International Association of Geodesy Symposia manuscript No.
(will be inserted by the editor)

Virtual Quake: Statistics, Co-Seismic Deformations and Gravity Changes for Driven Earthquake Fault Systems

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Received: xxxxx, 2014 / Accepted: xxxxx, 2015

Abstract With the ever increasing number of geodetic monitoring satellites, it is vital to have a variety of geophysical simulations produce synthetic datasets. Furthermore, just as hurricane forecasts are derived from the consensus among multiple atmospheric models, earthquake forecasts cannot be derived from a single comprehensive model. Here we present the functionality of Virtual Quake (formerly known as Virtual California), a numerical simulator that can generate sample co-seismic deformations, gravity changes, and InSAR interferograms in addition to producing probabilities for earthquake scenarios.

Virtual Quake is now hosted by the Computational Infrastructure for Geodynamics. It is available for download and comes with a user manual. The manual includes a description of the simulator physics, instructions for generating fault models from scratch, and a guide to deploying the simulator in a parallel computing environment.

http://geodynamics.org/cig/software/vq/

Keywords GENAH · Virtual California · Virtual Quake · earthquake forecasting

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1 Introduction

The purpose of this paper is to introduce a subset of the wide range of applications of the Virtual Quake (VQ) earthquake simulator, originally developed by Rundle [1988a,b]. Virtual Quake has been developed to characterize the space-time patterns and correlations that emerge from the underlying stress and strain dynamics. Virtual Quake simulates hundreds of thousands of years of interactions on any fault system given information about the fault geometry and the observed long-term slip rates.

Over the past decade, Virtual Quake has been part of a multi-disciplinary effort to better understand, predict, and respond to earthquake hazards known as QuakeSim [Donnellan et al., 2006]. As part of the QuakeSim team, Virtual Quake was selected as a co-winner of NASA's 2012 Software of the Year award. Tullis et al. [2012a] confirmed that the Virtual Quake (then called Virtual California) simulations are consistent with both the observed seismicity in California and with independent California fault system simulations. The analysis compared the simulated earthquake catalogs to the observed earthquake rates in California and to observed scaling relations for magnitude, rupture area, and mean slip. Yikilmaz et al. [2010] also showed that Virtual Quake simulations of the Nankai Trough in Japan are consistent with observed earthquake sequences.

The output of a Virtual Quake simulation is a physics-based catalog of earthquakes. In section 4 we discuss how this simulated earthquake catalog can be used to compute conditional probabilities of earthquake scenarios. In section 3 we show how the simulated co-seismic slip distributions can be used to generate spacetime patterns using Green's functions. These surface patterns can be compared to geodetic data like InSAR interferograms, co-seismic displacements, and co-seismic gravity changes.

2 Virtual Quake Simulator

Virtual Quake (VQ) is a boundary element code that simulates realistically driven fault systems based on stress interactions. VQ is designed to explore the long term statistical behavior of topologically complex fault networks [Rundle, 1988a,b, Rundle et al., 2006a,b,c]. The most recent version of VQ is a modern scientific code that simulates earthquakes in a high performance computing environment [Sachs et al., 2012, Heien and Sachs, 2012]. VQ simulates many thousands of years of seismic history on fault networks with any physically realizable geometry.

VQ consists of three major components: a fault model, simulation physics, and an event model. The fault model is the simulation input and it can be changed to any arbitrary fault geometry and function properly the simulation physics and event model. The simulation physics is based on fault stress via the accumulation of a slip deficit — the amount of slip each fault should move in a given time period given the long term slip rate. Actual values of stress are computed by a set of quasi-static elastic interactions given in Rundle et al. [2006a].

VQ initiates simulated earthquakes using both static and dynamic friction laws. Slip during a simulated earthquake is triggered by the stress on a fault element reaching the failure threshold as specified in the fault model, this is static failure. Elements on the same fault are also allowed to slip — even if they have not reached failure stress — if the stress is at least 50% of the threshold value. This dynamic failure condition can be tuned and is used to control rupture propagation for simulated earthquakes.

Hereafter Virtual Quake simulations of the California fault system are referred to as Virtual California (VC). For example, if we simulated the Japan fault system then we would refer to that application of Virtual Quake as Virtual Japan. For a detailed explanation of the properties of modern earthquake simulators and a comparison of Virtual California to other simulators see Tullis et al. [2012b]. For a comparison of Virtual California simulations to those of other earthquake simulators using the same fault model see Tullis et al. [2012a].

