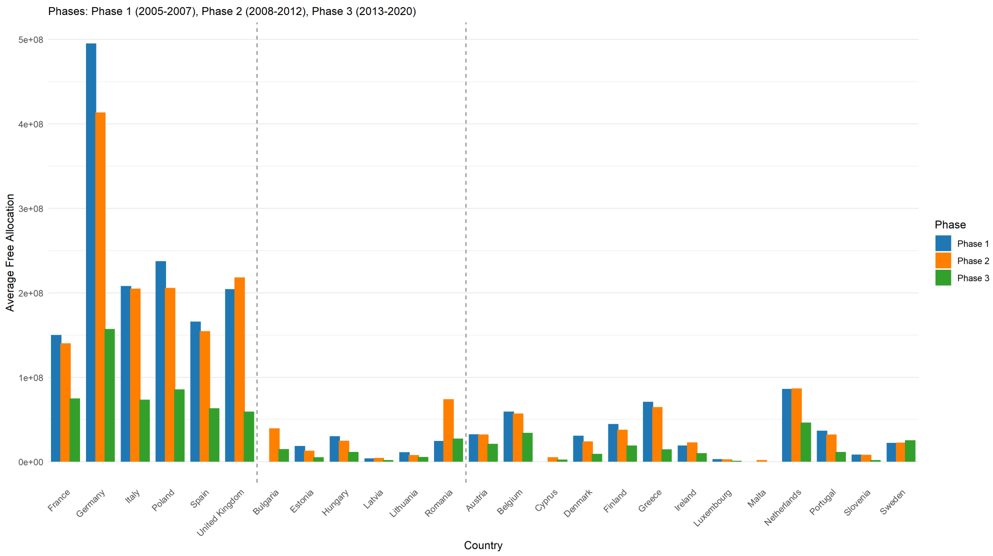
5.5 Conclusion

This chapter has provided an in-depth analysis of the allocation of free emission allowances under the EU ETS by employing clustering and regression techniques. The clustering of EU Member States based on economic and energy indicators revealed three distinct groups, each with unique characteristics influencing their allocation of free allowances.

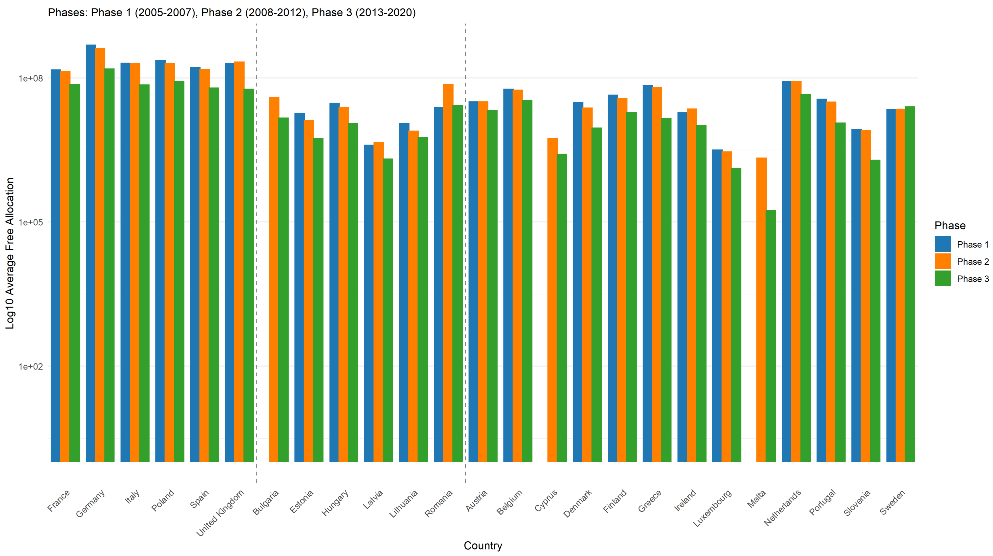
Our findings highlight that:

- **The Allocation Mechanism is Multifaceted**: The allocation of free allowances is not solely based on emissions levels but also incorporates considerations of economic size, energy intensity, and fairness principles.
- **Verified Emissions are a Key Determinant**: Across all clusters and phases, verified emissions are a strong predictor of free allocations, aligning with the economic efficiency principle by incentivizing reductions where they are most impactful.
- **Fairness Considerations are Evident**: Higher per capita allocations to less affluent, more energy-intensive countries suggest that the allocation mechanism accounts for differing capacities and challenges among Member States.
- **Policy Evolution Reflects Shifting Priorities**: The decreasing trend in free allocations and increasing significance of energy efficiency indicators indicate a shift towards stricter emissions control and a greater emphasis on sustainable energy practices.

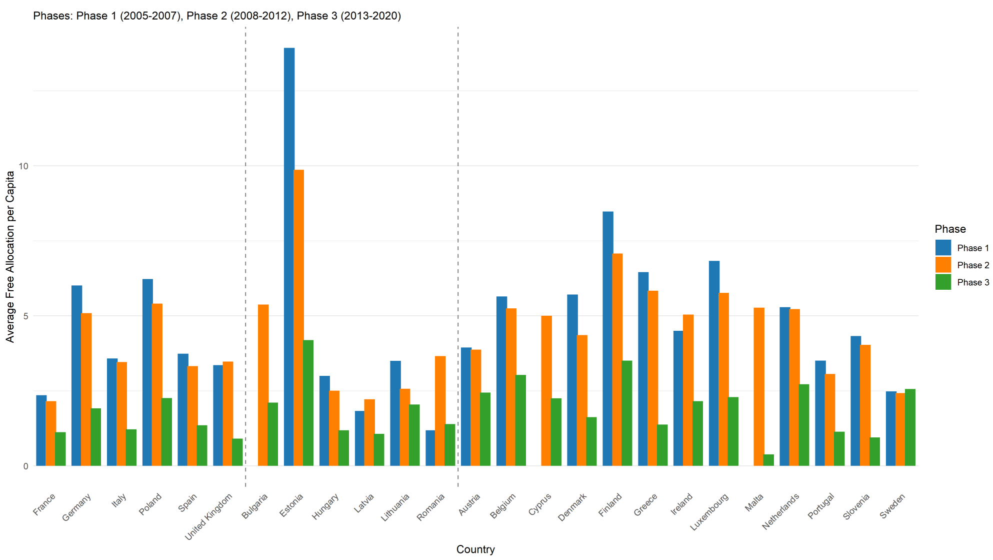
Overall, the allocation of free allowances under the EU ETS appears to be guided by a nuanced approach that balances efficiency and equity.



(a) Average Free Allocation Across Countries and Phases.

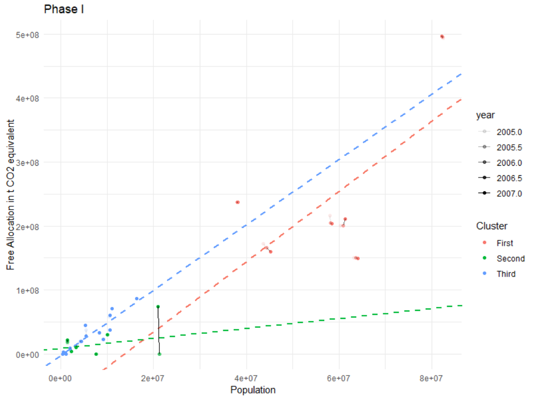


(b) Average Free Allocation Across Countries and Phases (Log Scale).

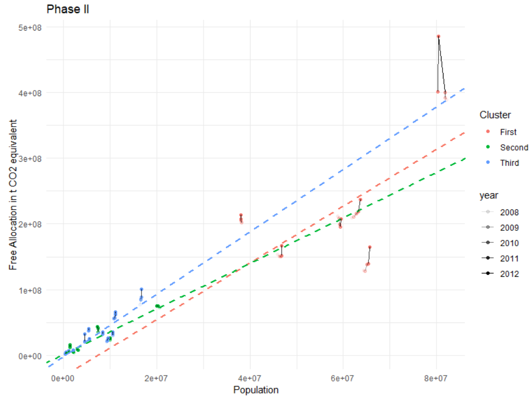


(c) Average Free Allocation per Capita Across Countries and Phases.

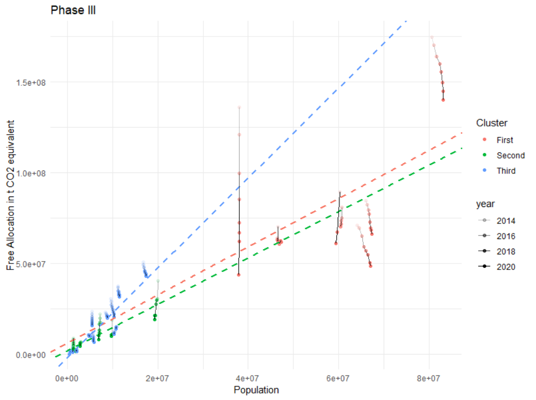
Figure 5.3: Comparison of Free Allocations Across Countries and Phases.



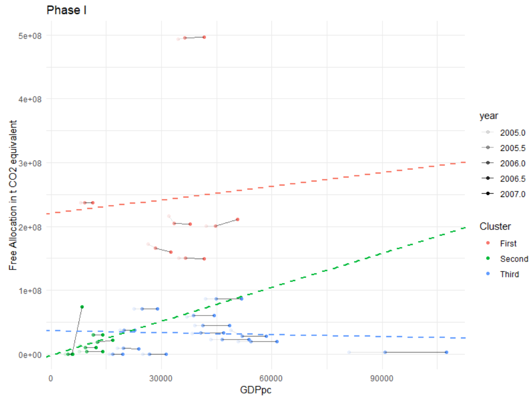
(a) Plot 1: Free Allocation vs Population for Phase I



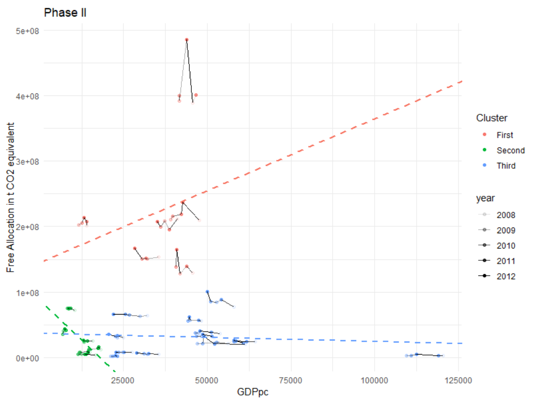
(b) Plot 2: Free Allocation vs Population for Phase II



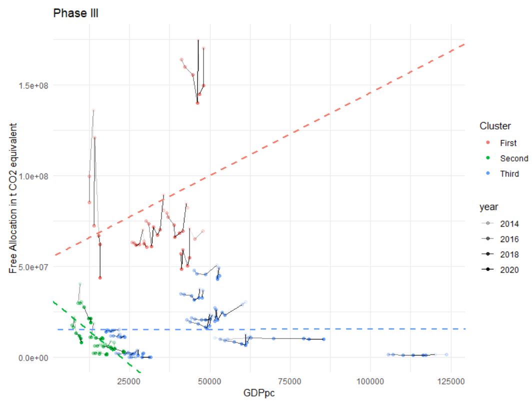
(c) Plot 3: Free Allocation vs Population for Phase III



(d) Plot 4: Free Allocation vs GDP per capita for Phase I

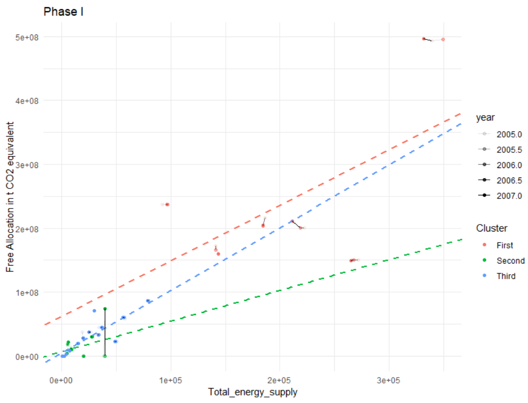


(e) Plot 5: Free Allocation vs GDP per capita for Phase II

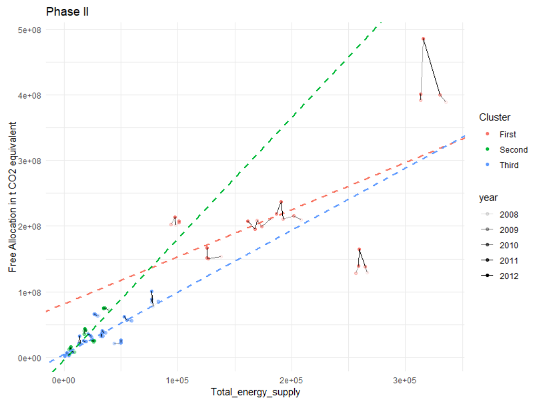


(f) Free Allocation vs GDP per capita for Phase III

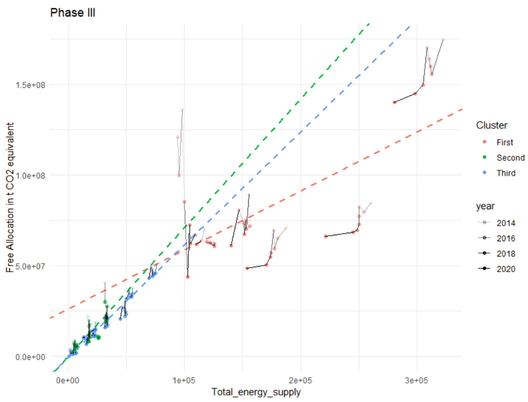
Figure 5.4: Free Allocation vs Population and GDP per capita



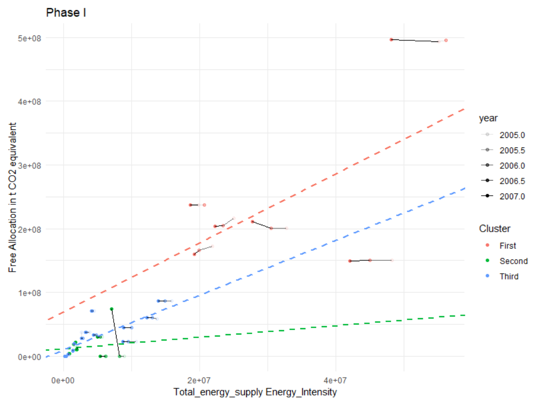
(a) Plot 1: Free Allocation vs Total Energy Supply for Phase I



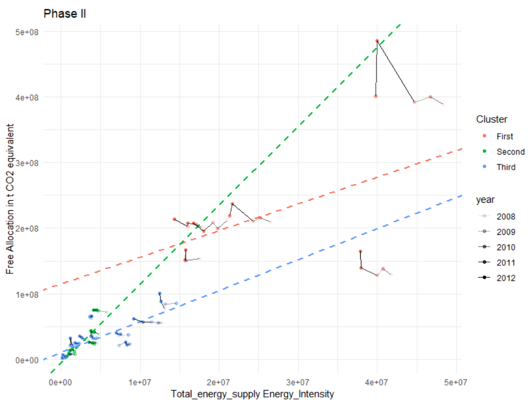
(b) Plot 2: Free Allocation vs Total Energy Supply for Phase II



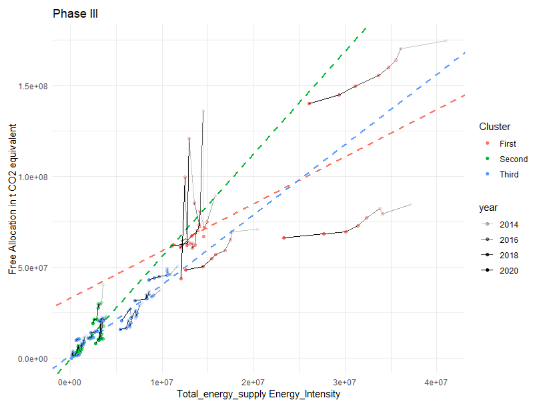
(c) Plot 3: Free Allocation vs Total Energy Supply for Phase III



(d) Plot 4: Free Allocation vs Total Energy Supply times Energy Intensity for Phase I



(e) Plot 5: Free Allocation vs Total Energy Supply times Energy Intensity for Phase II



(f) Free Allocation vs Total Energy Supply times Energy Intensity for Phase III

Figure 5.5: Free Allocation vs Total Energy Supply or Total Energy Supply times Energy Intensity

Chapter 6

Allowance Allocation as an Optimization Problem

6.1 Introduction

In this chapter, we transition from analyzing the fairness and efficiency of current allocation practices to proposing an optimized allocation model for the European Union Emissions Trading System (EU ETS). Based on the findings from previous chapters, particularly the clustering and regression analyses, we aim to balance the principles of fairness and economic efficiency in the allocation of free emission allowances.

The primary contribution of the optimization model is to standardize the allocation criteria by expressing all factors on an equal basis, effectively attempting to balance social welfare across Member States. The model seeks to integrate both country-centric and sector-based approaches, recognizing that viewing the allocation of free permits solely as a country-centric problem is not entirely unjustifiable. By combining these approaches within an optimization framework, we can capture the nuances of both perspectives.

