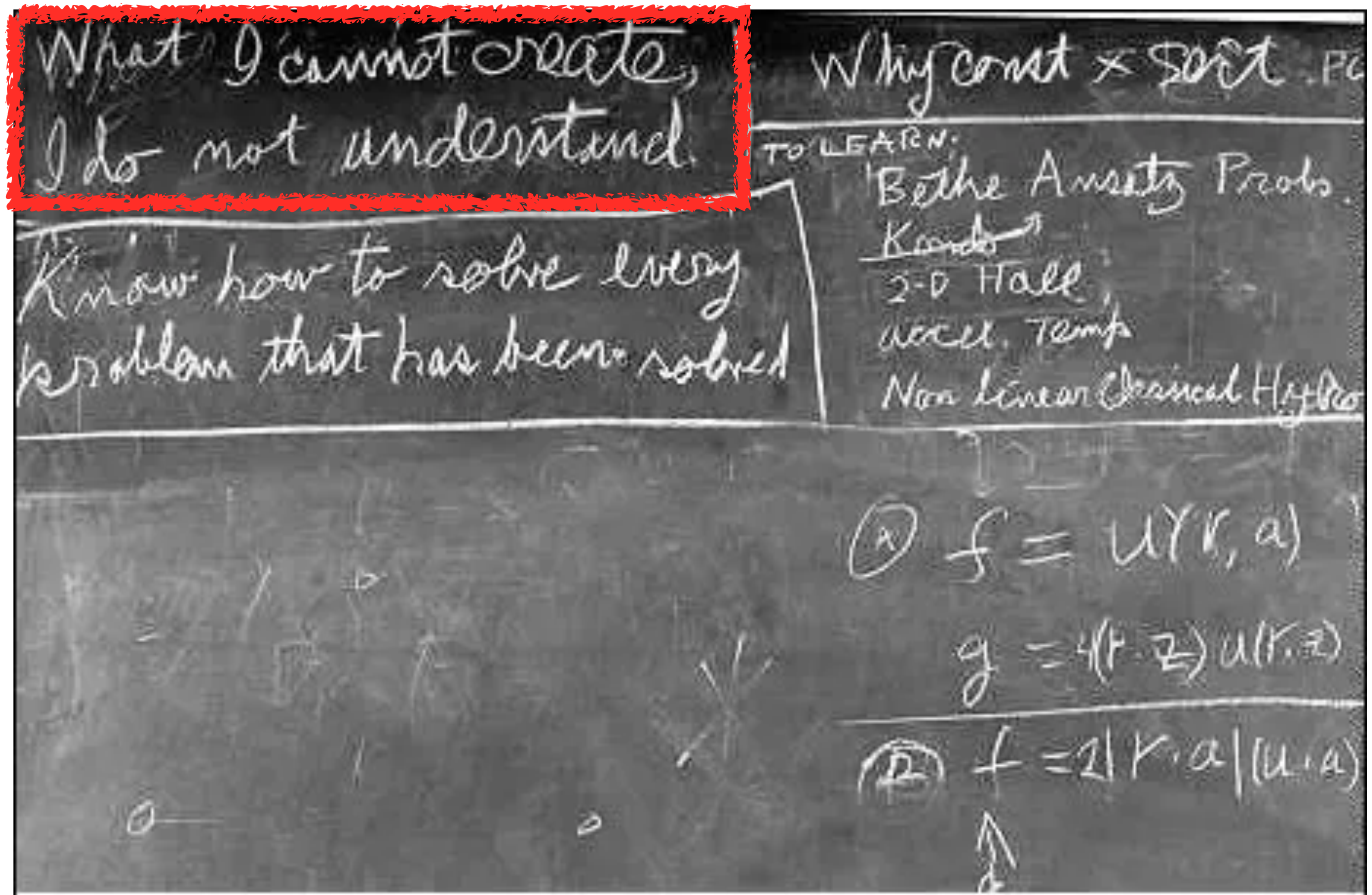
Deep Learning Basics

Lecture 9: Generative Models Part 1

최성준 (고려대학교 인공지능학과)

