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Modeling Intertemporal Trading of Emission Permits Under Market Power

Panagiotis Koromilas[1] · Angeliki Mathioudaki[1] · Sotirios Dimos[1] · Dimitris Fotakis[1]

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Abstract
In this work, we examine the effects of inter-temporal trading on the permit price and the existence of market power in the EU ETS. We contribute to the literature by dynamically modeling an emission trading system, taking into account (a) the interplay between dominant firms (leaders) and their competitive counterparts (followers), (b) intertemporal emission trading and (c) linkage between the permit and the output market. We provide equilibrium conditions for both leaders and followers. We conclude that leaders can affect followers' decisions both in the permit and the output market, since they control the overall emission abatement. Leaders' dominance may push followers to decrease their business-as-usual emissions and consequently their production levels by adjusting their overall abatement across periods. Further investigation through the actual emission data from the EU Transaction Log (EUTL) supports our theoretical findings. Finally, to examine model's accuracy, we perform simulations recreating realistic scenarios. The simulation results show that our modeling reproduces the system's behaviour and captures the changes in price, banking amount and emission levels with respect to variations on the output product and initial allocation.

Keywords Emission trading systems · EU ETS · EUA · Market power · Inter-temporal trading

✉ Angeliki Mathioudaki
tmathiou@corelab.ntua.gr

Panagiotis Koromilas
pa.koromilas@gmail.com

Sotirios Dimos
sdimos@corelab.ntua.gr

Dimitris Fotakis
fotakis@cs.ntua.gr

[1] School of Electrical and Computer Engineering, National Technical University of Athens, Iroon Polytechniou 9, Athens 157 80, Greece

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1 Introduction

Emission trading scheme (ETS) is a market-based approach that allows participants to control pollution and achieve pollution reduction targets in a cost-effective way (Montgomery 1972; Rubin 1996). The institutional framework of the scheme creates a market, through which, firms are compelled to buy or sell allowances in order to emit and hence the carbon price is directly determined by the allowance market. In most existing emission trading systems, the ability to transfer unused permits from one period to the next (i.e. banking) is permitted. Many studies suggest that this inter-temporal link reduces the overall compliance costs and stabilizes carbon prices across phases (Ellerman et al. 2016; Hintermann et al. 2016; Schleich et al. 2006; Tietenberg 1999).

The European Union Emission Trading System (EU ETS), a key instrument to Europe's climate and energy policy, has already become a field of global interest for its emission abatement efficiency, mostly due to its large size - as it covers more than 11.000 industrial installations across European countries - as well as its international market. The main goal of the system is not only to reduce GhG emissions in a way that is effective, both environmentally as well as economically, but also to promote investments in low carbon technology (Commission 2015; Hintermann et al. 2016).

Even though EU ETS is recognized as an innovative policy instrument (Ellerman et al. 2010), there have been several drawbacks, mainly concerning the first and the second Phase of its operation, which have been well criticized and studied. At the end of 2007 (during Phase I), the EUA (EU Allowance) price collapsed and almost reached zero, mainly due to the overallocation of emission permits and banking restrictions (Brink et al. 2016; Ellerman et al. 2016; Hintermann et al. 2016; Daskalakis 2018). Hence, at the beginning of the second phase, banking of permits between different phases was allowed, as a possible remedy to this drawback. Although the permission of this intertemporal link helped the EUA price to remain at higher levels at the end of the second phase, the accumulation of a substantial amount of stocked permits created a remarkable surplus (Ellerman et al. 2016; The European Environment Agency (EEA) 2020). Since 2008, the amount of allowances issued every year was greater than the actual need, which was one of the reasons responsible for this surplus, along with the rapid drop of the production levels across the economic recession that affected Europe (2008 to 2012), and the insertion of carbon reduction credits coming from Clean Development Mechanism (Koch et al. 2014; De Perthuis and Trotignon 2014; Ellerman et al. 2016).

To mitigate the effects of this surplus and absorb some of the excess allowances, EU proposed to backload the auctioning of 900 million allowances at the beginning of Phase III. In addition to this backloading measure, in 2017 the Commission set in operation the Market Stability reserve Mechanism, which allows for several adjustments in the annual allocation, in cases where the total amount of allowances is above a certain, predefined level (Commission 2015; Chaton et al. 2018). Although Market Stability Reserve seems to respond pretty well in increasing the permit price, there are still some issues mainly depending on the extend of the output product demand variations (Chaton et al. 2018). In a recent work in the context of MSR, (Osorio et al. 2021) argue that MSR cancellations increase price uncertainty and that investigating the impact of firms applying market power is an interesting direction for future work.

Apart from the structural malfunctions concerning the legislative framework and the operation of the EU ETS in the previous years, the fact that some firms have the ability to exercise market power over the permit market, is also an important issue. Several studies have stated that in such markets a form of market power exists (Rico 1995; Hahn 1984; Haita 2014). The initial allocation is based on past emission levels and the firms' ability to invest in abatement technology. Therefore there is a high degree of differentiation among the polluters (Ellerman et al. 2010). Besides, the firms that operate under an emission trading system participate in two markets: the permit and the output market. The adoption of advanced abatement technology requires high capital investment and thus it is more likely that firms which dominate the output market may have such ability, which in turn may lead to less abatement costs and more liquidity on the number of the available permits (Keohane 2002). Moreover a large firm in the output market along with a generous allocation of freely distributed permits may own a dominant position in the permit market (Hintermann 2011). In addition to that, recession period also emphasized the importance of the output product, as a significant reduction in the production levels had a great impact on the EUA prices and on the efficiency of the policy in general (Ellerman et al. 2016). The linkage between output product and emissions, creates an interaction between the two markets and that is the main reason why one cannot model the former without considering the latter.

All of these factors - initial allocation, intertemporal trading, market power, linkage with the output market - have been proved critical for the overall system performance in the past. They contribute immensely to the efficient operation of the system, and thus underline the need for a simple theoretical model, that exemplifies the complex relations between crucial system variables and parameters. That model will be of great use both by the regulator, to improve system performance, as well as the participants, in order to construct a better reaction strategy in the permit market. To the best of our knowledge, none of the existing studies take into consideration all of the aforementioned characteristics.

This paper contributes to the literature on market power in emissions permit markets. In this work we extend the analysis of several theoretical models found in the literature in order to encompass a variety of important factors. We dynamically model an emission trading system under the EU ETS rules, that is more inclusive and formulates the strategic interaction between dominant firms (leaders) and their competitive counterparts (followers), under inter-temporal emission trading. Polluters differ with respect to their emission rates and subsequently to their marginal abatement costs. To the best of our knowledge this is the first study that considers market power exertion by multiple firms that are Leaders over multiple followers. Along with the inclusion of the output product, an amount of freely distributed permits is also considered. Our model is meant to investigate the basic concepts of a trading system and explore the interaction between crucial system aspects.

Furthermore, in order to observe how the theoretical model responds, a variety of simulations on realistic data is performed. The theoretical formulation as well as the simulation process provide many considerably interesting results that coordinate perfectly with the literature, indicating that the presented model is powerful enough to capture complex relations between crucial system variables.

Initially, we mathematically describe the theoretical model and solve it to provide the equilibrium conditions both for the leaders' and followers' subsystems. Furthermore, based on the followers subsystem's equilibrium, it is observed that leaders are key players in the permit market which translates to a twofold conclusion: (a) leaders control the overall emission abatement while pushing the followers not to abate and buy the permits they need and (b) leaders can adjust the amount of emissions abatement across periods, which will also push the followers to decrease their business-as-usual emissions leading to a decrease on their output product. This is a core result of our paper and indicates that leaders can affect followers' decisions both in the permit and in the output market.

By solving the model a permit price function that is in the form of the marginal abatement cost of the followers, is derived. The permit price is a function of the leaders' permit purchase decisions as well as the initial allocation and the permit banking amount. Both of these factors have been well discussed in the literature with respect to their significance as price determinants.

Finally, by the simulation process we manage to recreate interesting scenarios based on real life events that occurred throughout the EU ETS operation and validate the overall system's performance. The simulation results showed that our mathematical modeling reproduces the system's operation and captures the changes in price as well as in emissions and banking amount with respect to changes on the output product and initial allocation.

The remainder of the paper is structured as follows. Section 2 provides a literature overview on existed mathematical models. In Sect. 3 the theoretical model is formed and proven equilibrium conditions are provided. At the end of that section evidence supporting the predictions of the model from the actual emissions of the energy sector is provided. In the next Sect. (4) we use some real data together with the produced mathematical model to create artificial data, which is then used to run simulations on the produced model, leading to results and discussion. Finally, in Sect. 5 there is a presentation on the outcomes of our work and some policy implications together with some suggested directions for future work. The Appendix contains the complete mathematical proof of the equilibrium conditions ("Appendix A"), the proof of the uniqueness of the zero abatement solution for the followers ("Appendix B") and the the artificial data ("Appendix D"). "Appendix C" provides more information and discussion on evidence from actual emissions covered by EU ETS.

2 Literature Review

Although the temporal dimension in multiple year emission trading systems i.e. banking (and borrowing) of allowances, have been well established, theoretical analysis on this area remain scarce. As a first step Rubin (1996) shows that an intertemporal equilibrium exists and intertemporal trading allows firms to equalize marginal abatement cost and thus permit price grows at the discount rate (Hotelling's rule). One of the few works that form a mathematical model and allow firms to bank their unused permits is that of Chaton et al. (2015). By considering symmetric firms they form a static model of intertemporal trading and extend it to describe the backloading measure as a predefined mandatory banking. However, in their model firms are not endowed with free allowances, whose role is central in firms' decision problem.

In every economical system, uncertainty is used to model complex information and lead to more robust results. The work of Schennach (2000) is a first attempt to include output market uncertainty on firms' abatement decisions across time and suggests that the higher the expected electricity price, the lower the emissions in earlier periods. Zhang et al. (2013) explores the effect of uncertainty in an intertemporal emission trading system and suggests that increased output or input price uncertainty induces larger emission reductions and higher allowance prices.

A missing factor in the above studies is the existence of market power in permit markets. In his influential work, Hahn (1984) introduced the idea of market power in emission trading systems. In a static context he demonstrated the effects of allocating permits to an agent able to exert market power. However his study did not include two main aspects of the system: the banking of permits, which directly affects the permit price, and the connection with the output market. Although he considered only one leader, his work was a great first approach considering the concept of market power. Liski and Montero (2005) study the effect of market power on the equilibrium of a permit market by introducing a large dominant firm and a competitive fringe. Their analysis shows that the large agent can accumulate the entire stock of permits and thus manipulate the market, while there is an incentive to exchange tradable permits with the fringe, in order to store allowances for the next period. Both banking and borrowing are allowed without restrictions in a continuous time setting. Chevallier (2011) develops a differential Stackelberg game with two types of non-cooperative agents: a large dominant firm and a competitive fringe the size of which are exogenously determined. Strategic interactions are modeled on an intra-industry permit market where agents can freely bank and borrow permits. His study is one of the few to use proper and realistic abatement cost functions. All of the above works include only one leader. Instead, Phaneuf and Requate (2016) composes one of the few models that involve a greater number of leaders. In a more recent work Wang et al. (2021), introduce a two-stage game model to capture the strategic behaviors between agents in an Emission Trading System. They state that there are several directions for future work on this topic by including the interaction between the permit and output market as well as the potential for intertemporal trading.

As one can observe, the so far presented works, do not include the output product into the formulation of the models. Although, Hong et al. (2017) take under consideration the output product in firms' production costs, they suppose that agents are symmetric and competitive equilibrium is found by a genetic algorithm.

One of the few works that considers market power in both permit and output market is that of Hintermann (2011). He creates a model of a dominant firm and a follower and he finds the allocation threshold, above which the leader will increase the permit price and maximize profits in both markets. In a follow-up work Hintermann (2017) uses a simplified version of his previous model and by considering the interaction between the permit and the ouput market he shows that firms with excess permit holdings may use their market power for price manipulation. A more recent study in market power literature which also takes into consideration the output market is that of André and Castro (2020). They also consider just one leader and a follower in the permit market and they create different structures on the output market, including Cournot and Stackelberg model. They conclude that when the leader is allocated with additional permits, then it tends to increase the output product less and decrease the abatement more than the follower in the same scenario. Another interesting result is that the total output product is decreasing and the permit price is increasing to the amount of the permits allocated to the dominant firm.

Finally, there is a limited number of studies that combines the interaction between the permit and the output market under market power and intertemporal trading. That is the case for Chen and Tanaka (2018), whose work, however, does not consider the important system factor of initial allocation. This absence together with the use of a specific emission rate coefficient instead of a proper abatement cost function, emphasizes the need for a more inclusive theoretical model.

3 Theoretical Model

In this section, first we present our mathematical model and the basic assumptions (Sect. 3.1). We then show the solution of our model in three basic scenarios: (1) when neither banking nor trading is allowed, (2) when only banking is allowed and (3) when trading is allowed without the potential of banking (Sect. 3.2). In Sect. 3.3, the solution procedure as well as the equilibrium conditions of our general model are described in a high-level manner, and finally a discussion on the interpretation of the theoretical results and their practical implications is given in Sect. 3.4.

