ΕΘΝ. ΜΕΤΣΟΒ. ΠΟΛΥΤΕΧΝΕΙΟΝ

ΑΠΛΕ

ΠΡΟΜΗΘΕΥΣ ΠΥΡΦΟΡΟΣ

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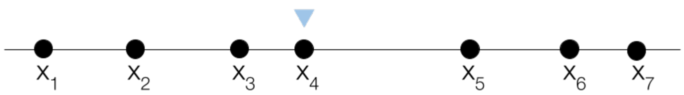
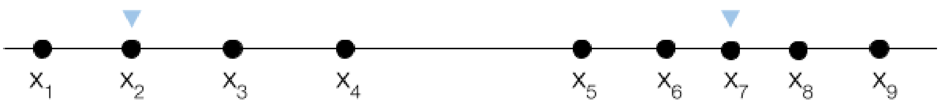
ΠΡΟΜΗΘΕΥΣ ΠΥΡΦΟΡΟΣ

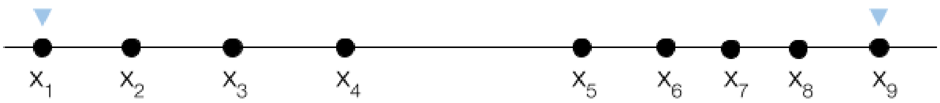
n

$knkkkk$

$(X,d) d : X \times X \rightarrow \mathbb{R}_+ \forall x,y,z \in Z d(x,x) = 0 d(x,y) = d(y,x) d(x,z) \leq d(x,y) + d(y,z)$
$\vec{x} = (x_1, ..., x_n)$
$M \vec{x} k (c_1, ..., c_k) M \vec{x} k (c_1, ..., c_k)$
$cost(x_i, M(\vec{x})) = min_{1 \leq j \leq k} \{d(x_i, c_j)\} M cost(\vec{x}, M(\vec{x})) = \sum_{i=1}^{n} cost(x_i, \vec{C})$







$(n-2)$

$\vec{x} = (x_1, ... x_n)$

$\bullet i \in N c_1 x_i$

$\bullet d_j = d(c_1, x_j) j j \frac{d_j}{\sum_{k \in N} d_k} c_2 x_j$

$k \geq 3$

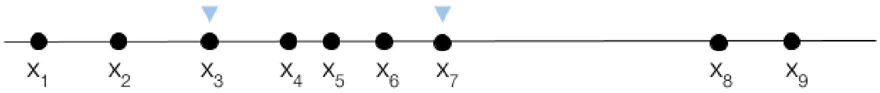
$k \geq 3$

$\vec{p}$

$\vec{p}$ $\vec{p} = (p_1, ..., p_k) 0 \leq p_1 \leq ... \leq p_k \leq 1 j p_j$

$$c_j = x_{i_j} \ : \ i_j = \lfloor (n-1) \cdot p_j \rfloor + 1$$

$(0.25, 0.75) \lfloor 8 \cdot 0.25 \rfloor + 1 = 3 \lfloor 8 \cdot 0.75 \rfloor + 1 = 7$



$k \geq 2 k \geq 3 (0, 1)$

$n/2 k n = k + 1 k n$

	$k=1$	$k=2$	$k \geq 3$
		$n-2$	∞
			n

k

$[0, \infty)$

NP

k $X d : X \times X \to [0, \infty) \vec{C} = (C_1, C_2, ..., C_k) k c_1, ..., c_k k$

$$\sum_{i=1}^{k} \left(\sum_{x \in C_i} d(c_i, x) \right)$$

$\gamma \gamma \geq 1$

γ $\gamma \geq \gamma (X, d)(X, d')$

$$d'(x, y) \in \left[\frac{1}{\gamma} d(x, y), d(x, y) \right]$$

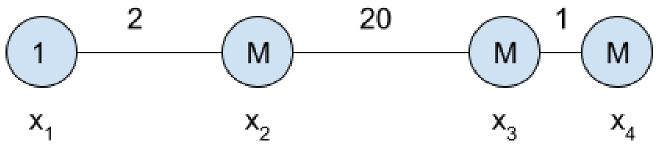
γ $(X, d) \gamma \Phi \vec{C} = C_1, ..., C_k \gamma (X, d') \vec{C}$

γ $\gamma \geq 1 \gamma C_1, ..., C_k c_1, ..., c_k \gamma C_i C_j x_i \in C_i$

$$d(x_i, c_j) > \gamma d(x_i, c_i)$$

γ $\gamma \geq 2 \gamma C_1, ..., C_k c_1, ..., c_k \gamma - x \in C_i y \notin C_i$

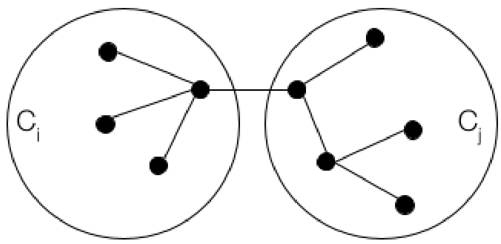
$$d(x, y) > (\gamma - 1) d(x, c_i)$$



$C_1, ..., C_k \gamma \gamma \geq 2 x_i, x_i' \in C_i x_j \in C_j (i \neq j)$

$$d(x_i, x_j) > \frac{(\gamma - 1)^2}{2\gamma} d(x_i, x_i')$$

$nk\vec{x} = (x_1, x_2, x_3, x_4) x_1 M$
$x_2 x_3 x_4 x_1 x_2 x_3 x_4 M$
kk



$kk - 1$

$\gamma \vec{x} \gamma \geq 2$

$\gamma \gamma \geq 2 + \sqrt{3} x_i$

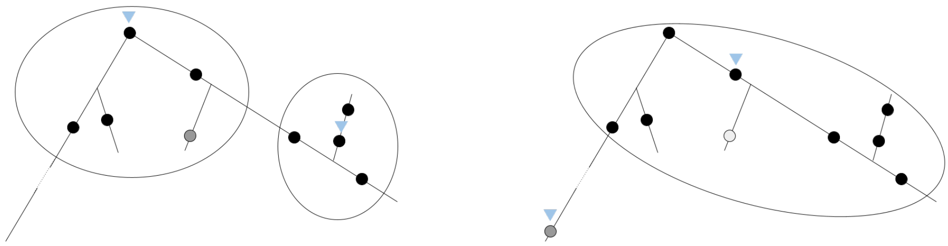
$$d(x_i, x_j) \geq d(x_i, x_i') \; \forall x_i, x_i; \in C_i \;\; x_j \in C_j$$

