

# Deep Learning Basics

## Lecture 4: Convolutional Neural Networks

**최성준** (고려대학교 인공지능학과)

# Convolution

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- Continuous convolution

$$(f * g)(t) = \int f(\tau)g(t - \tau)d\tau = \int f(t - \tau)g(\tau)d\tau$$

- Discrete convolution

$$(f * g)(t) = \sum_{i=-\infty}^{\infty} f(i)g(t - i) = \sum_{i=-\infty}^{\infty} f(t - i)g(i)$$

- 2D image convolution

$$(I * K)(i, j) = \sum_m \sum_n I(m, n)K(i - m, j - n) = \sum_m \sum_n I(i - m, j - n)K(m, n)$$

# Convolution

K (3x3 filter)

$K_{11}$	$K_{12}$	$K_{13}$
$K_{21}$	$K_{22}$	$K_{23}$
$K_{31}$	$K_{32}$	$K_{33}$

\*

I (7x7 image)

$I_{11}$	$I_{12}$	$I_{13}$	$I_{14}$	$I_{15}$	$I_{16}$	$I_{17}$
$I_{21}$	$I_{22}$	$I_{23}$	$I_{24}$	$I_{25}$	$I_{26}$	$I_{27}$
$I_{31}$	$I_{32}$	$I_{33}$	$I_{34}$	$I_{35}$	$I_{36}$	$I_{37}$
$I_{41}$	$I_{42}$	$I_{43}$	$I_{44}$	$I_{45}$	$I_{46}$	$I_{47}$
$I_{51}$	$I_{52}$	$I_{53}$	$I_{54}$	$I_{55}$	$I_{56}$	$I_{57}$
$I_{61}$	$I_{62}$	$I_{63}$	$I_{64}$	$I_{65}$	$I_{66}$	$I_{67}$
$I_{71}$	$I_{72}$	$I_{73}$	$I_{74}$	$I_{75}$	$I_{76}$	$I_{77}$

=

Output (5x5)

$O_{11}$	$O_{12}$	$O_{13}$	$O_{14}$	$O_{15}$
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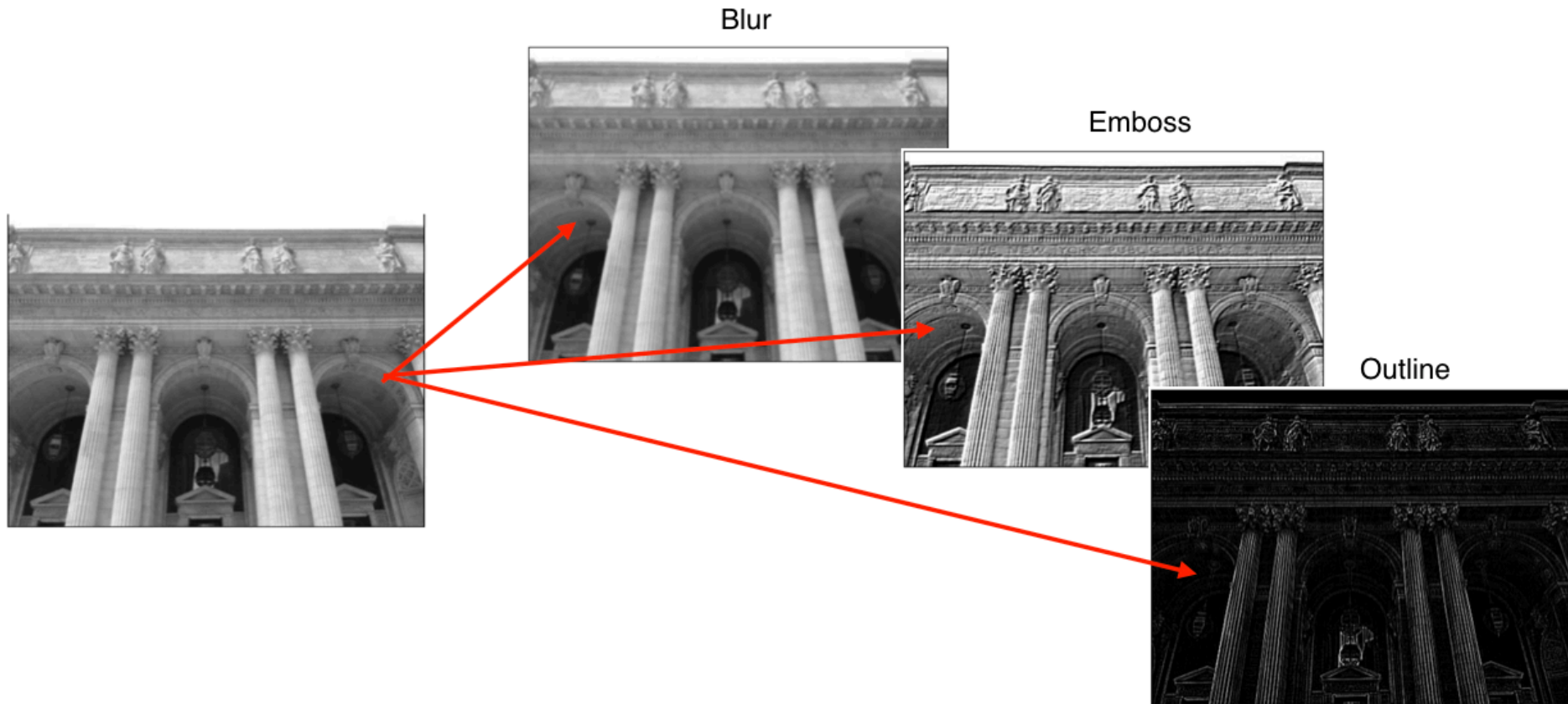
$$O_{13} = I_{13}K_{11} + I_{14}K_{12} + I_{15}K_{13} + I_{23}K_{21} + I_{24}K_{22} + I_{25}K_{23} + I_{33}K_{31} + I_{34}K_{32} + I_{35}K_{33} + bias$$

$$O_{14} = I_{14}K_{11} + I_{15}K_{12} + I_{16}K_{13} + I_{24}K_{21} + I_{25}K_{22} + I_{26}K_{23} + I_{34}K_{31} + I_{35}K_{32} + I_{36}K_{33} + bias$$