3.1 Mathematical Description

Suppose there is a set of n firms and a regulator that operate for multiple periods under an emission trading system. We divide firms into small firms $i = 1, ..., m$ and large firms $i = m + 1, ..., n$.

Output Product and Production Costs: During a specific time period t, firm i is involved in an output market by producing a quantity $q_{i,t}$ of product. The output $q_{i,t}$ is different for each company and it is produced at a cost $C^P_{i,t}(q_{i,t})$. Production costs are assumed to be increasing in the output product.

Emissions and Permit Allocation: At the end of every period t, the firm i will have to provide the regulator with $z_{i,t}$ permits, where $z_{i,t}$ is the amount of firm's emissions measured in tones. During each period, the total volume of emissions is limited by the amount A_t, which represents the initial allocation provided by the regulator ($A_t \geq A_{t+1}\ \forall\ t$). At the beginning of each period, every company receives a share of the period's initial allocation A_t. We denote the firm i and time t endowment by $z^f_{i,t}$.

Abatement Technology and Abatement Costs: In normal conditions under laisser-faire, the firm would use its standard production technologies and fuel, which have a specific emission rate, denoted by $r_{i,t}$. In such a situation the maximal emission level i.e. business as usual emissions, is denoted by $z^u_{i,t} = r_{i,t}q_{i,t}$.

The abatement cost function depends on the amount of abatement, which is a combination of the business as usual and the actual emissions. More specifically, let z be the amount of actual emissions, then the abatement cost function is denoted by $C^A(\alpha) = C^A(z^u - z) = C^A(z^u, z)$, where $\alpha = z^u - z$ corresponds to the level of abatement i.e. the difference between the actual emissions and the business-as-usual emissions. The abatement cost function $C^A(z^u, z)$ must satisfy the usual properties (Phaneuf and Requate 2016): If a firm is not compelled to reduce its emissions below its business as usual level z^u, the abatement cost is zero i.e. $C^A(0) \equiv C^A(z^u, z^u) = 0$. Conversely, $C^A(z^u, z)$ is positive for any emission level $z < z^u$, since the reduction of emissions comes at a cost i.e. $C^A(z^u, z) > 0,\ \forall z < z^u$. Since a marginally reduction in emissions marginally increases cost, the marginal abatement cost function is given by:

$$MAC = -\frac{\partial C^A(z^u, z)}{\partial z} > 0, \quad z < z^u$$

and is measured in $€/tCO_2$. Moreover abatement costs are lower for smaller abatement, while the marginal abatement cost increases when emissions are reduced i.e. $\frac{\partial^2 C^A(z^u,z)}{\partial z^2} > 0$ for $z < z^u$. Therefore, $C^A(z^u, z)$ constitutes a strictly convex function on z.

Trading and Banking: The firms can store a proportion of permits for use in the future. The permits that company i banked during period t are denoted by $\bar{z}_{i,t}$. The emission permits can also be traded and sold in the permit market. The amount of permits bought ($z^b_{i,t} > 0$) or sold ($z^b_{i,t} < 0$), at a permit price σ_t, by the firm i is given by $z^b_{i,t} = z_{i,t} - (z^f_{i,t} + \bar{z}_{i,t-1}) + \bar{z}_{i,t}$. That is, the permits which have to be bought at period t are equivalent to the difference between the permits that the firm must have at period t ($z_{i,t}$ for compliance and $\bar{z}_{i,t}$ for banking gives $z_{i,t} + \bar{z}_{i,t}$ in total) from the permits that it has at the beginning of the period ($z^f_{i,t} + \bar{z}_{i,t-1}$).

A complete listing of the above variables together with the cost functions can be found on Table 1.

Market Participants as Leaders and Followers: Considering the existence of market power, firms are divided into two types, the m Followers and the $n - m$ Leaders. This differentiation allows us to assume the following:

1. As stated by Keohane (2002), is that the Leaders are more likely to have adopted advanced abatement technologies leading to less abatement costs. For this reason, in the presented model the abatement cost function will be different between the two different types, while for simplicity they will remain the same for firms of the same type. Thus in the following Sections the abatement cost function of the Followers will be indicated by $C^A_{F,t}$ and for the Leaders by $C^A_{L,t}$.

Table 1 Definitions of model variables and functions for firm i and period t

Notation	Definition
n	Total amount of firms
m	Total amount of small firms
$q_{i,t}$	Amount of output product
$z_{i,t}$	Emissions measured in tones (or permits to be surrendered).
$r_{i,t}$	Emission rate under standard technologies and fuel (i.e. no abatement)
$z^u_{i,t} = r_{i,t}q_{i,t}$	Business as usual emissions
$\alpha_{i,t} = z^u_{i,t} - z_{i,t}$	Amount of abatement
$z^f_{i,t}$	Free allowances from the regulator
$\bar{z}_{i,t}$	Amount of permits banked for future use
$z^b_{i,t} = z_{i,t} - (z^f_{i,t} + \bar{z}_{i,t-1}) + \bar{z}_{i,t}$	Amount of permits bought ($z^b_{i,t} > 0$) or sold ($z^b_{i,t} < 0$)
σ_t	Permit price
β_t	Discount factor from period t to period $t + 1$
$p_{i,t}$	Price of the output product
$C^P_{i,t}(q_{i,t})$	Production cost for $q_{i,t}$ output products
$C^A_{i,t}(\alpha_{i,t})$	Abatement cost for $\alpha_{i,t}$ abatement
$\mathcal{H}_{i,t}(\alpha_{i,t})$	Marginal abatement cost (MAC) function
$\Pi_{i,t}(q_{i,t}, z_{i,t}, r_{i,t}, z^f_{i,t}, \bar{z}_{i,t}, \Pi_{i,t+1})$	Profit function
A_t	Total cap of allowances
B_t	Total permits remained unused
$E[X_{\alpha,\beta}\|\alpha]$	The average or expectation of quantities $X_{\alpha,\beta}$ for a specific α and all β
$\lfloor X \rfloor$	The largest integer not greater than the quantity X

2. Leaders are firms that find it easier to abate and receive a generous endowment from the regulator (Hintermann 2011), which help them accumulate more permits and control the greatest proportion of the market's supply. On the contrary, the Followers are companies that, according to Dasgupta and Heal (1979), cannot effectively engage on permit trading due to limited financial and human resources. This situation makes them net-buyers for the system, since they form the greatest proportion of the demand. Therefore, we consider the m small firms to be *price takers* as their actions do not affect the price and the $n - m$ large firms as *price setters* as they can control the permit price in the sense that their abatement will specify the number of permits they buy or sell. Technically, this information can be modeled by letting the variable $\partial\sigma_t / \partial z_{i,t}$ exist for the Leaders but not for the Followers. Market power is assumed to exist only in the permit market, but not in the output market.

Firm's Maximization Problem. Each firm seeks to maximize its profits. Firms' profit function consists of:

- The revenues from selling the output product: $p_t q_{i,t}$.
- The production cost of the output product: $C^P_{i,t}(q_{i,t})$.
- The cost to abate emissions $C^A_{i,t}(\alpha_{i,t})$.
- The expenditures (or revenues) from buying (or selling) the permits: $\sigma_t z^b_{i,t}$.

The dynamic nature of cap-and-trade systems along with the inclusion of banking allow firms to earn profits from the price spread across periods. For this reason the firms must make decisions that optimize their inter-temporal profits. For instance, if the permit price increases in period $t + 1$, then it may be wise for the participants to buy more permits in period t. This aspect of the system can be included in the model only by considering dynamic profit functions of the form:

$$\begin{aligned}
&\Pi_{i,t} = p_t(q_{i,t}) \cdot q_{i,t} - C^A_{i,t}(z^u_{i,t} - z_{i,t}) - \sigma_t(z^b_{i,t}) - C^P_{i,t}(q_{i,t}) + \beta_t \Pi_{i,t+1} \implies \\
&\Pi_{i,t}(z_{i,t}, \bar{z}_{i,t}, \bar{z}_{i,t-1}, q_{i,t}) = p_t(q_{i,t}) \cdot q_{i,t} - C^A_{i,t}(r_{i,t}q_{i,t} - z_{i,t}) - \sigma_t(z_{i,t} - z^f_{i,t} + \\
&+ \bar{z}_{i,t} - \bar{z}_{i,t-1}) - C^P_{i,t}(q_{i,t}) + \beta_t \Pi_{i,t+1}(z_{i,t+1}, \bar{z}_{i,t+1}, \bar{z}_{i,t}, q_{i,t+1})
\end{aligned} \tag{1}$$

Although the profit function (1) is constructed for multiple periods, for ease of calculation in the rest of the paper we consider a two-period model. Whichever modeling is considered banking at the end of the last period is assumed to be 0, thus the general behaviour we seek to understand can be captured by a two-period model. For two periods the following profit function is formed.

$$\begin{aligned}
\Pi_{i,1} &= p_1(q_{1,t}) \cdot q_{i,1} - C^A_{i,1}(r_{i,1}q_{i,1} - z_{i,1}) - \sigma_1(z_{i,1} - z^f_{i,1} + \bar{z}_{i,1}) - C^P_{i,1}(q_{i,1}) + \\
&+ \beta_1[p_2(q_{2,t}) \cdot q_{i,2} - C^A_{i,2}(r_{i,2}q_{i,2} - z_{i,2}) - \sigma_2(z_{i,2} - z^f_{i,2} - \bar{z}_{i,1}) - C^P_{i,2}(q_{i,2})]
\end{aligned} \tag{2}$$

Therefore, the two-period profit maximization problem can be formed as follows:

$$\begin{aligned}
\max \quad & \Pi_{i,1}(z_{i,1}, q_{i,1}, \bar{z}_{i,1}, z_{i,2}, q_{i,2}) \\
\text{s.t.} \quad & 0 \le z_{i,1} \le r_{i,1}q_{i,1} \\
& 0 \le z_{i,2} \le r_{i,2}q_{i,2} \\
& 0 \le \bar{z}_{i,1}
\end{aligned} \tag{3}$$

In Problem (3) the emissions cannot be negative or greater than the business as usual emissions, for both periods, i.e. $0 \leq z_1 \leq r_1 q_1$ and $0 \leq z_2 \leq r_2 q_2$, and banking should be positive since borrowing is not considered in our model, i.e. $0 \leq \bar{z}_1$.

Following the profit function above, each firm in order to hold at least as many permits as actual emissions at a lower cost, has to consider four strategies:

1. Emissions reduction, by decreasing $z_{i,t}$, which leads to a greater abatement cost.
2. Purchase of emission permits, by paying σ_t price for more permits.
3. Reduction of the output product, by decreasing $q_{i,t}$, which leads to less profits from the output market.
4. A mix of the above three compliance strategies.

The objective of each firm is to maximize the profit function (2). By taking the derivatives of this function over each decision variable (i.e. $q_{i,1}, z_{i,1}, \bar{z_{i,1}}, q_{i,2}, z_{i,2}$) a system of equations is formed for every firm. Each firm seeks to solve this system in order to decide the best strategy profile that maximizes its profit, given the strategies of the other firms.

In the first step the Leaders must carefully decide on the price they set. For this reason in our analysis, Leaders will first solve the problem of the Followers in order to find out a price function that captures the best reactions of the latter. Ultimately, the permit price paid by the followers will depend on the choices maid by the leaders. Large firms set the permit price by determining the amount of permits that is left for use by the small firms.

For simplicity, we assume that followers are symmetric and their abatement costs are equal and are denoted by index F. Similarly we assume that Leaders are also symmetric and their abatement costs are also equal and are denoted by index L.

In Sect. 3.3 we provide the equilibrium conditions for both Leaders and Followers. The complete mathematical solution of the system of equations lies in "Appendix A". For simplicity, in the following sections we refer to the marginal abatement cost function by $\mathcal{H}_{i,t}(r_{i,t}q_{i,t} - z_{i,t}) = -\partial C^A_{i,t}/\partial z_{i,t}$.

3.2 Simplified Model without Banking or Trading

In this section we provide three introductory scenarios for better understanding: (1) when neither banking nor trading are allowed, (2) when only banking is allowed and (3) when only trading is allowed. For simplicity, in all three scenarios we consider two firms, a leader and a follower (i.e. one firm with lower marginal abatement costs and another with higher marginal abatement costs).