$k - 1\gamma$
$k - 1$

$2 + \sqrt{3}$

$C_i x_i' C_i$
$x_i \gamma x_i x_i'$

$(\gamma, \epsilon)\epsilon$



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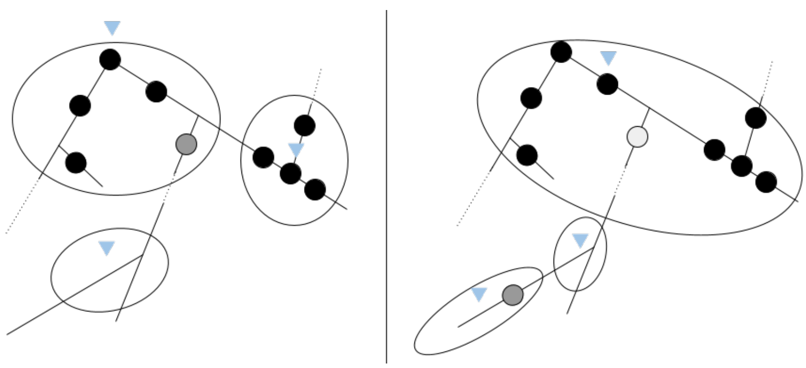
k
$\vec{x}k$
$(C_1, \ldots, C_k)c_iC_i$
$\big(\exists i \in [k] |C_i| = 1\big)\big(\exists\ x_i, x'_i \in Cix_j, x'_j \in C_j\{d(x_i, x'_i), d(x_j, x'_j)\} \geq d(x_i, x_j)\big)$

$|k(c_1, \ldots, c_k)$



k
$kknn-2n/2kn=k+1n$

	$k=1$	$k=2$	$k\geq 3$
		$n-2$	∞
			n

k

$k4k$

$NPkkNP$
$\gamma\gamma\gamma$

$k(2+\sqrt{3})5kk\geq 3kk\geq 3(2-\delta)\delta>0$
$k2+\sqrt{3}$

$Nn\mathcal{A}{\succ_i}\mathcal{A}L\mathcal{A}f : L^n \to \mathcal{A}$

$f\exists a \in \mathcal{A}\forall b \in \mathcal{A}i \in Na \succ_i bf(\succ_1, ..., \succ_n) = a$

$f\forall a \in \mathcal{A}\exists \vec{x} \in L^n f(\vec{x}) = a$

$ff(\vec{x}) = a\vec{x} \in L^n \nexists b \in \mathcal{A}b \succ_i \forall i \in N$

$f{\succ_1}, ..., {\succ_n} i {\succ'_i} f(\succ_1, ..., \succ_i, ..., \succ_n) \succ_i f(\succ_1, ..., \succ'_i, ..., \succ_n)$

$f\pi(\succ_1, ..., \succ_n) \in L^n f(\succ_1, ..., \succ_n) = f(\succ_{\pi(1)}, ..., \succ_{\pi(n)})$

$if{\succ_1}, ..., {\succ_n} \in Lf(\succ_1, ..., \succ_n) = aa \succ_i b, \forall b \in \mathcal{A}b \neq a$

3

$f\mathcal{A}\mathcal{A} \geq 3f$

$a \in \mathcal{A}x < y < a \implies y \succ xa < x < y \implies x \succ y$

$fa_1, a_2, ..., a_n \in [0, 1](x_1, ..., x_n) \in \mathbb{R}$

ixy

$N = \{1, .., n\}(X, d)d : X \times X \to \mathbb{R}_{\geq 0} dXd(x, x) = 0x \in Xd(x, y) = d(y, x)x, y \in X$ $d(x, z) \leq d(x, y) + d(x, z)x, y, z \in Xi \in Nx_i \in X\vec{x} = (x_1, ...x_n)$

$\vec{x}i\vec{x}_{-i}\vec{x}x_iS\vec{x}_S = (x_i)_{i\in S}S\vec{x}_{-S} = (x_i)_{i\notin S}\vec{x}S$
$MMk\vec{x}k\vec{c} = (c_1, ..., c_k) \in X^k M(\vec{x})M\vec{x}M\vec{x}k(c_1, ..., c_k) \in X^k$
$\vec{x}Micost(x_i, M(\vec{x})) = min_{1\leq j\leq k}\{d(x_i, c_j)\}Mcost(\vec{x}, M(\vec{x})) = \sum_{i=1}^{n} d(x_i, M(\vec{x}))$

$\vec{x}iy$

$$cost(x_i, M(x)) < cost(x_i, M(x_{-i}, y))$$

$\vec{x}S \subset N\vec{x}' = (\vec{x}_{-S}, x'_S)i \in S$

$$cost(x_i, M(\vec{x})) \leq cost(x_i, M(\vec{x}'))$$

$Mi\vec{x}_{-i}$

$$I_i(\vec{x}_{-i}) = \{a \in X : \exists\, y \in X \ with \ M(\vec{x}_{-i}, y)\}$$