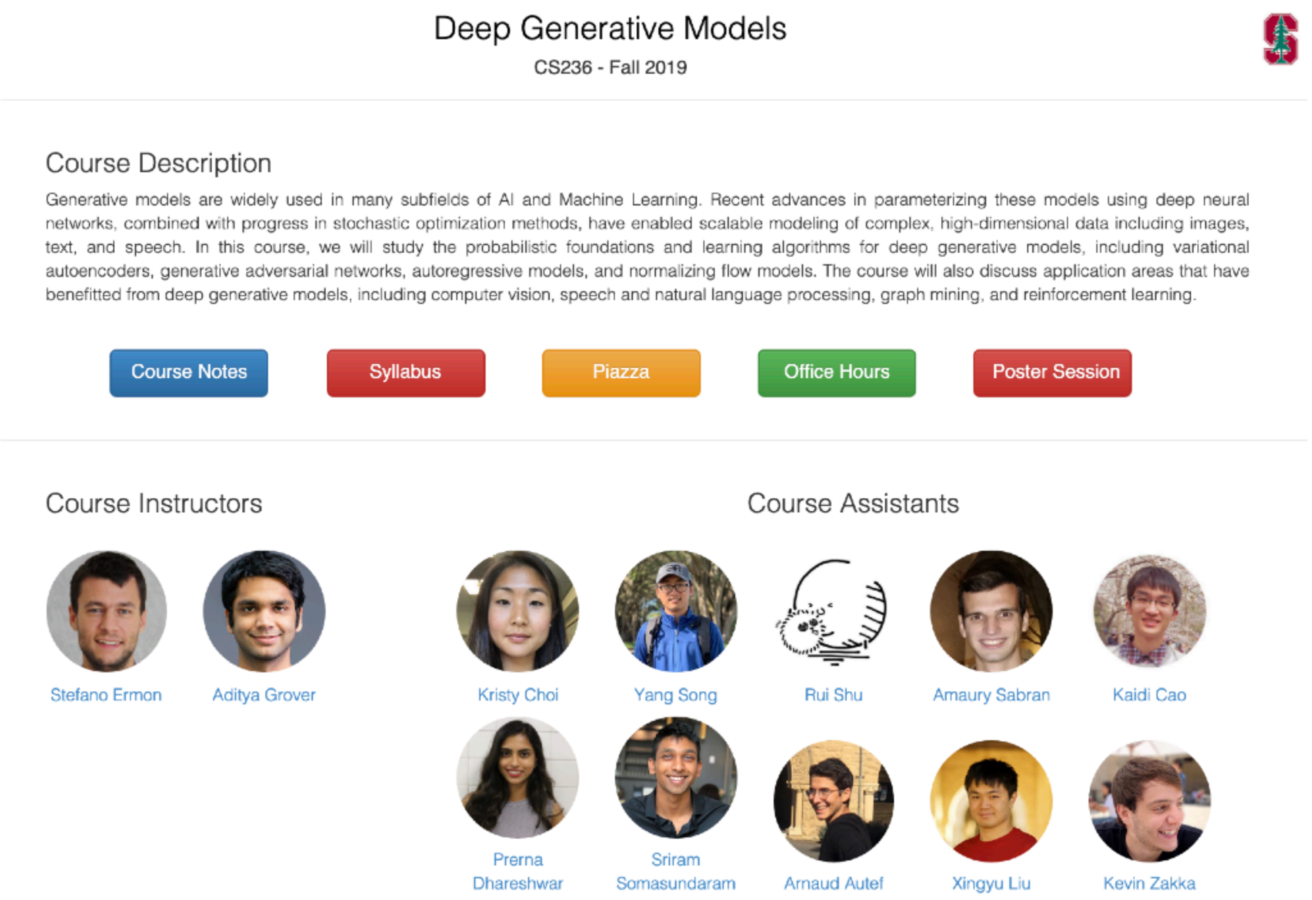
Introduction

Introduction



Richard Feynman
(1918~1988)