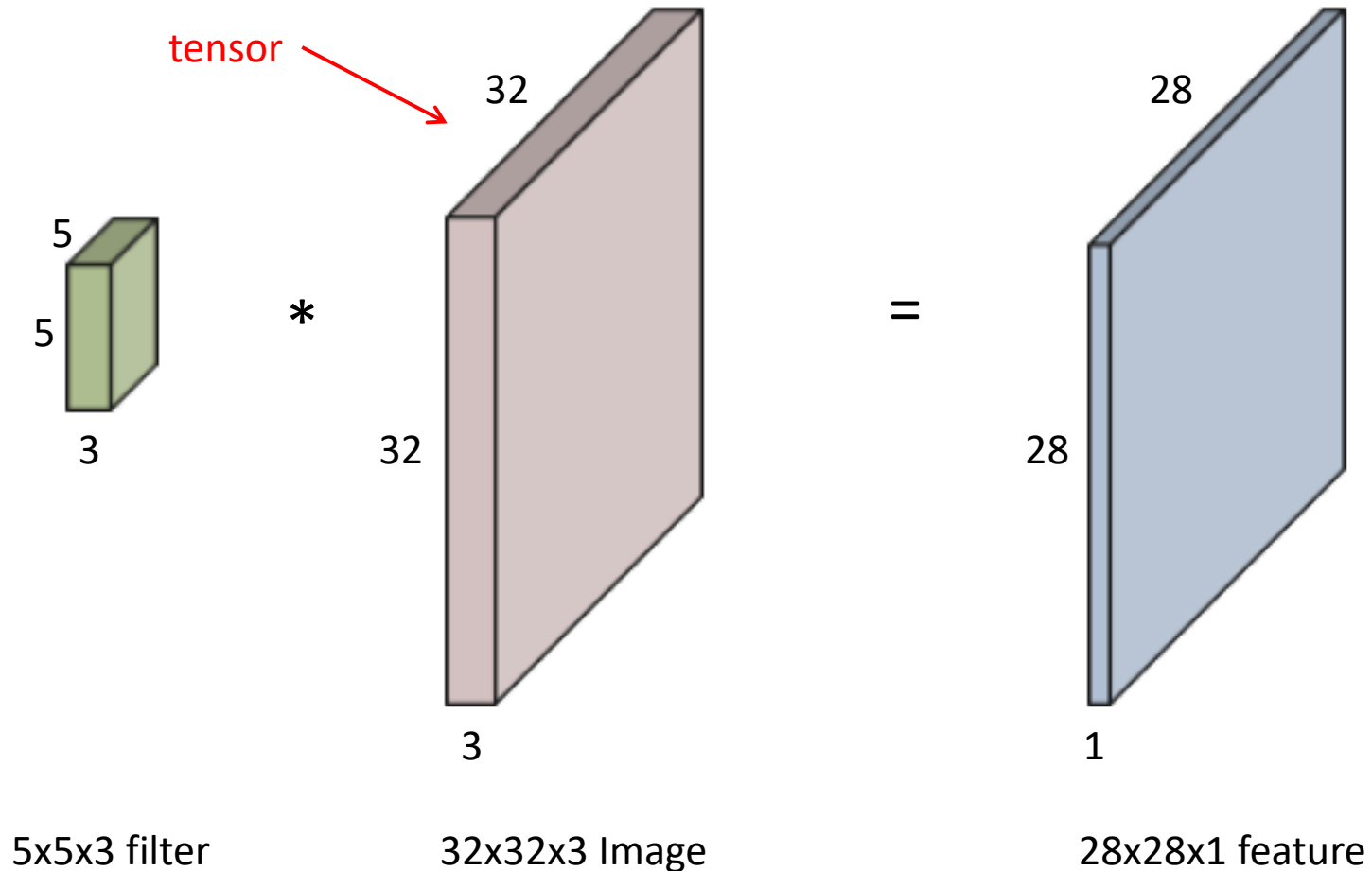
# Convolution

- 2D convolution in action

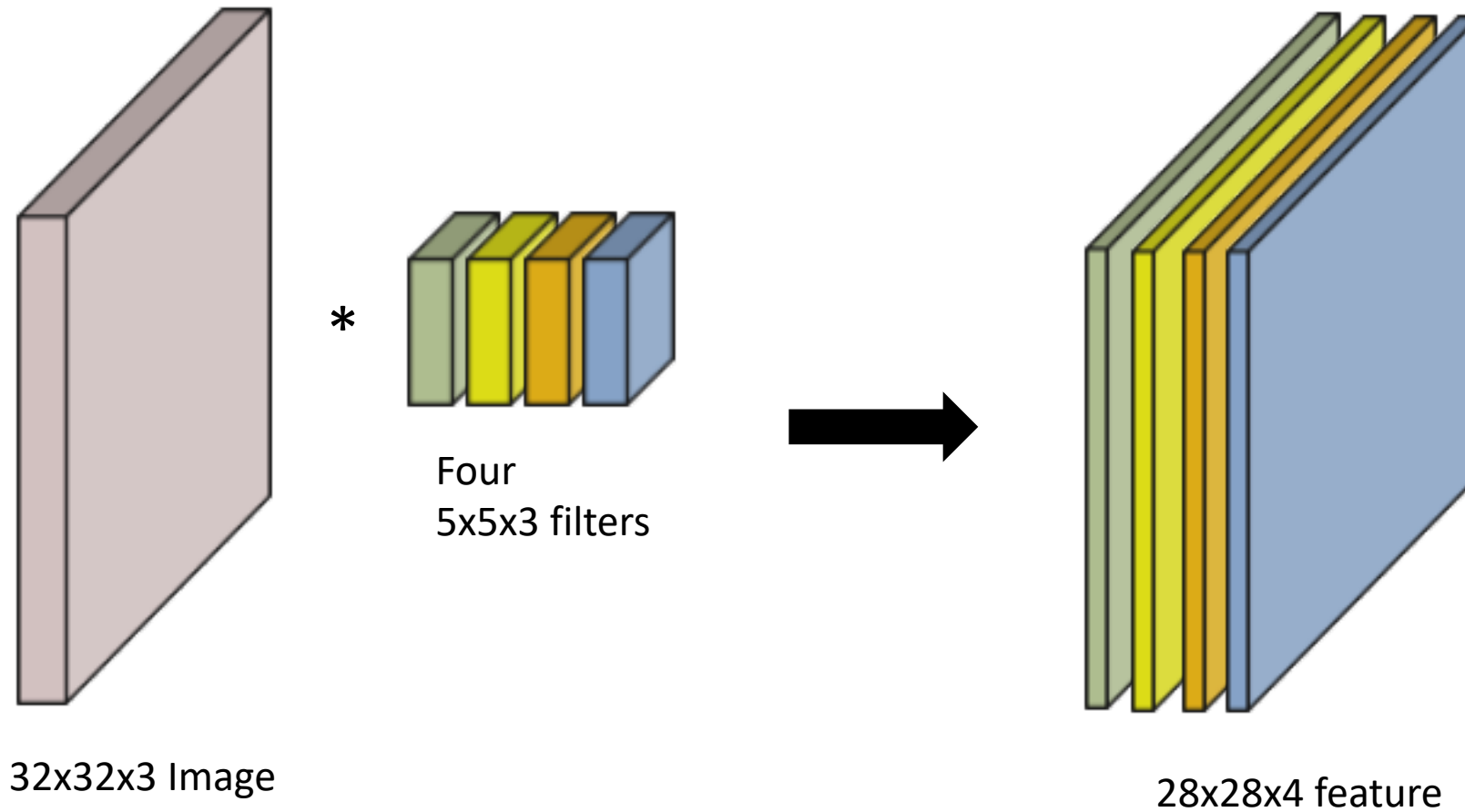




# RGB Image Convolution

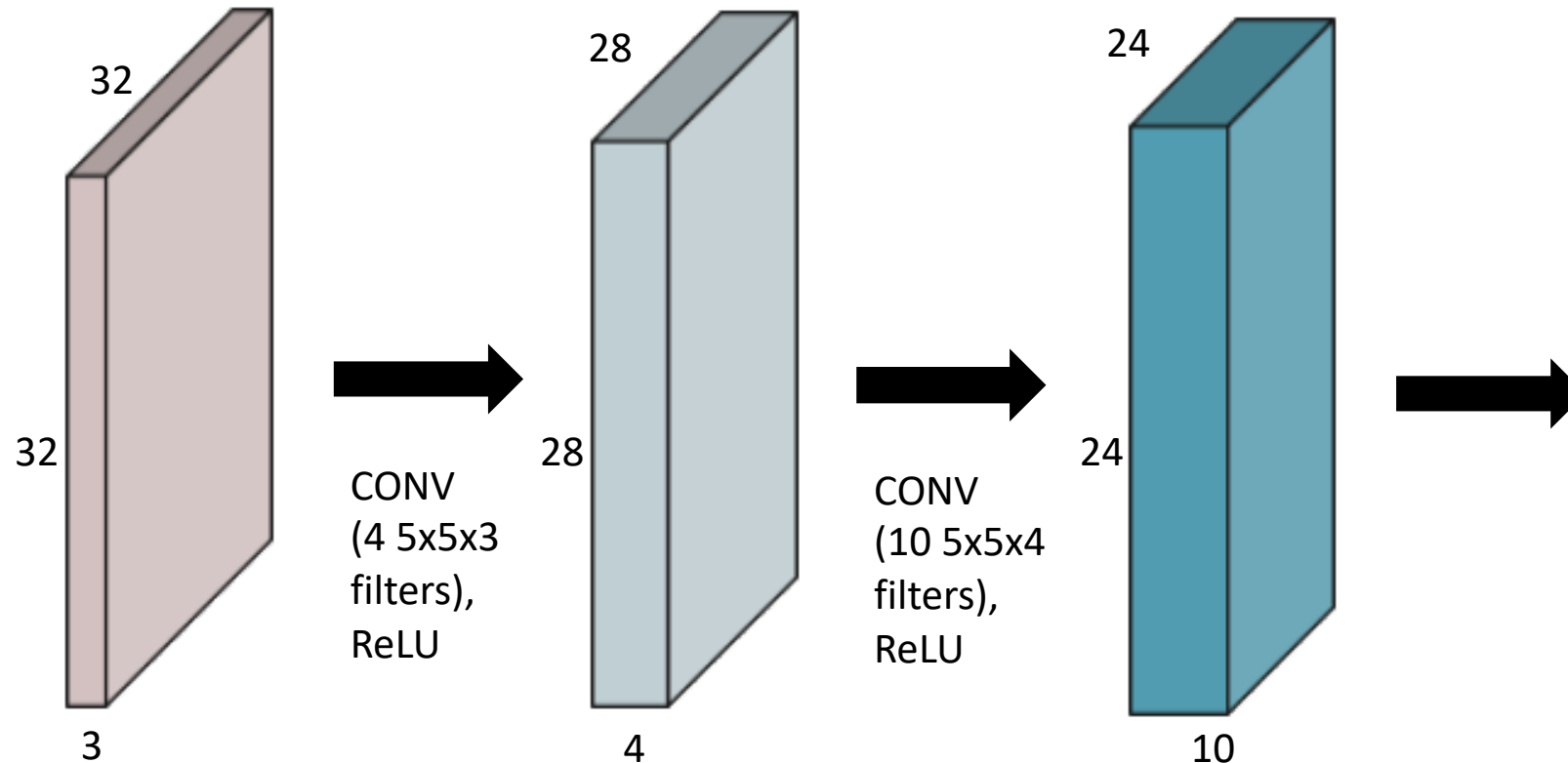


# RGB Image Convolution



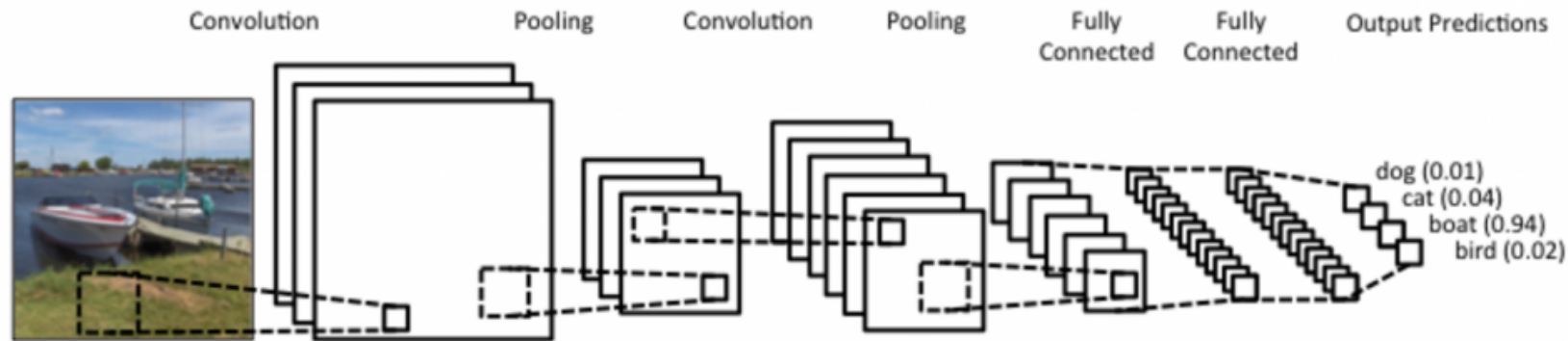
# Stack of Convolutions

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# Convolutional Neural Networks

- CNN consists of convolution layer, pooling layer, and fully connected layer.