$MI_i(\vec{x}_{-i})$

$Mk\vec{x} \in X^n i \in N$

$$cost(y, M(\vec{x}_{-i}, y)) = _{a \in I_i(\vec{x}_{-i})}\{d(a, y)\}$$

$\vec{x}\,' = (\vec{x}_{-i}, y) a \in M(\vec{x}\,') a^* \in I_i(\vec{x}_{-i}) d(a^*, y) < d(a, y) y^* a^* \in M(\vec{x}_{-i}, y^*) i y^* cost(y, M(\vec{x}_{-i}, y))$
$d(a^*, y) M$

$MS\vec{x}_{-S}$

$$I_S(\vec{x}_{-S}) = \{a \in X : \exists\, \vec{y} \in X^{|S|} \ with \ M(\vec{x}_{-S}, \vec{y})\}$$

$Mk\vec{x} \in X^n S \subset N \vec{y} \vec{y} = (y, ..., y)$

$$cost(y, M(\vec{x}_{-S}, \vec{y})) = _{a \in I_S(\vec{x}_{-S})}\{d(a, y)\}$$

$N = \{1, ..., n\} x_i \in \mathbb{R} \vec{x} = (x_1, ..., x_n) x_1 \leq x_2 \leq ... \leq x_n \vec{x} lt(\vec{x}) = min_{i \in N}\{x_i\} rt(\vec{x}) = max_{i \in N}\{x_i\} d(x, y) = |x - y|$
$\vec{x}$

$med(\vec{x}) n x_{(n+1)/2} n [x_{n/2}, x_{n/2+1}] x_{n/2} x d(x, med(\vec{x})) d(x, med(\vec{x})) x$
$med(\vec{x}) i x_i x_i' x_i < med(\vec{x}) x_i' < med(\vec{x}) x_i' > med(\vec{x})$

$\vec{c} = (c_1, c_2) c_1 \leq c_2 cost(x, \vec{c}) = min\{d(x, c_1), d(x, c_2)\}$
$\vec{x} c_1 c_2 c_1 L(\vec{x}) R(\vec{x}) L(\vec{x}) R(\vec{x}) c_1 c_2 \vec{x} L(\vec{x}) R(\vec{x})$
$L(\vec{x}) R(\vec{x}) L(\vec{x}) R(\vec{x}) (n-2) 2$

$lt(\vec{x}) rt(\vec{x}) (n-2)$

$n n - 2 \epsilon > 0 \epsilon 1 SC^* = \epsilon(n-2) \epsilon = (n-2) SC^* (n-2) \epsilon$
$\vec{x} [lt(\vec{x}), rt(\vec{x})] x' \in (-\infty, lt(\vec{x})) \cup (rt(\vec{x}), \infty)$

$1.5 2 (n-1)/2 n - 2$

$\vec{x} = (x_1, ... x_n)$

- $i N c_1 x_i$
- $d_j = d(c_1, x_j) j j \frac{d_j}{\sum_{k \in N} d_k} x_j$

$cost_k(x_i, M(\vec{x})) i x_k cost_k(x_k, M(\vec{x})) = 0 i$

$$cost(x_i, M(\vec{x})) = \frac{1}{n} \sum_{k=1}^{n} cost_k(x_i, M(\vec{x})) = \frac{1}{n} \sum_{k \neq i} cost_k(x_i, M(\vec{x}))$$

$\vec{x}\,' = (\vec{x}_i, x_i') i k \neq i$

$$cost(x_i, M(x)) < cost(x_i, M(\vec{x}\,'))$$

$x_k i x_k$

$$cost(c_2, x_i) = \sum_{j=1}^{n} Pr[c_2 = x_j] \cdot d(x_i, x_j) = \sum_{j=1}^{n} \frac{d_j}{\sum_{k=1}^{n} d_k} d(x_i, x_j) = \frac{\sum_{j=1}^{n} d_j \cdot d(x_i, x_j)}{\sum_{j=1}^{n} d_j}$$

$i x_k$

$$\begin{aligned} cost_k(x_i, M(\vec{x})) &= \left\{ d_i, \frac{\sum_{j=1}^{n} d_j \cdot d(x_i, x_j)}{\sum_{j=1}^{n} d_j} \right\} \\ &= \frac{\sum_{j=1}^{n} d_j \left\{ d_i, d(x_i, x_j) \right\}}{\sum_{j=1}^{n} d_j} \\ &= \frac{\sum_{j \neq i} d_j \left\{ d_i, d(x_i, x_j) \right\}}{\sum_{j=1}^{n} d_j} \end{aligned}$$

$d'_i = d(c_1, x'_i) i$

$$cost_k(x_i, M(\vec{x}\,')) = \frac{\sum_{j \neq i} d_j \left\{ d_i, d(x_i, x_j) \right\}}{\sum_{j=1}^{n} d_j + (d'_i - d_i)} + \frac{d'_i \left\{ d_i, d(x_i, x'_i) \right\}}{\sum_{j=1}^{n} d_j + (d'_i - d_i)}$$

$cost_k(x_i, M(\vec{x})) cost_k(x_i, M(\vec{x}\,'))$

$$cost_k(x_i, M(\vec{x}\,')) = \frac{cost_k(x_i, M(\vec{x})) \sum_{j=1}^{n} d_j}{\sum_{j=1}^{n} d_j + (d'_i - d_i)} + \frac{d'_i \left\{ d_i, d(x_i, x'_i) \right\}}{\sum_{j=1}^{n} d_j + (d'_i - d_i)}$$

$d'_i \leq d_i cost_k(x_i, M(\vec{x})) < cost_k(x_i, M(\vec{x}\,')) \frac{\sum_{j=1}^{n} d_j}{\sum_{j=1}^{n} d_j + (d'_i - d_i)} > 1$