In this model, economic efficiency is encapsulated in the objective function, while fairness is represented through the constraints. Our goal is to provide a versatile tool to analyze different allocation principles using a straightforward allocation mechanism. By adjusting the constraints and parameters, the model can simulate various fairness principles and assess their impact on the allocation outcomes.

At the end of this chapter, we will present different formulations of the problem to illustrate how alternative fairness considerations can be incorporated into the allocation mechanism.

6.2 Mathematical Formulation

6.2.1 Variables and Parameters

To formalize the optimization problem, we define the following variables and parameters:

1. **Countries** $i \in C$: The set of EU Member States participating in the EU ETS.

2. **Sectors** $j \in S$: The set of sectors subject to emission allowances.

3. **Years** t: The time periods under consideration.

4. **Percentage of Free Allocation** $v_{i,j,t}$: The percentage of the total free allocation assigned to country i, sector j, in year t. The year t will be omitted when context allows, and $t-1$ will be used to refer to the previous year.

5. **Gross Domestic Product** $GDP_{i,j,t}$: The GDP produced by sector j in country i, in year t.

6. **Verified Emissions** $e_{i,j}$: The verified emissions of sector j in country i, in year t.

7. **Purchasing Power Standards Multiplier** PPS_i: A factor similar to the Purchasing Power Parity (PPP), used to convert euros into purchasing power in country i, in year t.

8. **Aggregate Free Allocation** v_i: The total percentage of free allocation given to country i.

9. **Aggregate Free Allocation** v_j: The total percentage of free allocation given to sector j.

10. **Multipliers** α_k: Parameters used to encapsulate fairness and efficiency principles, controlling the allowable deviations in allocations.

6.2.2 Objective Function

The optimization model aims to maximize the overall economic efficiency of the allocation, measured by the ability of countries and sectors to transform allowances into economic value adjusted for purchasing power. The objective function is defined as:

$$\text{maximize} \quad Z = \sum_{i \in C} \sum_{j \in S} v_{i,j,t} \cdot \frac{GDP_{i,j,t-1}}{e_{i,j,t-1}} \cdot PPS_{i,t} \tag{6.1}$$

This function rewards allocations to sectors and countries that can generate more GDP per unit of emissions, adjusted for purchasing power, thereby promoting economic efficiency, aligning with Moulin's fitness principle 4.

6.2.3 Constraints

The optimization is subject to several constraints incorporating fairness principles and practical considerations.

Total Cap Constraint

The total allocation must not exceed the EU's emission cap, which is normalized to 1 in terms of percentages.

$$\sum_{i \in C} \sum_{j \in S} v_{i,j} = 1 \tag{6.1}$$

Country and Sector Allocation Constraints

The total allocation for each country is the sum of allocations across all sectors, and similarly for each sector:

$$v_i = \sum_{j \in S} v_{i,j} \quad \forall i \in C \tag{6.2}$$

$$v_j = \sum_{i \in C} v_{i,j} \quad \forall j \in S \tag{6.3}$$

Historical Deviation Bounds

To maintain stability and prevent abrupt changes in allocations, we impose bounds on the allowable deviation from the previous year's allocations, aligning with Moulin's Principle of Compensation 1.

For countries:

$$\alpha_1 \cdot v_{i,t-1} \leq v_{i,t} \leq \alpha_2 \cdot v_{i,t-1} \quad \forall i \in C \tag{6.4}$$

For sectors:

$$\alpha_3 \cdot v_{j,t-1} \leq v_{j,t} \leq \alpha_4 \cdot v_{j,t-1} \quad \forall j \in S \tag{6.5}$$

Population-Based Fairness

To incorporate fairness based on population, we constrain the country allocations to be proportional to their share of the total EU population, aligning to Moulin's Principle of Exogenous Rights 3:

$$v_i \approx \frac{\text{Population}_i}{\sum_{i \in C} \text{Population}_i} \quad \forall i \in C \tag{6.6}$$

This can be formulated as an equality or an inequality with acceptable deviation bounds.

Economic Activity Proportionality

To ensure that sectoral allocations within each country reflect the sector's contribution to the country's economy, aligning with Moulin's Principle of Reward 2, we include:

$$v_{i,j} \approx \frac{GDP_{i,j}}{GDP_i} \cdot v_i \quad \forall i \in C, \forall j \in S \tag{6.7}$$

Again, this can be formulated with acceptable deviations.

Definition of Multipliers

The multipliers α_k are parameters that control the allowable deviations and can be adjusted to simulate different fairness principles.

Example definitions:

$$\alpha_1 = \min\left(0.8, \frac{\overline{GDP}}{GDP_i}\right) \tag{6.8}$$

$$\alpha_2 = \max\left(1.2, \frac{\overline{GDP}}{GDP_i}\right) \tag{6.9}$$

Where $\overline{GDP}$ is the average GDP across all countries. These definitions allow countries with lower GDP to have smaller allowable decreases and more significant allowable increases in allocations.

6.2.4 Explanation of the Constraints

The constraints serve to balance fairness and efficiency:

- **Total Cap Constraint** ensures that the total allocation does not exceed the cap set by the EU ETS.
- **Country and Sector Allocation Constraints** to maintain consistency in aggregating allocations across countries and sectors.
- **Historical Deviation Bounds** prevent sudden changes in allocations that could disrupt economies or industries, allowing for gradual transitions. This aligns with the Compensation principle of Moulin 1. The notion here is that the agents are not responsible for their different emissions in the past, and thus are compensated in their journey to convergence.
- **Population-Based Fairness** aligns allocations with the principle of equal per capita entitlements, supporting vertical equity. Thus, it's fundamentally the "Equal right to Pollute", which sits under the "Exogenous Rights" of the Moulin's Principles 3.
- **Economic Activity Proportionality** ensures that sectors contributing more to the economy receive allocations commensurate with their economic significance. Here, this sum of utility is captured in the Fitness Principle or Under the Reward Principle of Moulin 4, 2.
- **Multipliers** α_k allow for flexibility in the model to incorporate different fairness principles, such as ability to pay or historical responsibility.

6.3 Solution and Methodology

6.3.1 Algorithm Selection

The optimization model is a linear programming (LP) problem, as both the objective function and the constraints are linear in the decision variables $v_{i,j}$. Linear programming is suitable for efficiently solving such problems, even with a large number of variables and constraints.

In cases where non-linear constraints or integer variables are introduced, the problem may become a Mixed Integer Programming (MIP) or a convex optimization problem, requiring appropriate solution methods.

6.3.2 Data Inputs

The data required for the model include:

- **GDP Data** ($GDP_{i,j}$): Sourced from national statistics and Eurostat, representing the economic output of each sector in each country.
- **Verified Emissions** ($e_{i,j}$): Obtained from the European Environment Agency (EEA), reflecting the actual emissions reported by sectors.
- **Population Data**: From the World Bank or Eurostat, used in the population-based fairness constraint.
- **Purchasing Power Standards** (PPS_i): Provided by Eurostat, used to adjust GDP for purchasing power differences between countries.
- **Historical Allocations** ($v_{i,t-1}$, $v_{j,t-1}$): Past allocation data required for the historical deviation constraints.

These data inputs are consistent with those used in previous analyses, ensuring continuity and comparability.

6.3.3 Data Limitations and Model Simplification

In our effort to implement the optimization model with detailed sectoral data, we encountered significant challenges due to inconsistencies between the coding systems used by different data sources. The EU ETS database utilizes specific activity codes for sectors, whereas Eurostat and other statistical agencies use the NACE (Nomenclature statistique des activités économiques dans la Communauté européenne) codes. Although these two classification schemes overlap, aligning them precisely proved difficult.

NACE Definition The Statistical Classification of Economic Activities in the European Community, commonly referred to as **NACE** (for the French term *nomenclature statistique des activités économiques dans la Communauté européenne*), is the industry standard classification system used in the European Union. The current version is revision 2 and was established by Regulation (EC) No 1893/2006. It is the European implementation of the UN classification ISIC, revision 4.

Sector Classification Mapping Despite efforts to reconcile the sector classifications, we were only able to compile five broad supersectors by grouping codes from both the EU ETS and NACE systems. The following table illustrates the mapping between the ETS activity codes, their descriptions, and the corresponding NACE codes and definitions:

Table 6.1: Mapping between EU ETS Activity Codes and NACE Codes

ETS Code	EU ETS Activity Type	NACE Code	NACE Definition	Supersector
22	Production of coke	C19.10	Manufacture of coke and refined petroleum products	C19
3	Coke ovens	C19.10	Manufacture of coke and refined petroleum products	C19
2	Mineral oil refineries	C19.20	Manufacture of refined petroleum products	C19
21	Refining of mineral oil	C19.10	Manufacture of coke and refined petroleum products	C19
— Manufacture of Coke and Refined Petroleum Products —				
23	Metal ore roasting or sintering	C24.41	Precious metals production	C24
24	Production of pig iron or steel	C24.10	Manufacture of basic iron and steel and of ferro-alloys	C24
25	Production or processing of ferrous metals	C24.10	Manufacture of basic iron and steel and of ferro-alloys	C24
26	Production of primary aluminium	C24.42	Aluminium production	C24
27	Production of secondary aluminium	C24.42	Aluminium production	C24
28	Production or processing of non-ferrous metals	C24.4	Manufacture of basic precious and other non-ferrous metals	C24
— Production of Basic Metals —				
29	Production of cement clinker	C23.51	Manufacture of cement	C23

Continued on next page

ETS Code	EU ETS Activity Type	NACE Code	NACE Definition	Supersector
30	Production of lime or calcination of dolomite	C23.52	Manufacture of lime and plaster	C23
31	Manufacture of glass	C23.1	Manufacture of glass and glass products	C23
32	Manufacture of ceramics	C23.3 C23.4	Manufacture of clay building materials Manufacture of other porcelain and ceramic products	C23
33	Manufacture of mineral wool	C23.99	Manufacture of other non-metallic mineral products n.e.c.	C23
34	Production or processing of gypsum or plasterboard	C23.52	Manufacture of lime and plaster	C23
— Manufacture of Other Non-Metallic Mineral Products —				
35	Production of pulp	C17.11	Manufacture of pulp	C17
36	Production of paper or cardboard	C17.12	Manufacture of paper and paperboard	C17
— Manufacture of Paper and Paper Products —				
37	Production of carbon black	C20.14	Manufacture of other organic basic chemicals	C20
38	Production of nitric acid	C20.15	Manufacture of fertilizers and nitrogen compounds	C20
— Manufacture of Chemicals and Chemical Products —				

Limitations While this mapping provides a general correspondence between the ETS activity types and NACE codes, it is not precise enough for detailed sectoral analysis required in our optimization model. The aggregation into broad supersectors limits the granularity of the data, making it unsuitable for accurately capturing the sector-specific dynamics within each country.

Due to these data limitations, we concluded that the available sectoral data are not sufficient for implementing the optimization model at the desired level of detail.

Model Simplification

To proceed with our analysis despite the lack of detailed sectoral data, we simplify the model by focusing solely on country-level information, removing the sector dimension. This adjustment allows us to continue exploring the optimization framework using the available data, while acknowledging the limitations imposed by the data constraints.

6.3.4 Reformulated Optimization Model

By eliminating the sectoral index j, the optimization model is reformulated as follows:

Variables and Parameters

- **Countries** $i \in C$: The set of EU Member States.
- **Years** t: The time periods under consideration.
- **Percentage of Free Allocation** $v_{i,t}$: The percentage of the total free allocation assigned to country i in year t.
- **Gross Domestic Product** GDP_i: The total GDP of country i.
- **Verified Emissions** e_i: The total verified emissions of country i.
- **Purchasing Power Standards Multiplier** PPS_i: The PPS adjustment factor for country i.
- **Multipliers** α_k: Parameters controlling deviation tolerances.

Objective Function

$$\text{maximize} \quad Z = \sum_{i \in C} v_i \cdot \frac{GDP_i}{e_i} \cdot PPS_i \tag{6.1}$$

Constraints

1. **Total Cap Constraint**:
$$\sum_{i \in C} v_i = 1 \tag{6.2}$$

2. **Historical Deviation Bounds**: Compensation Principle 1
$$\alpha_1 \cdot v_{i,t-1} \leq v_{i,t} \leq \alpha_2 \cdot v_{i,t-1} \quad \forall i \in C \tag{6.3}$$

3. **Population-Based Fairness**: Exogenous Rights Principle 3
$$v_i \approx \frac{\text{Population}_i}{\sum_{i \in C} \text{Population}_i} \quad \forall i \in C \tag{6.4}$$

This simplified model retains the essential structure of the optimization problem while operating at the country level. It allows us to analyze the allocation of free allowances among countries based on their economic efficiency and fairness considerations, despite the limitations in sectoral data.

6.4 Example Runs

To illustrate the application of the model, we present example runs under different scenarios.

6.4.1 Scenario 1: Base Case

In the base case, we set the multipliers α_k to allow for moderate deviations and use 2017 data to compute the 2018 allocation and compare them:

$$\alpha_1 = 0.8$$
$$\alpha_2 = 1.2$$

We solve the LP problem using the data inputs and observe the allocation results. Which are the following:

Forecasted Allocation Observations

- **Unreasonable Increase**: Romania (**47.56%**) and Estonia (**13.74%**) show high allocation growth, but this is partially attributed to the reduction they actually had in 2018.
- **Moderate Growth**: Countries like Sweden (21.07%), Ireland (15.87%), and France (19.76%) maintain steady increases, reflecting effective resource utilization.
- **Notable Decreases**: Germany (-5.62%), Spain (-21.97%), and Belgium (-21.03%) experience allocation reductions, suggesting areas for efficiency improvements.
- **Efficiency Correlation**: Higher efficiency scores generally align with positive allocation changes, supporting the notion of effective resource distribution.
- **Stable Allocations**: Luxembourg, Slovenia, and Cyprus maintain low but stable allocation percentages, ensuring consistent support for smaller allocations.

6.4.2 Scenario 2: Increased Flexibility &

In this scenario, we allow for greater deviations to explore how allocations change, in addition, we include a constraint about the Population:

$$\alpha_1 = 0.5$$
$$\alpha_2 = 2$$
$$\alpha_3 = 0.5$$
$$\alpha_4 = 2$$

Table 6.2: Forecasted Allocation of 2018, case 1

Country	Calculated Efficiency	This Year Allocation	Next Year Allocation	Forecasted	Change
Sweden	11.9490	3.29 %	3.27 %	3.95 %	21.07 %
Ireland	11.0789	1.39 %	1.44 %	1.67 %	15.87 %
France	10.8103	9.71 %	9.73 %	11.66 %	19.76 %
Latvia	10.2065	0.26 %	0.25 %	0.31 %	24.02 %
Luxembourg	9.9518	0.18 %	0.18 %	0.21 %	16.99 %
Denmark	9.9225	1.16 %	1.13 %	1.39 %	22.96 %
Lithuania	9.1099	0.77 %	0.77 %	0.93 %	20.39 %
Austria	9.0483	2.73 %	2.79 %	3.28 %	17.56 %
United Kingdom	8.8856	7.60 %	7.70 %	9.12 %	18.49 %
Romania	8.6489	3.67 %	2.99 %	4.41 %	47.56 %
Italy	7.9020	9.54 %	9.81 %	11.45 %	16.63 %
Slovenia	7.5470	0.25 %	0.25 %	0.29 %	18.37 %
Hungary	7.4713	1.44 %	1.48 %	1.72 %	16.32 %
Germany	6.2988	20.71 %	20.95 %	19.77 %	-5.62 %
Spain	6.0700	8.29 %	8.50 %	6.63 %	-21.97 %
Finland	5.7170	2.48 %	2.44 %	1.98 %	-18.93 %
Belgium	5.6981	4.53 %	4.59 %	3.62 %	-21.03 %
Portugal	4.8252	1.53 %	1.57 %	1.22 %	-22.40 %
Netherlands	4.1503	6.10 %	6.31 %	4.88 %	-22.61 %
Poland	3.7234	9.64 %	9.37 %	7.71 %	-17.68 %
Cyprus	2.5220	0.32 %	0.31 %	0.25 %	-17.85 %
Bulgaria	2.4126	1.79 %	1.68 %	1.43 %	-14.64 %
Greece	1.9529	1.98 %	2.04 %	1.58 %	-22.47 %
Estonia	1.6987	0.66 %	0.46 %	0.53 %	13.74 %

1. Country Allocation Deviation Constraint: This constraint ensures that each country's allocation $v_{i,t}$ for the current period does not deviate significantly from last year's allocation $v_{i,t-1}$, with bounds set by α_1 and α_2:

   $$\alpha_1 \cdot v_{i,t-1} \leq v_{i,t} \leq \alpha_2 \cdot v_{i,t-1} \quad \forall i \in C$$

   This implies that the allocation for each country in the current period should fall within a range determined by a fraction of the previous allocation, where $\alpha_1 = 0.5$ and $\alpha_2 = 2$. This keeps the allocation changes within a range of 50% to 200% of the previous year's allocation.

2. Population-Based Fairness Constraint

   This constraint ensures that each country's allocation $v_{i,t}$ is within a range based on its share of the total EU population. Let p_i represent the population share of country i as a fraction of the total EU population:

$$\alpha_3 \cdot p_i \leq v_{i,t} \leq \alpha_4 \cdot p_i \quad \forall i \in C$$

where: $p_i = \frac{\text{Population}_i}{\sum_{i \in C} \text{Population}_i}$, $\alpha_3 = 0.5$ and $\alpha_4 = 2$.