Introduction

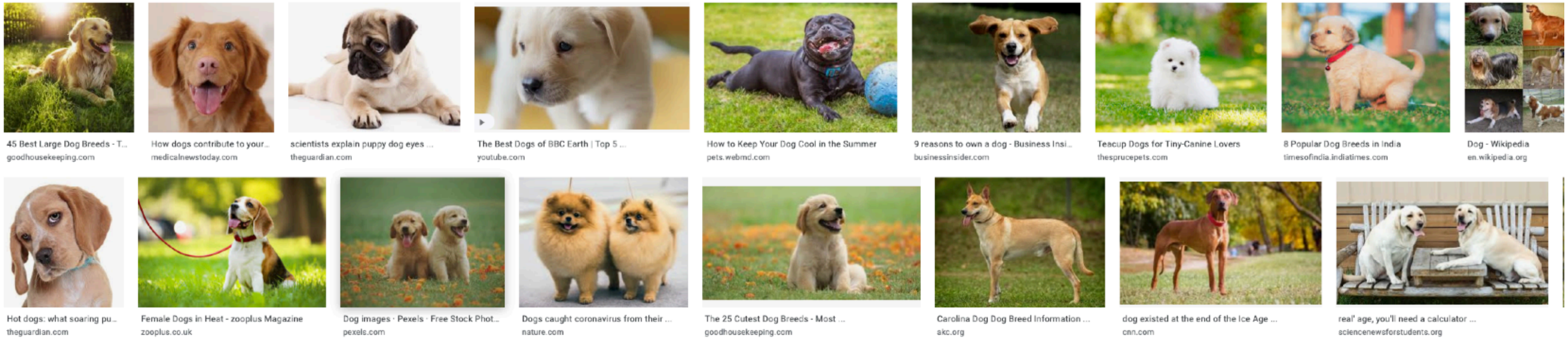


https://deepgenerativemodels.github.io/

Introduction

What does it mean to learn a **generative model**?

Learning a Generative Model



Google Search: Dog

- Suppose we are given images of dogs.

Learning a Generative Model

- Suppose we are given images of dogs.
- We want to learn a probability distribution $p(x)$ such that
  - **Generation:** If we sample $x_{new} \sim p(x)$, x_{new} should look like a dog (sampling).
  - **Density estimation:** $p(x)$ should be high if x looks like a dog, and low otherwise (anomaly detection).
    - Also known as, explicit models.
  - **Unsupervised representation learning:** We should be able to learn what these images have in common, e.g., ears, tail, etc (feature learning).
- Then, how can we represent $p(x)$?

Basic Discrete Distributions

- **Bernoulli distribution:** (biased) coin flip
  - $D = \{\text{Heads, Tails}\}$
  - Specify $P(X = \text{Heads}) = p$. Then $P(X = \text{Tails}) = 1 - p$.
  - Write: $X \sim \text{Ber}(p)$.

- **Categorical distribution:** (biased) m-sided dice
  - $D = \{1, \cdots, m\}$
  - Specify $P(Y = i) = p_i$, such that $\sum_{i=1}^{m} p_i = 1$.
  - Write: $Y \sim \text{Cat}(p_1, \cdots, p_m)$

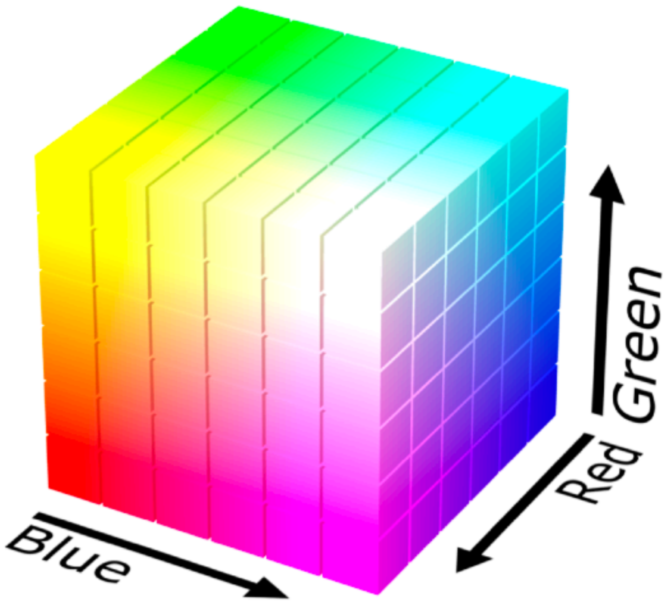
Example

- Modeling an RGB joint distribution (of a single pixel)
  - $(r, g, b) \sim p(R, G, B)$
  - Number of cases?

$$256 \times 256 \times 256$$

  - How many parameters do we need to specify?

$$255 \times 255 \times 255$$



https://en.wikipedia.org/wiki/RGB_color_space

Example

- Suppose we have $X_1, \ldots, X_n$ of n binary pixels (a binary image).
- How many possible states?

$$2 \times 2 \times \cdots \times 2 = 2^n$$

- Sampling from $p(x_1, \ldots, x_n)$ generates an image.
- How many parameters to specify $p(x_1, \ldots, x_n)$?

$$2^n - 1$$

Structure Through Independence

- What if $X_1, \ldots, X_n$ are independent, then

$$p(x_1, \ldots, x_n) = p(x_1)p(x_2)\cdots p(x_n)$$

- How many possible states?

$$2^n$$

- How many parameters to specify $p(x_1, \ldots, x_n)$?

$$n$$

- 2^n entries can be described by just n numbers! But this independence assumption is too strong to model useful distributions.