3.2.1 Without Banking and Without Trading

Given an initial endowment z_i^f and without the potential of banking or the ability to trade permits, the profit function 2 will be transformed as follows:

$$\Pi_i = p(q_i) \cdot q_i - C_i^P(q_i) - C_i^A(r_i q_i - z_i)$$

and the first order conditions are formed as follows:

$$-\frac{\partial C_i^A}{\partial z_i} \neq 0$$
$$\frac{\partial C_i^P}{\partial q_i} + \frac{\partial C_i^A}{\partial q_i} = p(q_i) + \frac{\partial p}{\partial q_i} q_i$$

Without the ability to trade or store allowances firms are not compelled to reduce their emissions below their initial endowment ($z_i < z_i^f$) otherwise the cost will increase. Since Leaders may be firms with a generous initial endowment, therefore $z_i^f \geq z_i^u$, then without the banking potential and the ability to engage in permit trading, the excess allowances will remain unused. On the other hand, followers may find it expensive to abate. Then, without the ability to buy the allowances they need for compliance, followers may resort in decreasing their output product levels.

Each firm chooses the quantity q_i that maximizes its profit. Therefore a decrease in follower's production levels due to the inability to reduce its emissions to the specified level, will decrease its profit which in turn will lead to a decrease in the total profit.

3.2.2 Allowing Banking but No Trading

Given an initial endowment z_i^f and assuming that only the potential of banking is allowed, the profit function 2 is as follows:

$$\Pi_{i,t} = p_1(q_{i,1}) \cdot q_{i,1} - C_{i,1}^P(q_{i,1}) - C_{i,1}^A(r_{i,1}q_{i,1} - z_{i,1}) + \\ + \beta_1[p_2(q_{i,2}) \cdot q_{i,2} - C_{i,2}^P(q_{i,2}) - C_{i,2}^A(r_{i,2}q_{i,2} - z_{i,2})]$$

Since banking is allowed, then $z_{i,1} = z_{i,1}^f - \bar{z}_{i,1}$ and $z_{i,1} = z_{i,2}^f + \bar{z}_{i,1}$. Consequently, the first order conditions are:

$$-\frac{\partial C_{i,t}^A}{\partial z_{i,t}} \neq 0$$
$$\frac{\partial C_{i,1}^A}{\partial \bar{z}_{i,1}} = \beta_1 \cdot \frac{\partial C_{i,2}^A}{\partial \bar{z}_{i,1}}$$
$$\frac{\partial C_{i,t}^P}{\partial q_{i,t}} + \frac{\partial C_{i,t}^A}{\partial q_{i,t}} = p_t(q_{i,t}) + \frac{\partial p_t}{\partial q_{i,t}} q_{i,t}$$

If the abatement cost is lower in the first period ($t = 1$), then the firms will reduce their emissions below their initial endowment, i.e. $z_{i,1} < z_{i,1}^f$ to store the excess allowances for use in the next period. Therefore their aggregate abatement costs will be lower than the respective costs in case banking was not allowed (Sect. 3.2.1). At the same time, since followers may find it difficult to abate, they may reduce their output production levels either in one or in both periods to reach their initial endowment levels. However, the total profit of the firms is improved when banking is allowed compare to the case where banking is not allowed (Sect. 3.2.1).

3.2.3 Allowing Trading but No Banking

In our discussion thus far we have assumed that trading is not allowed and therefore firms have to absorb a cost of abatement in order to reach their emissions targets. Considering the case of permit trading without the potential of banking (following the analysis in Section 8.3 of Phaneuf and Requate (2016)), the profit function 2 will be transformed as follows:

$$\Pi_i = p(q_i) \cdot q_i - C_i^P(q_i) - C_i^A(r_i q_i - z_i) + \sigma \cdot (z_i^f - z_i)$$

Starting by analyzing the behavior of the Follower, the respective first order conditions are:

$$-\frac{\partial C_F^A}{\partial z_F} = \sigma$$
$$\frac{\partial C_F^P}{\partial q_F} + \frac{\partial C_F^A}{\partial q_F} = p_F(q_F) + \frac{\partial p_F}{\partial q_F} q_F$$

If $-\frac{\partial C_F^A}{\partial z_F} = \mathcal{H}_F(r_F q_F - z_F)$, then the the Follower's emissions are $z_F = r_F q_F - \mathcal{H}_F^{-1}(\sigma)$. Then, the demand for permits is formed as $D = z_F - z_F^f = r_F q_F - \mathcal{H}_F^{-1}(\sigma) - z_F^f$.

Since the amount of the demand must equal the residual left by the Leader, we can rewrite D as:

$$r_F q_F - \mathcal{H}_F^{-1}(\sigma) = A - z_L$$

where A is the initial total permit endowment.

Therefore the inverse demand function for permits can be written as:

$$\sigma = \mathcal{H}_F(z_L + r_F q_F - A)$$

By definition, Leader set the permit price by determining the permits left for use by the Follower. By considering σ, Leader's first order conditions result to the following marginal abatement cost function:

$$\mathcal{H}_L(r_L q_L - z_L) = \mathcal{H}_F'(z_L + r_F q_F - A) \cdot (z_L - z_L^f) + \mathcal{H}_F(z_L + r_F q_F - A)$$

From the above it is clear that the leader will set the permit price at the marginal abatement cost of the follower, and thus the follower is indifferent if it will reduce its emissions or buy the allowances from the leader. This essentially allows the followers to exploit the lower abatement costs of the leaders, with the leaders benefit from their improved abatement costs, through the permit market. However this outcome also improves the total profit of the firms compare to the scenario when neither trading nor banking was allowed (Sect. 3.2.1), since the excess permit holdings of the leader will not remain unused.

3.3 General Case: Allowing Banking and Trading

In Sect. 3.2, we presented three simplified scenarios to better understand the improvement in the total profit of the firms when either banking or trading was allowed. In this

Section we present our general model, where we focus in the existence of market power when inter-temporal trading is allowed.

Given the assumptions in Sect. 3.1, to study the general case equilibrium, we start by analyzing the behavior of the m followers. By the followers's profit maximization problem, since they are price takers, we arrive to the familiar first order conditions (see "Appendix A") that marginal abatement costs for both periods, equal the permit price.

By further analysing followers demand for permits, we conclude to a price function of the form of their marginal abatement costs. Their demand for permits must equal the residual left for use by the leaders. What is left in each period is the cap, minus the actual emissions of the leaders minus the banking, divided equally to the m symmetric followers. In other words the permit price is a function of the leaders' decisions on emissions (i.e. $z_{L,t}, z^b_{L,t}$)

$$\sigma_{t\in\{1,2\}} = \mathcal{H}_{F,t}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,t}q_{i,t} - \underbrace{\left(A_t - \sum_{i=m+1}^{n} z_{i,t} + (2t-3)B_1\right)}_{\sum_{i=1}^{m} z_{i,t}}\right]\right) \tag{4}$$

where A_t is the cap and $B_1 = \sum_{i=1}^{n} \bar{z}_{i,1}$ is the total banking for the first period.

The $n-m$ leaders, then, set the permit price by determining the emission permits left for use by the followers, considering the followers inverse demand function (4).

Following the analysis in "Appendix A", an overall system of equations, indicating system equilibrium, is constructed. In the next sections we provide the equilibrium both for the Followers' and the leaders' subsystems.

3.3.1 Equilibrium Conditions: General Case

As stated in Sect. 3.1, the objective of each firm is to solve Problem (3). In order for the leaders to carefully decide the price level, we start our analysis by solving the followers' maximization problem. From the solution, we extract a price function (4) which incorporates followers' reactions on leader's decisions on emissions. Members of the latter group set the permit price, σ_t by determining the residual emission permits.

Followers' Equilibrium. The m followers' equilibrium is characterized by the following equations:

$$z_{i,t\in\{1,2\}} = r_{i,t}q_{i,t} - \frac{1}{m}\left(\sum_{j=1,j\neq i}^{m} r_{j,t}q_{j,t} - \sum_{j=1,j\neq i}^{m} z_{j,t}\right) \tag{5}$$

$$\mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right]\right) = \beta_1\mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right]\right) \tag{6}$$

$$\frac{\partial C^P_{i,t}}{\partial q_{i,t}} + \frac{\partial C^A_{F,t}}{\partial q_{i,t}} = p_t(q_{i,t}) + \frac{\partial p_t}{\partial q_{i,t}}q_{i,t} \tag{7}$$

Equation (5) contains information about the permit decisions in period 1 and 2 respectively, while Eq. (7) contains information about the output production decisions for both periods.

The main result in followers' solution lies in Eq. (5), which refer to their optimal permit decisions. Intuitively one can observe that if none of the followers choose to abate i.e. $\sum_{j=1,j\neq i}^{m} z_{j,1} = \sum_{j=1,j\neq i}^{m} r_{j,1}q_{j,1}$, then follower i will also choose not to abate i.e. $z_{i,1} = r_{i,1}q_{i,1}$. Conversely, if the rest of the followers choose to abate, i.e. $\sum_{j=1,j\neq i}^{m} z_{j,1} < \sum_{j=1,j\neq i}^{m} r_{j,1}q_{j,1}$, then i will also choose to abate.By assuming that the total abatement made each follower is x, then i abates $\frac{m-1}{m}x < x$. This means that the rest will adopt this exact behavior which will lead to a continuous decrease to the followers' abatement. Therefore, the only solution for the followers is not to abate. A detailed proof is presented in the "Appendix B".

By (7), one can observe the decrease in the product price and the increase in the revenue in the output market by producing one additional unit of product, for both periods (1 and 2).

The existence of banking entails Eq. (6), where the discounted marginal abatement costs are equalized in both periods. Firms have an incentive to "smooth" their abatement expenditures through time. This behavior has been well discussed in the literature and it provides an important aspect since it stabilizes the permit prices across periods, leading to a more robust system.

Leaders' Equilibrium. As mentioned in the beginning of the section, leaders' permit decisions affect the permit price via followers' response. In order for the leaders to maximize their profits, they consider Eq. (4). The Leaders equilibrium conditions are presented below:

$$\mathcal{H}_{L,t}(r_{i,t}q_{i,t} - z_{i,t}) = \frac{1}{m}\mathcal{H}'_{F,t}\left(K_t\right)\left(z_{i,t} - z^{f}_{i,t} - \bar{z}_{i,t} - \lambda_i\beta_1\right) + \mathcal{H}_{F,t}\left(K_t\right) \tag{8}$$

$$\mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 + B_1\right]\right) = \beta_1\mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 - B_1\right]\right) \tag{9}$$

$$\frac{\partial C^P_{i,t}}{\partial q_{i,t}} + \frac{\partial C^A_{L,t}}{\partial q_{i,t}} = p_t(q_{i,t}) + \frac{\partial p_t}{\partial q_{i,t}}q_{i,t} \tag{10}$$

Where K_t is the average abatement of the Followers in both periods, i.e. $K_1 = \frac{1}{m}\left(\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right)$ and $K_2 = \frac{1}{m}\left(\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right)$. These quantities, appear in leaders equilibrium conditions, since by definition leaders will set the permit price by determining the residual amount of permits for use by the followers.

Equation (8) refers to the permit and abatement decisions for both periods. Equation (10) is related with the output market equilibrium conditions in both periods, and refer to the decrease in the product price and the increase in the revenue in the output market by producing one additional unit of product. The two periods are connected through the banking condition, which here is illustrated on (9).

From Eq. (8), it is clear that the Leaders consider some variables which are associated with the Followers in order to form their marginal abatement costs i.e. $\mathcal{H}_{L,1}(r_{i,1}q_{i,1} - z_{i,1})$ and $\mathcal{H}_{L,2}(r_{i,2}q_{i,2} - z_{i,2})$. More specifically, K_1 and K_2 together with the abatement cost functions of the Followers ($\mathcal{H}_{F,1}$ and $\mathcal{H}_{F,2}$) are included in these equations. The Lagrange multiplier λ_i, is associated with the banking constraint.

Solving (8) with respect to banking $\bar{z}_{i,1}$, we obtain that:

$$\bar{z}_{i,1} = m\frac{\mathcal{H}_{L,1}(\alpha_{i,1})}{\mathcal{H}'_{F,1}(K_1)} - m\frac{\mathcal{H}_{F,1}(K_1)}{\mathcal{H}'_{F,1}(K_1)} + (z^f_{i,1} - z_{i,1}) - \lambda_i \tag{11}$$

$$\bar{z}_{i,1} = -m\frac{\mathcal{H}_{L,2}(\alpha_{i,2})}{\mathcal{H}'_{F,2}(K_2)} + m\frac{\mathcal{H}_{F,2}(K_2)}{\mathcal{H}'_{F,2}(K_2)} - (z^f_{i,2} - z_{i,2}) - \beta_1\lambda_i \tag{12}$$

Using (7) and equalizing (11) and (12), we obtain that:

$$m\frac{\mathcal{H}_{L,1}(\alpha_{i,1})}{\mathcal{H}'_{F,1}(K_1)} + (z^f_{i,1} - z_{i,1}) = -m\frac{\mathcal{H}_{L,2}(\alpha_{i,2})}{\mathcal{H}'_{F,2}(K_2)} - (z^f_{i,2} - z_{i,2}) + 2m\frac{\mathcal{H}_{F,1}(K_1)}{\mathcal{H}'_{F,1}(K_1)} - \lambda_i(\beta_1 + 1) \tag{13}$$

The Lagrange multiplier λ_i is a free variable for the system. That means, the produced equilibrium conditions, as described above, signify infinite equilibrium solutions and each of them can be indicated by a λ_i value.