This constraint ensures that each country's allocation is proportional to its population, constrained within 50% to 200% of its share of the total population. This helps to balance allocations fairly across countries based on demographic size.

These constraints aim to stabilize the allocations by preventing abrupt shifts year-to-year, preserving consistency while allowing for measured adjustments based on economic and efficiency goals.

Results

Forecasted Allocation Observations

- **Major Increases**: Romania (**145.94%**), France (**99.60%**), Latvia (**106.70%**), and Denmark (**104.94%**) exhibit the highest allocation growth.
- **Significant Decreases**: Germany (**-50.57%**), Belgium (**-50.64%**), Netherlands (**-51.63%**), and Spain (**-44.24%**) face substantial allocation reductions, indicating potential areas for efficiency improvements.
- **Efficiency Alignment**: Higher calculated efficiency scores generally correlate with positive allocation changes, as seen in Sweden (**11.94**) and Ireland (**11.0789**), reinforcing effective resource utilization.
- **Stable Allocations**: Smaller nations like Luxembourg (**9.95**) and Cyprus (**2.52**) maintain low but stable allocation percentages, ensuring consistent support without major fluctuations.

6.4.3 Scenario 3: Inverse GDP per capita

Proposed Fairness Constraint: Development-Based Equity

Notion behind fairness criterion : To promote fairness, allocate emission permits inversely proportional to GDP per capita. This ensures that countries with lower economic development receive a fair share of permits, acknowledging their development needs and limited resources to invest in low-emission technologies. This idea aligns with the COmpensation Principle of Moulin 1

Let $\tilde{GDP}_i$ be the normalized GDP per capita for the country i, calculated as:

Country	Calculated Efficiency	This Year Allocation	Next Year Allocation	Forecasted	Change
Sweden	11.9490	3.29 %	3.27 %	4.09 %	25.31 %
Ireland	11.0789	1.39 %	1.44 %	1.96 %	35.61 %
France	10.8103	9.71 %	9.73 %	19.43 %	99.60 %
Latvia	10.2065	0.26 %	0.25 %	0.52 %	106.70 %
Luxembourg	9.9518	0.18 %	0.18 %	0.24 %	33.32 %
Denmark	9.9225	1.16 %	1.13 %	2.31 %	104.94 %
Lithuania	9.1099	0.77 %	0.77 %	1.15 %	49.27 %
Austria	9.0483	2.73 %	2.79 %	3.58 %	28.34 %
United Kingdom	8.8856	7.60 %	7.70 %	15.20 %	97.48 %
Romania	8.6489	3.67 %	2.99 %	7.35 %	145.94 %
Italy	7.9020	9.54 %	9.81 %	12.99 %	32.30 %
Slovenia	7.5470	0.25 %	0.25 %	0.21 %	-15.49 %
Hungary	7.4713	1.44 %	1.48 %	1.00 %	-32.86 %
Germany	6.2988	20.71 %	20.95 %	10.36 %	-50.57 %
Spain	6.0700	8.29 %	8.50 %	4.74 %	-44.24 %
Finland	5.7170	2.48 %	2.44 %	1.24 %	-49.33 %
Belgium	5.6981	4.53 %	4.59 %	2.26 %	-50.64 %
Portugal	4.8252	1.53 %	1.57 %	1.05 %	-33.45 %
Netherlands	4.1503	6.10 %	6.31 %	3.05 %	-51.63 %
Poland	3.7234	9.64 %	9.37 %	4.82 %	-48.55 %
Cyprus	2.5220	0.32 %	0.31 %	0.16 %	-48.66 %
Bulgaria	2.4126	1.79 %	1.68 %	0.89 %	-46.65 %
Greece	1.9529	1.98 %	2.04 %	1.09 %	-46.38 %
Estonia	1.6987	0.66 %	0.46 %	0.33 %	-28.91 %

Table 6.3: Forecasted Allocation of 2018, case 2

$$\tilde{GDP}_i = \frac{GDP_{i,\text{per capita}}}{\sum_{k \in C} GDP_{k,\text{per capita}}}$$

We define an inverse economic capacity index D_i:

$$D_i = \frac{1}{\tilde{GDP}_i}$$

To ensure fairness, we allocate permits proportional to D_i, within bounds set by coefficients β_1 and β_2:

$$\beta_1 \cdot \frac{D_i}{\sum_{k \in C} D_k} \leq v_i \leq \beta_2 \cdot \frac{D_i}{\sum_{k \in C} D_k} \quad \forall i \in C$$

Where: $\beta_1 = 0.5$ $\beta_2 = 2$

This constraint ensures that countries with lower GDP per capita receive a larger share of emission permits relative to their economic size, promoting equitable development opportunities.

This allocation is rather simplified, as, for it to work, it would have to be adjusted to align with the Historical Deviations Bounds constraint section 6.2.3. To avoid it, we have to relax its multipliers a lot, to get a semi/un-reasonble result.

$$\alpha_1 = 0.25$$
$$\alpha_2 = 15$$
$$\beta_1 = 0.5$$
$$\beta_2 = 2$$

Forecasted Allocation Observations with Development-Based Fairness

- **Significant Allocation Increases**: Countries such as Latvia (**1450.26%**), Luxembourg (**1362.43%**), Romania (**269.49%**), and Cyprus (**590.99%**) experience substantial increases in allocations. These increases likely reflect their larger population shares, ensuring allocations are proportionate to their demographic sizes.
- **Moderate Allocation Growth**: Sweden (**91.69%**), Ireland (**189.33%**), and Denmark (**420.61%**) show significant but more moderate increases, aligning with their population proportions while maintaining fairness across the EU.
- **Notable Allocation Decreases**: Countries such as Slovenia (**-15.49%**), Hungary (**-32.86%**), Germany (**-75.28%**), Spain (**-75.61%**), Netherlands (**-75.82%**), Poland (**-70.46%**), Cyprus (**-48.66%**), Bulgaria (**-46.65%**), Greece (**-46.38%**), and Estonia (**-28.91%**) face significant allocation reductions. These decreases may indicate adjustments to better align with their population shares or to address over-allocation in previous years.
- **Fairness in Allocation Distribution**: The allocation changes demonstrate a balanced approach to fairness by ensuring that countries with larger populations receive allocations proportional to their demographic weight. This approach prevents disproportionate allocations that could disadvantage smaller or larger nations unfairly.
- **Consistency with Population-Based Fairness**: The overall allocation adjustments reflect the implemented fairness constraints, ensuring that allocations are equitable and directly related to each country's population. This promotes an equitable distribution of resources across the EU member states.

6.5 Conclusion

In this chapter, we have developed an optimization model that integrates both efficiency and fairness principles into the allocation of emission allowances under the EU ETS. By formulating

Country	Calculated Efficiency	This Year Allocation	Next Year Allocation	Forecasted	Change
Sweden	11.95	3.29 %	3.27 %	6.26 %	91.69 %
Ireland	11.08	1.39 %	1.44 %	4.17 %	189.33 %
France	10.81	9.71 %	9.73 %	7.34 %	-24.56 %
Latvia	10.21	0.26 %	0.25 %	3.90 %	1450.26 %
Luxembourg	9.95	0.18 %	0.18 %	2.66 %	1362.43 %
Denmark	9.92	1.16 %	1.13 %	5.87 %	420.61 %
Lithuania	9.11	0.77 %	0.77 %	9.67 %	1153.91 %
Austria	9.05	2.73 %	2.79 %	6.01 %	115.61 %
United Kingdom	8.89	7.60 %	7.70 %	7.14 %	-7.26 %
Romania	8.65	3.67 %	2.99 %	11.04 %	269.49 %
Italy	7.90	9.54 %	9.81 %	2.38 %	-75.70 %
Slovenia	7.55	0.25 %	0.25 %	2.22 %	792.62 %
Hungary	7.47	1.44 %	1.48 %	2.77 %	86.59 %
Germany	6.30	20.71 %	20.95 %	5.18 %	-75.28 %
Spain	6.07	8.29 %	8.50 %	2.07 %	-75.61 %
Finland	5.72	2.48 %	2.44 %	1.72 %	-29.58 %
Belgium	5.70	4.53 %	4.59 %	1.62 %	-64.73 %
Portugal	4.83	1.53 %	1.57 %	2.48 %	57.51 %
Netherlands	4.15	6.10 %	6.31 %	1.53 %	-75.82 %
Poland	3.72	9.64 %	9.37 %	2.77 %	-70.46 %
Cyprus	2.52	0.32 %	0.31 %	2.12 %	590.99 %
Bulgaria	2.41	1.79 %	1.68 %	3.82 %	127.89 %
Greece	1.95	1.98 %	2.04 %	2.85 %	39.69 %
Estonia	1.70	0.66 %	0.46 %	2.42 %	422.55 %

Table 6.4: Forecasted Allocation with Development-Based Fairness Constraint

the allocation as a linear programming problem, we can systematically explore how different constraints and parameters influence the distribution of allowances.

By balancing social welfare through standardized efficiency metrics and incorporating fairness constraints, the model offers a comprehensive approach to designing allocation strategies that align with the EU's environmental and economic objectives.

Chapter 7

Uniform Linear Mechanism for Allocation

7.1 Introduction to the Uniform Linear Mechanism Model

This chapter addresses a critical issue with the optimization model from Chapter ??: the lack of a comparative baseline. To resolve this, we incorporate the Uniform Linear Mechanism (ULM) for permit allocation, as presented in the paper "Allocating Emission Permits Efficiently via Uniform Linear Mechanisms" by [26]. This model offers a structured approach to emission permit allocation that maximizes efficiency within the EU ETS framework, thus providing a point of comparison.

7.2 Definition and Structure of the Uniform Linear Mechanism

In this section, we present the mathematical formulation of the Uniform Linear Mechanism, following the model described by [26].

7.2.1 Model Description

In this model, we have N firms operating in a single-period, homogeneous product market under Cournot competition, where each firm's production generates emissions. Firms must comply with an emissions trading system (ETS) by acquiring enough permits to cover their emissions. The model aims to determine an efficient allocation of permits that maximizes consumer surplus and minimizes the social cost of pollution.

Production and Emission Decision

Each firm i produces a quantity q_i and emits a corresponding level of pollution x_i. Without any abatement, all units produced generate pollution in a fixed proportion, normalized to one-to-one. Hence, producing q_i units without abatement would result in $x_i = q_i$.

Let:

- q_i: the production level of firm i,
- x_i: the emission level of firm i,

- $f_i(\cdot)$: the abatement cost function for the firm i.

Revenue and Production Cost

The total revenue R_i for each firm i depends on the aggregate market production $Q = \sum_{j=1}^{N} q_j$ and is given by:

$$R_i(q_i, Q) = p(Q) \cdot q_i, \tag{7.1}$$

where $p(Q)$ is the inverse demand function, assumed to be decreasing and concave. For instance, with a linear demand function, we have:

$$p(Q) = b - aQ, \tag{7.2}$$

where $a, b > 0$.

Abatement Cost Function

Each firm has an abatement cost function $f_i(q_i - x_i)$, which represents the cost for reducing emissions from q_i to x_i. The abatement cost function is assumed to be strictly convex and non-decreasing, i.e., $f_i(\cdot) \geq 0$ and $f_i'(\cdot) \geq 0$, implying that abatement becomes more costly as emissions reductions increase. In their work, [18] suggests a quadratic marginal abatement cost function, and [10] uses a quadratic MAC in his work. Following their work, we consider third-order abatement cost functions to reflect a quadratic marginal abatement cost:

$$f_i(q_i - x_i) = c_{i1}(q_i - x_i) + c_{i2}(q_i - x_i)^2 + c_{i3}(q_i - x_i)^3, \tag{7.3}$$

where $c_{i1}, c_{i2}, c_{i3} > 0$ are firm-specific abatement cost coefficients.

Permit Allocation Mechanism

The regulator allocates permits to each firm i based on its production q_i. Let $\Phi_i(q)$ denote the number of permits allocated to firm i as a function of the production output vector $\vec{q} = (q_1, \ldots, q_N)$.

In our case, we have multiple sectors, and each sector s has its own allocation factor α_s. The mechanism is modified to be:

$$\Phi_i(q) = \alpha_{s(i)} \cdot q_i, \tag{7.4}$$

where $s(i)$ denotes the sector to which firm i belongs.

In some experiments, the free allocation might become a separate function $\Phi_i(q) = \phi_i(q_i)$, allowing for more flexibility in the allocation mechanism.

Permit Trading Cost

If a firm emits more than the permits it holds, it needs to purchase additional permits in the market. Conversely, if it emits less, it can sell excess permits. The trading cost or revenue for firm i is:

$$\tau \cdot (x_i - \Phi_i(q)), \tag{7.5}$$

where τ is the market-clearing permit price.

Profit Function

The profit Π_i of firm i is defined as:

$$\Pi_i = p(Q) \cdot q_i - f_i(q_i - x_i) - \tau \cdot (x_i - \Phi_i(q)). \tag{7.6}$$

Firm's Optimization Problem

Each firm chooses q_i and x_i to maximize its profit:

$$\max_{q_i \geq 0,\, x_i \in [0, q_i]} \left(p\left(\sum_{j=1}^{N} q_j\right) \cdot q_i - f_i(q_i - x_i) - \tau \cdot (x_i - \Phi_i(q)) \right). \tag{7.7}$$

Market Equilibrium Conditions

In equilibrium:

1. Each firm maximizes its profit by choosing optimal q_i and x_i.
2. The permit market clears:

$$\sum_{i=1}^{N} x_i = \sum_{i=1}^{N} \Phi_i(q). \tag{7.8}$$

Regulator's Objective

The regulator's objective is to maximize the **adjusted consumer surplus** (consumer surplus minus the pollution cost):

$$\mathrm{ACS}(\Phi) = \mathrm{CS}(Q) - S(K), \tag{7.9}$$

where:

- $Q = \sum_{i=1}^{N} q_i$ is the aggregate production,
- $K = \sum_{i=1}^{N} x_i$ is the total emissions,
- $S(K)$ is a strictly increasing function representing the social cost of pollution.

7.3 Mock Data Generation

To implement and test the Uniform Linear Mechanism model, we need to generate mock data that simulate the behavior of firms in the market. We consider multiple sectors, each with its own demand function and allocation factor.

7.3.1 Assumptions for Data Generation

We make the following assumptions for generating the mock data:

- There are multiple sectors, each with a specific inverse demand function $p_s(Q_s)$.
- Each sector has its own allocation factor α_s.
- Firms have abatement cost functions of the third power to reflect quadratic marginal abatement costs.
- Firms are associated with countries, introducing geographical considerations.

7.3.2 Data Generation Procedure

We utilize object-oriented programming concepts to represent the entities in our model. The classes used are:

- **Firm**: Represents an individual firm with attributes such as name, sector, country, production cost function, abatement cost function, actual output, emission, and profit.
- **Sector**: Represents a sector with attributes such as name, price-demand function, and free emission multiplier.
- **Regulator**: Represents the regulator with attributes such as permit price and emission cap.

7.4 Implementation of the Uniform Linear Mechanism Model with Optimization

7.4.1 Optimization Framework

The optimization problem involves maximizing firms' profits while satisfying market-clearing conditions. The key steps are:

1. **Define Variables**:

- For each firm i, define the following decision variables:

$$\begin{aligned} q_i &: \text{Output produced by firm } i, \\ ab_i &: \text{Abatement level by firm } i, \\ x_i = q_i - ab_i &\quad : \text{Emissions produced by firm } i. \end{aligned}$$

2. **Formulate Profit Functions**:

   - The profit function for each firm i is given by:

   $$\Pi_i = p(Q) \cdot q_i - C_A(ab_i) - \tau \cdot (x_i - \Phi_i(q_i)), \tag{7.1}$$

   where:

     - $p(Q)$ is the price of output as a function of total market output $Q = \sum_{i=1}^{n} q_i$.
     - $C_A(ab_i)$ represents the abatement cost for firm i.
     - τ is the permit price for emissions.
     - $\Phi_i(q_i)$ denotes the baseline emissions function for firm i.

3. **Derive First-Order Conditions**:

   - To maximize profits, derive the first-order conditions by taking partial derivatives of Π_i with respect to q_i and ab_i:

   $$\frac{\partial \Pi_i}{\partial ab_i} = -C_A'(ab_i) + \tau = 0 \quad \forall i, \tag{7.2}$$

   $$\frac{\partial \Pi_i}{\partial q_i} = p'(q_i)q_i + p(q_i) - \tau + \tau\Phi_i'(q_i) = 0 \quad \forall i. \tag{7.3}$$

4. **Set Up Optimization Problem**:

   - Formulate the optimization problem using a suitable solver (e.g., Gurobi). Incorporate the first-order conditions derived in the previous step as constraints:

   $$-C_A'(ab_i) + \tau \leq 0 \quad \forall i, \tag{7.4}$$

   $$p'(q_i)q_i + p(q_i) - \tau + \tau\Phi_i'(q_i) \leq 0 \quad \forall i. \tag{7.5}$$

5. **Market-Clearing Condition**:

   - Ensure that the total emissions across all firms meet the emission cap Cap:

   $$\sum_{i=1}^{n} q_i - \sum_{i=1}^{n} ab_i = \text{Cap}. \tag{7.6}$$

6. **Solve the Problem**:

   - Utilize an optimization solver (e.g., Gurobi) to determine the equilibrium outputs q_i and abatement levels ab_i that maximize the firms' profits while satisfying all constraints.