- Convolution and pooling layers: feature extraction
- Fully connected layer: decision making (e.g., classification)

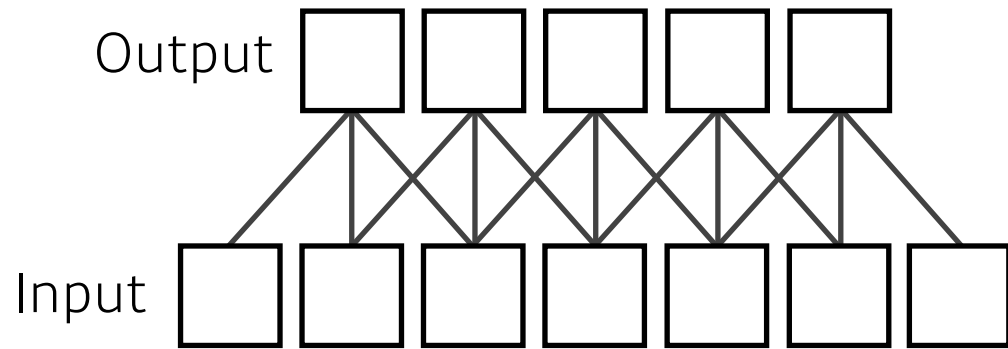


# Convolution Arithmetic (of GoogLeNet)

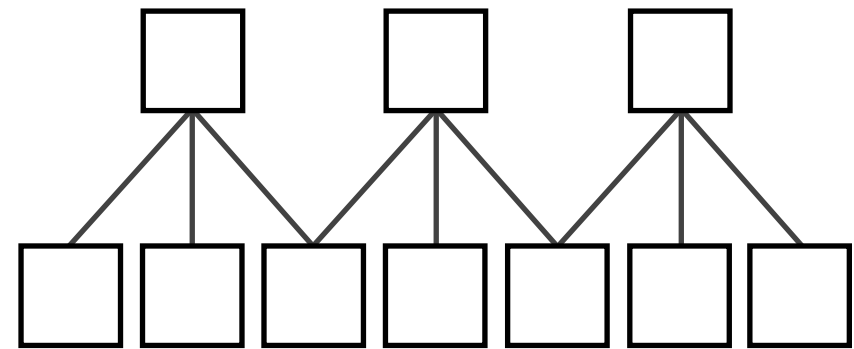
	Stride	Channel	#param								
type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

# Stride

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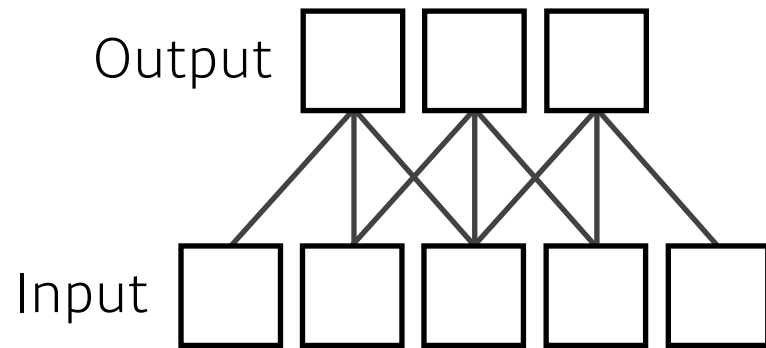