For a fixed λ_i, the quantity $2m\frac{\mathcal{H}_{F,1}(K_1)}{\mathcal{H}'_{F,1}(K_1)} - \lambda_i(\beta_1 + 1)$ is constant, since K_1 is a constant quantity. If we now define the variable $T_{i,t} = m\frac{\mathcal{H}_{L,t}(\alpha_{i,t})}{\mathcal{H}'_{F,t}(K_t)} + (z^f_{i,t} - z_{i,t})$, Eq. (13) can be written as:

$$T_{i,1} + T_{i,2} = const \tag{14}$$

The quantity $\frac{\mathcal{H}_{L,t}(\alpha_{i,t})}{\mathcal{H}'_{F,t}(K_t)}$ is an increasing function in α_i, since $\frac{\partial\mathcal{H}_{L,1}(\alpha_{i,t})}{\partial\alpha_{i,t}} < 0$ and $\mathcal{H}'_{F,t}(K_t) < 0$ and thus $\frac{1}{\mathcal{H}'_{F,t}(K_t)}\frac{\partial\mathcal{H}_{L,t}(\alpha_{i,t})}{\partial\alpha_{i,t}} > 0$.

Note that the quantities $T_{i,1}$ and $T_{i,2}$ contain the initial permit endowment $z^f_{i,t}$ for both periods and that $z^f_{i,1} > z^f_{i,t}$. Let us now consider an increase in the abatement made by a Leader. In such case both $\frac{\mathcal{H}_{L,t}(\alpha_{i,t})}{\mathcal{H}'_{F,t}(K_t)}$ and $(z^f_{i,t} - z_{i,t})$ would increase leading $T_{i,t}$ to rise. In the exact same way, an abatement drop will result in a lower $T_{i,t}$. Therefore, by (14), one can observe that an abatement reduction in the first period will cause second period's abatement to increase. Consequently, the Leaders can adjust the overall two-period abatement among periods.

3.4 Overall Interpretation and Theoretical Results

Observing the equilibrium conditions both for the leaders' and the followers' subsystems, our core results can be summarized below:

- The only solution for the followers in the equilibrium is to avoid abatement (proof in "Appendix B").
- Leaders are responsible for the total abatement.
- Considering that firms seek to maximize their profit, leaders should cover the followers' demand for permits across periods. Therefore, they can adjust their period's abatement decisions as long as the total abatement fulfill the followers' need for permits.

- In equilibrium permit prices are equal in both periods. Therefore, if leaders choose not to abate in a specific period, then followers have to decrease their output product in order to be able to meet their compliance requirements.

We observe that using business as usual emissions $(z_{i,1}, z_{i,2}) = (r_{i,1}q_{i,1}, r_{i,2}q_{i,2})$ satisfy the followers' equilibrium conditions. In "Appendix B", we prove that the only equilibrium strategy for the followers is not to abate (see also the discussion after the followers' equilibrium conditions). Thus in all solutions of the followers sub-system, there is no abatement.

From the above analysis it is inferred that emission reductions achieved under the existence of such a policy system are due to extensive abatement from the leader companies. This behaviour has been observed by Guo et al. (2020) on real data from the first two phases of the EU ETS.

In case the Leaders choose to sell a small amount of permits, the above solution indicates that the Followers will have to decrease their z^u emissions. This is possible only by decreasing their output product, which is similar with the findings of Hintermann (2011). If the leaders compete with the followers on the output market, our findings suggest that their decisions on the amount of permits that are left for use by the followers may also affect followers' output production levels. Hence the Leaders can transfer their market power across the two markets and gain more profit. That outcome is in perfect shape with what Hintermann (2011) and Chen and Tanaka (2018) have stated. Therefore, the Leaders excess market power to both permit and output market.

By viewing two separate periods, without the potential of banking Leader has to cover the demand for permits in both periods. If the abatement cost is higher in the first period, then without the potential of banking, this may lead to lower total profits, since the leader will not be able to take advantage of the lower cost and store permits for future use. Overall inter-temporal trading improves the total profit of the firms compared to the case where banking is not allowed (Sect. 3.2.3).

Furthermore, during the solution procedure we arrive to the price function (4) in the form of the marginal abatement cost of the Followers. The permit price is a function of the emissions that the Leaders are going to produce. In fact it is shown that the large firms form the supply of permits and set the permit price depending on their abatement decisions. It is also shown that the permit price will depend on the Followers' business as usual emissions, which is reasonable since this quantity has an important role on the demand for permits.

From this function we can get some more dependencies that have been stated in the literature. Maeda (2004) and Hintermann et al. (2016) discuss that banking and permit price change in the exact same directions. This can be easily seen in Eq. (4), since it's obvious that σ and B are positive correlated. On the other hand, the negative relation between permit price and the initial period allocation, is widely discussed in the literature (Ellerman et al. 2016; Goulder and Schein 2013) and this information is also included in our permit function (4), since, σ and A are negative correlated.

3.5 Evidence from EU Emission Trading System

In this section we provide evidence from the actual verified emissions that justify the plausibility of our model. In other words we discovered patterns in the EUTL registry (European Union Transaction Log (EUTL) - Union Registry 2020) that are in accordance with what our model suggests (i.e. only Leaders abate—see Sect. 3.4). This implies that "small" companies are expected to exhibit relatively stable emissions over time, while “large” companies are expected to reduce them.

However, since the EUTL includes data concerning neither the EUA price, nor the output product, our findings provide strong evidence rather than actual proof.

Furthermore, the actual system has experienced some shocks and failures such as the initial over-allocation of permits, the economical crisis and the VAT fraud (Guegan et al. 2011) to name a few.

Before proceeding, the firms (Account Holders in EUTL terminology) must be classified as “small” and “large” implying Follower and Leader roles. We do so by considering the size in terms of emission amount, assuming that a Leader possess a lot and/or big installations and as a result the majority of emissions comes from them.

For each firm i and year $t \in [2005, 2018]$, we retrieve the verified emissions, $z_{i,t}$, for all its installations. The firms are classified according to the natural logarithm of the average annual verified emissions (see quantity (15)).

$$\ln\left(E[z_{i,t}|i]\right) \tag{15}$$

More specifically, interval (16) is divided into 5 equal sub-intervals grouped into three regions. The first region consists of the two first sub-intervals. The following two sub-intervals form the second region and the fifth sub-interval is identical to the third region. Each firm is classified according to the region that the quantity (15) falls in. The defined classes in order of emission size are labeled as “Negligible”, “Follower” and “Leader” (see Fig. 1).

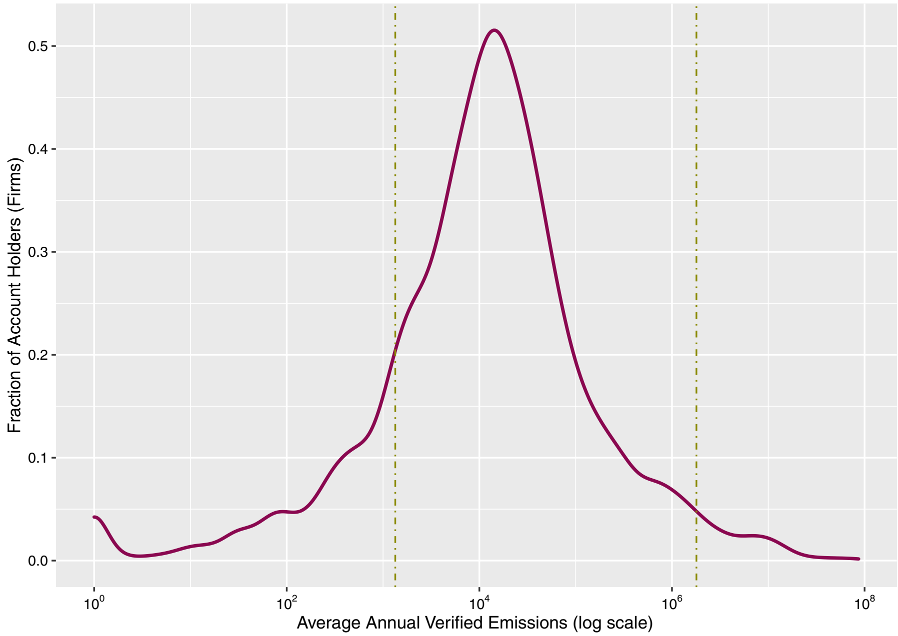


Fig. 1 Distribution of the annual verified emissions of the Account Holders associated with stationary installations of sector 20 (named as “Combustion of fuels” - a superset of the energy sector). The horizontal axis is in log scale. Dashed vertical lines partition Account Holders into three classes in terms of the size of verified emissions. These classes are from left to right: “Negligible”, “Follower” and “Leader”

$$\left[0\,,\ \max_i \left\lfloor \ln\left(E[z_{i,t}|i]\right)\ \right\rfloor\right] \tag{16}$$

Class “Negligible” is associated with a few and very small installations emitting a negligible amount of pollution, less than 1340 tones of CO_2. In contrast “Followers” emit hundreds of thousands of tones of CO_2 per year (see table 2 in “Appendix C”). Thus we restrict our attention to the classes “Follower”, “Leader”. More on the classification of firms can be found in the “Appendix C”.

Figures 2 and 3 show a clear reduction in emissions from “Leaders” of sector 20. The only exception is the years following the economic crisis of 2009. A possible explanation may be due to the economic recovery in many European countries (European Environmental Agent 2012). All of the dashed green lines in Fig. 2, correspond to a 1.74% reduction from the previous year. Given that the cap during phase III (2013–2020) was reducing by 1.74% every year, green dashed lines can be thought as a naive, but a simple minimum respond to the cap reduction. More evidence for the justification of our model can be drawn from Figs. 10 and 11 in the “Appendix C”.

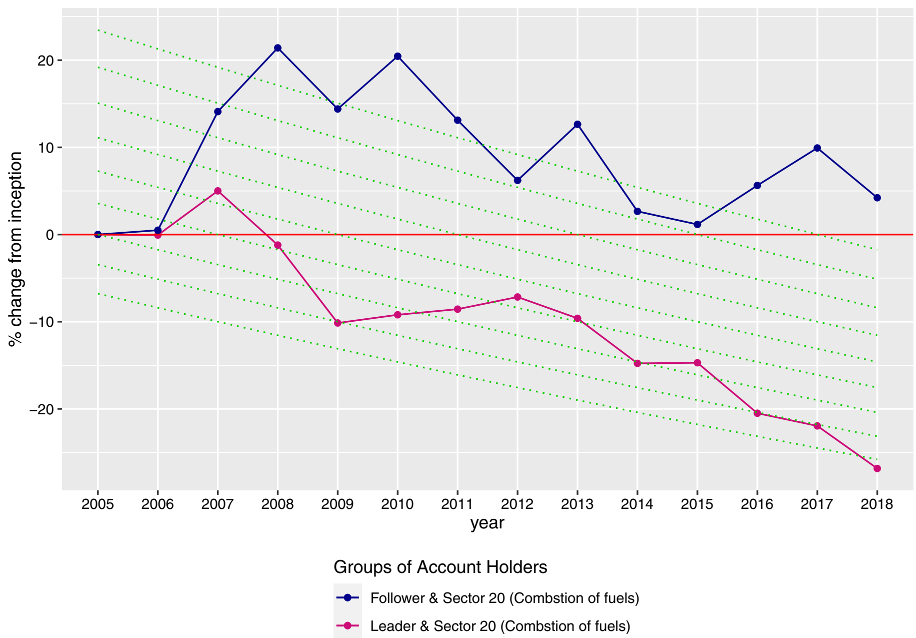


Fig. 2 Normalized total verified annual emissions of installations of sector 20 (Combustion of fuels) according to Account Holder’s emission size class. Annual emissions are represented as a percentage change from the first observation i.e., year 2005. The dashed green lines correspond to an annual emission reduction of 1.74% from the previous year. The “Leaders” clearly reduce their emissions over time. The total relative reduction is greater than the cap reduction during the 3rd phase (2013–2020). On the other hand “Followers” emit in total no less emissions than the inception of EU ETS