$d'_i > d_i cost_k(x_i, M(\vec{x}))$

$$cost_k(x_i, M(\vec{x}\,')) - cost_k(x_i, M(\vec{x})) = \frac{-(d'_i - d_i) cost_k(x_i, M(\vec{x}))}{\sum_{j=1}^{n} d_j + (d'_i - d_i)} + \frac{d'_i \left\{ d_i, d(x_i, x'_i) \right\}}{\sum_{j=1}^{n} d_j + (d'_i - d_i)}$$

$$d'_i \left\{ d_i, d(x_i, x'_i) \right\} - (d'_i - d_i) cost_k(x_i, M(\vec{x})) \geq 0$$

$\left\{ d_i, d(x_i, x'_i) \right\} = d_i d_i \geq cost(x_i, M(\vec{x})) i x_k d'_i \geq d'_i - d_i$
$\left\{ d_i, d(x_i, x'_i) \right\} = d(x_i, x'_i) d'_i = d_i \geq cost(x_i, M(\vec{x})) d(x_i, x'_i) \geq d'_i - d_i$

$n/21.045$
$kk \geq 3k \geq 3n = k + 1nk$
$k - 1\vec{x} = (x_1 | x_2 | ... | x_k, x_{k+1})$

$Mk(k+1)\vec{x} M_1(\vec{x}) \leq x_2 M_k(\vec{x}) \geq x_k$

$Mk\vec{x} = (x_1 | x_2 | ... | x_k, x_{k+1}) M_k(\vec{x}) \in [x_k, x_{k+1}]$

$Mk\vec{x} = (x_1|x_2|...|x_k, x_{k+1}) M_k(\vec{x}) = x_k \vec{x}\,' = (\vec{x}_{-\{k,k+1\}}, x_k', x_{k+1}') x_k \geq x_k' M_k(\vec{x}\,') = x_k'$

$Mk\vec{x} = (x_1|x_2|...|x_k, x_{k+1}) M_k(\vec{x}) = x_{k+1} \vec{x}\,' = (\vec{x}_{-\{k,k+1\}}, x_k', x_{k+1}') x_{k+1} \leq x_{k+1}' M_k(\vec{x}\,') = x_{k+1}'$

$k(k+1)$

$Mkk \geq 2n = k+1 \vec{x} = (x_1, ..., x_{k+1}) x_1 \leq ... \leq x_{k+1} M_1(\vec{x}) = x_1 M_k(\vec{x}) = x_{k+1}$

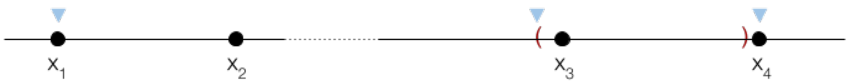
$kk \geq 3$

$k \geq 3kn \geq k+1$

$M\vec{x} = (x_1|x_2|x_3, x_4) x_1 x_4 x_3 x_4$

$\vec{x}$

$iI_i(\vec{x}_{-i}) I_i(\vec{x}_{-i}) i x_3 \vec{x} I_3(\vec{x}_{-3}) x_3 l r I_3(\vec{x}_{-3}) x_3 \vec{y} = (\vec{x}_{-3}, l+\epsilon) ll$



$\vec{y}$

$\vec{z} = (\vec{y}_{-4}, l) = (\vec{x}_{\{-3,4\}}, \{l, l+\epsilon\}) x_1 \leq x_2 \leq x_3 \leq x_4 \vec{y} \vec{z} M \vec{y} \vec{z} l + \epsilon \vec{z} l M l x_4 \vec{x} \vec{z} \vec{z}$

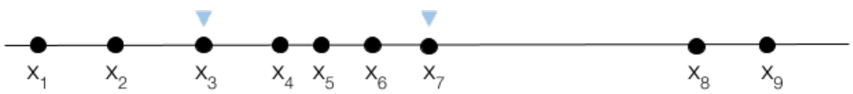
$\vec{z}$

$kk \geq 3k \geq 2k\vec{p} \in (0,1)^k \vec{p}$

$\vec{p}\ \vec{p} = (p_1, ..., p_k) 0 \leq p_1 \leq ... \leq p_k \leq 1 j p_j$

$$c_j = x_{i_j} \ : \ i_j = \lfloor (n-1) \cdot p_j \rfloor + 1$$

$\vec{x} = (x_1, ..., x_9) 9 (0.25, 0.75) c_1 = x_3 \lfloor 8 \cdot 0.25 \rfloor + 1 = 3 c_2 = x_7 \lfloor 8 \cdot 0.75 \rfloor + 1 = 7$



$(0.25, 0.75)$

$\vec{p}$

$k = 2k \geq 2\vec{c} = (c_1, c_2)\vec{c}' = (c_1', c_2')S \subset N\Delta_1 = c_1 - c_1'\Delta_2 = c_2' - c_2S$

$\Delta_1 \geq 0\Delta_2 > 0x_i \in (c_1, c_2)c_1c_2i$

$$\begin{aligned} cost(x_i, M(\vec{x}\,')) &= min(d(x_i, c_1'), d(x_i, c_2')) \\ &\geq min(d(x_i, c_1), d(x_i, c_2)) \\ &= cost(x_i, M(\vec{x})) \end{aligned}$$

$\Delta_1 \geq 0\Delta_2 < 0ix_i > c_2c_2$

$$cost(x_i, M(\vec{x}\,')) = x_i - c_2' \geq x_i - c_2 = cost(x_i, M(\vec{x}))$$

$\Delta_1 < 0\Delta_2 \geq 0$

$\Delta_1 < 0\Delta_2 < 0c_2$

$\Delta_1 = 0\Delta_2 = 0\Delta_1\Delta_2$

$9(0.25, 0.75)37x'$
$\vec{p} = (0, 1)\vec{p} \neq (0, 1)\vec{x} = (0, \epsilon, 1)\epsilon > 00(0, 0.6)\epsilon 1 - \epsilon$