Stride = 1

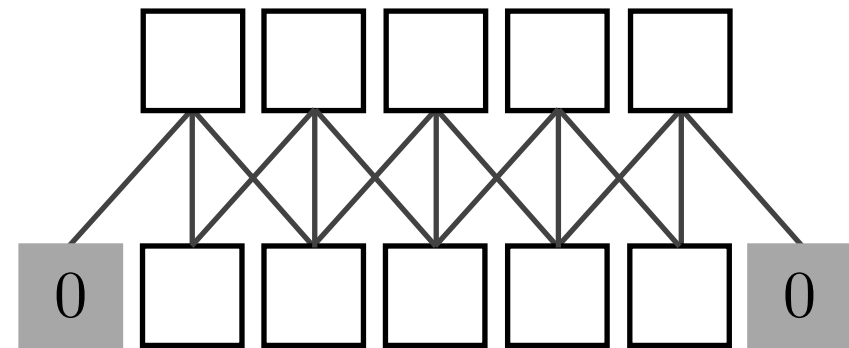


# Padding

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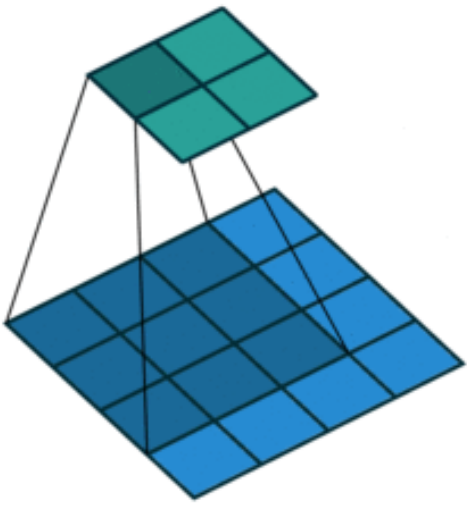
No padding (stride=1)



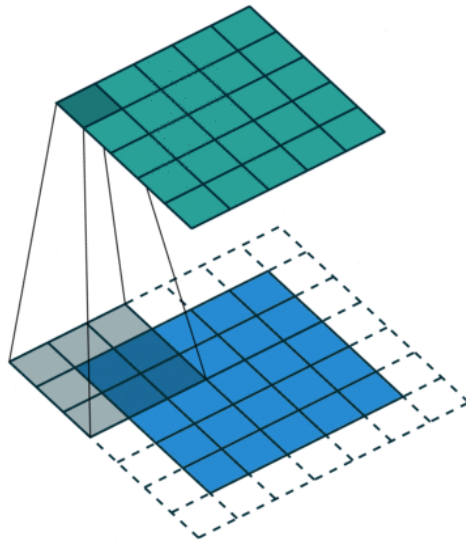
Zero padding (stride=1)

# Stride? Padding?

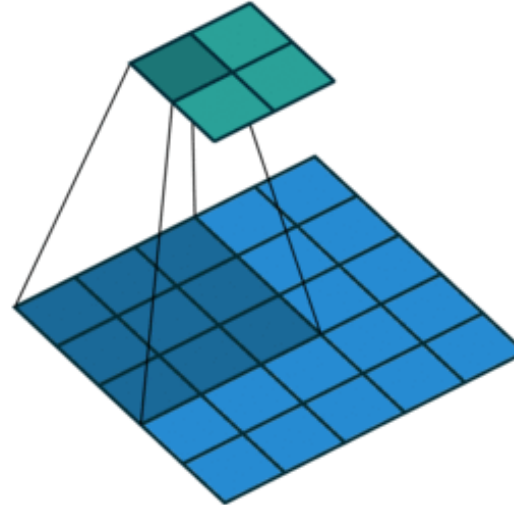
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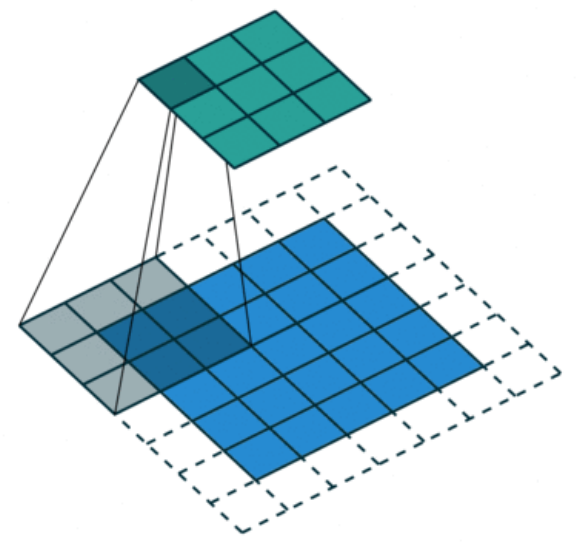
Stride (1)  
Padding (0)



Stride (1)  
Padding (1)



Stride (2)  
Padding (0)

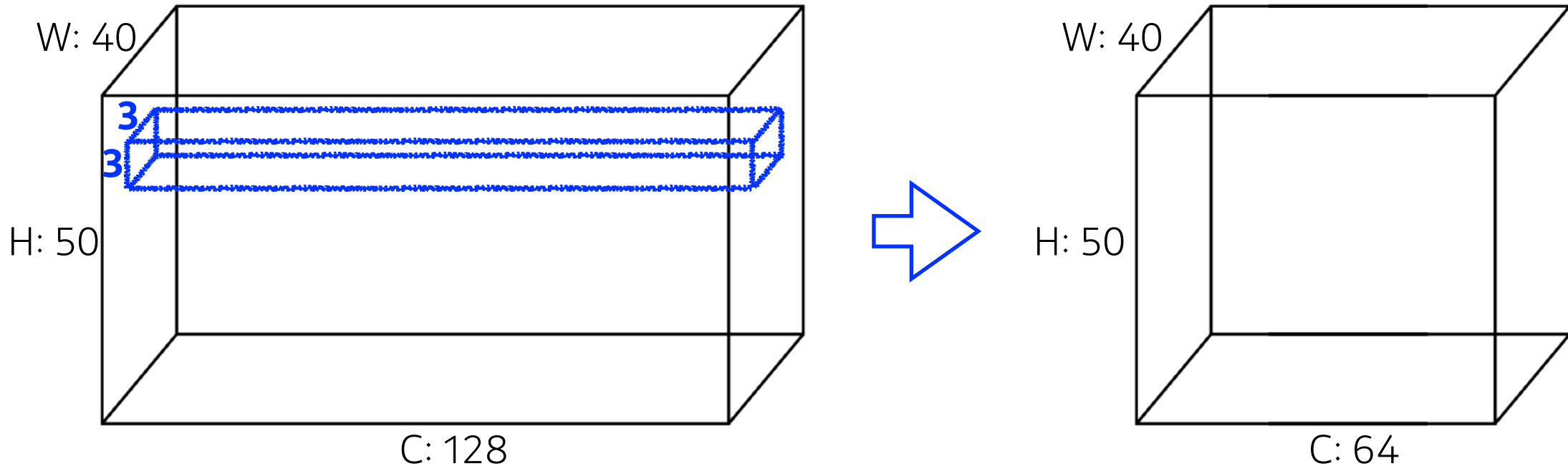


Stride (2)  
Padding (1)



# Convolution Arithmetic

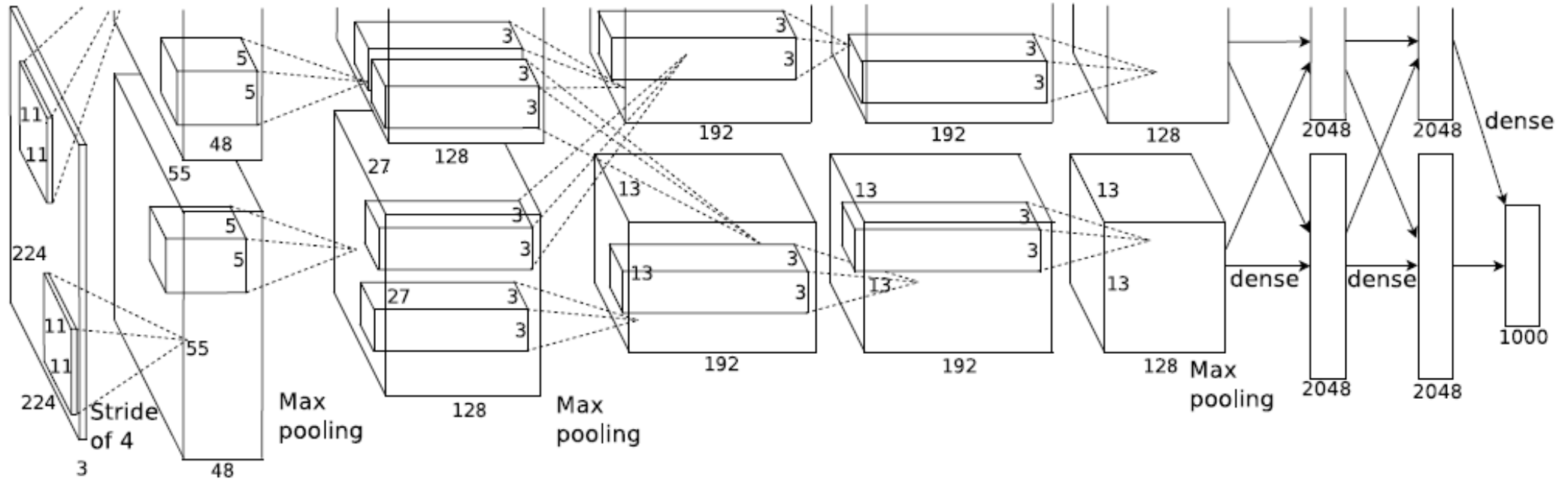
- Padding (1), Stride (1),  $3 \times 3$  Kernel