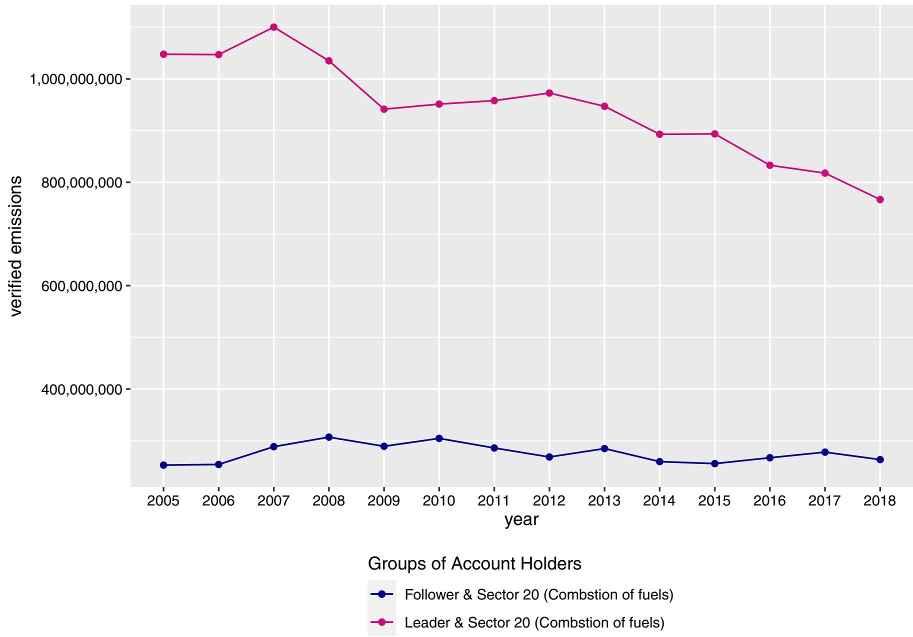


Fig. 3 Total verified annual emissions of installations of sector 20 (Combustion of fuels) according to Account Holder's emission size class. The "Leaders" clearly reduce their emissions over time, while the "Followers" keep them relatively stable

4 Simulations

In Sect. 3, we presented our theoretical model and described the equilibrium conditions both for the leaders and the followers. As discussed in Sect. 1, the overallocation of Phase I, along with the banning of banking between Phase I and II, pushed the EUA price to zero. Moreover, EU ETS was prone to abrupt changes of the output product at the beginning of the financial crisis of 2009. Hintermann (2011) discusses how the level of initial allocation can change the balance of the system leading to suboptimal system operation. Moreover, Ellerman et al. (2016) and Kettner et al. (2011) discuss how the economic crisis of 2009 and the overall reduction of the output product led the emissions to be reduced, and hence, the cap became not binding. Motivated by these studies and working towards supporting our theoretical model with empirical evidence, we test our model in real life situations and observe the trends and changes in variables such as the allowance price, emissions and banking, depending on changes in output product and initial allocation.

The simulations presented in this section, are divided into two categories: (a) variation in the output product and (b) variation in the initial allocation. A variable of interest i.e. emissions, permit price, banking, will be specified and the model will be run for different output levels and different initial allocation. In this way, we will be able to observe how the variable of interest reacts to changes in the initial allocation and in the production levels.

Data Construction and Sources: Due to the lack of relevant data concerning the firms' output product, for our simulations, we focus on the electricity sector, since it is easier to find reliable information about the volumes of power production. Price

Waterhouse Coopers (2008) provides a list of European power companies by carbon intensity per year until 2009. We are interested in the output product, the actual emissions, and the emission rate.

Moreover, to perform our simulations some EU ETS information, such as the free allocation, surrendered/verified allowances and banking volumes, are also needed. By using the European Union Transaction Log (EUTL) - Union Registry (2020) we gathered the relevant data concerning the trading volumes and the free allowances and by using the database developed by Dimos et al. (2020) we also gathered the banking volumes for each firm. In this way some Leaders can be clearly assorted. For example, in 2009, using the latter database, it is seen that Edison S.p.A. got 8,202,117 initial allowances and surrendered 6,991,207 without making a significant technological upgrade, as verified by the PWC study. Hence, the regulator supplied this company with 1.17 times more allowances than actually needed. This is in perfect line with what Hintermann (2011) and Chevallier (2011) stated about the Leaders, i.e. they receive a sufficiently large permit allocation. Adding that the same year, according to PWC, Edison S.p.A.'s emission rate r was 495 CO_2/MWh (which is considered small), it is pointed out that our criteria for the Leaders, as described in the Introduction, in paragraph 3 and in Rico (1995), are fulfilled. That is, the Leaders have more advanced technologies, which is denoted by the emission rate and also enjoy a generous initial allocation from the regulator.

The data construction procedure is described by the following steps.

1. Determine three Leaders and seven Followers from the databases.
2. Estimate the r and z^u of these companies from the PWC study.
3. Define a rule to allocate free allowances.
4. Derive plausible emissions data in the ways below:
   i The Followers emit their business as usual emissions.
   ii The Leaders abate a fixed percentage of their business as usual emissions.
5. Calculate the remaining allowances and use them as banking.

The produced 2-period dataset can be found in Tables 4 and 5 in "Appendix D".

Marginal Abatement Cost Curve: We use a MAC function of the type that Ellerman and Decaux (1998) developed using the MIT EPPA model, because it gives a more general approach to the dependence between abatement of emissions and its cost. This function, as used by Chevallier (2011) at his numerical simulations, has the following functional form:

$$Y = aX^2 + bX + c$$

where Y is the marginal abatement cost and X is the extent of abatement in million metric tons of carbon (Mton).

To produce the coefficients, we apply (4) to our dataset, and try some different combinations of them in the range of those used by Chevallier (2011) (i.e., $a \in [0.0001, 0.2]$ and $b \in [0.0, 2]$). For each combination, a MAC function is formed, which is then provided to our models' equations together with the generated data. If the equations hold, then the produced data and the MAC function, respect the EU ETS rules and our modeling.

The MAC coefficients that was used can be found in Table 6 of "Appendix D".

Abatement Levels One more aspect of the simulation process is to examine how different first periods' abatement levels for the Leaders affect significant system variables, such as banking, price and emissions.

To that end, the system is examined under different strategies of the Leaders. As described in Sect. 3, Eq. (8) contains the Lagrangian multiplier λ_i, which is a free variable for the system. Hence, we can choose more than one strategy and compare different equilibrium scenarios. For example, if we want a company to adopt a specific abatement level, i.e. 10% of z^u, we can input the value of emissions to Eq. (8) for the first period, i.e. $z_{1_i} = 0.9z^u$, solve for λ_i, and then use that Lagrangian multiplier to get the emissions for the second period.

We set four different abatement levels for the Leaders:

1. Abatement 0% of business as usual emissions.
2. Abatement 10% of business as usual emissions.
3. Abatement 25% of business as usual emissions.
4. Abatement according to Followers demand. That is, the Leaders abate in order to provide the Followers with the exact amount of permits they need.

4.1 Variation in Output Product

Primarily we examine how the variables of interest, correspond to changes in the total output product.

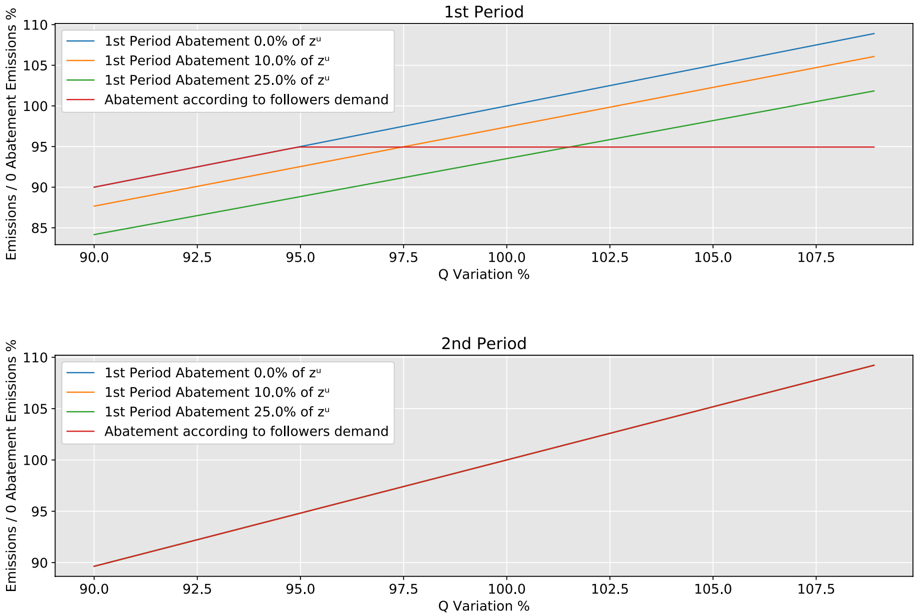


Fig. 4 Comparison per period on emissions as a percentage of 0 Abatement Emissions, under variation in output product

Emissions: In order to clearly picture how different abatement strategies affect the emissions, we can plot, for every different abatement level, the overall emissions as a percentage of the emissions made under no abatement (i.e. 0%).

In Fig. 4, one can see that for the three fixed abatement levels (i.e. 0%, 10% and 25%) the emissions grow as the output product grows in both periods. This makes absolute sense since the more product is produced, the greater the $z^u = rq$ emissions are and under a fixed level of abatement, let us say l, the firm emits $z = (1 - l)rq$ tons. That concludes that, under specified abatement levels, the emissions variate proportionally to the change of the output product. Moreover, as expected, the overall emissions get smaller for bigger abatement levels.

It is interesting to highlight the differences between the two periods. As it illustrated in second period's graph, the emission levels are the same for every abatement scenario. Since Leaders are responsible for the overall abatement (see Sect. 3.3.1), coupled with a generous allocation, they still hold more allowances even in a 10% product growth. So they choose to not abate and sell their surplus at a greater price, since the overall gain is the same (equilibrium) and the second period is the last one.

It is important to note that while for the second period, there is no abatement under all different strategies, the first and second period curves are not identical in the case of 0% abatement. This is due to the reduction of the cap in the second period and the difference between the two periods' overall product.

Another remarkable difference occur when the Leaders abate according to the Followers demand. For less than 95% of the output product, Leaders can handle the total Followers demand using their surplus. After that point, the Followers demand start to rise (i.e. $rq - z_f$ grows), and the Leaders ensure to abate exactly the requested amount. That is the Followers' and Leaders' emissions are inversely proportional amounts. The outcome is a fixed level of emissions independent on the variation in the output product. This strategy stabilizes the systems emissions to 95% of the level that occurs for business as usual emissions under 100% of the product.

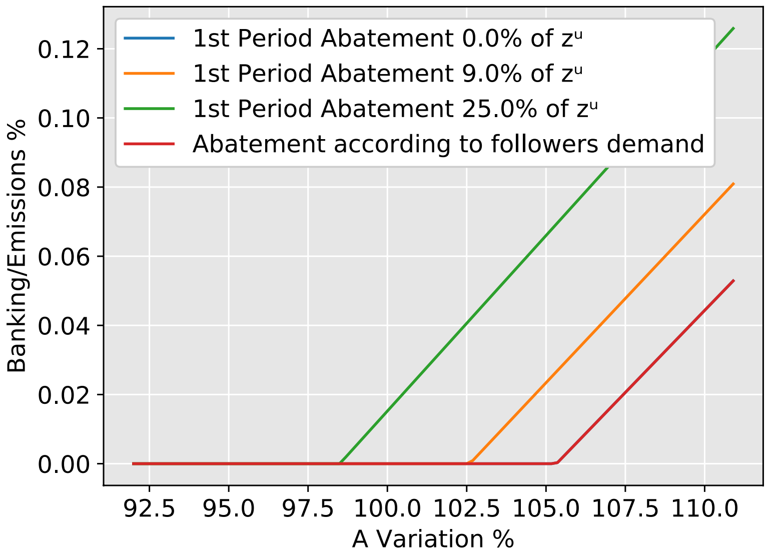


Fig. 5 Banking as a percentage of Emissions, under variation in output product

Banking: Moving forward to the banking variable as it is illustrated in Fig. 5, it can be noted that when output product varies, the only variable of interest is the percentage of banking over emissions, since the emissions change with q.

In Fig. 5, we observe that while the output product decreases, the percentage of banking over emissions rises. That is because, while the product drops, the need for permits is fulfilled by the initial allocation, arriving to a point where a surplus of permits is accumulated, which the companies choose to bank for future use. This point occurs when the Leaders over-allocation's surplus plus the permits saved due to abatement equals the demand of the Followers. This point is different for separate abatement levels, since the greater the abatement is, the larger the surplus gets.

One last thing to notice about this graph is the absence of the curve that corresponds to 0% abatement. In the sense of banking/emissions this case is exactly the same as when Leaders abate according to the Followers' demand. This is because, for less than 95% of the product, the Leaders don't need to abate and store all of their surplus. For 95% and beyond the dominant firms abate in order to fulfill the demand, but they do not store the permits. That is also the case for 0% and that is why these two curves are identical.

Permit price: Proceeding to Figs. 6 and 7, we can extract information about the system's most important aspect, the permit price.

Firstly, it is obvious that the greater the product is, the larger the price gets. That is because the allocation cap is the same across strategies, and while the product rises, the demand for permits grows and pushes the price upwards. We can also see that for smaller abatement levels, the first period's price is more prone to an output product change. That means that if an incentive for abatement exists, the system will be more stable.