$kk = 3$
$n_00n_11n_21 + x1 + x + yn_00y = 100x = 10^{100}n_1 = 50n_2 = 4$
$1 + x1 + x + yyx = 10^{100}1 + x1 + x1 + x$
$k4kk\ell = 1, ..., kC_\ell\ell$

$\vec{x} = (x_1, ...x_n)$

- $iNx_iC_1 = \{x_i\}$
- $\ell\ell = 2, ..., kd_\ell = d(x_\ell, C_{\ell-1})\ell\ell\frac{d_\ell}{\sum_{k\in N} d_k}\ell x_\ell x_\ell C_\ell = C_{\ell-1} \cup \{x_\ell\}$

$xT_1T_1d(x, med(\vec{x}))d(x, med(\vec{x}))x$

$2[0, \infty)OOS_3b_1b_2b_3(x, b_l)S_3x \geq 0(x, b_l)(x', b_{l'})S3|x - x'|l = l'x + x'$

$n \geq 3S_3$

$(S^1, d)S^1 \subset \mathbb{R}^2d(x, y)x, y \in S^1xy$
$n - 1n - 1xyxd(x, y)y(n - 1)d(x, y)$

$2 - \frac{2}{n}$

y

$$
\begin{aligned}
cost(\vec{x}, M(\vec{x})) &= \frac{1}{n}\sum_{i\in N}\sum_{j\neq i} d(x_i, x_j) \\
&= \frac{1}{n}\sum_{i\in N}\sum_{j\neq i} d(x_i, y) + d(y, x_j) \\
&= \frac{1}{n}\sum_{i\in N}\Big((n-1)d(x_i, y) + SC^* - d(x_i, y)\Big) \\
&= \frac{1}{n}\sum_{i\in N}\Big((n-2)d(x_i, y) + SC^*\Big) \\
&= SC^* + \frac{n-2}{n}SC^*
\end{aligned}
$$

$\vec{x} = (x_1, ...x_n) x_1 \hat{x}_1 x_1 x_1 \hat{x}_1 \mathcal{L}\mathcal{R}\mathcal{A}\mathcal{B}\mathcal{L}\mathcal{R} x_1 \hat{x}_1 \mathcal{A}\mathcal{A}\cap\mathcal{B} = \emptyset d_A = max_{i\in\mathcal{A}} d(x_1, x_i) d_B = max_{i\in\mathcal{B}} d(x_1, x_i)$
$\mathcal{B} d_B = 0$

- $d_A < d_B c_2 \mathcal{R} min\{max\{d_B, 2d_A\}, 1/2\} c_1$
- $d_A \geq d_B c_2 \mathcal{L} min\{max\{d_A, 2d_B\}, 1/2\} c_1$

$2n-1$

$n-1 \vec{x} = (x_1, ..., x_n) d(x_1, x_2) = d(x_1, x_3) = 0.1 x_3, ...x_n x_2 x_3 x_1 x_1 x_2 x_3 x_1 (n-1) 0.1$
$L_2 x_1, ..., x_n \in R^m$

$$med = {}_{y\in\mathbb{R}^2}\left\{\sum_{i=1}^{n} d(x_i, y)\right\}$$

$X = \mathbb{R}^m a_1, ..., a_k \in (\mathbb{R}\cup\{-\infty, \infty\})^m \vec{x} \in \mathbb{R}^m j = 1, 2, ..., m$

$$M^j(\vec{x}) \coloneqq (x_1^j, x_2^j, ..., x_n^j, a_1^j, ..., a_k^j)$$

$X = \mathbb{R}^2 M$

$X = \mathbb{R}^m \sqrt{m} n$

k

$NPNP$

$k\ \ Xd : X \times X \to [0, \infty)\vec{C} = (C_1, C_2, ..., C_k)$

k

$$\sum_{i=1}^{k} \left(\sum_{x \in C_i} d(c_i, x) \right)$$

k

$$_{x_i \in C_i} d(x_i, c_i)$$

kk

$\gamma\gamma \geq 1$

$\gamma\ \ \gamma \geq \gamma(X, d)(X, d')$

$$d'(x, y) \in \left[\frac{1}{\gamma} d(x, y), d(x, y) \right]$$

$\gamma\ \ (X, d)\gamma\Phi\vec{C} = C_1, ..., C_k\gamma(X, d')\vec{C}$

$\gamma d' d' \gamma\gamma\gamma\gamma\gamma\gamma\gamma\gamma$

$\gamma\ \ (X, d)\gamma\Phi\vec{C} = C_1, ..., C_k\gamma(X, d')\vec{C}$

γ

$\gamma\ \ \gamma \geq 1\gamma C_1, ..., C_k c_1, ..., c_k\gamma C_i C_j x_i \in C_i$

$$d(x_i, c_j) > \gamma d(x_i, c_i)$$

$C_i C_j x_i \in C_i\gamma C_i C_j\gamma C_i C_j c_i C_i c_j C_j x_i C_i d'(x_i, c_j) > d'(x_i, c_i) d'(x_i, c_j) = \frac{1}{\gamma} d(x_i, c_j) d'(x_i, c_i) = d(x_i, c_i) d(x_i, c_j) > \gamma d(x_i, c_i)$

$\gamma\gamma - 1$

$\gamma\ \ \gamma \geq 2\gamma C_1, ..., C_k c_1, ..., c_k\gamma - x \in C_i y \notin C_i$

$$d(x, y) > (\gamma - 1) d(x, c_i)$$

$x C_i y C_j$

$d(y, c_j) \geq d(x, c_i) d(y, c_i) \leq d(y, x) + d(x, c_i) d(y, c_i) > \gamma d(y, c_j) > \gamma d(x, c_i)\gamma d(x, y) \geq d(y, c_i) - d(x, c_i) > \gamma d(x, c_i) - d(x, c_i) = (\gamma - 1) d(x, c_i)$