We implement the Uniform Linear Mechanism model using the Gurobi optimizer, solving the optimization problem by setting the first-order conditions to zero.

7.4.2 Algorithm Implementation

The implementation is encapsulated in a method of the `Regulator` class. The algorithm is as follows: algorithm 4

7.5 Implementation of the Uniform Linear Mechanism Model with Modified Best Response

We also implement a modified best response algorithm, combining elements of best response dynamics and gradient descent.

7.5.1 Algorithm Overview

The algorithm iteratively adjusts the permit price and firms' outputs to meet the emission cap:

Algorithm 5: Find Optimal Permit Price

```
while Permit price interval not within tolerance do
    Set permit price τ to midpoint of interval;
    Call optimize_them_all to update firms' outputs;
    Compute total emissions;
    if Total emissions > Emission Cap then
        Adjust lower bound of permit price;
    else
        Adjust upper bound of permit price;
    end if
end while
```

7.5.2 Firms' Output Optimization

The `optimize_them_all` method updates firms' outputs using a mix of best response and gradient descent:

Algorithm 6: Optimize Firms' Outputs

```
while Not Converged do
    foreach sector do
        foreach firm in sector do
            Calculate optimal output and emission;
            Update firm's output and emission with step size a;
        end foreach
    end foreach
    Check convergence based on maximum differences;
    if Diverging then
        Adjust step size or restart;
    end if
end while
```

7.5.3 Equilibrium Testing

To validate the results, we use the `equilibrium_tester` method, which checks the first and second-order conditions for profit maximization:

1. Ensure that the first order derivatives are close enough to 0.

2. Ensure that the second order derivatives are negative (indicating a maximum)

3. Ensure that the Hessian determinant is positive.

7.6 Example Runs

We conduct simulations to demonstrate the model.

7.6.1 Scenario 1: 2 Sectors with only one receiving free allocation

In this scenario, we'll explore the dynamics of a small economy where the Regulator grows increasingly biased, handing out free allocation permits to only one sector while forcing the other to purchase theirs. Needless to 'write', these results should be taken as purely qualitative insights rather than actual quantities. All the plots present the mean of the first and second sectors (normalized to 1 company). The x-axis is always the percentage of free allocation sector 1 receives as a percentage of its output, which means that at 100%, there might be permits to sell as well.

Data Overview

Table 7.1: Regulator Information for ULM scenario 1

Regulator Name	Emission Cap	Permit Price
Regulator	80% of BAU emissions	to be determined by equilibrium

Table 7.2: Sector Information for ULM scenario 1

Sector Name	**Price Demand Function**	**Free Emission Multiplier**
Cement	$p(x) = 100 - 0.1x$	from 0 to 1
Steel	$p(x) = 150 - 0.1x$	0

Table 7.3: Firm Information for ULM scenario 1

Firm Name	**Sector**	**Firm ID**	**Abatement Cost Function**
Firm1	Cement	1	$10x + 2x^2 + 0.1x^3$
Firm2	Cement	2	$11x + 3x^2 + 0.2x^3$
Firm3	Cement	3	$5x + 4x^2 + 5x^3$
Firm4	Steel	1	$7x + 5x^2 + 3x^3$
Firm5	Steel	2	$1x + 6x^2 + 2x^3$
Firm6	Steel	3	$2x + 7x^2 + 3x^3$

Results and Observations

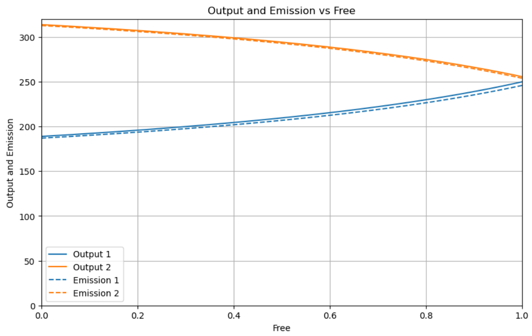
Qualitative Observations

1. **Abatement Costs**

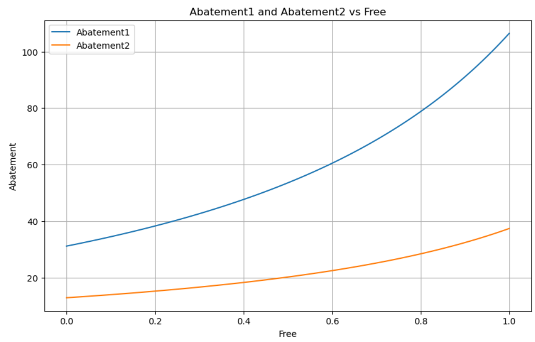
    - Sector 1's (Cement) abatement cost increases significantly as the percentage of free allocation rises, suggesting that the surplus permits might allow higher emissions or increased production flexibility.
    - Sector 2 (Steel), receiving no free permits, has a slower increase in abatement, possibly limited by the need to purchase permits, reducing available funds for abatement.

2. **Money Spent on Permits**

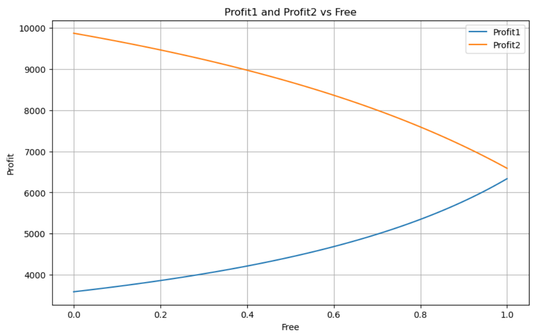
    - Sector 1's permit spending decreases as free allocations increase, eventually reaching zero at 100% allocation.
    - Sector 2 must continuously purchase permits, with expenditure rising as Sector 1's allocation increases, suggesting that the permit price equilibrium may shift due to the allocation imbalance.



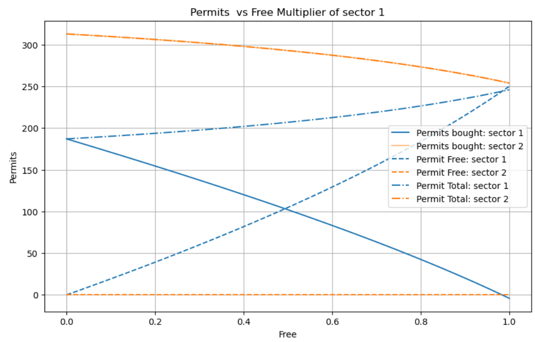
(a) Output and Emissions



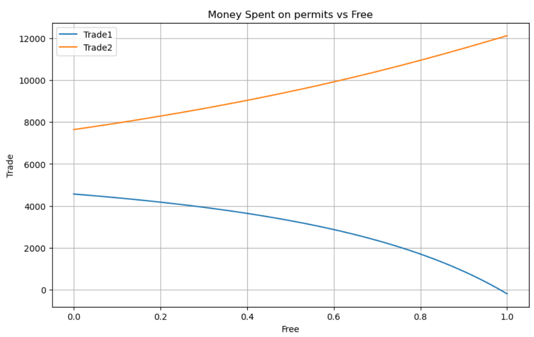
(b) Abatement



(c) Profits



(d) The number of permits present



(e) Money spent on permits

(f) Permit price

Figure 7.1: ULM simulation Scenario 1, x-axis represents the percentage of free allocation of sector 1

3. **Output and Emissions**

    - Sector 1's output increases with more free permits, likely due to reduced permit costs allowing greater production. Emissions for Sector 1 increase proportionally, possibly due to a cap or marginal emission costs.
    - Sector 2's output and emissions decrease with increased allocation to Sector 1, indicating a potential competitive disadvantage from the need to buy permits.

4. **Permit Allocation**

    - Sector 1's permits bought decrease as free allocation rises, while free and total permits increase, giving more operational flexibility.
    - Sector 2 has stable permit purchases but no free permits, leading to competitive asymmetry that could distort market dynamics.

5. **Permit Price**

    - Permit price increases with Sector 1's free allocation, possibly due to heightened demand pressure on Sector 2 for permits, indicating a market price sensitivity to permit distribution.

6. **Profit Analysis**

    - Sector 1's profit increases with free allocations due to cost savings on permits, which may be reinvested in production and abatement.
    - Sector 2's profit decreases as Sector 1's allocation rises, likely due to increased permit costs and limited flexibility, reflecting a market disadvantage.

Summary This scenario suggests that biased allocation of free permits favors Sector 1, enhancing its production and reducing permit costs. Sector 2 faces higher costs and reduced profits, likely due to competitive disadvantage. Rising permit prices further intensify this imbalance, hinting at a need for regulatory adjustments to stabilize competition.

7.6.2 Scenario 2: Repetition of Scenario 1 at different Emission Caps

In this scenario, we repeat the scenario 1, on different caps. Specifically, we analyze the outcomes on profits and other variables across different cap levels (70%, 80%, and 90% of BAU emissions).

The setup includes:

- **Emission Caps**: 70%, 80%, and 90% of BAU emissions
- **Total Firms**: 9, across 3 sectors
- **Sectors**:

- Sector 1: Receives gradually increasing free permits, up to its production level (q).
- Sector 2: Receives no free permits.
- Sector 3: A much larger sector in terms of sales.

Data Overview

Table 7.4: Regulator Information for ULM scenario 2

Regulator Name	Emission Cap	Permit Price
Regulator22	Variable (70%, 80%, 90% of BAU emissions)	Determined by equilibrium

Table 7.5: Sector Information for ULM scenario 2

Sector Name	**Price Demand Function**	**Free Emission Multiplier**
Cement	$p(x) = 100 - 0.1x$	Varies from 0 to production (q)
Steel	$p(x) = 150 - 0.1x$	0
Paper	$p(x) = 200 - 0.02x^{1.5}$	0

Table 7.6: Firm Information for ULM scenario 2

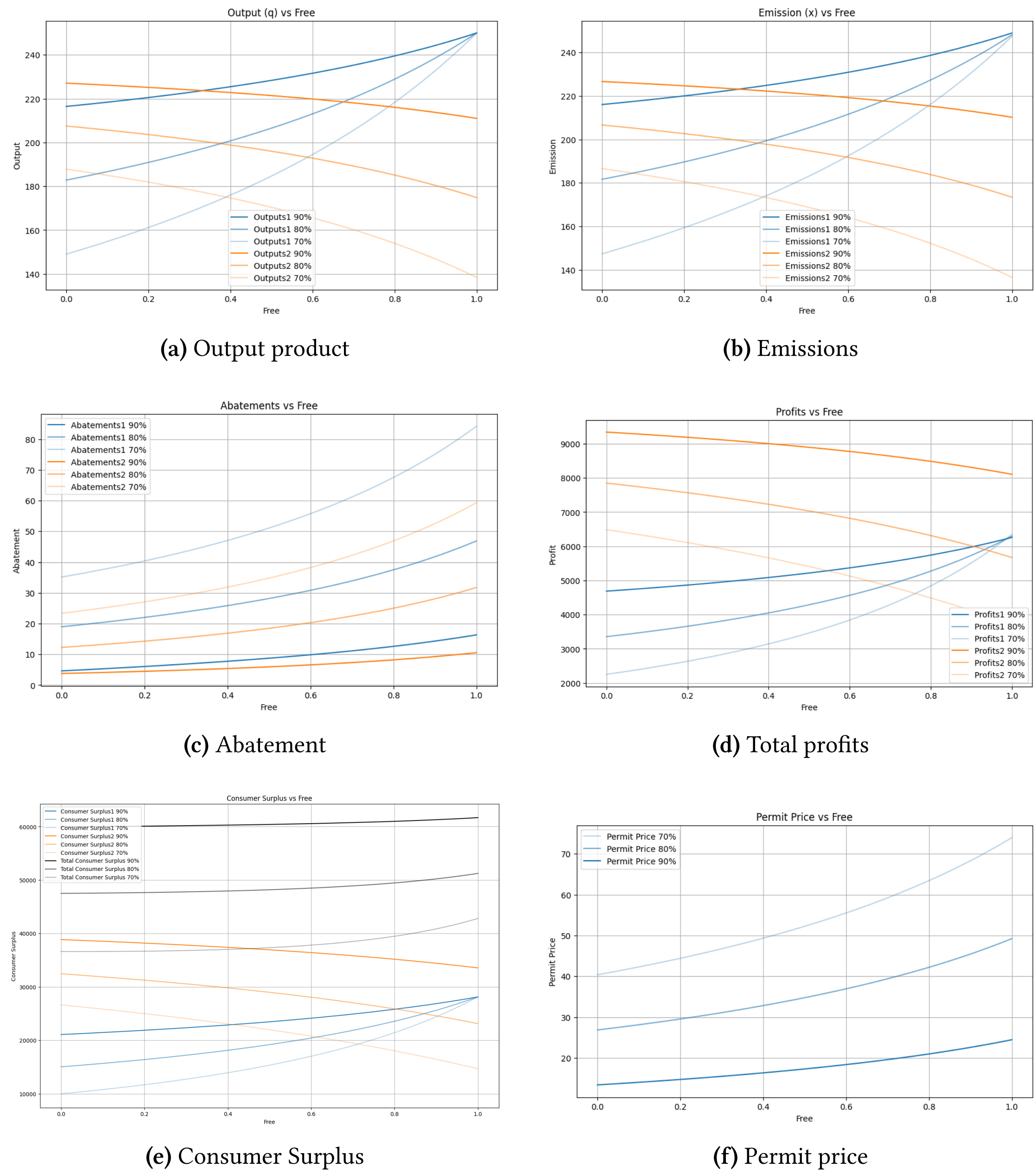
Firm Name	**Sector**	**Firm ID**	**Abatement Cost Function**
Firm1	Cement	1	$10x + 2x^2 + 2x^3$
Firm2	Cement	2	$11x + 3x^2 + 2x^3$
Firm3	Cement	3	$5x + 4x^2 + 5x^3$
Firm4	Steel	4	$7x + 5x^2 + 3x^3$
Firm5	Steel	5	$1x + 6x^2 + 2x^3$
Firm6	Steel	6	$2x + 7x^2 + 3x^3$
Firm7	Paper	7	$3x + 8x^2 + 4x^3$
Firm8	Paper	8	$4x + 9x^2 + 10x^3$
Firm9	Paper	9	$5x + 10x^2 + 11x^3$

Results and Key Observations

Key Observations on Emission Caps (70%, 80%, and 90% of BAU)

1. **Abatement and Costs**

   - The 70% cap requires the most abatement and leads to higher regulatory costs, as firms must invest more to comply with the strict emission limits.



(a) Output product

(b) Emissions

(c) Abatement

(d) Total profits

(e) Consumer Surplus

(f) Permit price

Figure 7.2: ULM simulation Scenario 2, x-axis represents the free allocation of sector 1

- As the cap increases to 80% and 90%, abatement requirements and regulatory costs decrease, providing firms with greater operational flexibility.

2. **Consumer and Market Impact**

   - Consumer surplus and output increase with higher caps, as firms face fewer production constraints and can meet more market demand. The 90% cap yields the highest consumer surplus. It is important to mention here that here we do not account for the damage from the emissions.

3. **Permit Prices and Profits**

   - Permit prices are highest under the 70% cap due to higher demand for scarce allowances, and they decrease with higher caps.
   - Profits improve as the cap increases, with the 90% cap maximizing profitability by reducing regulatory costs and enabling higher output.

Summary The 70% cap imposes stringent emission limits, resulting in high regulatory costs, lower consumer surplus, and limited profits. Looser caps (80% and 90%) allow firms to operate with reduced regulatory burdens, leading to greater output, higher consumer surplus, and maximized profits under the 90% cap.

7.6.3 Scenario 3: Varying Emission Cap

In this scenario, we investigate the effects of varying the emission cap from 0% to 100% of Business as Usual (BAU) emissions on profits and other metrics across different sectors within an economy. This experiment aims to understand how progressively relaxed emission caps influence firm behavior.