It was reasonable to expect that for a great drop of the amount of output product, the price would drop equivalently. However, banking normalizes the price across time and

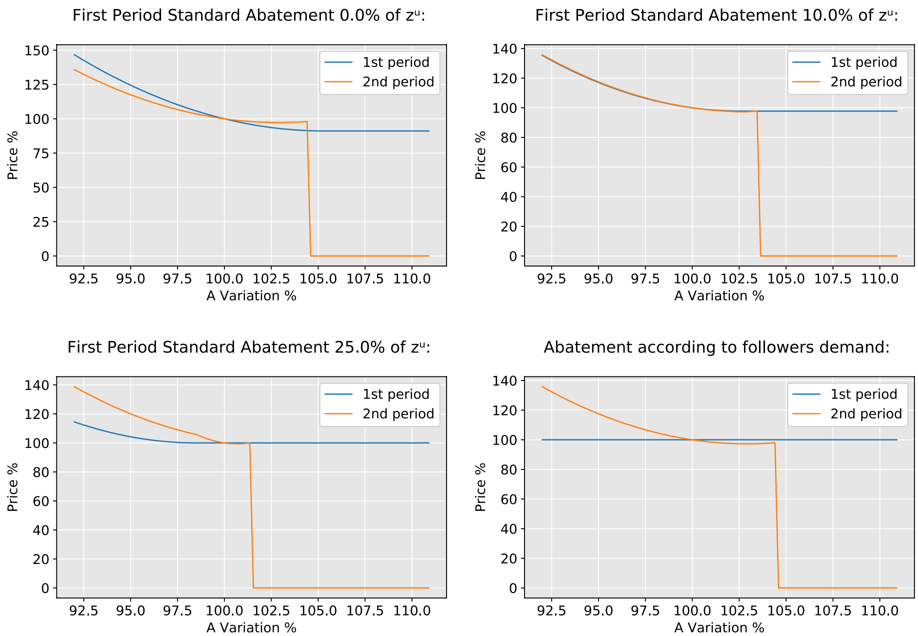


Fig. 6 Price variation, under variation in output product

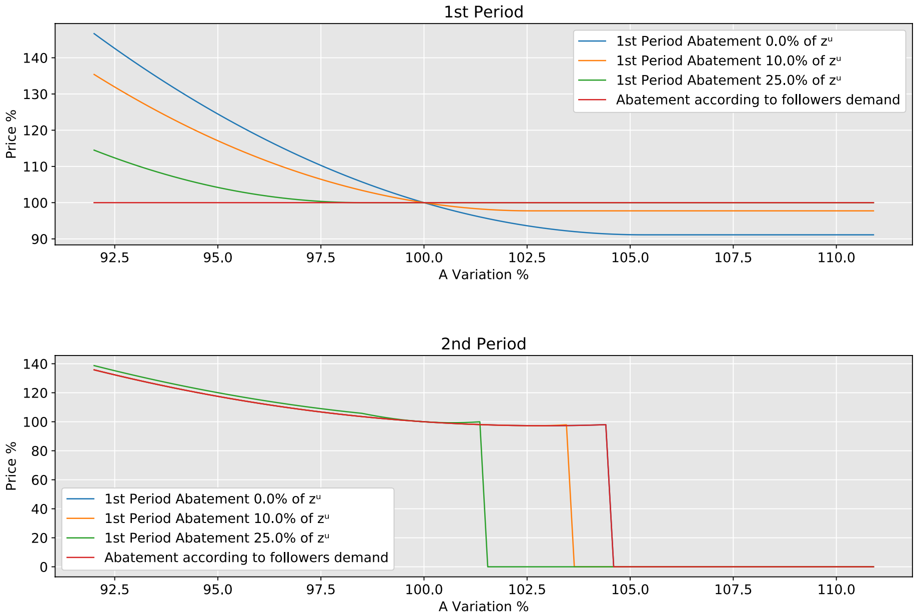


Fig. 7 Price variation comparison per period, under variation in output product

the consequences of big changes in system variables will be reflected to the price in the future. This is the reason why in Figs. 6 and 7, there is a point below which the second period's price equals zero. Yet, this point depends on the abatement level. For bigger abatement levels, a greater amount of permits is banked and used in the second period, increasing the supply to the market and leading to earlier price drop. So it is inferred that an increase in banking leads to better system functionality in the present, but makes the system more unstable in the future.

As stated before, the Leaders decide to avoid abatement in the second period. That means that from a point on, the supply of permits from the Leaders will be the same, regardless the abatement level in the first period, giving the same price across all different cases. Before this point we notice that the price drops abruptly to zero. Of course, as mentioned, the greater the banking, the more unstable the system will be in the future. Therefore for bigger abatement levels, the price drops to zero for smaller product changes. Although a rather smoother price drop was expected, this abrupt change may be caused due to the marginal abatement used, since its second coefficient is zero (Table 6, "Appendix D").

4.2 Variation in Initial Allocation

Considering the variations in the initial allocation of permits, Fig. 8, is a mirrored version of Fig. 6. The variation in the initial allocation has exactly the opposite effect than the variation in the output product. This means that if the allocation increases, the surplus will grow, making the firms to store the extra permits and thus rising inter-temporal trading.

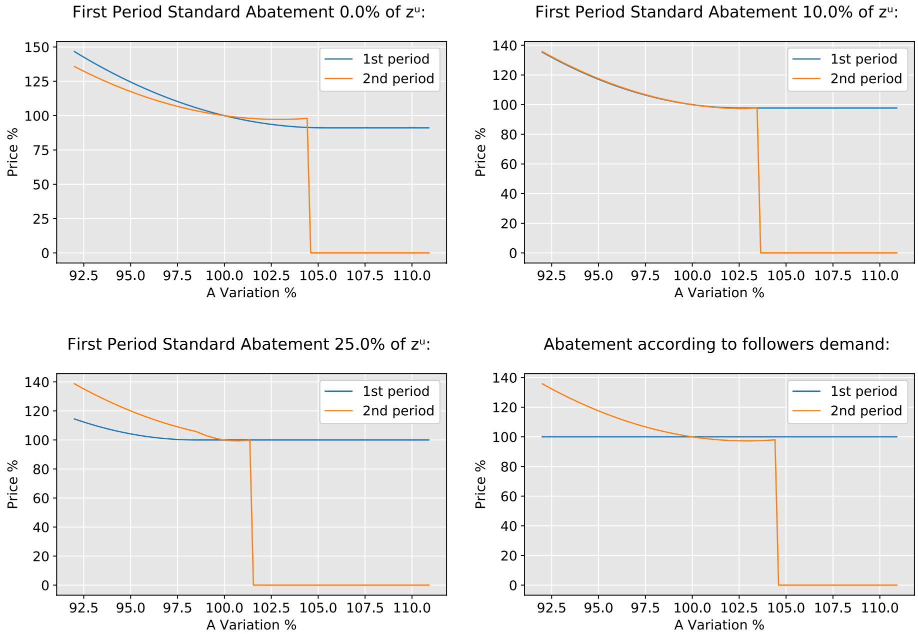


Fig. 8 Price variation, under variation in initial allocation

It should be noted that while the cap grows, the price drops. This is because a greater allocation increases the permit supply of the Leaders, it shrinks the demand of the Followers and vice versa.

It is also worth noting that a bigger abatement level can stabilize the price, so it can become neutral in cap changes. The firms that made great abatement are more immune to smaller allocations, while they will store more permits, in case of a cap rise. That behavior keeps the price stable across time in case of a cap reduction, but drops it rapidly in case of a cap rise.

As an overall conclusion, we can point out that under a cap reduction, the system is more stable for greater abatement levels, while for a cap rise, it works better under small abatement levels. This information can be useful for the regulator in finding an optimized allocation level based on past abatement data.

4.3 Overall Interpretation and Simulations Results

From the simulations, it is clear that the permit price reaches zero after an overallocation of allowances, which was a real situation in the EU ETS (Brink et al. 2016; Ellerman et al. 2016; Hintermann et al. 2016; Daskalakis 2018. Furthermore, it is shown that the aspect of banking can stabilize the system and improve its performance under such extraordinary cases. This behaviour has been noticed in real systems and discussed in the literature, see e.g., (Ellerman et al. 2016; The European Environment Agency (EEA) 2020). Finally, as happened with Europe's economic recession across 2008 and 2012 (Koch et al. 2014; De Perthuis and Trotignon 2014; Ellerman et al. 2016), reductions of the production levels lead the permit price to fall.

Furthermore there is a performance drawback since, when the participants abate more, the system is more stable in the first period, while this behavior may cause an abrupt price drop to zero in the second period. Overall, the robustness of the system under changes in free allocations and production levels is strongly connected to abatement levels and the amount of banked permits.

5 Conclusions

Even though initial allocation, inter-temporal trading, market power and linkage with the output market, are factors that have been proved crucial for the efficiency and the overall performance of the EU ETS, none of the existing theoretical models includes them all. There is a sizable literature about market power in permit markets and numerous studies that examine inter-temporal trading and the effects that this linkage has on the permit price. However, to the best of our knowledge, there is a gap considering the combination of those topics in theoretical modeling.

In this work, we dynamically built a model of an emission trading system, taking into consideration the existence of market power in the permit market. Firms differ with respect to their marginal abatement costs, and thus some hold a dominant position over the others. Along with the inclusion of the output product, we also considered that firms are endowed with an amount of permits. Moreover, we performed a variety of simulations and by using realistic data, we managed to recreate interesting real-life situations and observe how the various system variables respond.

First, we solved the two-period model and we provided the equilibrium conditions for both the followers and the leaders. Our core theoretical result provides us with a twofold conclusion: (a) In equilibrium, the only solution for the followers is not to abate, and therefore the leaders are responsible for the overall abatement; and (b) the leaders can adjust the amount of the abatement across periods, as long as they can cover for the overall followers' demand, leading to a decrease on followers' business-as-usual emissions, and thus, on their output product. In such a case, the leaders can affect followers' decisions in both markets.

Initial allocation plays a key role in the excess allowance holdings. Assuming that leaders are generally firms with more advanced technology and more financial and human resources, a generous initial endowment automatically gives them the ability to hold market power and adjust the permit residual for their own gain.

Moreover, by solving the model, an equilibrium permit price, in the form of the marginal abatement cost of the followers, was derived. The permit price function is positively correlated with the amount of the permit banking, and negatively correlated with the initial allocation, which is consistent with the existing literature.

We also provided three introductory scenarios to further enhance the comprehensibility of our general model. The three scenarios that we presented are: (1) a simplified model where neither banking nor trading were allowed, (2) a model where only banking was allowed and (3) a model where only trading was allowed. Overall we showed that intertemporal trading improves the total profit compared to all three simplified scenarios. The potential of banking allows firms to adjust their output product if the demand increases, by storing allowances for future use while trading allows followers to buy the allowances they need from leaders and consequently exploit leaders' advanced abatement technology.

Further investigation through the actual emission data from the European Union Transaction Log (EUTL) support our theoretical findings. Although EUTL does not contain data for either the permit price or the firms' output product, our evidence suggests that smaller companies exhibit relatively stable emission patterns and larger companies seem to exhibit declining emission patterns on average.

Finally, through the simulation procedure, we validated the overall system's performance. Our simulation results showed the changes in permit price, banking and emission levels with respect to changes in the output product and initial allocation. As expected, permit price is maintained in higher levels, as the output product decreases, due to inter-temporal trading, while from some point and below, the second period's price equals zero depending on the abatement level. The amount of banking increases as the output product decreases and this behavior is observed in every considered abatement level. Furthermore, the variations of the initial allocation have the exact opposite effect than the variations of the output product. As expected, a cap rise decreases the permit price, while a cap reduction increases the price. In a nutshell, we can observe that under a cap reduction the system performs better under greater abatement levels, while for a cap rise the performance is better under small abatement levels.

Overall, our mathematical modeling captures the complex relations between crucial system variables and parameters. Along with the simulation process, it can be of great use by the regulator to improve system's performance and minimize policy implications. Different scenarios based on the variation of the cap, as well as the variation of the output product can be implemented to envisage the effects that future structural changes may bring to system's variables and to avoid future malformations. Such simulations can result in determining an optimal allocation level or observe how different measures, e.g. back-loading, may work.

The present study was a first approach to model inter-temporal trading in emission trading systems under the existence of market power. In the follow up of our work, it would be interesting to investigate the impact of uncertainty in the prices both in the permit and in the output market. Finally, another interesting research direction could be to apply real data and evaluate the behavior of the theoretical model on the actual EU ETS evolution.

Appendix

A: Proof of Equilibrium Conditions

Each firm has to maximize its profit, therefore solve problem (3). Without loss of generality, we assume that abatement costs are equal among the followers denoted by index F and that abatement costs are also equal among the leaders denoted by index L.