$d(y, c_j) < d(x, c_i) d(x, c_j) \leq d(x, y) + d(y, c_j) \gamma d(x, c_j) > \gamma d(x, c_i) d(x, y) \geq d(x, c_j) - d(y, c_j) > \gamma d(x, c_i) - d(x, c_i) = (\gamma - 1) d(x, c_i)$

$C_1, ..., C_k \gamma \gamma \geq 2 x_i, x_i' \in C_i x_j \in C_j (i \neq j)$

$$d(x_i, x_j) > \frac{(\gamma-1)^2}{2\gamma} d(x_i, x_i')$$

$\vec{C} \vec{x} x_i, x_i' \in C_i c_i x_j \in C_j (i \neq j) c_j$

$$\gamma d(x_i, c_i) < d(x_i, c_j) < d(x_i, c_i) + d(c_i, c_j) \implies (\gamma - 1) d(x_i, c_i) < d(c_i, c_j)$$

γ

$$\begin{aligned} d(c_i, c_j) &< d(c_i, x_i) + d(x_i, x_j) + d(x_j, c_j) \\ &< \frac{2}{\gamma-1} d(x_i, x_j) + d(x_i, x_j) \\ &< \frac{\gamma+1}{\gamma-1} d(x_i, x_j) \end{aligned}$$

γ

$$\begin{aligned} d(x_i, x_i') &\leq d(x_i, c_i) + d(c_i, x_i') \\ &\overset{()}{<} \frac{1}{\gamma-1} d(x_i, x_j) + \frac{1}{\gamma-1} d(c_i, c_j) \\ &\overset{()}{<} \frac{1}{\gamma-1} d(x_i, x_j) + \frac{\gamma+1}{(\gamma-1)^2} d(x_i, x_j) \\ &= \frac{2\gamma}{(\gamma-1)^2} d(x_i, x_j) \end{aligned}$$

$\gamma = 3 \epsilon > 0 x_1 - x_2 x_2 - x_3 x_3 - x_4 \gamma \frac{2}{3} + 44 + \epsilon 3$
$\gamma \gamma$

$\gamma \ (X, d) \vec{C} = C_1, ..., C_k \gamma \Phi x_i \in C_i$

$$d(x_i, c_j) > \gamma d(x_i, c_i)$$

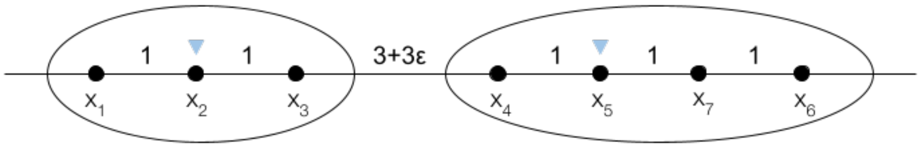
$2 \gamma \geq 2 + \sqrt{3} \approx 3.7 n 1 k \vec{x} = (x_1, x_2, x_3, x_4) 3 x_1 M >> 1$
$x_2, x_3 x_4 x_1 x_2 2 x_3 x_4 M \gamma \gamma M$
$\gamma n 1 k k T$

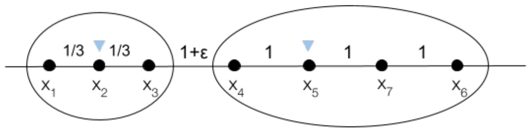
$k \binom{n-1}{k-1} k - 1 k T$

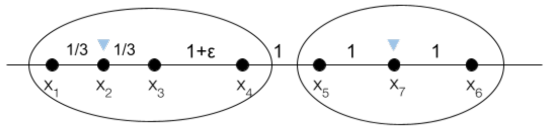
$C, C' d_{min}(C, C') A, B \subset X$

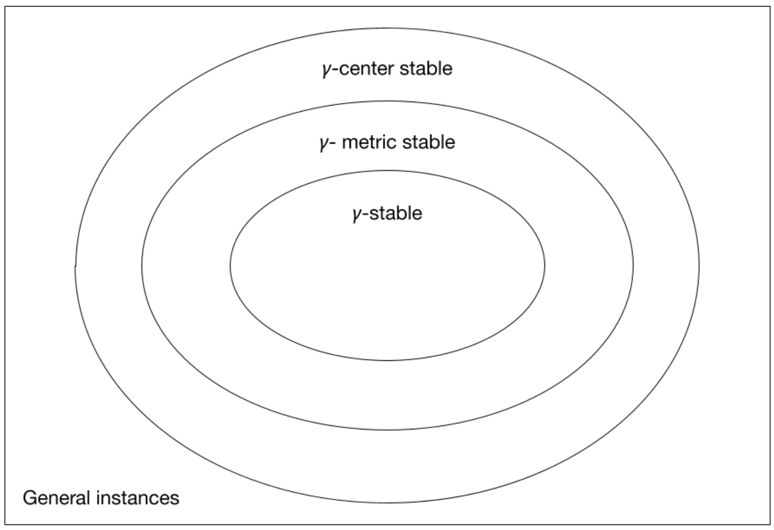
$$d_{min}(A, B) = min\{d(a, b) | a \in A, b \in B\}$$

$32 + \sqrt{3} 2$









$d_{min} 2$

$C_i T a \in C_i c_i T C_i b (a, b) a c_i (a, b) d(a, b) < d(a, c_i)(a, c_i)(a, b) b C_i a c_i C_i$