The setup includes:

- **Emission Cap**: Ranges from 0% to 100% of BAU emissions
- **Total Firms**: 9, distributed across 3 sectors
- **Sectors**:
  - Sector 1: Cement
  - Sector 2: Steel
  - Sector 3: Paper, which is significantly larger in terms of sales compared to the other sectors

Data Overview

Table 7.7: Regulator Information for ULM scenario 3

Regulator Name	Emission Cap	Permit Price
Regulator26	Variable (0% to 100% of BAU emissions)	Determined by equilibrium

Table 7.8: Sector Information for ULM scenario 3

Sector Name	**Price Demand Function**	**Free Emission Multiplier**
Cement	$p(x) = 100 - 0.1x$	0
Steel	$p(x) = 150 - 0.1x$	0
Paper	$p(x) = 200 - 0.02x^{1.5}$	0

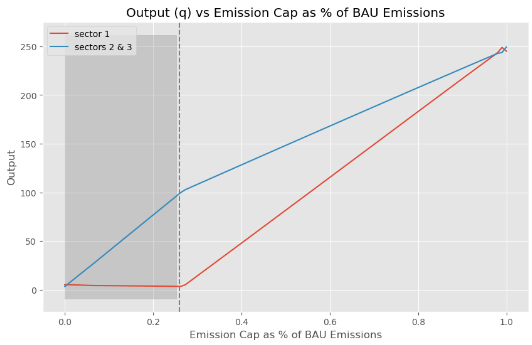
Table 7.9: Firm Information for ULM scenario 3

Firm Name	**Sector**	**Firm ID**	**Abatement Cost Function**
Firm1	Cement	1	$2x + 2x^2 + 2x^3$
Firm2	Cement	2	$3x + 3x^2 + 2x^3$
Firm3	Cement	3	$1x + 3x^2 + 2x^3$
Firm4	Steel	1	$7x + 5x^2 + 3x^3$
Firm5	Steel	2	$1x + 6x^2 + 2x^3$
Firm6	Steel	3	$2x + 7x^2 + 3x^3$
Firm7	Paper	1	$3x + 8x^2 + 4x^3$
Firm8	Paper	2	$4x + 9x^2 + 10x^3$
Firm9	Paper	3	$5x + 10x^2 + 11x^3$

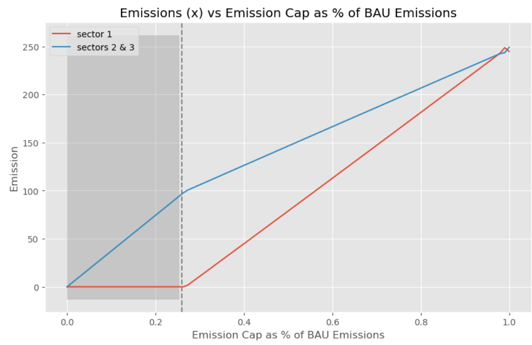
Results and Observations

Key Observations on Emission Caps from 0% to 100% of BAU Emissions

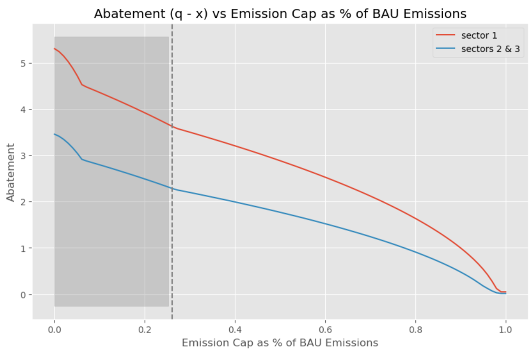
1. **Abatement and Costs**
   - At lower cap levels (closer to 0% of BAU), firms incur high abatement costs as they must significantly reduce emissions to comply.
   - As the cap increases towards 100%, both abatement efforts and costs decline, as firms are allowed to emit closer to their BAU levels.
2. **Emissions, Permit Price, and Regulatory Costs**
   - Emissions steadily increase with a higher cap, reaching near-BAU levels as the cap approaches 100%.



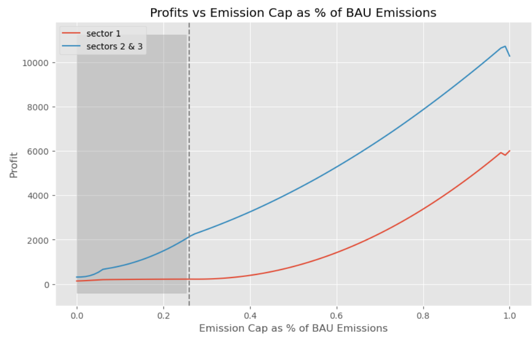
(a) Output product



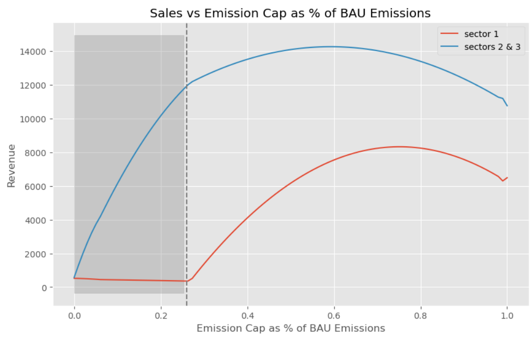
(b) Emissions



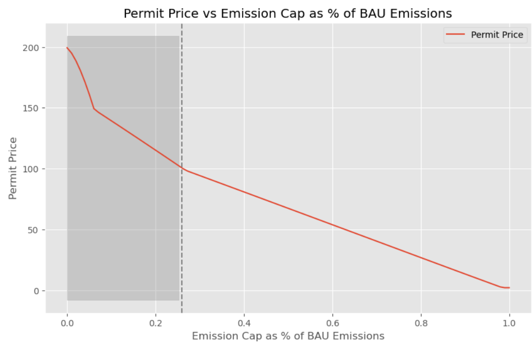
(c) Abatement



(d) Total profits



(e) Revenue purely from sales



(f) Permit price

Figure 7.3: ULM simulation Scenario 3, x-axis represents the Emission Cap as a Percentage of BAU Emissions.

- Permit prices are highest at low cap levels, reflecting scarcity, but decrease as the cap loosens, reducing firms' regulatory expenses.

3. **Sales Revenue and Profitability**

    - Sales revenue peaks at an intermediate cap level, where firms produce at prices closer to monopoly levels, maximizing revenue per unit.
    - Total profits, however, continue to increase with the cap, as lower regulatory costs and reduced permit spending outweigh any gains from monopoly pricing, leading to maximum profitability at a 100% cap.

Summary This scenario demonstrates that stricter caps drive up regulatory costs and abatement, constraining profits despite higher sales revenue per unit. As the cap loosens, firms benefit from reduced costs and greater flexibility, with maximum profitability achieved at 100% of BAU emissions, balancing economic output and regulatory expenses.

7.7 Conclusion

This chapter examined the Uniform Linear Mechanism for emission permit allocation, detailing its structure and implementation. The next chapter will compare this mechanism with the previously proposed allocation system.

Algorithm 4: Optimization Concave Formulation with Abatement Constraints

Initialize the Gurobi model for optimization;
Define symbolic and Gurobi variables for each firm's output q_i, emissions x_i, and abatement ab_i;
foreach *firm* i **do**
  Define symbolic variables q_i, x_i, and ab_i in SymPy for firm i;
  Create Gurobi variables qq_i (output) and ab_i (abatement) with lower bounds;
  Map SymPy variables to Gurobi variables for consistent use in the model;
  Add constraints $q_i \geq ab_i$ and $ab_i \geq 0$ to the Gurobi model;
end foreach
Define the permit price variable τ as both symbolic and Gurobi variables, with a lower bound;
Initialize the objective function as zero;
foreach *firm* i **do**
  Calculate the revenue for firm i based on sector demand and total output;
  Calculate the abatement cost function for firm i;
  Compute the trading cost based on permit price τ and the firm's abatement;
  Set firm i's profit as the sum of its revenue, abatement cost, and trading cost;
  Derive the first-order conditions with respect to q_i (output) and ab_i (abatement);
  Add these conditions as constraints to the Gurobi model, ensuring non-negativity for optimality;
  Update the objective function by subtracting the first-order conditions for each firm;
end foreach
Add the market-clearing constraint:

$$\sum_i (q_i - ab_i) = \text{Emission Cap}$$

Set the objective function to minimize the negative sum of the first-order conditions, promoting equilibrium;
Solve the optimization problem using Gurobi;
if *solution is optimal* **then**
  Output the results for each firm's output, emissions, and permit price;
end if
if *print output is requested* **then**
  Print each firm's final output and emissions, and the optimal permit price;
end if
return the optimized model m;

Chapter 8

Comparison of Allocation Models

8.1 Problem Definition

In this chapter, we compare two models for allocating emission permits within the European Union Emissions Trading System (EU ETS): the Uniform Linear Mechanism (ULM) and our proposed optimization model. The ULM, as discussed in the literature, is proven to be optimal under certain assumptions, while our model seeks to incorporate additional sector-specific and country-specific measures to enhance the allocation process.

The primary objective is to assess how each model performs in allocating permits when dealing with multiple sectors and countries, all within the same regulatory framework. Both models aim to distribute emission allowances efficiently and fairly, but they differ in their methodologies and underlying assumptions. By comparing these models, we aim to identify the strengths and limitations of each approach and provide insights into their practical applicability in the EU ETS.

8.2 Common Data and Parameters

To facilitate a comprehensive comparison between the Uniform Linear Mechanism (ULM) and our proposed optimization model, it is essential to establish a common dataset and set of parameters. This ensures that both models operate under identical conditions, allowing for an accurate assessment of their respective performances. Below, we outline the key components of the dataset, including the regulator, countries, sectors, and firms involved in the European Union Emissions Trading System (EU ETS).

8.2.1 Limitations

t is essential to emphasize that we do not assert the realism of the mock data used in this study. We aim to simulate an economy to understand its dynamics. This is not a quantitative analysis.

8.2.2 Regulator

The EU ETS is governed by a single regulator responsible for overseeing the allocation of emission permits, monitoring compliance, and ensuring the overall effectiveness of the system. In our models, we represent the regulator as follows:

- **Regulator Name**: ETS

8.2.3 Countries

The allocation models consider five countries, each characterized by their GDP per capita and the percentage of their economy dedicated to industry. These attributes influence the allocation of emission permits, reflecting the economic capacity and industrial intensity of each country.

Table 8.1: Country Attributes

Country	GDP per Capita	Industry Percentage (%)
Atlantis	400	30
Omashu	350	25
Hogsmeade	450	20
The Court of Miracles	300	15
Lilipoupoli	250	10

The five countries considered in this analysis range from highly industrialized to less industrialized economies. Their GDP per capita and industrial percentage are as follows:

- **Atlantis**: The most industrialized country with a GDP per capita of 400 and 30% of its economy dedicated to industry.
- **Omashu**: A moderately industrialized country with a GDP per capita of 350 and 25% industrial economy.
- **Hogsmeade**: The wealthiest country with the highest GDP per capita of 450 and 20% industrial economy.
- **The_Court_of_Miracles**: A less industrialized country with a GDP per capita of 300 and 15% industrial economy.
- **Lilipoupoli**: The least industrialized country with a GDP per capita of 250 and 10% industrial economy.

8.2.4 Sectors

The models encompass six sectors, each with a distinct price-demand function that determines the relationship between the price of the product and the quantity demanded. These functions are crucial for simulating market dynamics and the economic impacts of emission permit allocations.

Table 8.2: Sector Information

Sector Name	Price-Demand Function	Free Emission Multiplier
Steel	$p(x) = 200 - 0.1x$	Varies per model
Cement	$p(x) = 150 - 0.05x$	Varies per model
Paper	$p(x) = 100 - 0.02x$	Varies per model
Chemicals	$p(x) = 250 - 0.15x$	Varies per model
Automotive	$p(x) = 300 - 0.2x$	Varies per model
Textiles	$p(x) = 80 - 0.01x$	Varies per model

8.2.5 Firms

A total of thirty firms operate across the six sectors and five countries. Each firm is characterized by its name, sector, country of operation, and abatement cost function. The abatement cost function represents the cost incurred by a firm to reduce emissions and is modeled as a cubic function to capture the quadratic marginal abatement costs, as proposed [18], and implemented by [25], [10].

Table 8.3: Firm Information

Firm Name	Sector	Country	Abatement Cost Function
Sector 1: Steel			
S1_F1	Steel	Atlantis	$2x + 3x^2 + x^3$
S1_F2	Steel	Atlantis	$3x + 2x^2 + 2x^3$
S1_F3	Steel	Omashu	$4x + x^2 + 3x^3$
S1_F4	Steel	Hogsmeade	$2x + 2x^2 + 2x^3$
S1_F5	Steel	The_Court_of_Miracles	$3x + 3x^2 + x^3$
Sector 2: Cement			
S2_F1	Cement	Atlantis	$x + 2x^2 + 3x^3$
S2_F2	Cement	Atlantis	$2x + x^2 + 2x^3$
S2_F3	Cement	Omashu	$3x + x^2 + x^3$
S2_F4	Cement	Hogsmeade	$x + 3x^2 + x^3$
S2_F5	Cement	The_Court_of_Miracles	$2x + 2x^2 + x^3$
Sector 3: Paper			
S3_F1	Paper	Hogsmeade	$x + x^2 + x^3$
S3_F2	Paper	The_Court_of_Miracles	$x + 2x^2 + x^3$
S3_F3	Paper	Lilipoupoli	$x + x^2 + 2x^3$
S3_F4	Paper	Omashu	$2x + x^2 + x^3$
S3_F5	Paper	Atlantis	$x + 2x^2 + 2x^3$
Sector 4: Chemicals			
S4_F1	Chemicals	Atlantis	$3x + 4x^2 + x^3$
S4_F2	Chemicals	Atlantis	$4x + 3x^2 + 2x^3$
S4_F3	Chemicals	Omashu	$2x + 5x^2 + x^3$

Firm Name	Sector	Country	Abatement Cost Function
S4_F4	Chemicals	Hogsmeade	$3x + 3x^2 + 3x^3$
S4_F5	Chemicals	The_Court_of_Miracles	$2x + 2x^2 + 4x^3$
Sector 5: Automotive			
S5_F1	Automotive	Atlantis	$5x + 4x^2 + x^3$
S5_F2	Automotive	Omashu	$4x + 5x^2 + 2x^3$
S5_F3	Automotive	Hogsmeade	$3x + 3x^2 + 2x^3$
S5_F4	Automotive	The_Court_of_Miracles	$2x + 4x^2 + 3x^3$
S5_F5	Automotive	Lilipoupoli	$x + 3x^2 + 4x^3$
Sector 6: Textiles			
S6_F1	Textiles	Lilipoupoli	$x + x^2 + x^3$
S6_F2	Textiles	The_Court_of_Miracles	$x + 2x^2 + x^3$
S6_F3	Textiles	Hogsmeade	$x + x^2 + 2x^3$
S6_F4	Textiles	Omashu	$2x + x^2 + x^3$
S6_F5	Textiles	Atlantis	$x + 2x^2 + 2x^3$

8.2.6 Common Parameters

Both allocation models operate under the following shared parameters:

1. **Emission Cap**: The total emission cap is set at 80% of the business-as-usual (BAU) emissions.

2. **Free Allocation Percentage**: 40% of the emission cap is allocated as free permits to firms.

3. **Permit Allocation Mechanism**: Both models distribute permits based on production levels and sector-specific factors, ensuring that allocations are reflective of industrial activity and economic capacity.

4. **Common Environmental Damage function**: The most important benefit of the common emission cap is the same damage introduced to the environment. This enables us to compare freely the models, using other metrics like consumer surplus, as the environmental damage cancels out.

8.3 New Assumptions for the Uniform Linear Mechanism Model

While the ULM provides a structured approach to permit allocation, the original formulation does not address the complexities introduced by multiple sectors. specifically, in the section 5 of [26] suggests using a non-uniform linear allocation, distributing more permits to firms with higher marginal production cost. On our model the production cost is hidden inside the demand function of the sector (as we assumed fixed and uniform marginal production cost inside each sector). To adapt the ULM to our multi-sector context, we introduce the following new assumptions:

1. **Sector-Specific Allocation Factors**: Since the original ULM does not specify how to allocate the free emission multiplier across multiple sectors, we assume that each sector can have its own allocation multiplier α_s.
2. **Optimization of Multipliers**: We formulate an optimization problem to determine the optimal combination of sector-specific multipliers α_s that maximizes consumer surplus while ensuring that the total free allocation remains at 40% of the emission cap.
3. **Objective Function**: The optimization aims to maximize the consumer surplus.
4. **Constraints**:
    - The sum of free allocations across all sectors must equal 40% of the emission cap.
    - $a_s \in [0, 1)$

These assumptions enable us to extend the ULM to a multi-sector setting and make it suitable for comparison with our proposed model.

8.4 New Assumptions and Limitations for the Proposed Optimization Model

Our proposed model incorporates a temporal component to incentivize firms to increase production and invest in abatement technologies over time. However, for this comparison, we focus on a single-period analysis. This limits out proposed model, since we cannot take advantage of the efficiency incentives it offers. Having this in mind, we can still compare the allocation and the models, even though the proposed one is slightly handicapped. The new assumptions and limitations are as follows:

1. **Single-Period Analysis**: We consider only one time period, acknowledging that without the temporal component, firms may have limited incentives for long-term abatement or production increases.
2. **Baseline Allocation Establishment**:
    - We use the ULM with zero free allocation to determine the baseline efficiency of firms based solely on their abatement capabilities.
    - This provides a reference point to assess how the allocation of free permits affects firm behavior in our model.
3. **Economic Indicators Computation**:
    - In our data creation, we assumed that we know the percentage the industry sales influence the GDP. Given that, GDP can be split into Industry, Services, and Agriculture.

- We calculate the GDP and population of each country using the sales data of companies, the industrialization percentage, and GDP per capita.
- These indicators are used to incorporate country-specific economic factors into the allocation mechanism.
- The proper way to calculate GDP is GDP (Y) is the sum of consumption (C), investment (I), government Expenditures (G) and net exports (X – M). [51]

$$Y = C + I + G + (X{-}M)$$

In our example, we assume that GDP is equal or at least proportional to C. With this assumption, we can substitute GDP with sales from firms.