What is the **number of parameters** of this model?

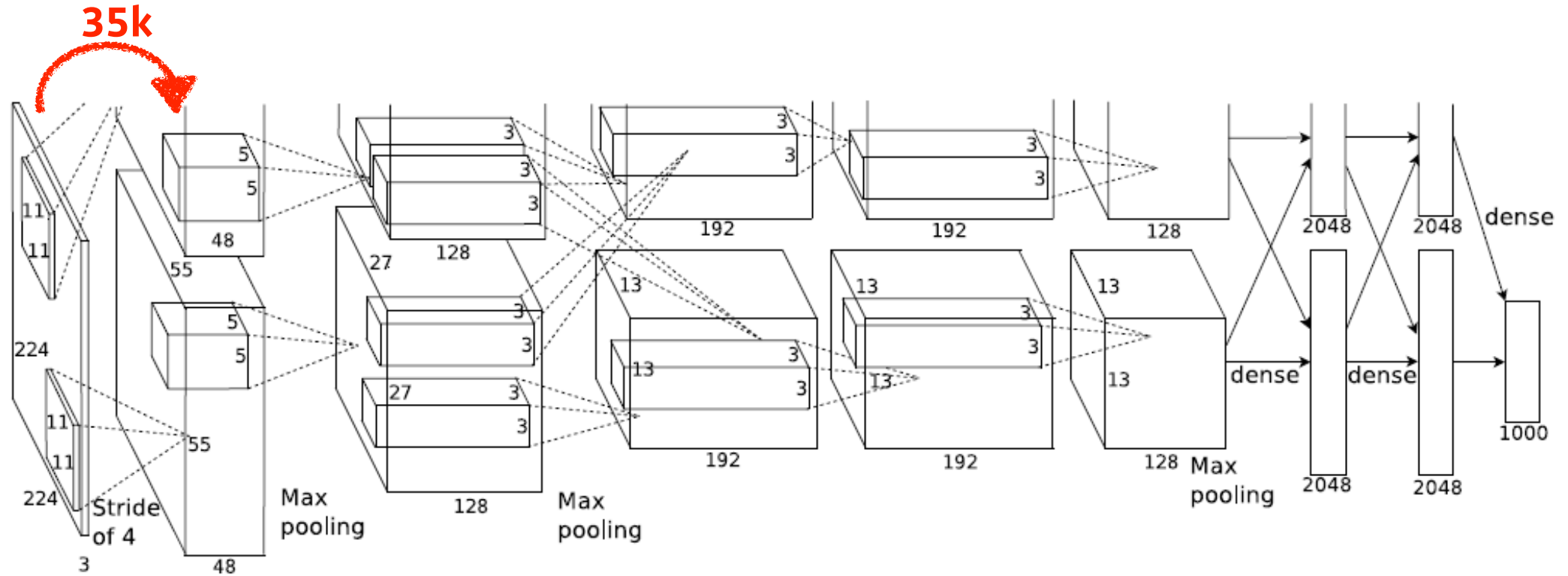
The answer is  $3 \times 3 \times 128 \times 64 = 73,728$

# Exercise



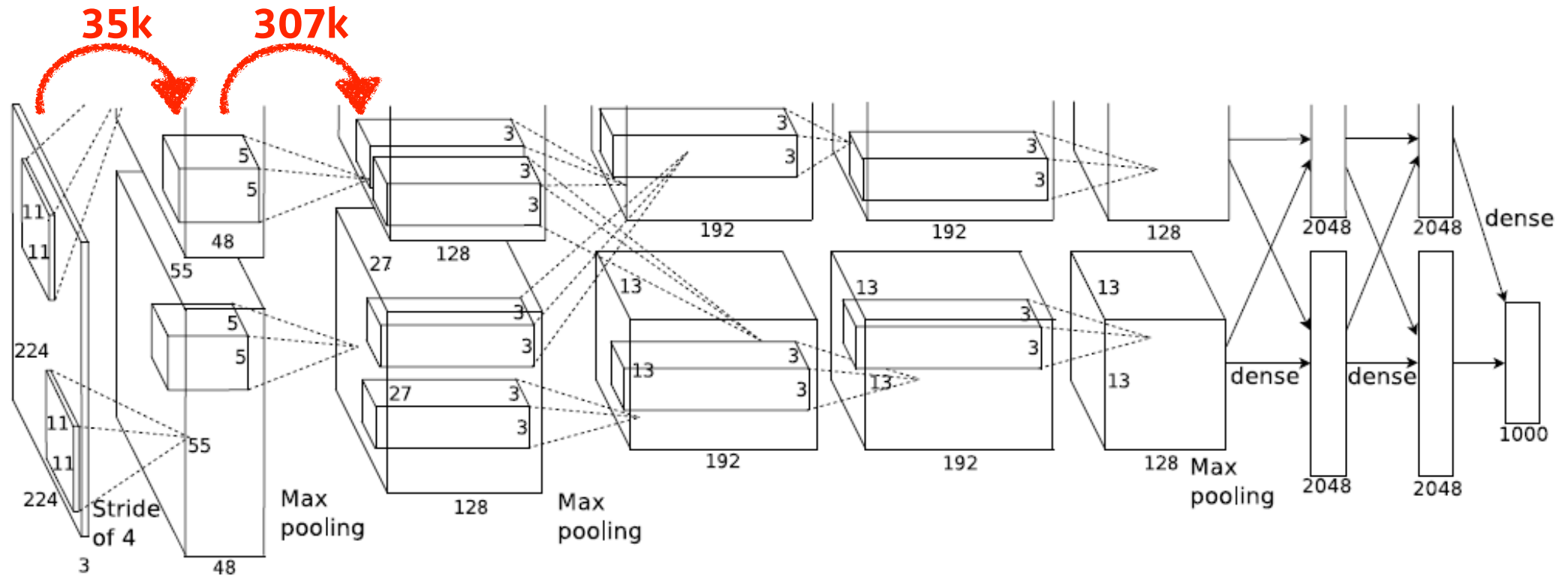
What is the **number of parameters** of this model?