$C, C' C \cup C' p r p r \mathbb{B}(p, r) := \{q : d(p, q) < r\}$

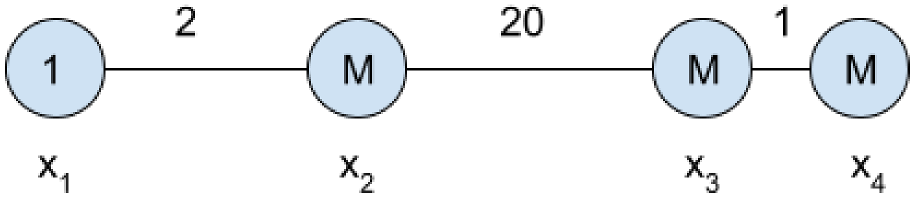
$d_S(C, C') C, C' \subset X d > 0 c \in C \cup C'$

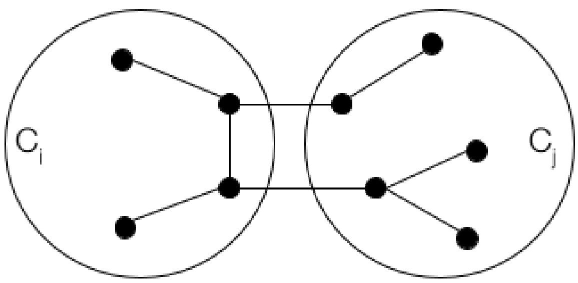
$\mathbb{B}(c, d) C C' C \cup C' \subseteq \mathbb{B}(c, d)$

$\mathbb{B}(c, d) \forall p \in \mathbb{B}(c, d), q \notin \mathbb{B}(c, d) d(c, p) < d(c, q)$

$d_S 1 + \sqrt{2}$

$\epsilon > 0 k(2 - \epsilon) NP NP = PR NP k(2 - \epsilon) \gamma\gamma\gamma$
$\gamma\gamma\gamma$





C_jT

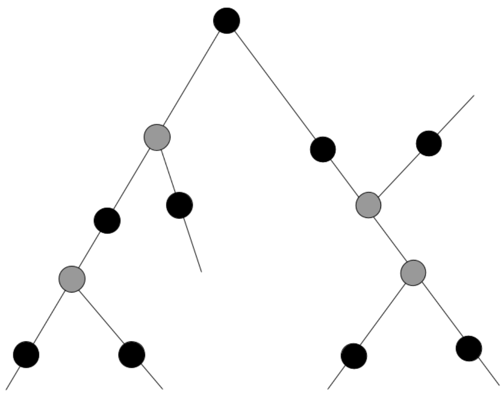
$k2(n-1)$
x_1

$\gamma\gamma$ $Nn(T,d)d\vec{x} = (x_1,...,x_n)\vec{z} = (z_1,...,z_m)\vec{x}\,' = (x_1',...,x_n') \in (T,d')\gamma\vec{x}\gamma xyxyx_i - z_jx_i - x_j$
$z_i - z_j$

$$d'(x,y) \in \left[\frac{1}{\gamma}d(x,y), d(x,y)\right]$$

$k\vec{x}\vec{C} = (c_1,...,c_k)\gamma\gamma\vec{x}\,'\vec{x}\vec{C}k$

$\vec{z} = (z_1,...z_m)\gamma\vec{x}\,'\vec{x}\gamma\vec{x}x_ix_j\gamma\gamma x_ix_jx_ix_j\gamma\gamma\gamma$



$\vec{x}\vec{z}$

$\gamma\gamma \geq 2T$

$\vec{x}\gamma\gamma \geq 2\vec{C}C_i$

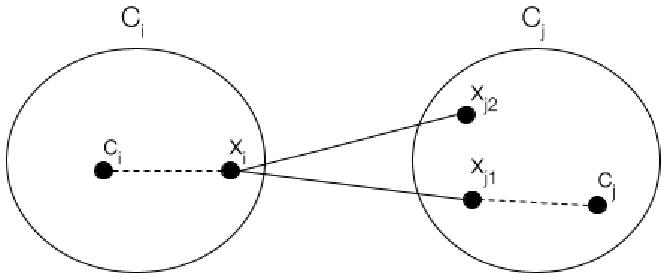
$C_jd(x_{j_2},x_i) > (\gamma-1)d(x_{j_2},c_j)d(x_{j_2},c_j) = d(x_{j2},x_i) + d(x_i,c_j)d(x_{j_2},c_j) > (\gamma-1)d(x_{j2},c_j) + d(x_i,c_j) \implies d(x_i,c_j) < (2-\gamma)d(x_{j_2},c_j)\gamma \geq 2d(x_i,c_j)$

γ
$(2+\sqrt{3})(2+\sqrt{3})\gamma\vec{x}x_i \in C_ix'\vec{x}\vec{x}\,'x'k - 1C_iC_jx_i\vec{x}\,'\gamma$
$k-1$

$(2+\sqrt{3})k$

$(2+\sqrt{3})\vec{C} = (C_1,...,C_k)x_i \in C_iy\vec{y} = (\vec{x}_{-i},y)\vec{Y}$

yC_i



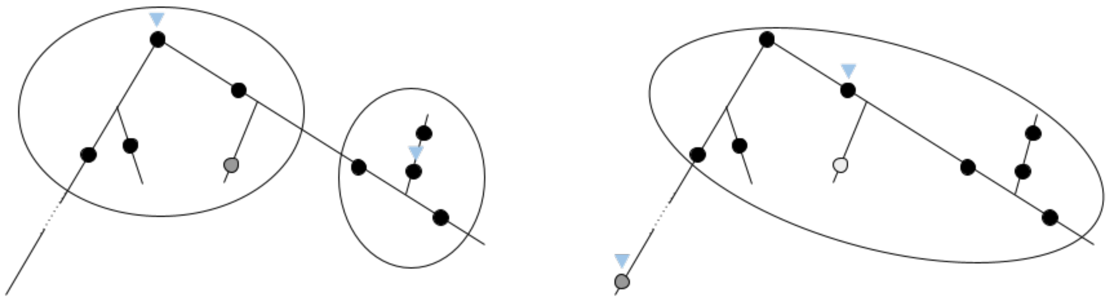
k

$k\vec{x}$

$(C_1, \ldots, C_k) c_i C_i$

$\big(\exists i \in [k] |C_i| = 1\big)\big(\exists\, x_i, x_i' \in C i x_j, x_j' \in C_j \{d(x_i, x_i'), d(x_j, x_j')\} \geq d(x_i, x_j)\big)$