3 Surface Patterns

Virtual Quake calculates co-seimic deformations, InSAR (Synthetic Aperture Radar Interferometry) patterns, co-seismic gravity changes and more for arbitrarily complex fault geometries [Rundle et al., 2006c, Sachs et al., 2012, Schultz et al., 2014]. VQ calculates these surface patterns by using fault geometry and co-seismic slips as input to a specific set of Green's functions. Since Virtual Quake partitions the fault system into finite boundary elements embedded in an elastic half-space, the Green's functions are a logical extension of the simulator's capability. For surface deformation and InSAR calculations we use the Green's functions presented in Okada [1992], and for gravity and potential changes we use the Green's functions presented in Okubo [1992].

These surface patterns reveal changes in the dynamic variables associated with the earthquake cycle that are inherently unobservable like the stress field. Rundle et al. [2000] developed a technique for describing the evolution of space-time patterns as a "pattern dynamics". Tiampo et al. [2002] applied this analysis on seismicity data in southern California and argued for the development of more sophisticated computer simulations to carry out a more systematic analysis. Our goal is to provide a tool for generating physics-based catalogs of geophysical fields arising only from fault-fault interactions across entire fault networks. Utilizing these catalogs, future studies carrying out a detailed pattern dynamics analysis may help reveal characteristic patterns associated with high seismic risk and help guide fault monitoring methods.

3.1 Gravity Green's Functions

Following Hayes et al. [2006] we have added a more generalized method for modeling co-seismic gravity changes for simulated earthquakes on arbitrary faults with Virtual Quake [Schultz et al., 2014]. We utilize a custom version of the gravity Green's functions presented in Okubo [1992] for faulting in an elastic half-space. Though VQ does not yet support a three dimensional layered earth, these gravity Green's functions have been extended for a dislocations in a three dimensional earth and are presented in Sun et al. [2009]. These equations have shown good agreement with observed co-seismic gravity signals observed by the GRACE satellite for the 2011 Tohoku-Oki earthquake [Matsuo and Heki, 2011] and for the 2010 Central Chile earthquake [Heki and Matsuo, 2010].

We have implemented a custom version of Okubo's gravity equations in Virtual Quake that allow modeling of gravity changes for arbitrary fault geometry and slip. For a detailed explanation of the Virtual Quake implementation of the Okubo's equations see Schultz et al. [2014].

Figure 1 shows the co-seismic gravity pattern produced by Virtual California for a M=7.69 simulated earthquake rupturing the northern San Andreas Fault. This is a model of the total gravity signal at the surface including contribution from surface displacement and from compression and dilatation. The dark black

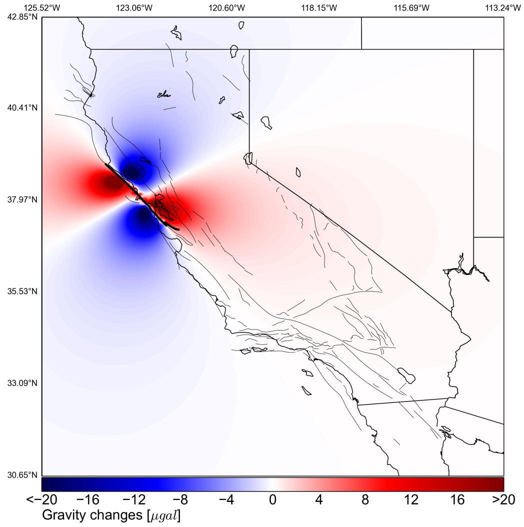

Fig. 1 Gravity Green's function solutions for co-seismic slips from a magnitude 7.69 VC simulated earthquake on the San Andreas Fault. The color unit is microGals ($10^{-8}\text{m}/s^2$), fault rupture is shown in dark black.