# Exercise



$$11 \times 11 \times 3 \times 48 * 2 \approx 35k$$

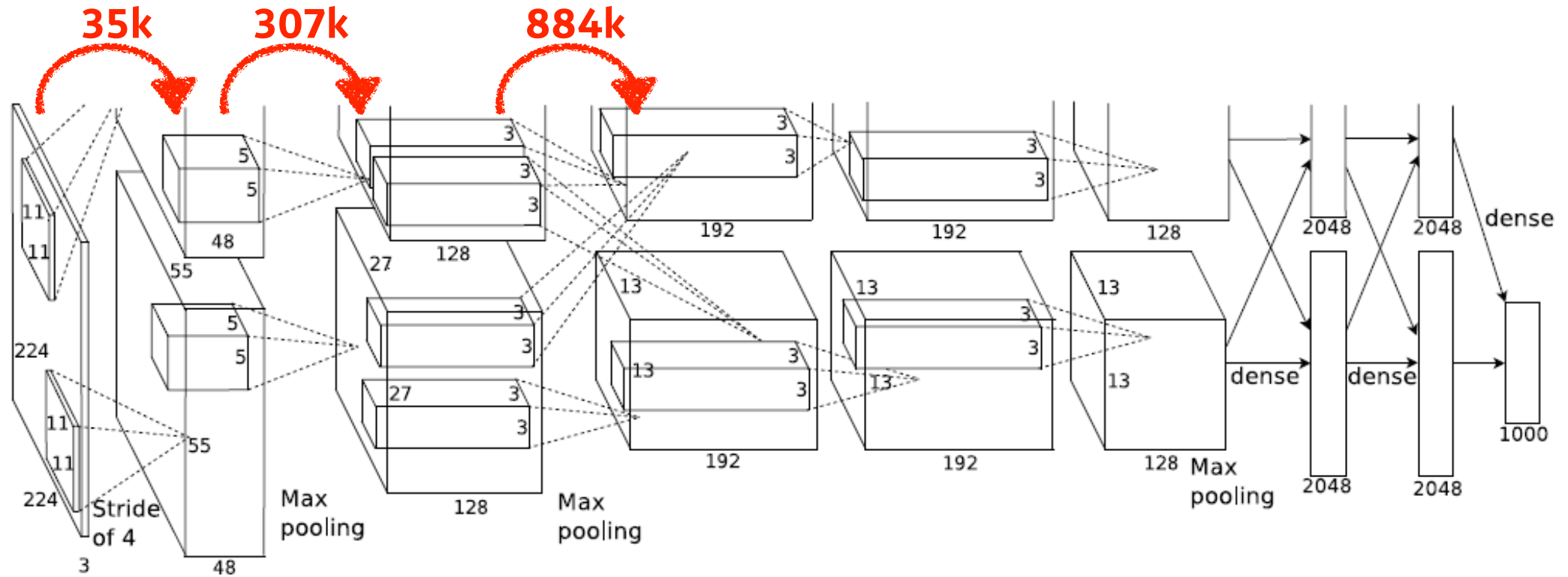
What is the **number of parameters** of this model?



$$5 \times 5 \times 48 \times 128 * 2 \approx 307k$$

What is the **number of parameters** of this model?

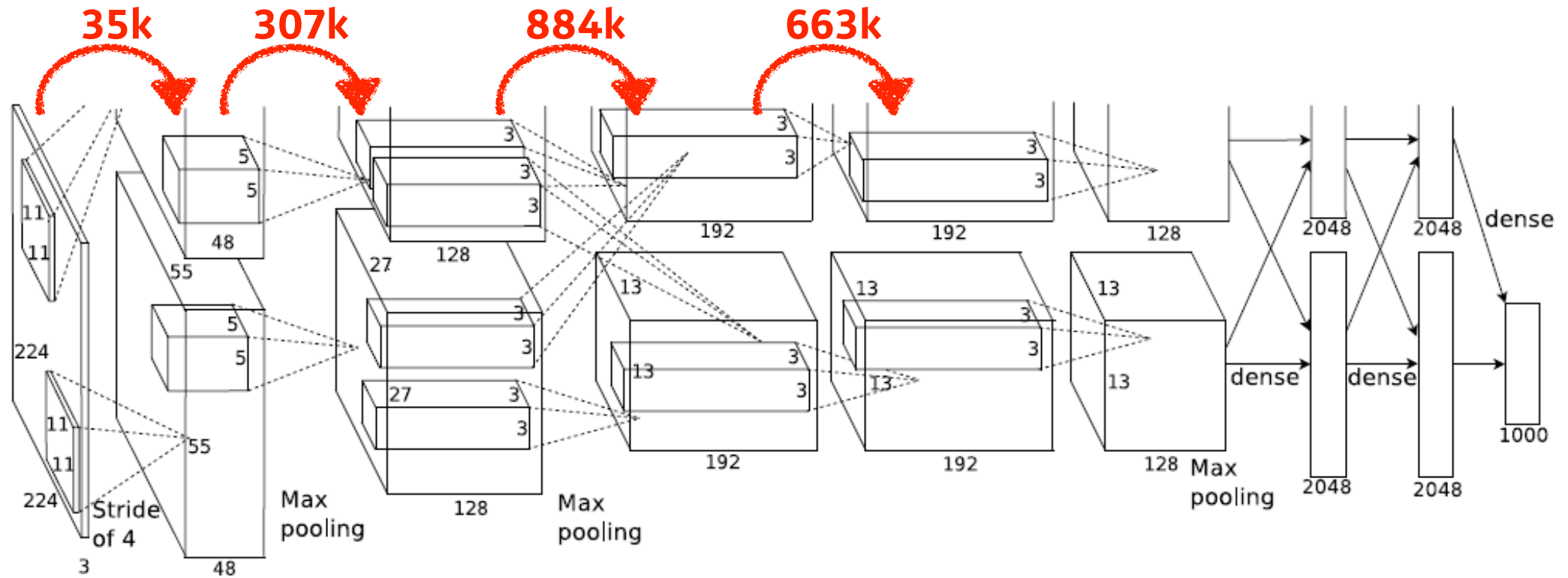
# Exercise



$$3 \times 3 \times 128 * 2 \times 192 * 2 \approx 884k$$

What is the **number of parameters** of this model?