Conditional Independence

- Three important rules
  - Chain rule:

$$p(x_1, \ldots, x_n) = p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_1, x_2)\cdots p(x_n \mid x_1, \cdots, x_{n-1})$$

  - Bayes' rule:

$$p(x \mid y) = \frac{p(x, y)}{p(y)} = \frac{p(y \mid x)p(x)}{p(y)}$$

  - Conditional independence:

If $x \perp y \mid z$, then $p(x \mid y, z) = p(x \mid z)$

Conditional Independence

- Using the chain rule,

$$p(x_1, \ldots, x_n) = p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_1, x_2) \cdots p(x_n \mid x_1, \cdots, x_{n-1})$$

- How many parameters?
  - $p(x_1)$: 1 parameter
  - $p(x_2 \mid x_1)$: 2 parameters (one per $p(x_2 \mid x_1 = 0)$ and one per $p(x_2 \mid x_1 = 1)$)
  - $p(x_3 \mid x_1, x_2)$: 4 parameters
  - Hence, $1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$, which is the same as before.
- Why?

Conditional Independence

- Now, suppose $X_{i+1} \perp X_1, \ldots, X_{i-1} \mid X_i$ (Markov assumption), then

$$p(x_1, \ldots, x_n) = p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_2)\cdots p(x_n \mid x_{n-1})$$

- How many parameters?

$$2n - 1$$

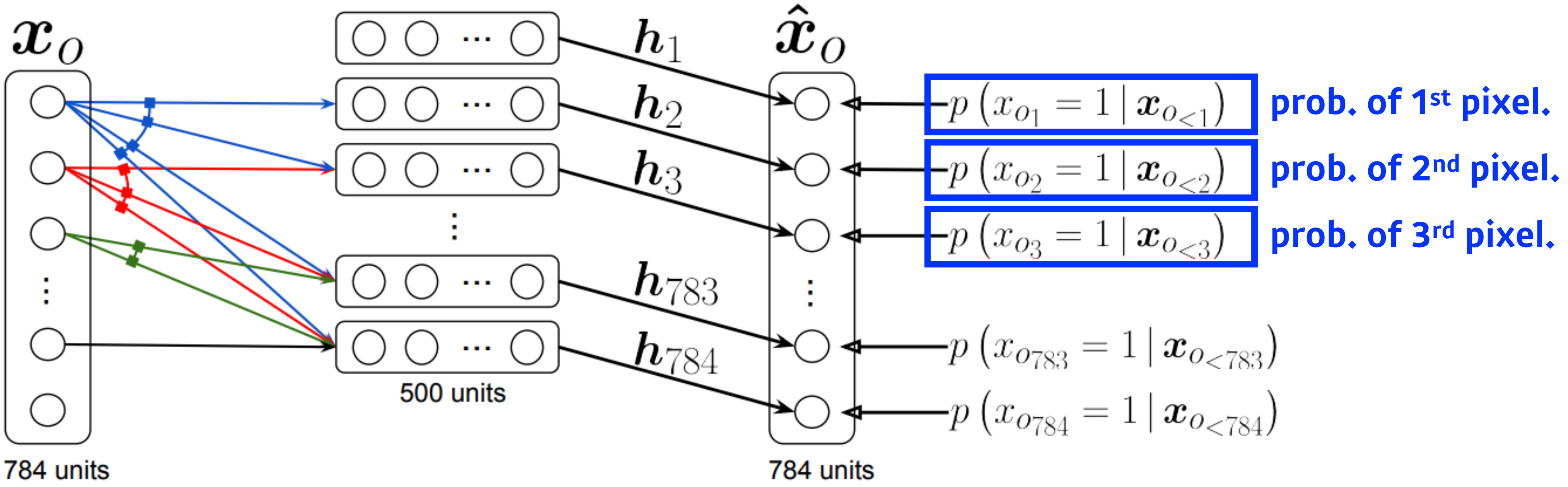
- Hence, by leveraging the Markov assumption, we get exponential reduction on the number of parameters.
- **Auto-regressive models** leverage this conditional independency.

Auto-regressive Model

Auto-regressive Model

- Suppose we have 28×28 binary pixels.
- Our goal is to learn $p(x) = p(x_1, \ldots, x_{784})$ over $x \in \{0,1\}^{784}$.
- How can we parametrize $p(x)$?
  - Let's use the chain rule to factor the joint distribution.
  - $p(x_{1:784}) = p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_{1:2}) \cdots$
  - This is called an autoregressive model.
  - Note that we need an ordering of all random variables.