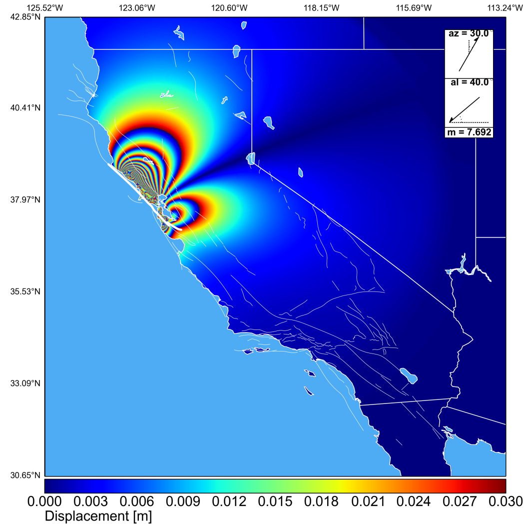

Fig. 2 Simulated InSAR interferogram given co-seismic slips at the surface from a magnitude 7.69 VC simulated earthquake on the San Andreas Fault. View angles: azimuth = 30°, elevation = 40°, fault rupture is shown with bold white lines.

line indicates the fault sections that slipped during the simulated event. Our model is a single layer elastic half-space model and it does not account for viscoelasticity, elastic discontinuities, or the three dimensional layered earth.

3.2 Displacement Green's Functions and InSAR

Virtual Quake employs a custom version of the displacement Green's functions given in Okada [1992] to produce simulated co-seismic deformation maps and InSAR interferograms. Using simulated co-seismic slip distributions and the Okada Green's functions, Virtual Quake can generate entire catalogues of InSAR interferograms and surface deformation maps for arbitrary observing angles. Figure 2 shows the modeled InSAR interferogram for the same $M = 7.69$ simulated earthquake shown in Figure 1, as seen by observing with an azimuthal angle of 30° and elevation angle of 40°.

4 Virtual California Forecasts

Van Aalsburg et al. [2007] and Van Aalsburg et al. [2010] first studied the feasibility of computing earthquake probabilities with an early version of Virtual California and compared the forecasting method with forecasts produced by the Working Group on California Earthquake Probabilities. Yikimaz et al. [2011] made a detailed comparison of observed northern California seismicity and simulated seismicity by adding background seismicity to an early Virtual California simulation.

Here we derive the probabilities for a northern California earthquake scenario using a recent 50,000 year Virtual California simulation of the allcal2 California fault system model. The fault model, which is partitioned into more than 14,000 $3km \times 3km$ square boundary elements, is described in detail in Tullis et al. [2012b]. We follow Rundle et al. [2005] and illustrate our approach using the Virtual California simulation to obtain recurrence statistics and construct the probability distributions. We then compare earthquake probabilities generated by Virtual California to an independent forecasting method.

4.1 Weibull distribution

A probability distribution that is used frequently for earthquake recurrence statistics is the Weibull distribution [Rundle et al., 2005]. Yakovlev et al. [2006] and Abaimov et al. [2008] showed that the Weibull distribution fit early Virtual California simulations of the San Andreas Fault. The Weibull distribution specifies the fraction of recurrence times $P(t)$ that are less than t, and is expressed as

$$P(t) = 1 - \exp\left[-\left(\frac{t}{\tau}\right)^{\beta}\right], \tag{1}$$

where β and τ are fitting parameters [Sieh et al., 1989, Sornette and Knopoff, 1997]. The Weibull distribution is also extended to a cumulative conditional distribution

$$P(t, t_0) = 1 - \exp\left[\left(\frac{t_0}{\tau}\right)^{\beta} - \left(\frac{t}{\tau}\right)^{\beta}\right]. \tag{2}$$

Equation 2 defines the cumulative conditional probability that an earthquake will have occurred at time t

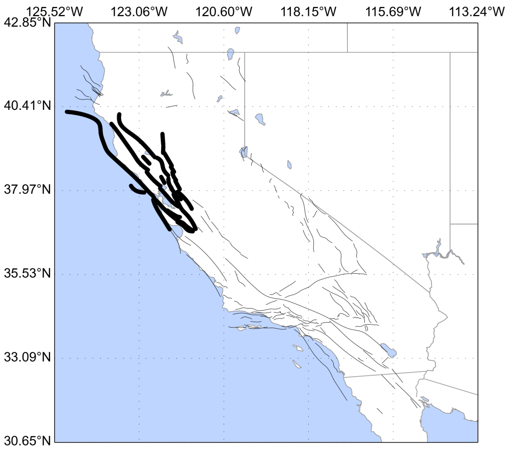

Fig. 3 Virtual California fault sections targeted for the northern California forecast, including the northern San Andreas fault.

