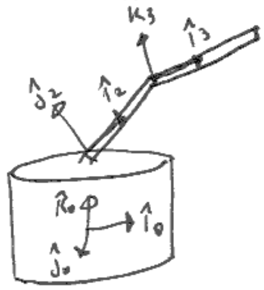


Frames

f_0 fixed
f_1 on base
f_2 on upper arm
f_3 on lower arm

arm length $= L$
base height $= H$
base radius $= R$

Kinematics

$$\omega_1 = \dot{\theta}_1 \hat{k}_1 \qquad V_{c1} = 0$$

$$\omega_2 = \dot{\theta}_2 \hat{j}_1 \qquad V_{c2} = \dot{\theta}_2 \tfrac{L}{2} \sin\theta_2 \, \hat{i}_1 - \dot{\theta}_2 \tfrac{L}{2} \cos\theta_2 \, \hat{k}_1$$

$$\omega_3 = \dot{\theta}_3 \hat{j}_1 \qquad V_{c3} = \left(\dot{\theta}_2 \tfrac{L}{2}(\sin\theta_2 + \sin\theta_3) + \dot{\theta}_3 \tfrac{L}{2}(\sin\theta_2 + \sin\theta_3)\right)\hat{i}_1$$

$$- \left(\dot{\theta}_2 \tfrac{L}{2}(\cos\theta_2 + \cos\theta_3) + \dot{\theta}_3 \tfrac{L}{2}((\cos\theta_2 + \cos\theta_3)\right)\hat{k}_1$$

Kinetic Energy

$$T = \tfrac{1}{2} M V^2 + \tfrac{1}{2} \omega \cdot (I_c \cdot \omega)$$

$$T_1 = \tfrac{1}{2} M_1 \cancel{V_{c1}^2}^{0} + \tfrac{1}{2} \omega_1 (I_c \cdot \omega_1) = 1/2 \, \dot{\theta}_1^2 M R^2$$

(used I_c for cylinder)

$$T_2 = \tfrac{1}{2} M_2 V_{c2}^2 + \tfrac{1}{2} \omega_2 (I_c \cdot \omega_2)$$

(for T_2 + T_3 used I_c for slender Rod)

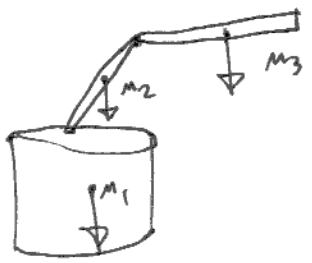
$$\tfrac{1}{2} M \left(\dot{\theta}_2^2 \tfrac{L^2}{4} \sin^2\theta_2 + \dot{\theta}_2^2 \tfrac{L^2}{4} \cos^2\theta_2 + \dot{\theta}_2^2 \tfrac{L^2}{4} \cos\theta_2 \sin\theta_2 \right) + \tfrac{1}{24} \dot{\theta}_2^2 M L^2$$

$$T_3 = \tfrac{1}{2} M \Big[\left(\dot{\theta}_2 \tfrac{L}{2}(\sin\theta_2 + \sin\theta_3) + \dot{\theta}_3 \tfrac{L}{2}(\sin\theta_2 + \sin\theta_3)\right)^2 + \left(\dot{\theta}_2 \tfrac{L}{2}(\cos\theta_2 + \cos\theta_3) + \dot{\theta}_3 \tfrac{L}{2}(\cos\theta_2 + \cos\theta_3)\right)^2$$

$$+ \left(\dot{\theta}_2 \tfrac{L}{2}(\sin\theta_2 + \sin\theta_3) + \dot{\theta}_3 \tfrac{L}{2}(\sin\theta_2 + \sin\theta_3)\right)\left(\dot{\theta}_2 \tfrac{L}{2}((\cos\theta_2 + \cos\theta_3) + \dot{\theta}_3 \tfrac{L}{2}(\cos\theta_2 + \cos\theta_3)\right) \Big]$$

$$+ \tfrac{1}{24} \dot{\theta}_3 M L^2$$

Potential Energy



$$PE = m_1 g\left(\tfrac{1}{2}H\right) + m_2 g\left(H + L\sin\theta_2\right) + m_3 g\left(H + L\sin\theta_2 + L\sin\theta_3\right)$$

~~$\mathcal{L} = KE - PE$~~ $\mathcal{L} = KE - PE$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_k}\right) - \frac{\partial \mathcal{L}}{\partial q_k} = \tau$$

θ_1

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} = \dot{\theta}_1 mR^2, \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) = \ddot{\theta}_1 mR^2$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = 0$$

$$\boxed{\ddot{\theta}_1 mR^2 = \tau_1}$$

θ_2

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = \frac{1}{2}M\left(\dot{\theta}_2 \frac{L^2}{2}\sin^2\theta_2 + \dot{\theta}_2 \frac{L^2}{2}\cos^2\theta_2 + \dot{\theta}_2 \frac{L^2}{2}\cos\theta_2 \sin\theta_2\right) + \frac{1}{12}\dot{\theta}_2 ML^2$$

$$= \frac{1}{4}ML^2\dot{\theta}_2\left(1+\cos\theta_2\sin\theta_2\right) + \frac{1}{12}\dot{\theta}_2 ML^2 + \frac{1}{4}ML^2\dot{\theta}_2^2\left((\sin\theta_2+\sin\theta_3)^2+(\cos\theta_2+\cos\theta_3)^2\right)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) = \ddot{\theta}_2\frac{1}{4}ML^2\left(1+\cos\theta_2\sin\theta_2\right) + \ddot{\theta}_2\frac{1}{12}ML^2 + \frac{1}{4}ML^2\ddot{\theta}_2^2\left((\sin\theta_2+\sin\theta_3)^2+(\cos\theta_2+\cos\theta_3)^2\right)$$

$\frac{\partial \mathcal{L}}{\partial \theta_2} =$ I cant solve this.

θ_3

Also too long to solve by hand.