4. **Objective function simplification**:

- As stated before, we substitute $GDP_{i,j}$ (The GDP produced by sector j in country i with. $\text{sales}_{i,j}$.
- PPS correction makes no sense for our data, since it's generated data, we could assume that the PPS is "hidden" in the GDP or the industrialization of each country.

$$\text{maximize} \quad Z = \sum_{i \in C} \sum_{j \in S} v_{i,j} \cdot \frac{\text{GDP}_{i,j}}{\text{Emission}_{i,j}} \cdot PPS_i \tag{8.1}$$

turns into:

$$\text{maximize} \quad Z = \sum_{i \in C} \sum_{j \in S} v_{i,j} \cdot \frac{\text{Sales}_{i,j}}{\text{Emission}_{i,j}} \tag{8.2}$$

5. **Scenario Analysis**:

- We run the optimization model under three different scenarios to explore how various allocation principles impact the distribution of permits.
- Each scenario adjusts certain parameters or constraints to reflect different fairness or efficiency considerations.

8.4.1 Limitations

The absence of a dynamic, multi-period analysis limits the ability to capture long-term incentives and investments in abatement technologies. Data limitations may affect the precision of economic indicators and the resulting allocation outcomes.

The equilibrium for both models is calculated in the same way.

- Set the first-order conditions equal to 0:

$$-C_A'(ab_i) + \tau = 0 \quad \forall i, \tag{8.1}$$

$$p'(q_i)q_i + p(q_i) - \tau + \tau\Phi_i'(q_i) = 0 \quad \forall i. \tag{8.2}$$

- Market-Clearing Condition: Ensure that the total emissions across all firms meet the emission cap Cap:

$$\sum_{i=1}^{n} q_i - \sum_{i=1}^{n} ab_i = \text{Cap}. \tag{8.3}$$

The issue arises with the term

$$\tau\Phi_i'(q_i)$$

, as in our comparison, free allocation is treated as a fixed value rather than a function of q_i. Consequently, the equilibrium solution becomes independent of the free allocation. This represents a significant limitation of the model, stemming from the absence of a temporal component (e.g., incorporating multiple periods). This constraint severely impacts the model's efficiency, as it eliminates any incentive for firms to increase abatement efforts or production levels.

These assumptions allow us to adapt our model for a fair comparison with the ULM while acknowledging the inherent limitations of a single-period analysis.

8.5 Experimental Design

To compare the two models effectively, we design a series of experiments that apply both allocation mechanisms under identical conditions. The experiments are structured as follows:

8.5.1 Optimization Model Scenarios

To evaluate the performance of our proposed optimization model, we consider three distinct scenarios: Core Constraints, Historical Deviation Bounds, and Inverse GDP per Capita Adjustment. Each scenario introduces specific constraints to balance efficiency and fairness in allocating emission permits.

1. **Scenario 1: Core Constraints**

   In the core scenario, the model optimizes the allocation of emission permits based solely on the efficiency of firms and sectors without additional fairness considerations. The core is in accordance with the Fitness Principle of Moulin 4.

$$\text{maximize} \quad Z = \sum_{i \in C} \sum_{j \in S} v_{i,j} \cdot \frac{\text{Sales}_{i,j}}{\text{Emission}_{i,j}} \tag{8.1}$$

$$\sum_{i \in C} \sum_{j \in S} v_{i,j} = 1 \tag{8.2}$$

$$v_i = \sum_{j \in S} v_{i,j} \quad \forall i \in C \tag{8.3}$$

$$v_j = \sum_{i \in C} v_{i,j} \quad \forall j \in S \tag{8.4}$$

Here, $v_{i,j}$ represents the proportion of free permits allocated to country i and sector j, $\text{Sales}_{i,j}$ denotes the sales of firms in country i and sector j, and $\text{Emission}_{i,j}$ is the total emissions from country i and sector j.

2. **Scenario 2: Historical Deviation Bounds**

   This scenario builds upon the core constraints by introducing bounds based on historical emission data to ensure a fair distribution relative to each country-sector pair's emission share. This scenario introduces a constraint that aligns with the Compensation Principle outlined by Moulin 1.

   - **Coefficient Definition**:
   $$a_1 = 0.5, \quad a_2 = 1.5$$
   - **Sum of Emissions**:
   $$\text{Sum\_of\_Emissions} = \sum_{k \in C} \sum_{l \in S} \text{Emission}_{k,l}$$
   - **Emissions per (Country, Sector)**:
   $$\text{Emission}_{i,j} = \sum_{\text{firm} \in (i,j)} \text{Emission}_{\text{firm}}$$
   - **Constraints**:
   $$a_1 \times \frac{\text{Emission}_{i,j}}{\text{Sum\_of\_Emissions}} \leq v_{i,j} \leq a_2 \times \frac{\text{Emission}_{i,j}}{\text{Sum\_of\_Emissions}} \quad \forall i \in C, \forall j \in S \tag{8.5}$$

   These constraints ensure that the allocation $v_{i,j}$ for each country-sector pair remains within 50% to 150% of its historical emission share, promoting a balanced distribution of permits.

3. **Scenario 3: Inverse GDP per Capita Adjustment**

   The final scenario incorporates economic capacity by adjusting allocations based on the inverse GDP per capita of each country-sector pair, introducing a small deviation to further enhance fairness. This adheres to the "Exogenous Rights" Principle as outlined by Moulin 3. .

- **Coefficient Definition**:

$$a_3 = 0.25, \quad a_4 = 4$$

- **Inverse GDP per Capita Calculation**:

$$\text{Inverse\_GDP}_{i,j} = \frac{\text{Population}_i}{\sum \text{Sales}_{i,j}}$$

$$\text{Sum\_of\_Inverse\_GDP} = \sum_{i \in C} \sum_{j \in S} \text{Inverse\_GDP}_{i,j}$$

- **Proportion Calculation**:

$$\text{Proportion}_{i,j} = \frac{\text{Inverse\_GDP}_{i,j}}{\text{Sum\_of\_Inverse\_GDP}}$$

- **Constraints**:

$$a_3 \times \text{Proportion}_{i,j} \leq v_{i,j} \leq a_4 \times \text{Proportion}_{i,j} \quad \forall i \in C, \forall j \in S \tag{8.6}$$

These constraints ensure that the allocation $v_{i,j}$ is proportional to the inverse GDP per capita of each country-sector pair, within a relaxed bound of 25% to 400% of the calculated proportion. This adjustment accounts for the economic capacity of each country, promoting equitable support for less affluent nations.

8.5.2 Uniform Linear Mechanism Optimization

1. **Optimization of Sector Multipliers**

    - Implement an optimization problem to find the optimal set of sector-specific free emission multipliers α_s.
    - The objective is to maximize consumer surplus while keeping the total free allocation at 40% of the emission cap.

2. **Comparative Runs**

    - Run the ULM with the optimized multipliers and compare the results with those from the proposed model under each scenario.

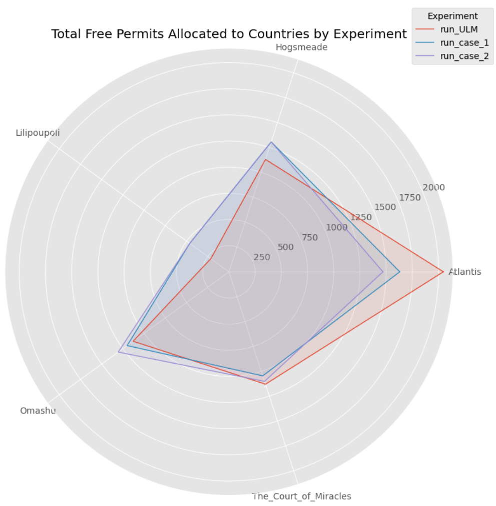
8.6 Results

8.6.1 Free Permits allocation

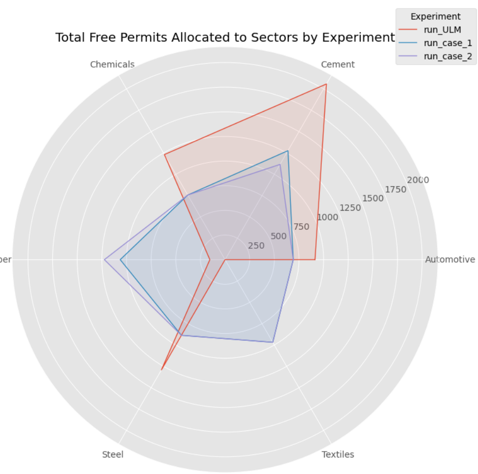
Table 8.4: Free Permits Allocation by Sector and Experiment

Execution sector name	ULM	Base Run	Case 1	Case 2
Automotive	912.62	0.00	691.95	691.95
Cement	2058.39	0.00	1276.81	1115.99
Chemicals	1231.22	0.00	756.85	756.85
Paper	154.38	0.00	1064.02	1224.85
Steel	1291.77	5644.44	886.46	886.46
Textiles	0.00	0.00	968.34	968.34
Total	5648.37	5644.44	5644.44	5644.44

This Table 8.4 illustrates that the Base Run of the Linear Problem is ineffective, as it allocates all permits to a single sector in one country, resulting in disproportionately high profit gains for that sector. This scenario serves only as a baseline. In the following sections, we will examine the outcomes of the remaining three cases.



(a) Per Country



(b) Per Sector

Figure 8.1: Free Allocation

Table 8.5: Free Permits Allocation by Country and Experiment

Execution Country name	ULM	Case 1	Case 2
Atlantis	2045.95	1630.30	1469.48
Hogsmeade	1129.67	1305.85	1305.85
Lilipoupoli	213.40	462.08	462.08
Omashu	1129.67	1200.23	1305.69
The Court of Miracles	1129.67	1045.98	1101.34
Total	5648.37	5644.44	5644.44

8.6.2 Profits

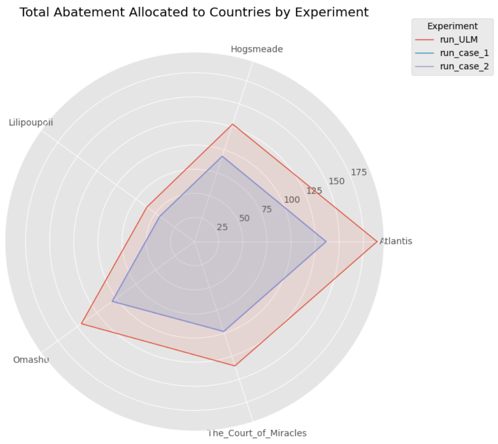
Table 8.6: Sum of Profits by Country and Experiment

Execution Country name	ULM	Case 1	Case 2
Atlantis	94972.42	119532.95	116030.03
Hogsmeade	61266.54	84828.76	84828.76
Lilipoupoli	27536.56	38794.96	38794.96
Omashu	61271.82	82529.70	84826.75
The Court of Miracles	61266.45	79167.22	80373.09
Total	306313.79	404853.59	404853.59

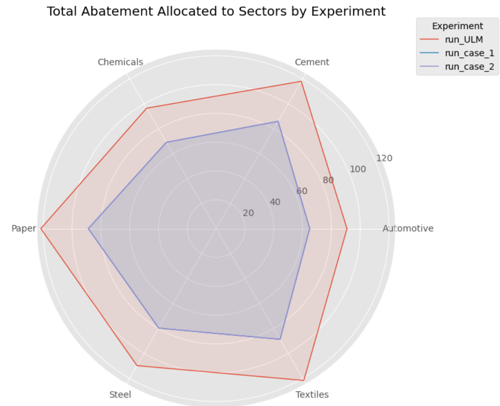
Table 8.7: Sum of Profits by Sector and Experiment

Execution Sector name	ULM	Case 1	Case 2
Automotive	59791.26	68890.95	68890.95
Cement	59262.95	73586.37	70083.45
Chemicals	56694.04	64782.43	64782.43
Paper	38611.60	65789.53	69292.46
Steel	52687.59	63510.38	63510.38
Textiles	39266.36	68293.93	68293.93
Total	306313.79	404853.59	404853.59

8.6.3 Abatement



(a) Per Country



(b) Per Sector

Figure 8.4: Abatement

Table 8.8: Sum of Abatement by Country and Experiment

Execution Country name	ULM	Case 1	Case 2
Atlantis	189.17	136.43	136.43
Hogsmeade	128.02	93.00	93.00
Lilipoupoli	60.93	44.46	44.46
Omashu	144.85	105.39	105.39
The Court of Miracles	135.53	97.98	97.98
Total	658.49	477.25	477.25

Table 8.9: Sum of Abatement by Sector and Experiment

Execution Sector name	ULM	Case 1	Case 2
Automotive	90.81	64.95	64.95
Cement	118.06	85.99	85.99
Chemicals	96.49	69.15	69.15
Paper	121.67	88.72	88.72
Steel	109.79	79.73	79.73
Textiles	121.67	88.72	88.72
Total	658.49	477.25	477.25

8.6.4 Output

(a) Per Country

(b) Per Sector

Figure 8.6: Output

Table 8.10: Sum of Actual Output by Country and Experiment

Execution Country name	ULM	Case 1	Case 2
Atlantis	3918.61	3810.01	3810.01
Hogsmeade	2833.49	2832.00	2832.00
Lilipoupoli	1748.38	1854.00	1854.00
Omashu	2833.49	2832.00	2832.00
The Court of Miracles	2833.49	2832.00	2832.00
Total	14167.47	14160.02	14160.02

Table 8.11: Sum of Actual Output by Sector and Experiment

Execution Sector name	ULM	Case 1	Case 2
Automotive	1221.60	1159.25	1159.25
Cement	2431.13	2136.99	2136.99
Chemicals	1373.39	1267.88	1267.88
Paper	3099.61	3259.14	3259.14
Steel	1621.05	1485.16	1485.16
Textiles	4420.70	4851.60	4851.60
Total	14167.47	14160.02	14160.02

8.6.5 Consumer Surplus

Experiment Sector Name	Output ULM	Output Case 1	Output Case 2	CS ULM	CS Case 1	CS Case 2
Automotive	1,221.60	1,159.25	1,159.25	149,229.77	134,385.33	134,385.33
Cement	2,431.13	2,136.99	2,136.99	147,759.76	114,167.88	114,167.88
Chemicals	1,373.39	1,267.88	1,267.88	141,464.88	120,564.87	120,564.87
Paper	3,099.61	3,259.14	3,259.14	96,075.96	106,219.62	106,219.62
Steel	1,621.05	1,485.16	1,485.16	131,389.41	110,285.07	110,285.07
Textiles	4,420.70	4,851.60	4,851.60	97,712.85	117,690.29	117,690.29
Total	14,167.47	14,160.02	14,160.02	763,632.63	703,313.05	703,313.05

8.7 Key Observations

- **Total Economic Output**
  - The total economic output remains virtually unchanged across all experiments; this aligns with our expectations as the cap is the same, and the higher abatement on ULM is not enough to justify a big change.:
    * **ULM**: 14,167.47
    * **Case 1 & Case 2**: 14,160.02
- **Consumer Surplus (CS)**
  - ULM achieves a higher total consumer surplus compared to Case 1 and Case 2:
    * **ULM**: 763,632.63
    * **Case 1 & Case 2**: 703,313.05
- **Abatement Levels**

  - ULM results in greater total abatement. ULM incentivizes abatement, and it's clear:
    * **ULM**: 658.49
    * **Case 1 & Case 2**: 477.25

- **Profits**
  - Total profits are significantly lower under ULM:
    * **ULM**: 306,313.79
    * **Case 1 & Case 2**: 404,853.59

- **Free Permits Allocation**
  - The total number of free permits allocated is consistent across all experiments:
    * **ULM**: 5,648.37
    * **Case 1 & Case 2**: 5,644.44
  - Distribution of free permits varies significantly:
    * **ULM**: Atlantis receives a disproportionately large share (2,045.95 permits).
    * **Case 1 & Case 2**: Permits are more evenly distributed among countries.

- **Sector-Specific Insights**
  - Under ULM, sectors such as Automotive, Cement, Chemicals, and Steel exhibit higher outputs compared to Case 1 and Case 2.
  - Conversely, the Paper and Textiles sectors show increased outputs in Case 1 and Case 2 relative to ULM.
  - Abatement is more effective in key sectors (Automotive and Cement) under ULM.

- **Country-Specific Abatement and Permits**
  - All countries achieve higher abatement levels under ULM compared to Case 1 and Case 2.
  - Smaller countries like Lilipoupoli receive fewer free permits under ULM, potentially impacting their economic performance.

- **Efficiency of Abatement**
  - ULM demonstrates greater abatement efficiency per permit, achieving higher abatement levels with a comparable number of permits.

- **Economic Efficiency vs. Profit Maximization**
  - ULM prioritizes higher consumer surplus and greater abatement, enhancing environmental and consumer welfare but resulting in lower profits.
  - Case 1 and Case 2 focus more on profitability, potentially at the expense of reduced abatement and consumer benefits.

In summary, ULM appears to outperform in terms of efficiency, economic output, and abatement. However, much of this advantage can be directly attributed to the absence of temporal components in the system. On the other hand, the Linear Problem tends to distribute permits more uniformly, following fairness constraints.