NADE: Neural Autoregressive Density Estimator



- The probability distribution of i-th pixel is

$$p(x_i \mid x_{1:i-1}) = \sigma(\alpha_i \mathbf{h}_i + b_i) \text{ where } \mathbf{h}_i = \sigma(W_{<i} x_{1:i-1} + \mathbf{c})$$

https://arxiv.org/abs/1605.02226

NADE: Neural Autoregressive Density Estimator

- NADE is an explicit model that can compute the density of the given inputs.
- How can we compute the density of the given image?
  - Suppose we have a binary image with 784 binary pixels, $\{x_1, x_2, \ldots, x_{784}\}$.
  - Then, the joint probability is computed by

$$p(x_1, \ldots, x_{784}) = p(x_1)p(x_2 \mid x_1) \cdots p(x_{784} \mid x_{1:783})$$

where each conditional probability $p(x_i \mid x_{1:i-1})$ is computed independently.

- In case of modeling continuous random variables, a mixture of Gaussian can be used.

https://arxiv.org/abs/1605.02226

Pixel RNN

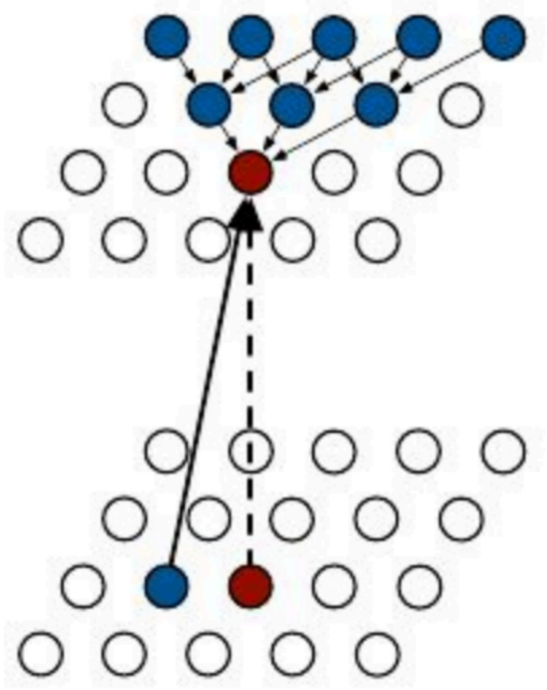
- We can also use **RNNs** to define an auto-regressive model.
- For example, for an $n \times n$ RGB image,

$$p(x) = \Pi_{i=1}^{n^2} \underbrace{\boxed{p(x_{i,R} \mid x_{<i})}}_{\text{Prob. i-th R}} \underbrace{\boxed{p(x_{i,G} \mid x_{<i}, x_{i,R})}}_{\text{Prob. i-th G}} \underbrace{\boxed{p(x_{i,B} \mid x_{<i}, x_{i,R}, x_{i,G})}}_{\text{Prob. i-th B}}$$

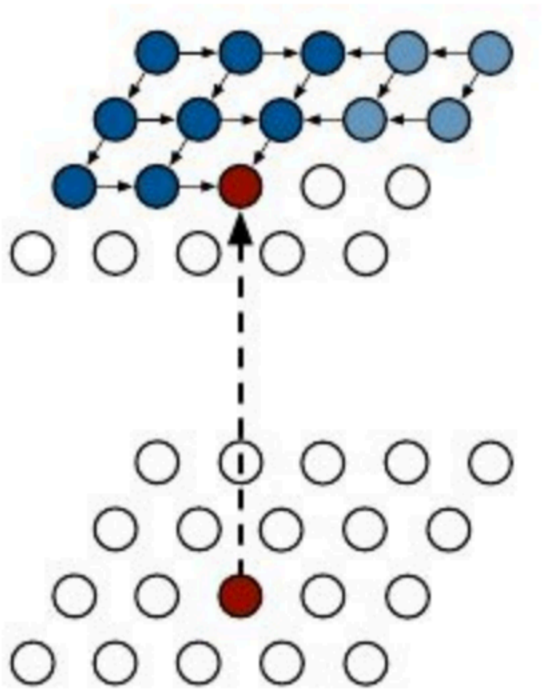
- There are two model architectures in Pixel RNN based on the **ordering** of chain:
  - Row LSTM
  - Diagonal BiLSTM

https://arxiv.org/abs/1601.06759

Pixel RNN



Row LSTM



Diagonal BiLSTM

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Thank you for listening