$|k(c_1, \ldots, c_k)$



$y\vec{Y}$

$y\vec{C}$

$d(x_i, \vec{C}) < d(x_i, \vec{Y})$

$y\vec{C} C_j C_l \vec{Y} C_j \cup C_l \cup \{y\} C_j \cup C_l \vec{C} \vec{C} \vec{Y} C_j 2 + \sqrt{3} C_j \vec{x} \setminus C_j$

$y\vec{Y}$

$y C_j C_j \vec{x} \setminus C_j \vec{Y}$

$y C_j C_j \vec{x} \setminus C_j \vec{Y}$

$y\vec{Y}$

$y C_j y y C_j C_l y C_j$

$\vec{Y} y C_{j-1} C_j$

$\vec{Y} C_j C_j C_j \cup \{y\} \vec{C} \vec{Y} x_i \vec{Y} C_j \cup \{y\} y d(x_i, \vec{C}) > d(x_i, \vec{Y}) \vec{Y} \vec{y}$

$$\begin{aligned} cost(\vec{y}, \vec{C}) &> cost(\vec{y}, \vec{Y}) \iff \\ cost(\vec{x}, \vec{C}) + d(y, \vec{C}) - d(x_i, \vec{C}) &> cost(\vec{x}, \vec{Y}) + d(y, \vec{Y}) - d(x_i, \vec{Y}) \iff \\ d(y, \vec{C}) - d(y, \vec{Y}) &> cost(\vec{x}, \vec{Y}) - cost(\vec{x}, \vec{C}) + d(x_i, \vec{C}) - d(x_i, \vec{Y}) \end{aligned}$$

$i y d(x_i, \vec{C}) - d(x_i, \vec{Y}) > 0$

$$\begin{aligned} d(y, \vec{C}) - d(y, \vec{Y}) &> cost(\vec{x}, \vec{Y}) - cost(\vec{x}, \vec{C}) \\ &= cost(C_j, \vec{Y}) - cost(C_j, \vec{C}) - cost(\vec{x} \setminus C_j, \vec{Y}) - cost(\vec{x} \setminus C_j, \vec{C}) \end{aligned}$$

$\gamma \vec{x}\,' \vec{x} C_j \gamma \vec{C} \vec{x}\,' cost(\vec{x}\,', \vec{C}) < cost(\vec{x}\,', \vec{Y}) C_j \cup \{y\} \vec{x} \setminus C_j \vec{C} \vec{Y}$

$$\begin{aligned} cost(\vec{x}\,', \vec{C}) &= cost(C_j, \vec{C}) + \frac{1}{\gamma} cost(\vec{x} \setminus C_j, \vec{C}) \\ cost(\vec{x}\,', \vec{Y}) &= cost(C_j, \vec{Y}) + \frac{1}{\gamma} cost(\vec{x} \setminus C_j, \vec{Y}) \end{aligned}$$

$cost(\vec{x}\,', \vec{C}) < cost(\vec{x}\,', \vec{Y}) \gamma \geq 2 \frac{1}{\gamma} \leq 1 - \frac{1}{\gamma}$

$$\begin{aligned} cost(C_j, \vec{C}) - cost(C_j, \vec{Y}) &< \frac{1}{\gamma} \Big(cost(\vec{x} \setminus C_j, \vec{Y}) - cost(\vec{x} \setminus C_j, \vec{C}) \Big) \\ &\leq (1 - \frac{1}{\gamma}) \Big(cost(\vec{x} \setminus C_j, \vec{Y}) - cost(\vec{x} \setminus C_j, \vec{C}) \Big) \end{aligned}$$

$$cost(\vec{x}, \vec{Y}) - cost(\vec{x}, \vec{C}) > \frac{1}{\gamma} \Big(cost(\vec{x} \setminus C_j, \vec{Y}) - cost(\vec{x} \setminus C_j, \vec{C}) \Big)$$

$C_j \cup \{y\} Y_{j_1} y C_j x_j \in Y_{j_1} y \vec{Y} C_j \vec{Y} y \vec{Y} y c_j y C_j y C_j \vec{Y} y C_j y C_j$

$$\begin{aligned} d(y, \vec{C}) - d(y, \vec{Y}) &\leq cost(C_j, \vec{C}) - cost(C_j, \vec{Y}) \overset{()}{\Rightarrow} \\ &\leq \frac{1}{\gamma} \Big(cost(\vec{x} \setminus C_j, \vec{Y}) - cost(\vec{x} \setminus C_j, \vec{C}) \Big) \overset{()}{\Rightarrow} \\ &< cost(\vec{x}, \vec{Y}) - cost(\vec{x}, \vec{C}) \end{aligned}$$

$C_j C_l Y_{j_1} Y_{j_2} \vec{Y} C_j Y_{j_1} C_j C_l x_1 \in Y_{j_1} \cap C_j z \in Y_{j_1} \cap C_l x_2 \in Y_{j_2} \cap C_j d(x_1, z) \geq D_{x_1} D_{x_1} x_1 x_1 x_2 \vec{C}$ $d(x_1, x_2) < D_{x_1} d(x_1, z) > d(x_1, x_2)$

$\gamma \geq 2 + \sqrt{3}$

$(\gamma, \epsilon) \epsilon n \gamma$

k