Followers

By the followers' profit maximization problem and if $\mathcal{H}_{i,t}(r_{i,t}q_{i,t} - z_{i,t}) = -\partial C^A_{i,t}/\partial z_{i,t}$, we have:

$$\begin{aligned}
&\sigma_1 = -\frac{\partial C^A_{F,1}}{\partial z_{i,1}} \implies z_{i,1} = r_{i,1}q_{i,1} - \mathcal{H}^{-1}_{F,1}(\sigma_1)\\
&\sigma_1 = \beta_1 \sigma_2\\
&\sigma_2 = -\frac{\partial C^A_{F,2}}{\partial z_{i,2}} \implies z_{i,2} = r_{i,2}q_{i,2} - \mathcal{H}^{-1}_{F,2}(\sigma_2)\\
&\frac{\partial C^P_{i,1}}{\partial q_{i,1}} + \frac{\partial C^A_{F,1}}{\partial q_{i,1}} = p_{i,1}(q_{i,1}) + \frac{\partial p_{i,1}}{\partial q_{i,1}} q_{i,1}\\
&\frac{\partial C^P_{i,2}}{\partial q_{i,2}} + \frac{\partial C^A_{F,2}}{\partial q_{i,2}} = p_{i,2}(q_{i,2}) + \frac{\partial p_{i,2}}{\partial q_{i,2}} q_{i,2}
\end{aligned}$$

Considering the first m companies are followers, while the rest $n - m$ are leaders, the following demand for permits is being formed.

$$D_1 = \sum_{i=1}^{m} (z_{i,1} - z^f_{i,1} + \bar{z}_{i,1}) \tag{17}$$

If, as mentioned before, $\frac{\partial C^A_{i,1}}{\partial z_{i,1}} = \frac{\partial C^A_{j,1}}{\partial z_{j,1}}$ then all companies act equivalently. So from (17) we get,

$$D_1 = \sum_{i=1}^{m} r_{i,1}q_{i,1} - m\mathcal{H}^{-1}_{F,1}(\sigma_1) - \sum_{i=1}^{m} z^f_{i,1} + \sum_{i=1}^{m} \bar{z}_{i,1} \tag{18}$$

Since the number of permits used by the followers is equal to the residual left by the leaders, we can rewrite the demand fucntion as:

$$\begin{aligned}
&\sum_{i=1}^{m} r_{i,1}q_{i,1} - m\mathcal{H}^{-1}_{F,1}(\sigma_1) + \sum_{i=1}^{m} \bar{z}_{i,1} = A_1 - \sum_{i=m+1}^{n} z_{i,1} - \sum_{i=m+1}^{n} \bar{z}_{i,1}\\
&\implies \mathcal{H}^{-1}_{F,1}(\sigma_1) = \frac{1}{m}\left(\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 + \sum_{i=1}^{n} \bar{z}_{i,1} \right)\\
&\implies \sigma_1 = \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 + B_1 \right] \right)
\end{aligned} \tag{19}$$

Where $B_1 = \sum_{i=1}^{n} \bar{z}_{i,1}$ is the total banking for the first period. As we can see a permit price function is created, in the form of the marginal abatement cost of the followers.

Working in the exact same fashion we get a permit price function for the second period:

$$\implies \sigma_2 = \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 - B_1\right]\right) \tag{20}$$

Based on the the banking condition (i.e. $\sigma_1 = \beta_1\sigma_2$), we can derive the following general system condition, which equalizes marginal abatement costs across periods:

$$\begin{aligned}\mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 + B_1\right]\right) = \\ \beta_1\mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 - B_1\right]\right)\end{aligned} \tag{21}$$

We can also get the solution for the followers permits $z_{i,1}$ and $z_{i,1}$, which depend on the leaders' emissions and consequently on the amount of permits that will be available on the market. So we have:

$$z_{i,1} = r_{i,1}q_{i,1} - \frac{1}{m}\left(\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 + B_1\right) \tag{22}$$

$$z_{i,2} = r_{i,2}q_{i,2} - \frac{1}{m}\left(\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 - B_1\right) \tag{23}$$

Since the banking variable connects the two periods by transferring permits from the first period to the second, the overall demand for permits must equal the total residual left by the leaders. Every firm cannot own permits neither at the beginning nor at the end of the systems' duration, therefore:

$$A_1 + A_2 = \sum_{i=1}^{m} z_{i,1} + \sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} z_{i,2} + \sum_{i=m+1}^{n} z_{i,2} \tag{24}$$

Using Eq. (23) on (24) we get:

$$\begin{aligned}A_1 + A_2 &= \sum_{i=1}^{m} z_{i,1} + \sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=m+1}^{n} z_{i,2} \\ &\quad - \sum_{i=1}^{m} r_{i,2}q_{i,2} + A_2 + B_1 + \sum_{i=m+1}^{n} z_{i,2} \\ \implies A_1 &= \sum_{i=1}^{m} z_{i,1} + \sum_{i=m+1}^{n} z_{i,1} + B_1 \\ \implies \sum_{i=m+1}^{n} z_{i,1} &= A_1 - \sum_{i=1}^{m} z_{i,1} - B_1\end{aligned} \tag{25}$$

Substituting Eqs. (25) and (22), enables us to express the followers permits as:

$$
\begin{aligned}
& z_{i,1} = r_{i,1}q_{i,1} - \frac{1}{m}\left(A_1 - \sum_{i=1}^{m} z_{i,1} - B_1 + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 + B_1\right) \\
\Longrightarrow\ & z_{i,1} = r_{i,1}q_{i,1} - \frac{1}{m}\left(-\sum_{i=1}^{m} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1}\right) \\
\Longrightarrow\ & z_{i,1} = r_{i,1}q_{i,1} + \frac{1}{m}\left(\sum_{j=1, j\neq i}^{m} z_{j,1} - \sum_{j=1, j\neq i}^{m} r_{1_j}q_{j,1}\right), \forall i \in [1, m]
\end{aligned}
\tag{26}
$$

Equation (26) provides a clear solution for the followers' emissions target. Working on the same fashion for the $z_{i,2}$ variables of the small firms, we derive:

$$
z_{i,2} = r_{i,2}q_{i,2} + \frac{1}{m}\left(\sum_{j=1, j\neq i}^{m} z_{j,2} - \sum_{j=1, j\neq i}^{m} r_{j,2}q_{j,2}\right), \forall i \in [1, m] \tag{27}
$$

Therefore, according to (21) and (25):

$$
\mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right]\right) = \beta_1 \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right]\right) \tag{28}
$$

Finally based on (26)), (27) and (28) we can write a system of equations for all small firms and get their emissions for the two periods by solving it.

Leaders

The leaders presume the followers' equilibrium conditions and to solve their profit maximization problem, the only Lagrange multiplier left is the one corresponding to the $\sigma_1 = \beta_1\sigma_2$ condition. That is because, in contrast with the followers' problem, the prices σ_1 and σ_2 are dependent from the emissions made by the leaders (i.e. $z_{i,1}$ and $z_{i,2}$). It is important to remind that since the followers are price-takers, the price is independent of their individual actions. Supposing now that λ is the Lagrange multiplier for the $\sigma_1 = \beta_1\sigma_2$ condition, we get the following first order conditions:

$$
\begin{aligned}
& \frac{\partial C^A_{L,1}}{\partial z_{i,1}} + \frac{\partial \sigma_1}{\partial z_{i,1}}(z_{i,1} - z^f_{i,1} + \bar{z}_{i,1} + \lambda) + \sigma_1 = 0 \\
& \sigma_1 = \beta_1\sigma_2 \\
& \frac{\partial C^A_{L,2}}{\partial z_{i,2}} + \frac{\partial \sigma_2}{\partial z_{i,2}}(z_{i,2} - z^f_{i,2} - \bar{z}_{i,1} - \lambda\beta_1) + \sigma_2 = 0 \\
& \frac{\partial C^P_{i,1}}{\partial q_{i,1}} + \frac{\partial C^A_{L,1}}{\partial q_{i,1}} = p_{i,1}(q_{i,1}) + \frac{\partial p_{i,1}}{\partial q_{i,1}}q_{i,1} \\
& \frac{\partial C^P_{i,2}}{\partial q_{i,2}} + \frac{\partial C^A_{L,2}}{\partial q_{i,2}} = p_{i,2}(q_{i,2}) + \frac{\partial p_{i,2}}{\partial q_{i,2}}q_{i,2}
\end{aligned}
$$

We can now use (19) and (25) to analyse the first equation of the above system.

$$
\begin{aligned}
&\nwarrow H_{L,1}(r_{i,1}q_{i,1} - z_{i,1}) + \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 - B_1\right]\right) \\
&\quad + \frac{\partial \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 - B_1\right]\right)}{\partial z_{i,1}}\left(z_{i,1} - z^{f}_{i,1} + \bar{z}_{i,1} + \lambda\right) = 0 \\
&\implies \mathcal{H}_{L,1}(r_{i,1}q_{i,1} - z_{i,1}) = \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right]\right) \\
&\quad + \frac{\partial \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 - B_1\right]\right)}{\partial z_{i,1}}\left(z_{i,1} - z^{f}_{i,1} + \bar{z}_{i,1} + \lambda\right) \\
&\implies \mathcal{H}_{L,1}(r_{i,1}q_{i,1} - z_{i,1}) = \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right]\right) \\
&\quad + \frac{1}{m}\mathcal{H}_{F,1}{}'\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,1} + \sum_{i=1}^{m} r_{i,1}q_{i,1} - A_1 - B_1\right]\right)\left(z_{i,1} - z^{f}_{i,1} + \bar{z}_{i,1} + \lambda\right) \\
&\implies \mathcal{H}_{L,1}(r_{i,1}q_{i,1} - z_{i,1}) = \mathcal{H}_{F,1}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right]\right) \\
&\quad + \frac{1}{m}\mathcal{H}_{F,1}{}'\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,1}q_{i,1} - \sum_{i=1}^{m} z_{i,1}\right]\right)\left(z_{i,1} - z^{f}_{i,1} + \bar{z}_{i,1} + \lambda\right)
\end{aligned}
\tag{29}
$$

In the same way, we analyse the third system equation:

$$
\begin{aligned}
&- \mathcal{H}_{L,2}(r_{i,2}q_{i,2} - z_{i,2}) + \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 + B_1\right]\right) \\
&\quad + \frac{\partial \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 + B_1\right]\right)}{\partial z_{i,2}}\left(z_{i,2} - z^{f}_{i,2} - \bar{z}_{i,1} - \lambda\beta_1\right) = 0 \\
&\implies \mathcal{H}_{L,2}(r_{i,2}q_{i,2} - z_{i,2}) = \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right]\right) \\
&\quad + \frac{\partial \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 + B_1\right]\right)}{\partial z_{i,2}}\left(z_{i,2} - z^{f}_{i,2} - \bar{z}_{i,1} - \lambda\beta_1\right) \\
&\implies \mathcal{H}_{L,2}(r_{i,2}q_{i,2} - z_{i,2}) = \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right]\right) \\
&\quad + \frac{1}{m}\mathcal{H}_{F,2}{}'\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{i,2} + \sum_{i=1}^{m} r_{i,2}q_{i,2} - A_2 + B_1\right]\right)\left(z_{i,2} - z^{f}_{i,2} - \bar{z}_{i,1} - \lambda\beta_1\right) \\
&\implies \mathcal{H}_{L,2}(r_{i,2}q_{i,2} - z_{i,2}) = \mathcal{H}_{F,2}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right]\right) \\
&\quad + \frac{1}{m}\mathcal{H}_{F,2}{}'\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{i,2}q_{i,2} - \sum_{i=1}^{m} z_{i,2}\right]\right)\left(z_{i,2} - z^{f}_{i,2} - \bar{z}_{i,1} - \lambda\beta_1\right)
\end{aligned} \tag{30}
$$

Finally, Eqs. (29) and (30) can solve the leaders' problem using the information gained from the followers optimization problem.