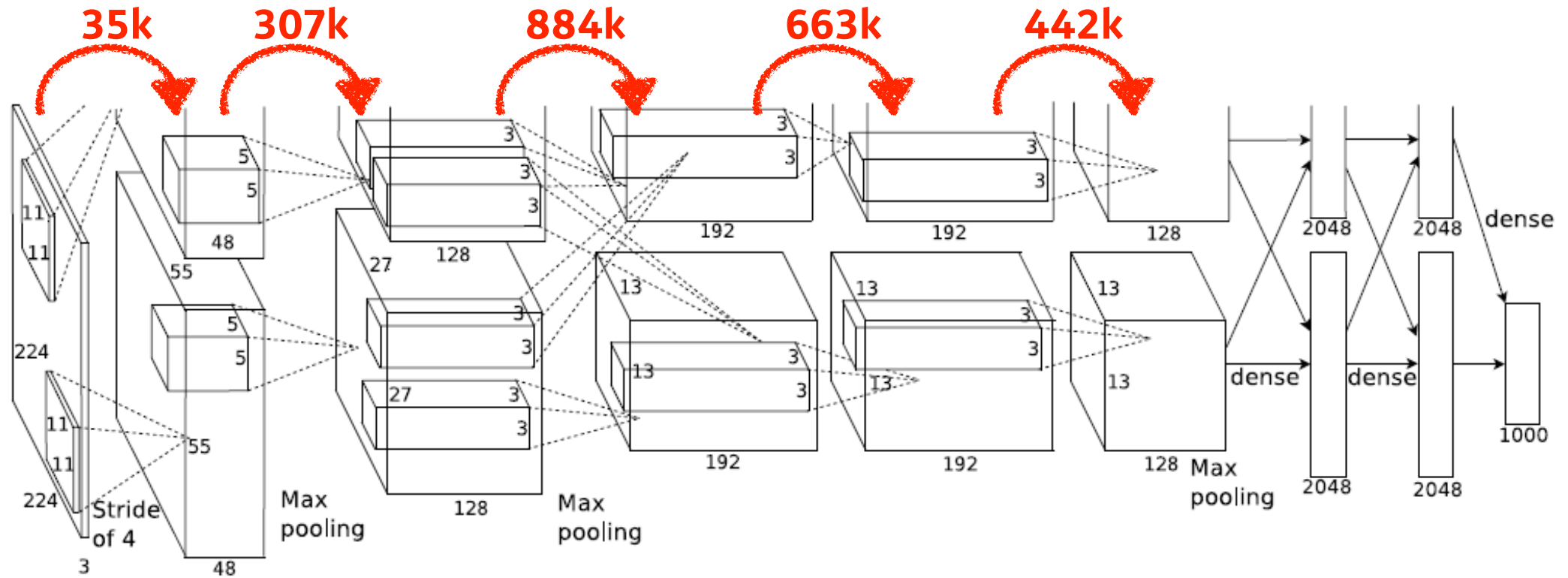
# Exercise



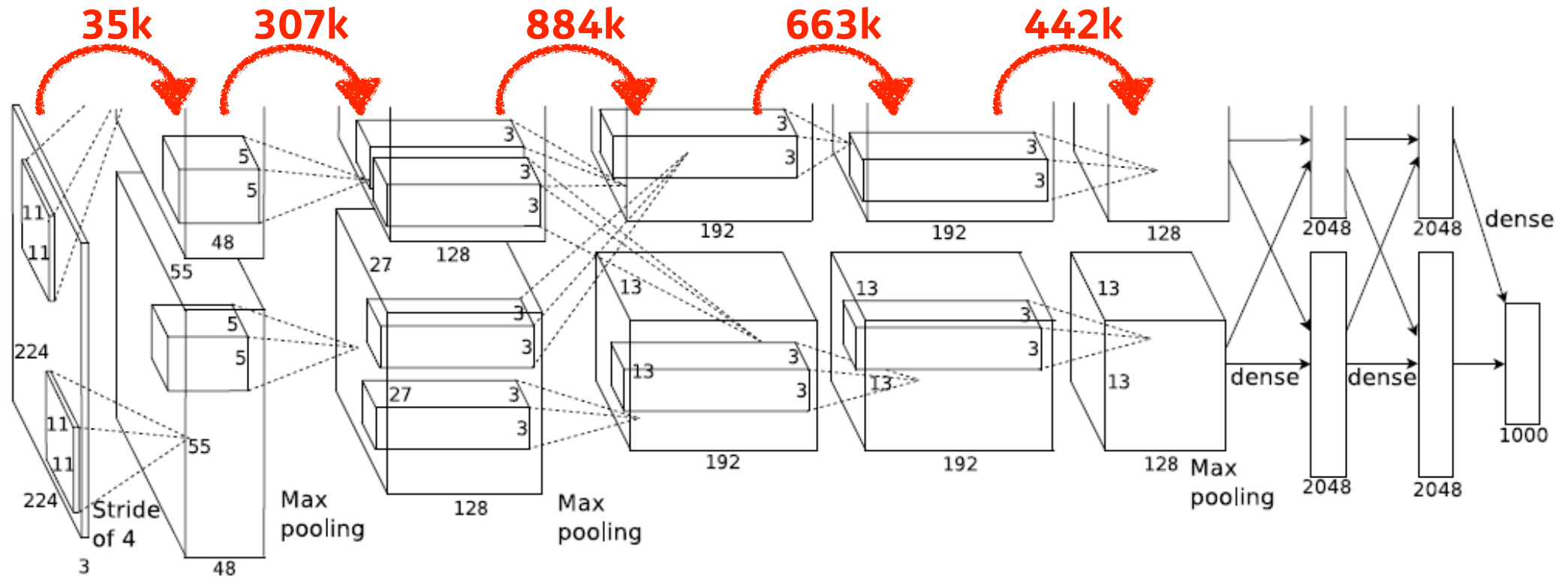
$$3 \times 3 \times 192 \times 192 \times 2 \approx 663k$$

What is the **number of parameters** of this model?

# Exercise



$$3 \times 3 \times 192 \times 128 * 2 \approx 442k$$

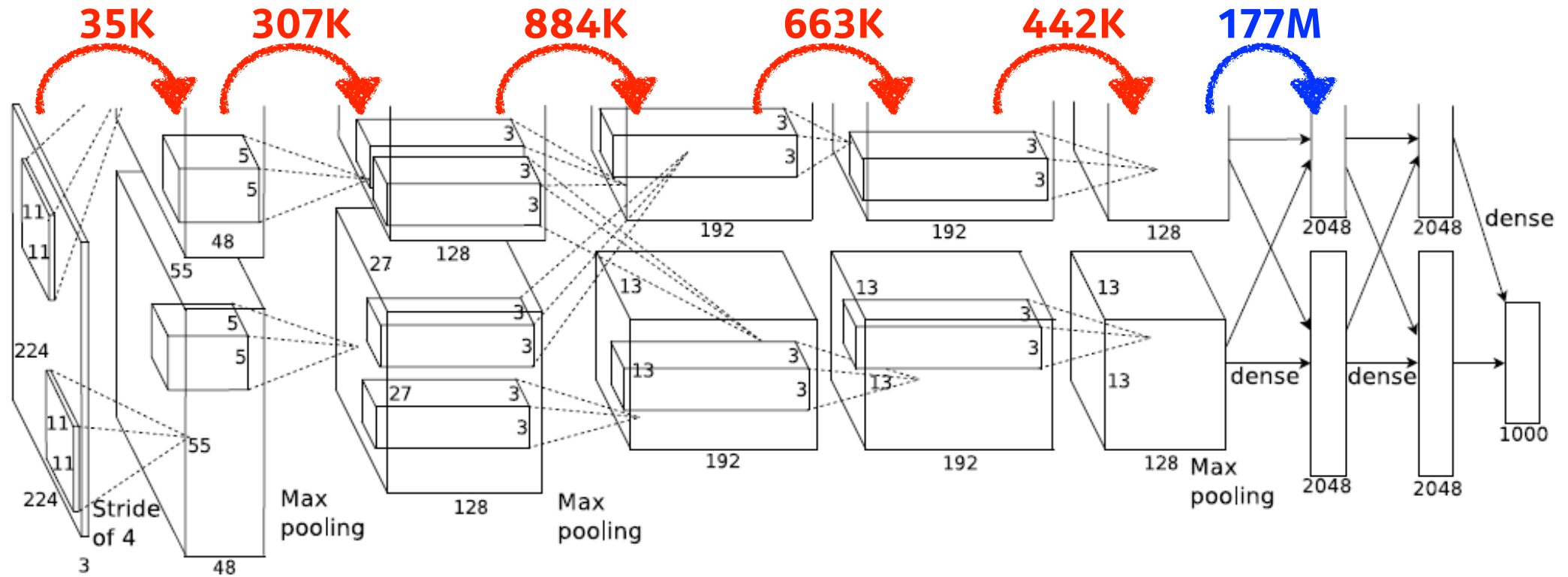


$$3 \times 3 \times 192 \times 128 * 2 \approx 442k$$

What is the **number of parameters** of this model?