8.8 Conclusion

In this chapter, we have conducted a comparative analysis of the Uniform Linear Mechanism and our proposed optimization model for allocating emission permits within the EU ETS. By applying both models to the same dataset and under identical conditions, we aim to highlight the strengths and limitations of each approach. The experiments and scenarios are designed to explore how different allocation principles impact economic efficiency, fairness, and overall welfare.

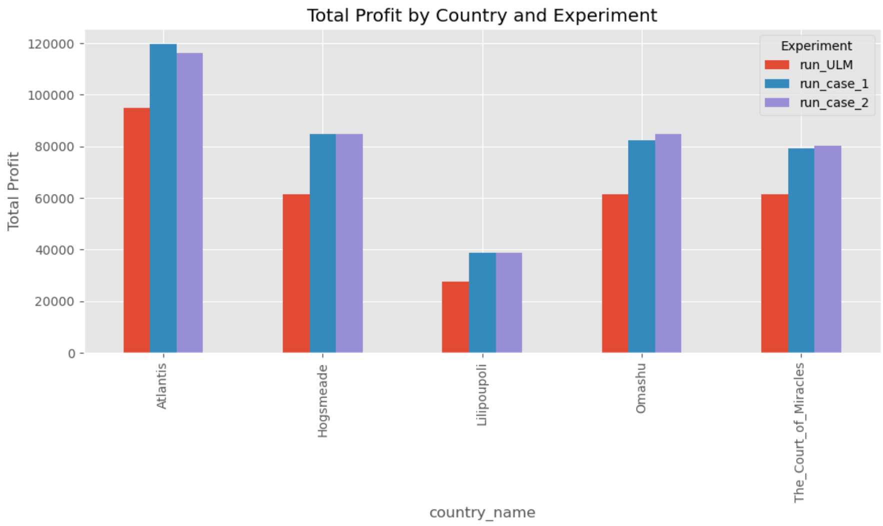


Figure 8.2: Profits per Country.

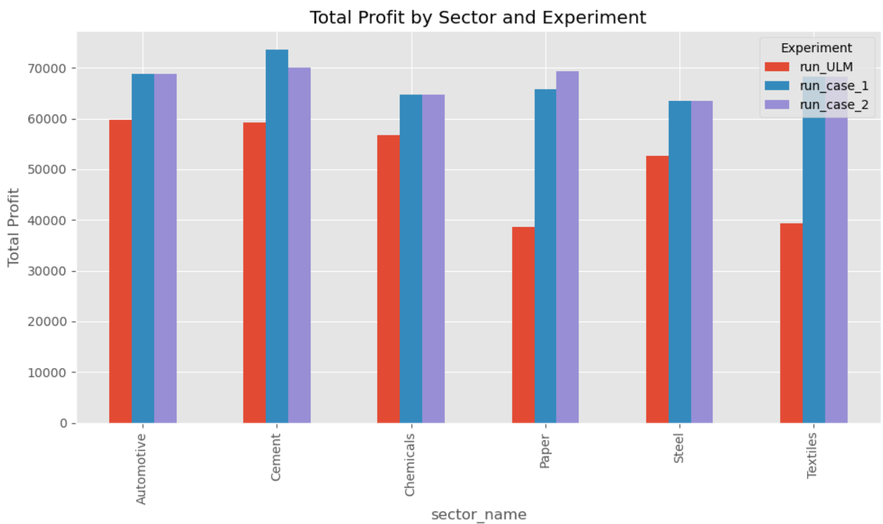


Figure 8.3: Profits per Sector.

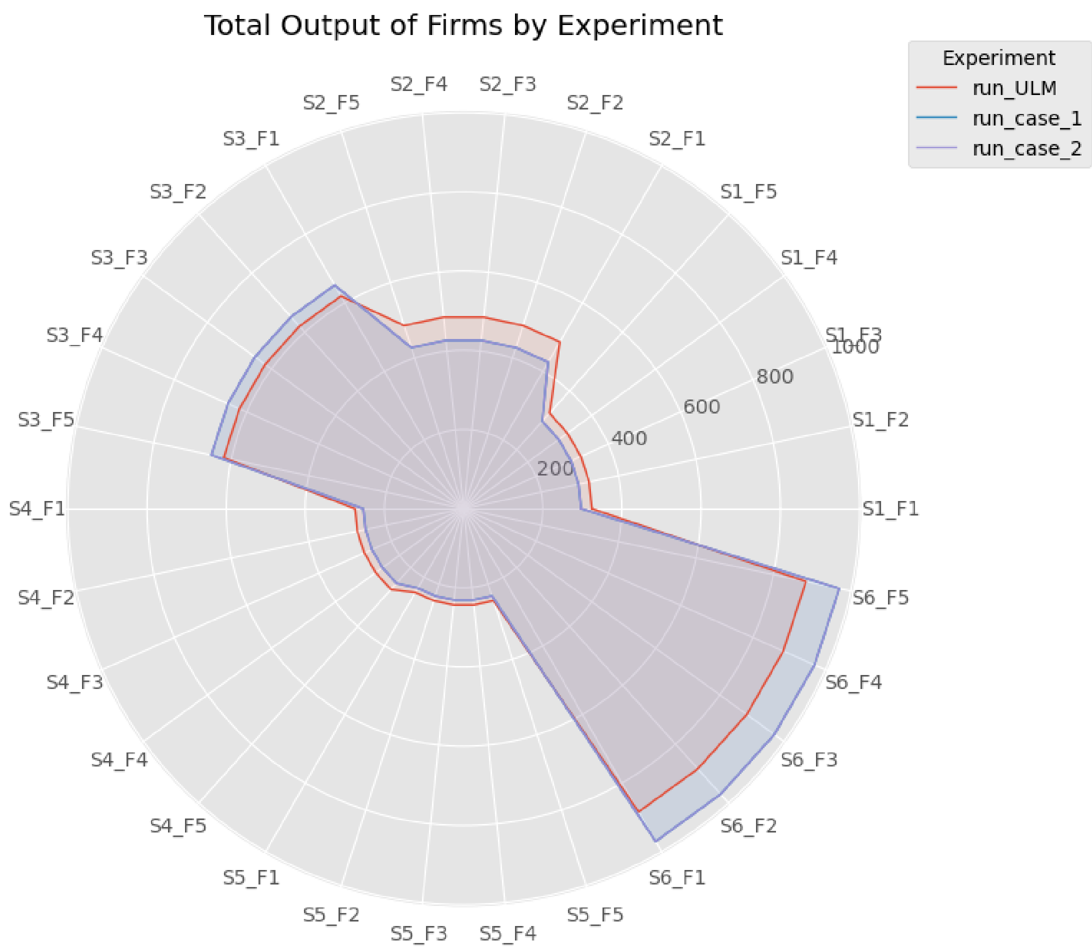
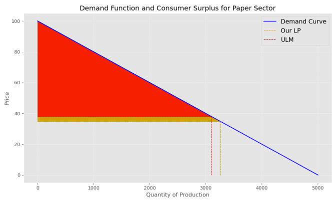
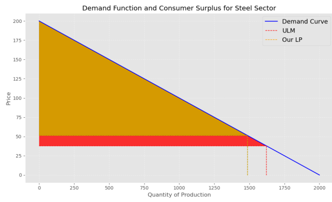


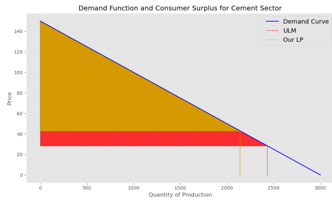
Figure 8.5: Output of all Firms



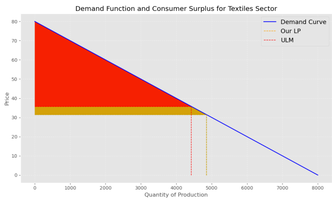
(a) Paper



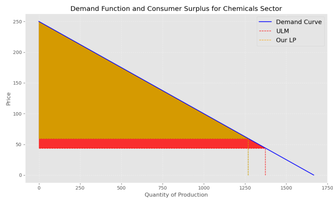
(b) Steel



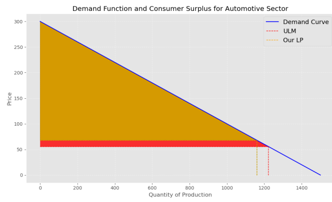
(c) Cement



(d) Textiles



(e) Chemicals



(f) Automotive

Figure 8.7: Consumer Surplus across Various Sectors.

Chapter 9

Future Work

The research presented in this thesis offers a foundation for improving the fairness and efficiency of emission permit allocation within the EU ETS. However, several avenues remain open for further exploration to enhance the models and methodologies developed. Future work could focus on the following areas:

- **Incorporating Production Cost Functions**: Integrate production cost functions into both the Uniform Linear Mechanism (ULM) and the linear programming (LP) models to more accurately reflect firms' economic behaviors and decision-making processes.
- **Exploring Additional Constraints in the LP Model**: Test and document a variety of new constraints within the LP framework, including those inspired by existing literature, to further balance fairness and efficiency in permit allocation.
- **Temporal Analysis with Multiple Periods**: Extend the models to include multiple time periods, allowing for the analysis of dynamic allocation strategies and their effects over time, which could enhance the incentive structures for emission reductions.
- **Enabling Banking and Borrowing of Permits**: Incorporate mechanisms for banking and borrowing emission permits into the models to simulate more realistic market conditions and assess their impact on firms' strategies and overall system efficiency.
- **Analyzing Discounted Abatement Costs**: Explore the use of discounted abatement costs, both globally and relative to firms' previous abatement efforts, to understand how discounting affects allocation decisions and long-term investment in emission reduction technologies.
- **Incorporating More Realistic Data**: Utilize empirical data to calibrate and validate the models, enhancing their applicability to real-world scenarios and improving the robustness of policy recommendations.

The proposed areas for future research offer substantial opportunities to advance the understanding and implementation of fair and efficient emission permit allocation mechanisms within the EU ETS and similar systems. By addressing these topics, future studies can build upon the foundation laid in this thesis, contributing to more equitable and effective climate policies that support global efforts to mitigate climate change.

Table of Symbols

Table 9.1: Table of Symbols

Symbol	Description	Units
q_i	Quantity produced by firm i	units
x_i	Emissions by firm i	kg
τ	Permit price	$/kg
C_A	Abatement cost function	$
E_i	Total emissions for firm i	kg
A_i	Allocation of free permits for firm i	permits
P	Market price of the product	$
$p(q_i)$	Price demand function for quantity q_i	$/unit
Π_i	Profit for firm i	$
BM	Benchmark emissions level per unit of output	kg/unit
R	Reduction factor applied to cap	-
σ	Cap on total emissions	kg
D	Demand level	units
η	Emission intensity (emissions per unit of output)	kg/unit
α	Correction factor (varies across sectors)	-
C	Set of EU Member States (Countries)	-
S	Set of Sectors	-
t	Time Periods	-
$v_{i,j,t}$	Percentage of Free Allocation assigned to country i, sector j, in year t	Percentage
$GDP_{i,j}$	Gross Domestic Product produced by sector j in country i	Currency (e.g., Euros)
$e_{i,j}$	Verified emissions of sector j in country i	Tons of CO_2 equivalent
PPS_i	Purchasing Power Standards Multiplier for country i	Dimensionless
v_i	Aggregate free allocation for country i	Percentage
v_j	Aggregate free allocation for sector j	Percentage
α_k	Multipliers controlling allowable deviations in allocations	Dimensionless

Symbol	Description	Units
Z	Objective function representing overall economic efficiency	Dimensionless
N	Set of agents (e.g., firms)	-
u_i	Utility function for agent i	Utility units
MMS_i	Maximin Share for agent i	Utility units
Y_i	Free allowance allocation for country i	Percentage
$D_{x_{ij}}$	Euclidean distance between countries i and j's attribute vectors	Number
$D_{Y_{ij}}$	Absolute difference in allocations between countries i and j	Number
D_{x_i}	Euclidean distance between country i and the median country	Number
D_{Y_i}	Absolute difference between country i's allocation and the median country's allocation	Number
$G\tilde{D}P_i$	Normalized GDP per capita for country i	Dimensionless
D_i	Inverse economic capacity index for country i	Dimensionless
$\Phi_i(q)$	Number of permits allocated to firm i as a function of production	Permits
Q	Aggregate production	Units
K	Total emissions	Tons of CO_2 equivalent
$S(K)$	Social cost of pollution	$
ab_i	Abatement level by firm i	kg

Table 9.2: Table of Abbreviations

Abbreviation	Full Meaning
PPS	Purchasing Power Standard
CAT	Cap and Trade system
EU ETS	European Union Emissions Trading System
GDP	Gross Domestic Product
CO_2	Carbon Dioxide
GHG	Greenhouse Gases
UN	United Nations
EU	European Union
UNFCCC	United Nations Framework Convention on Climate Change
HFCs	Hydrofluorocarbons
CORSIA	Carbon Offsetting and Reduction Scheme for International Aviation
ICAO	International Civil Aviation Organization

Abbreviation	Full Meaning
NACE	Statistical Classification of Economic Activities in the European Community
NDC	Nationally Determined Contributions
MSR	Market Stability Reserve
REPowerEU	European Union's energy resilience plan (REPowerEU)
CSCF	Cross-Sectoral Correction Factor
LRF	Linear Reduction Factor
NER 300	New Entrants Reserve (fund for renewable energy and carbon capture projects)
CDM	Clean Development Mechanism
SO_2	Sulfur Dioxide
NO_x	Nitrogen Oxides
ETS	Emissions Trading System
BAT	Best Available Techniques
CITL	Community Independent Transaction Log
EUTL	European Union Transaction Log
CLEF	Carbon Leakage Exposure Factor
CF	Correction Factor
ULM	Uniform Linear Mechanism
LP	Linear Programming
EF1	Envy-Freeness up to One Good
C&C	Contraction and Convergence
CDC	Common but Differentiated Convergence
EEX	European Energy Exchange
CS	Consumer Surplus
MSE	Mean Squared Error
NbClust	NbClust: R package for determining the optimal number of clusters
L-BFGS-B	Limited-memory Broyden-Fletcher-Goldfarb-Shanno with Bounds
BAU	Business as Usual
Gurobi	Gurobi Optimizer
SymPy	Symbolic Python library
ACS	Adjusted Consumer Surplus

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Chapter 10

Appendix

10.1 Data selection

Energy Intensity measures the amount of energy used per unit of GDP, indicating a country's efficiency in energy use relative to economic output. Lower energy intensity reflects greater energy efficiency, suggesting a reduced need for free allowances. This attribute helps ensure that allowance distribution aligns with the goal of incentivizing efficient energy usage and reducing overall emissions across the EU.

- Principle: Economic Efficiency
- File: nrg_ind_ei_linear.csv
- Source: `https://ec.europa.eu/eurostat/databrowser/view/NRG_IND_EI__custom_5726612/default/table?lang=en`
- Field in question on nrg_bal is "EI_GDP_PPS"
- Country: All countries
- Year: 1996 - 2021
- Unit: KGOE_TEUR_PPS

Verified Emissions directly quantify a country's annual emissions and represent its environmental impact. Including verified emissions allows for an accurate reflection of each country's contribution to total emissions, thus supporting a fair allocation of allowances. By aligning the allowances with verified emissions, the EU ETS ensures that countries receive allocations proportional to their emissions levels, thereby supporting a fair distribution that respects actual emissions data.

- Principle: Fairness
- File: Historical emissions_data.csv
- Source: `https://www.eea.europa.eu/data-and-maps/dashboards/emissions-trading-viewer-1`

- Country: All countries
- Year: 1990 - 2021
- Unit: K tons of Co2 equivalent

	min	25-quantile	median	75-quantile	max	Std
Austria	27.36	29.61	30.51	30.87	32.08	1.27
Belgium	45.03	45.17	45.99	46.29	55.46	3.16
Bulgaria	31.28	33.24	34.57	35.95	40.00	2.63
Cyprus	4.19	4.58	4.61	4.92	5.58	0.41
Denmark	15.50	17.05	19.46	23.71	26.55	4.05
Estonia	10.38	13.50	13.89	14.76	16.00	1.55
Finland	26.18	27.81	30.68	34.72	41.30	4.74
France	101.40	104.86	110.90	114.46	124.13	7.10
Germany	428.29	448.66	462.35	469.31	489.86	18.24
Greece	47.34	50.74	58.84	61.09	69.85	7.10
Hungary	20.08	21.23	22.40	22.61	27.24	1.91
Ireland	15.77	18.88	23.63	27.08	28.53	4.72
Italy	148.37	157.09	166.78	187.42	220.68	21.98
Latvia	2.43	2.59	2.74	2.96	3.24	0.26
Lithuania	5.61	5.92	6.23	6.65	7.56	0.59
Luxembourg	1.73	1.83	2.06	2.17	3.62	0.52
Malta	0.84	1.09	1.89	1.92	2.28	0.51
Netherlands	79.97	82.27	89.14	92.79	96.47	6.31
Poland	191.17	198.28	199.73	203.07	206.35	4.12
Portugal	24.17	25.64	26.99	28.75	31.42	2.23
Romania	40.53	42.21	43.07	48.72	63.82	6.83
Slovenia	6.18	6.60	7.45	8.03	8.86	0.91
Spain	121.48	128.26	132.69	140.52	163.46	11.31
Sweden	17.49	21.05	22.51	22.63	22.86	1.71
United Kingdom	141.76	172.90	220.88	236.57	265.06	42.22

Table 10.1: Verified emissions in G tons of CO_2 equivalent

Historical Emissions

- File: Historical emissions_data.csv
- Source: `https://www.eea.europa.eu/data-and-maps/dashboards/emissions-trading-viewe`
- Country: All countries
- Year: 1990 - 2021
- Unit: K tons of Co2 equivalent

GPD per capita reflects a country's economic wealth and ability to fund emissions reductions independently. Including this metric in the analysis acknowledges that wealthier countries have more financial capacity to invest in green technologies, potentially reducing their need for free allowances. Incorporating GDP per capita aligns with a fairness-based approach, as it considers vertical equity—ensuring that countries with lower economic resources are not disproportionately burdened in the transition to greener economies.