B: Proof of Followers' Unique Abatement Solution

In Table 1 it is seen that the amount of abatement of a firm is indicated by $\alpha_{i,t} = z^{u}_{i,t} - z_{i,t} \implies \alpha_{i,t} = r_{i,t}q_{i,t} - z_{i,t}$. Now, if $c_{i,t} = \sum_{j=1, j\neq i}^{m} \alpha_{j,t}$ from Eqs. (5) and (6) it is observed that the abatement solution for the followers can be written as:

$$
m\alpha_{i,t} = c_{i,t} \tag{31}
$$

In case of $i > 1$ we can analyse Eq. (31) as follows:

$$
\begin{aligned}
& m\alpha_{i,t} = c_{i,t} \\
\implies & m\alpha_{i,t} = c_{1,t} - \alpha_{i,t} + \alpha_{1,t} \\
\implies & \alpha_{i,t} = \frac{c_{1,t} + \alpha_{1,t}}{m+1}, \forall i \in [2, m]
\end{aligned} \tag{32}
$$

We can now try to find the $\alpha_{1,t}$ quantity by analysing Eq. (31) for $i = 1$:

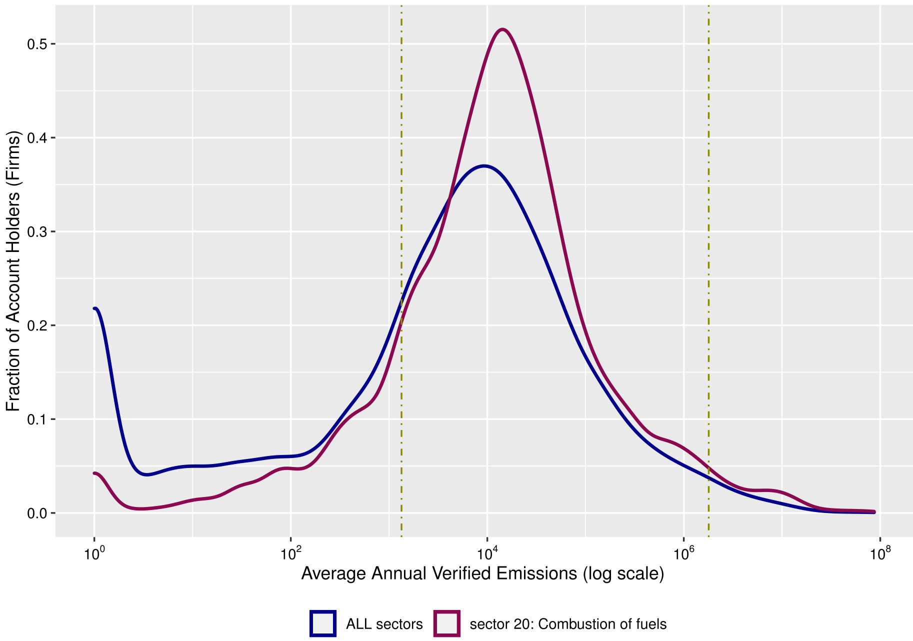
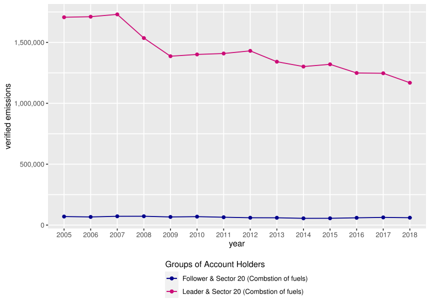


Fig. 9 Distribution of the annual verified emissions of the Account Holders associated with stationary installations of sector "20—Combustion of fuels" compared with the corresponding distribution of all Account Holders. The horizontal axis is in log scale. Dashed vertical lines partition Account Holders into three classes in terms of of the size of verified emissions. These classes are from left to right: "Negligible", "Follower" and "Leader"

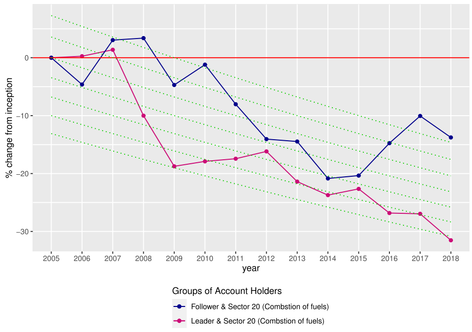
Table 2 Various statistics for the three considered classes of emissions size

Class of emission size	Max mean annual emissions max $E[z_{i,t}\|i]$	Number of Account Holders all sectors	Number of Account Holders sector 20
Negligible	1339.431	3503 (30.92 %)	622 (15.884 %)
Follower	1,794,074.773	7622 (67.28 %)	3187 (81.384 %)
Leader	86,718,268	204 (1.8 %)	107 (2.732 %)

$$
\begin{aligned}
& m\alpha_{1,t} = c_{1,t} \\
\implies & m\alpha_{1,t} = \sum_{j=2}^{m} \alpha_{j,t} \\
\overset{(32)}{\implies} & m\alpha_{i,t} = \sum_{j=2}^{m} \frac{c_{1,t} + \alpha_{1,t}}{m+1} \\
\implies & m\alpha_{1,t} = \frac{m-1}{m+1}(c_{1,t} + \alpha_{1,t}) \\
\implies & \alpha_{1,t} = \frac{m-1}{m^2+1} c_{1,t}
\end{aligned}
\tag{33}
$$



(a) Average annual verified emissions.



(b) Normalized average annual emissions. Annual emissions are represented as % change from the first observation i.e., year 2005. The dashed green lines correspond to an annual emission reduction of 1.74% from the previous year.

Fig. 10 Average verified emissions from facilities related to sector 20 (Combustion of fuels) grouped by emission-size classes of their Account Holders (firms). On average, emissions from an installation of a "Leader" energy firm are clearly declining

So from Eqs. (31) and (33) it is inferred that $\alpha_{1,t} = c_{1,t}/m$ and $\alpha_{1,t} = \frac{m-1}{m^2+1} c_{1,t}$. By equalizing these equations we get:

$$\frac{c_{1,t}}{m} = \alpha_{1,t} = \frac{m-1}{m^2+1} c_{1,t} \tag{34}$$

Now let us suppose that $c_{1,t} \neq 0$, then from Eq. (34) it is inferred that

$$\begin{aligned} \frac{1}{m} &= \frac{m-1}{m^2+1} \\ \implies m &= -1 \end{aligned} \tag{35}$$

So by assuming that $c_{1,t} \neq 0$ we arrive to a contradiction, since m must be a positive integer and thus it cannot equal -1 which is the inferred outcome in Eq. (35). So it is proven by contradiction that $c_{1,t} = 0$.

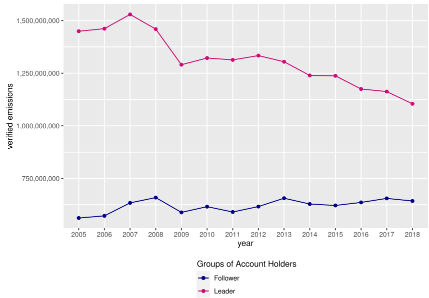
By using this outcome on Eq. (31) we get that $\alpha_{1,t} = 0$. Finally, from Eq. (32) it is inferred that $\alpha_{i,t} = 0, \forall i \in [2, m]$. Overall, we get:

$$\alpha_{i,t} = 0, \forall i \in [1, m] \tag{36}$$

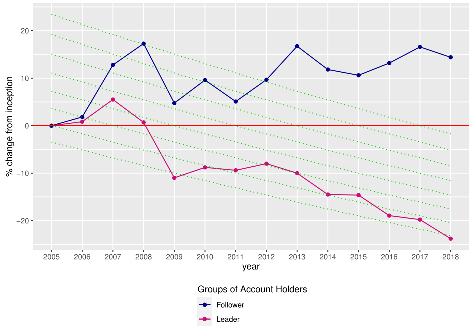
C: More on Evidence from EU ETS

In this appendix we provide supplementary material concerning Sect. 3.5 and the evidence from the EU ETS.

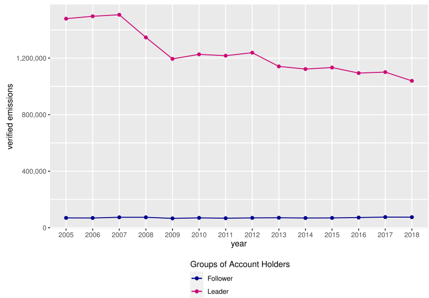
As depicted in Fig. 1 the distribution of quantity $\ln(E[z_{i,t} \, | i)$, which is the natural logarithm of average annual verified emissions, exhibits a "bell" shape in case of firms of



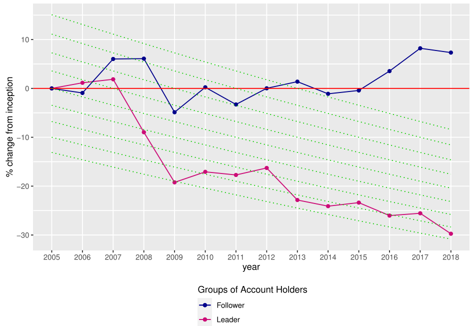
(a) Total annual verified emissions per emission-size class.



(b) Normalized total annual verified emissions per emission-size class.



(c) Average annual verified emissions from an installation of each emission-size class.



(d) Normalized average annual verified emissions from an installation of each emission-size class.

Fig. 11 Verified annual emissions from all regulated firms. Subfigures (b) and (d) show normalized emissions in the sense that the values are presented as a percentage change from 2005 emissions. The dashed green lines correspond to an annual emission reduction of 1.74% from the previous year

energy sector. This pattern is a pervasive one exhibited by many sectors. Figure 9 shows that "bell" shape distribution for sector 20 compared with the distribution of all firms.

Table 2 indicates that class "Negigible" make up an insignificant amount of the emissions with less than 1340 tones of CO_2 per year. On the other hand a firm classified as "Leader" emit tens of millions of tones of CO_2 every year.

Figures 2, 3, 10 and 11 show a clear reduction in emissions from "Leaders". All, but the last figure are restricted to sector 20. The difference between Figs. 2, 3 and 10 is that the last one shows the average emissions of an installation, while the first two depict the total emissions of all installations for each emission-size category. It is worth noting that neither all regulated countries were members, nor were all industrial sectors regulated by the ETS from the beginning (or from the same year). Consequently, the number of regulated installations increases over time. This leads to an upward trend in the amount of total emissions, neutralizing the downward trend due to abatement. One way to tackle this problem is to consider the average emissions instead of the total as in Figs. 10, 11c and d. Figure 11 depicts the emissions from all regulated firms. Again "Leader" clearly reduce their emissions opposed to what "Followers" do.

D: Data

In this appendix we provide the data we discussed on Sect. 4. Specifically, the data developed by Price Waterhouse Coopers (2008) together with the artificial data created for the needs of the present study (Tables 3, 4, 5).

Table 3 Part of Price Waterhouse Coopers study's data

Company	Production2008[1]	Emission2008[2]	CO2/MWh2008[3]	Production2009[1]	Emission2009[2]	CO2/MWh2009[3]
EDF Group	704	103.79	147	652	88.09	135
EDF Energy	27	21.9	805	72	23.8	330
Edison S.p.A	63	32.4	514	61	29.9	495
RWE Group	194	144.46	747	169	133.7	792
EnBW	67	17.0	254	66	15.9	241
NPower	38	25	665	27	16.6	622
E.ON Group	239	100.07	418	216	84.7	393
Enel Group	186	83	447	170	77.29	454
EDP Group	40	19.78	500	42	20.01	477
Vattenfall	178	81.72	459	175	79.05	452
CEZ	68	40.38	597	65	37.2	569
DEI	52	52.2	996	50	49.7	992
Fortum	53	2.16	41	49	2.02	41
Statkraft	53	1.6	30	57	1.6	28
Union Fenosa	18	7.26	398	29	9.48	330
Verbund	29	2.89	101	30	2.21	74
Drax	27	22.3	818	24	19.85	815
Dong	19	7.43	401	18	6.93	383
PVO	22	2.92	131	22	2.88	131

[a] Production measured in *TWh*

[b] Emissions in $MtCO_2$

[c] CO_2/MWh in *Kg*

Table 4 Companies data produced for period 1

ID	Name	isLeader	Q	RQ	Free	Surrendered	Banking	Previous banking
0	Leader1	True	651.999	88.02	105.624	61.614	6.049	0
1	Leader2	True	65.999	15.906	19.087	11.134	0.612	0
2	Leader3	True	71.999	23.759	28.512	16.632	0.668	0
3	Follower1	False	169.999	77.179	66.365	77.179	1.577	0
4	Follower2	False	41.999	20.034	17.227	20.034	0.389	0
5	Follower3	False	60.999	30.195	25.964	30.195	0.566	0
6	Follower4	False	64.999	36.985	31.802	36.985	0.603	0
7	Follower5	False	26.999	16.794	14.440	16.794	0.25	0
8	Follower6	False	168.999	133.848	115.093	133.848	1.568	0
9	Follower7	False	49.999	49.599	42.649	49.599	0.464	0

Table 5 Companies data produced for period 2

ID	Name	isLeader	Q	RQ	Free	Surrendered	Banking	Previous Banking
0	Leader1	True	650.102	87.764	105.316	81.638	0	6.049
1	Leader2	True	65.809	15.859	19.032	9.734	0	0.612
2	Leader3	True	71.790	23.691	28.429	17.565	0	0.668
3	Follower1	False	169.505	76.955	66.172	76.955	0	1.577
4	Follower2	False	41.877	19.976	17.177	19.976	0	0.389
5	Follower3	False	60.822	30.107	25.888	30.107	0	0.566
6	Follower4	False	64.811	36.877	31.709	36.877	0	0.603
7	Follower5	False	26.921	16.745	14.399	16.745	0	0.250
8	Follower6	False	168.508	133.458	114.758	133.458	0	1.568
9	Follower7	False	49.854	49.455	42.526	49.456	0	0.464

Table 6 MAC coefficients, discount factor and EUA price data

Discount factor	EUA Price	MAC function coefficients		
β	σ	a	b	c
3%	18€	0.1385	0.000	18.00

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