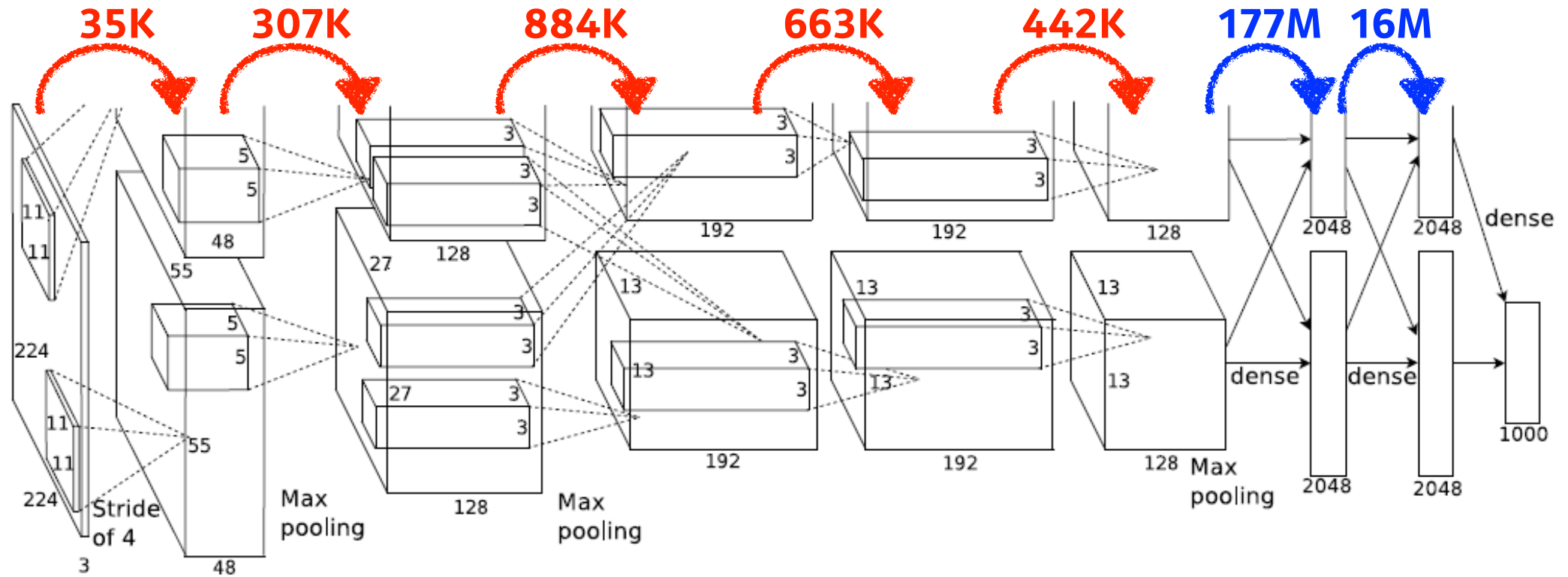
# Exercise



$$13 * 13 * 128 * 2 \times 2048 * 2 \approx 177M$$

What is the **number of parameters** of this model?

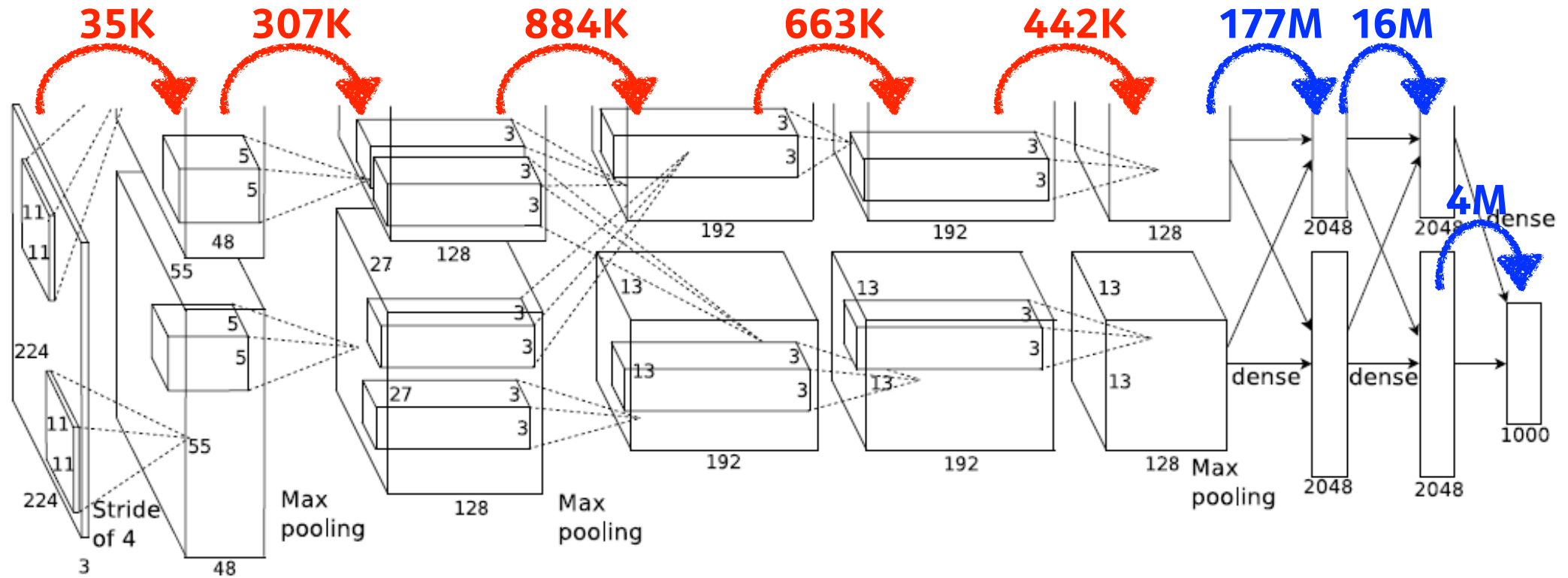
# Exercise



$$2048 * 2 \times 2048 * 2 \approx 16M$$

What is the **number of parameters** of this model?

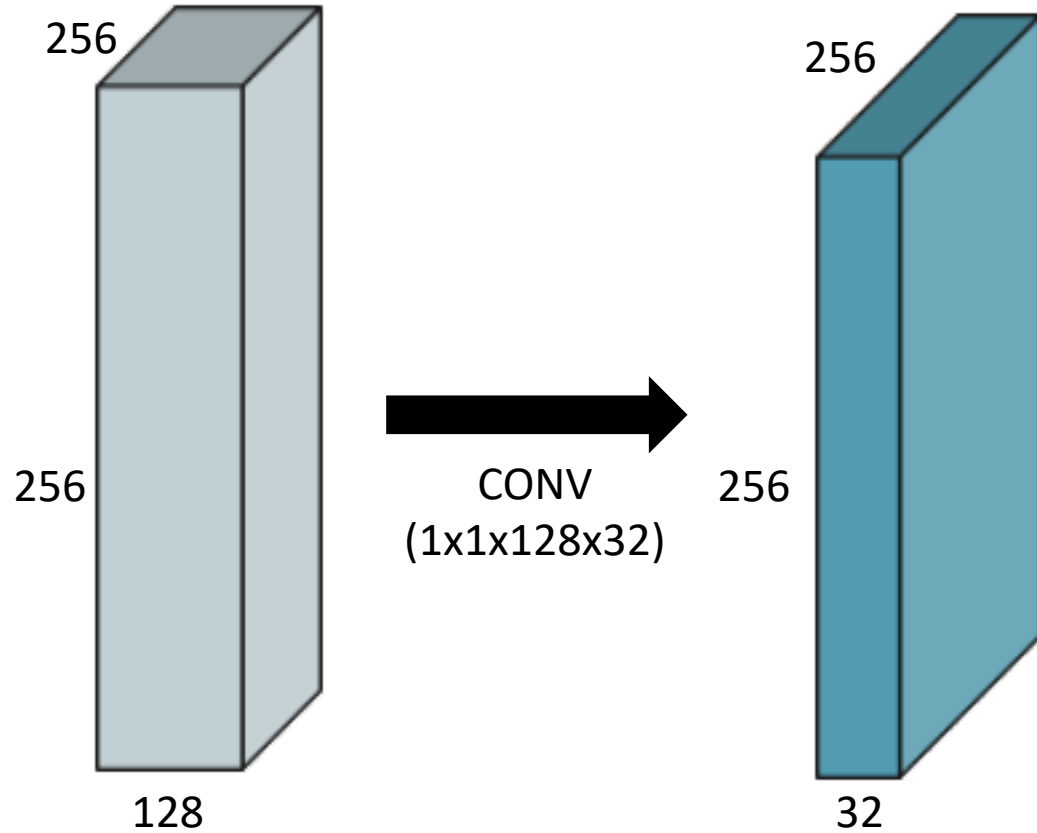
# Exercise



$$2048 * 2 * 1000 \approx 4M$$

What is the **number of parameters** of this model?

# 1x1 Convolution



## Why?

- Dimension reduction
- To reduce the number of parameters while increasing the depth
- e.g., bottleneck architecture

# Thank you for listening

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