- Principle: Fairness
- File: GDP_per_capita_1960_2021.csv
- Source: `https://data.worldbank.org/indicator/NY.GDP.PCAP.CD`
- Data: GDP per capita (current US$)
- Country: All countries
- Year: 1960 - 2021
- Unit: US$

Inflation affects purchasing power and the overall cost of living, influencing a country's economic stability and its ability to absorb additional costs associated with emissions trading. High inflation rates may signal economic vulnerabilities, making it harder for countries to manage fluctuations in emissions trading markets. By including inflation as an attribute, the analysis respects the fairness principle by accounting for economic conditions that might otherwise disadvantage certain countries.

- Principle: Fairness
- File: Inflation_1960_2021.csv
- Source: `https://data.worldbank.org/indicator/FP.CPI.TOTL.ZG`
- Data: Inflation, consumer prices (annual
- Country: All countries
- Year: 1960 - 2021
- Unit:

	min	25-quantile	median	75-quantile	max	Std
Austria	44.20	47.17	48.56	51.46	51.92	2.75
Belgium	41.01	44.19	44.76	47.48	48.30	2.43
Bulgaria	6.85	7.17	7.57	7.88	9.45	0.74
Cyprus	23.41	26.89	28.91	31.57	35.40	3.57
Denmark	53.25	57.83	58.51	61.67	64.32	3.39
Estonia	14.66	17.40	18.20	19.66	23.06	2.45
Finland	42.80	46.46	47.71	50.16	53.77	3.25
France	36.65	39.73	41.59	42.84	45.52	2.75
Germany	41.10	41.89	44.65	46.50	48.02	2.61
Greece	17.92	19.17	21.79	26.10	32.13	4.84
Hungary	12.72	13.09	13.72	14.46	16.43	1.21
Ireland	48.66	51.83	55.60	62.44	79.11	9.57
Italy	30.24	33.51	35.56	36.63	40.94	3.17
Latvia	11.42	13.56	14.33	15.72	17.87	1.87
Lithuania	11.82	14.32	14.94	16.14	19.19	2.11
Luxembourg	105.46	109.81	112.58	119.51	123.68	6.14
Malta	21.08	22.37	24.77	26.19	31.57	3.23
Netherlands	45.19	49.37	52.20	52.97	57.88	3.66
Poland	11.53	12.60	13.70	13.94	15.47	1.08
Portugal	19.25	21.03	22.10	23.18	24.95	1.68
Romania	8.21	8.76	9.55	10.24	12.40	1.23
Slovenia	20.89	23.07	23.53	24.96	27.60	1.92
Spain	25.74	28.25	29.50	31.11	35.51	2.74
Sweden	46.95	52.42	54.59	59.03	61.13	4.45
United Kingdom	38.95	41.18	42.69	44.56	47.79	2.92

Table 10.2: GDP per capita in thousands USD

Population is a fundamental indicator of a country's size and resource needs. Larger populations imply greater demand for energy and, subsequently, higher emissions, suggesting that allocation should consider the number of inhabitants to ensure an equitable distribution of allowances. Population-based allocation also aligns with fairness principles, as it supports the idea that countries with more people should have proportionate access to resources under a shared system.

- Principle: Fairness
- File: API_SP.POP.TOTL_DS2_en_csv_v2_4701113.csv
- Source: `https://data.worldbank.org/indicator/SP.POP.TOTL`
- Country: All countries
- Year: 1960 - 2021

Inflation	min	25-quantile	median	75-quantile	max	Std
Austria	0.87	1.71	1.85	2.01	2.30	0.45
Belgium	0.53	1.30	1.81	1.90	1.96	0.47
Bulgaria	0.07	1.23	3.32	4.52	8.10	2.45
Cyprus	-1.33	-0.66	1.00	1.65	4.73	1.74
Denmark	0.25	0.58	0.89	1.78	4.13	1.27
Estonia	-0.39	2.00	3.59	4.05	6.80	2.02
Finland	0.09	1.22	1.77	2.59	3.04	1.02
France	0.07	0.55	0.95	1.10	2.37	0.58
Germany	0.65	1.20	1.50	1.87	1.97	0.46
Greece	-2.05	-0.44	-0.18	0.62	4.34	1.85
Hungary	1.32	2.66	2.89	4.11	4.85	1.15
Ireland	-4.62	-0.28	0.71	1.22	7.70	3.10
Italy	0.44	0.92	1.13	1.58	2.40	0.54
Latvia	-9.67	0.49	1.92	3.77	11.65	5.17
Lithuania	-3.30	1.06	2.53	3.88	9.71	3.30
Luxembourg	-1.11	1.94	2.28	3.36	6.61	1.99
Malta	1.12	2.06	2.22	2.75	4.22	0.82
Netherlands	0.19	0.35	0.94	1.36	2.44	0.79
Poland	0.30	0.75	1.65	2.82	3.89	1.34
Portugal	-0.39	0.67	1.51	1.78	2.25	0.90
Romania	1.80	3.33	3.77	4.38	16.02	3.88
Slovenia	-1.03	0.68	1.04	1.86	4.47	1.49
Spain	-0.22	0.06	0.32	0.90	2.25	0.76
Sweden	0.93	1.04	1.74	2.25	3.24	0.75
United Kingdom	0.51	1.60	1.82	2.03	3.23	0.65

Table 10.3: Inflation between 2008-2018

- Unit: Persons

Total Energy Supply represents a country's energy consumption needs, which correlates with its emissions output. Countries with higher energy supply requirements typically have higher emissions, necessitating a proportional allocation to meet their demand. This attribute respects the fairness principle, as it aligns the allowances with the actual energy demand of each country, thereby supporting a distribution that reflects each country's energy usage and emissions potential.

- Principle: Fairness
- File: nrg_bal_s__custom_4143365_linear.csv
- Source: Eurostat

Population	min	25-quantile	median	75-quantile	max	Std
Austria	8.32	8.38	8.48	8.69	8.84	0.19
Belgium	10.71	10.97	11.16	11.30	11.43	0.24
Bulgaria	7.03	7.15	7.27	7.37	7.49	0.15
Cyprus	1.08	1.12	1.14	1.17	1.19	0.03
Denmark	5.49	5.56	5.61	5.71	5.79	0.10
Estonia	1.31	1.32	1.32	1.33	1.34	0.01
Finland	5.31	5.38	5.44	5.49	5.52	0.07
France	64.37	65.19	66.00	66.64	67.10	0.93
Germany	80.27	80.81	81.78	82.23	82.91	0.91
Greece	10.73	10.80	10.97	11.09	11.12	0.15
Hungary	9.78	9.83	9.89	9.99	10.04	0.09
Ireland	4.49	4.57	4.62	4.73	4.87	0.12
Italy	58.83	59.33	60.23	60.58	60.79	0.73
Latvia	1.93	1.97	2.01	2.08	2.18	0.08
Lithuania	2.80	2.89	2.96	3.06	3.20	0.13
Luxembourg	0.49	0.51	0.54	0.58	0.61	0.04
Malta	0.41	0.42	0.43	0.45	0.48	0.03
Netherlands	16.45	16.65	16.80	16.99	17.23	0.25
Poland	37.97	37.98	38.04	38.06	38.15	0.06
Portugal	10.28	10.34	10.46	10.56	10.57	0.12
Romania	19.47	19.76	19.98	20.20	20.54	0.33
Slovenia	2.02	2.05	2.06	2.06	2.07	0.01
Spain	45.95	46.46	46.58	46.68	46.80	0.24
Sweden	9.22	9.41	9.60	9.86	10.18	0.31
United Kingdom	61.81	63.01	64.13	65.36	66.46	1.55

Table 10.4: Population in millions 2008-2018

- Data tree : All data -> Environment and energy -> Energy -> Energy statistics -> quantities Energy statistics -> quantities, annual data -> Energy balances
- Data name on Eurostat: Simplified energy balances
- Data: Energy balance
- Country: All countries
- Year: 1990 - 2020
- Unit: Thousand tonnes of oil equivalent
- nrg_bal codes:
- Primary production -> PPRD

- Imports -> IMP
- Exports -> EXP
- Gross Available Energy -> GAE
- Total energy supply -> NRGSUP
- Available for final consumption -> AFC

	min	25-quantile	median	75-quantile	max	Std
Austria	32011.29	33031.17	33177.75	33562.30	34166.28	660.54
Belgium	52238.38	53015.79	55039.87	55272.03	59313.06	2255.37
Bulgaria	16923.38	17726.43	18234.79	18722.35	19823.90	813.92
Cyprus	1955.96	2121.56	2262.44	2440.09	2616.93	221.87
Denmark	16374.16	16838.22	17383.65	18304.45	19558.31	1128.72
Estonia	4399.62	5378.58	5648.48	5846.32	5978.41	496.84
Finland	32022.99	33165.71	33582.86	34469.06	36251.74	1195.84
France	248383.41	250072.60	256292.91	259845.13	266394.54	6206.33
Germany	305036.83	310746.75	313107.91	318940.66	335474.27	9403.81
Greece	22748.62	23240.71	23407.97	27231.17	30404.91	2854.85
Hungary	23652.51	24816.51	25609.63	26395.50	26900.70	1096.99
Ireland	12776.63	13293.33	13625.33	14225.86	15022.14	727.58
Italy	146769.88	152859.01	156093.49	168758.58	181736.20	10969.89
Latvia	4259.46	4307.46	4407.65	4465.42	4640.00	129.86
Lithuania	6946.51	7067.95	7290.53	7647.47	9553.50	819.46
Luxembourg	3682.05	3785.50	3948.40	4127.44	4212.94	195.48
Malta	594.33	687.68	780.25	834.21	881.48	97.20
Netherlands	71379.09	73882.75	75619.51	77123.41	82743.78	3132.34
Poland	93773.11	96236.75	97971.41	101131.11	108970.23	4555.35
Portugal	21439.29	22060.46	22651.14	23433.93	24716.20	1092.37
Romania	31378.66	31668.07	33454.78	34845.41	39485.27	2432.37
Slovenia	6473.51	6722.90	6875.85	7127.16	7982.69	419.35
Spain	114522.76	119335.11	125486.61	126434.02	138166.04	6411.76
Sweden	44092.81	48432.69	49103.98	49874.22	50118.89	1857.82
United Kingdom	174024.39	177000.66	186386.17	191789.70	208268.88	11384.30

Table 10.5: Total energy supply between 2008-2018

Sectoral GDP Composition (Agriculture, Industry, Manufacturing). The economic structure of a country, represented by the composition of its sectoral GDP, directly impacts its emissions profile. Countries with a higher reliance on industry or manufacturing tend to have greater emissions intensity, which should be considered in allocation. Analyzing sectoral GDP composition supports a fair allocation by recognizing that countries with emissions-intensive economies face distinct challenges compared to those with service-based economies. This attribute thus enhances vertical equity and ensures allowances are distributed in line with each country's economic activity type.

- Principle: Fairness

	min	25-quantile	median	75-quantile	max	Std
Austria	4.32	4.79	5.14	5.46	6.05	0.55
Belgium	2.99	3.22	3.39	3.62	3.88	0.27
Bulgaria	2.04	2.20	2.39	2.56	3.20	0.33
Cyprus	0.37	0.44	0.49	0.52	0.59	0.07
Denmark	2.68	3.02	3.87	4.49	5.41	0.91
Estonia	0.49	0.63	0.65	0.80	0.85	0.13
Finland	5.29	5.82	6.03	6.41	6.59	0.43
France	35.54	39.60	42.43	44.42	47.28	3.91
Germany	22.99	25.78	29.91	32.78	35.20	4.54
Greece	6.79	7.71	8.33	8.85	9.99	0.90
Hungary	4.01	4.87	5.31	5.51	5.67	0.60
Ireland	1.32	2.28	2.61	3.06	3.96	0.71
Italy	36.20	38.32	40.70	43.21	45.94	3.17
Latvia	0.85	0.96	0.97	1.07	1.24	0.11
Lithuania	0.95	1.38	1.55	1.66	1.71	0.24
Luxembourg	0.13	0.14	0.15	0.17	0.20	0.02
Malta	0.09	0.10	0.11	0.11	0.13	0.01
Netherlands	13.20	13.89	15.07	15.49	15.71	0.92
Poland	11.25	13.13	14.22	15.96	16.76	1.91
Portugal	4.16	4.38	4.64	4.77	5.17	0.32
Romania	7.87	8.47	9.88	10.88	13.51	1.74
Slovenia	0.89	0.91	0.92	1.01	1.22	0.10
Spain	31.90	34.18	34.84	36.07	39.19	2.23
Sweden	6.27	7.53	8.11	8.47	9.59	0.87
United Kingdom	14.74	16.11	17.05	18.33	22.84	2.28

Table 10.6: Agriculture as a percentage of GDP

	min	25-quantile	median	75-quantile	max	Std
Austria	96.16	102.24	106.03	111.06	116.48	6.50
Belgium	90.94	98.13	100.59	104.53	111.72	6.42
Bulgaria	12.04	13.16	13.62	14.01	14.87	0.87
Cyprus	2.03	2.45	2.95	3.63	4.98	0.90
Denmark	60.50	64.40	68.89	69.76	79.84	5.40
Estonia	4.61	5.63	5.97	6.42	7.36	0.81
Finland	54.66	61.38	63.49	65.70	84.47	7.78
France	431.14	459.74	479.72	506.08	551.30	36.97
Germany	843.80	934.68	1000.01	1014.25	1085.27	69.42
Greece	27.44	28.64	35.83	42.48	55.73	9.66
Hungary	32.23	32.95	33.99	36.13	40.79	2.98
Ireland	51.79	57.18	62.99	110.85	141.75	32.57
Italy	383.06	431.67	449.66	474.26	568.48	50.60
Latvia	4.90	5.37	5.81	6.01	7.81	0.78
Lithuania	9.38	11.12	12.22	13.05	13.97	1.52
Luxembourg	5.99	6.42	6.93	7.48	7.80	0.62
Malta	1.36	1.48	1.54	1.59	1.84	0.14
Netherlands	138.29	155.19	166.89	173.29	204.75	19.28
Poland	129.76	146.41	150.02	156.35	169.32	10.65
Portugal	38.81	41.82	43.54	47.46	53.70	4.42
Romania	54.81	61.02	63.42	67.76	78.81	7.80
Slovenia	12.08	12.77	13.77	13.95	16.62	1.31
Spain	240.11	268.98	278.66	328.16	429.02	57.54
Sweden	97.80	114.87	122.21	126.94	135.67	10.52
United Kingdom	448.57	473.78	494.10	522.75	583.44	40.55

Table 10.7: Industry as a percentage of GDP

	min	25-quantile	median	75-quantile	max	Std
Austria	63.75	66.61	70.28	72.47	76.57	4.24
Belgium	58.69	62.46	64.21	66.70	72.39	4.02
Bulgaria	-470.74	-453.31	-432.37	-420.07	-399.33	22.79
Cyprus	0.85	0.95	1.12	1.33	1.52	0.23
Denmark	35.21	37.47	40.47	41.64	46.70	3.45
Estonia	2.41	3.20	3.35	3.56	4.13	0.47
Finland	34.44	38.11	39.73	42.29	59.40	6.76
France	254.30	268.11	278.31	292.38	325.40	20.87
Germany	603.23	697.07	743.97	755.59	796.43	56.32
Greece	15.64	16.78	18.19	22.95	30.27	4.63
Hungary	22.45	24.56	25.39	27.39	29.71	2.23
Ireland	43.26	47.00	49.10	100.06	126.39	31.76
Italy	264.39	290.81	301.69	308.97	372.63	28.24
Latvia	2.57	2.85	3.20	3.32	3.63	0.32
Lithuania	5.65	7.20	8.00	8.15	8.91	0.96
Luxembourg	2.53	2.88	3.14	3.41	3.93	0.42
Malta	0.83	0.96	1.00	1.06	1.23	0.11
Netherlands	82.70	89.39	91.54	95.12	109.07	7.43
Poland	71.81	82.05	85.70	88.28	98.64	7.14
Portugal	24.20	25.62	27.24	27.71	31.35	2.12
Romania	35.95	37.70	39.72	44.53	49.91	4.70
Slovenia	8.43	8.69	9.28	9.90	11.00	0.87
Spain	135.09	148.07	155.06	166.17	207.17	19.84
Sweden	60.02	69.65	72.99	77.49	83.80	6.77
United Kingdom	218.33	242.14	251.42	268.16	286.56	20.19

Table 10.8: Manufacturing as a percentage of GDP