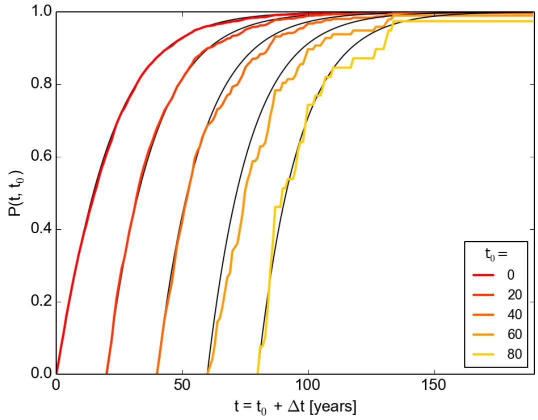

Fig. 4 Conditional cumulative probability $P(t, t_0)$ of a magnitude ≥ 7.0 earthquake on the forecasted faults (figure 3). The distribution is evaluated at $t = t_0 + \Delta t$, with the last earthquake occurring t_0 years ago, computed for various t_0. The red line shows the Weibull distribution fit to the data with $\beta = 1.089$ and $\tau = 20.92$ years.

after the last earthquake given that the last earthquake occurred a time t_0 years ago.

4.2 Northern California Forecast M $\geq$ 7.0

The Virtual California simulation data that are fundamental in generating earthquake probabilities are the recurrence times, defined as the time between successive large earthquakes. Following the methods outlined in Rundle et al. [2005], we compute the cumulative probability distributions and waiting times until the next earthquake on a selected subset of faults directly from a set of simulated recurrence times.

We consider earthquakes on the major faults in northern California, including the northern San Andreas and surrounding faults, shown in Figure 3. Over the 50,000 year simulation there are 2463 earthquakes with moment magnitudes $M \geq 7.0$ that cause slip on at least one of these faults. These earthquakes have an average recurrence time of 20.2 years, with a maximum of 190.4 years. From these data we construct the distribution of recurrence times t, defined as the time between successive earthquakes on the selected faults with $M \geq 7.0$. The cumulative conditional probability of an $M \geq 7.0$ earthquake occurring on the forecasted faults $P(t, t_0)$ at a time t given that the last $M \geq 7.0$ earthquake on these faults occurred t_0 years ago is plotted in Figure 4 for multiple values of t_0. The conditional Weibull distribution (2) is fit to the simulation-derived distribution and is shown as the smooth black curves in Figure 4, with $\beta = 1.089$ and $\tau = 20.92$ years.

In addition to the distribution of recurrence times, we compute the distribution of waiting times Δt until the next large earthquake. The waiting time Δt is measured forward from the present, such that $t = t_0 + \Delta t$. We express our results in terms of the conditional cumulative probability $P(t, t_0)$ that an earthquake will occur in the waiting time $\Delta t = t - t_0$, given that the last major earthquake occurred t_0 years ago, these distributions are shown in Figure 5 for waiting times with 25%, 50%, and 75% probability.

If we define the last observed earthquake with $M \geq 7.0$ in northern California to be the magnitude 7.0 Loma Prieta earthquake in 1989, then that determines $t_0 = 2015 - 1989 = 26$ years, shown as the vertical dashed line in Figure 5. The relatively constant waiting times as a function of t_0 combined with the fitted Weibull parameter $\beta = 1.089$ being nearly 1 implies that the $M \geq 7.0$ earthquakes on faults shown in Figure 3 are occurring randomly. A Weibull distribution with $\beta \sim 1$ behaves like a Poisson process with the time to the next event independent of previous events [Sornette and Knopoff, 1997]. This behavior is expected as the subset of faults is weakly correlated and contains multiple major faults capable of $M \geq 7.0$ earthquakes, each with its own recurrence interval.

4.3 Comparing to Independent Method

We have selected to use the earthquake forecasting website OpenHazards.com to compute earthquake probabilities and compare with our Virtual California forecast. The website computes the probabilities of earthquakes of various magnitudes within a selected region by employing the natural time Weibull method (NTW). This model uses small earthquakes to forecast the occurrence of large earthquakes. The basic idea of their method is to compute large earthquake probabilities using the number of small earthquakes (updated daily from the ANSS catalog) that have occurred in a region since the last large earthquake. This forecasting method

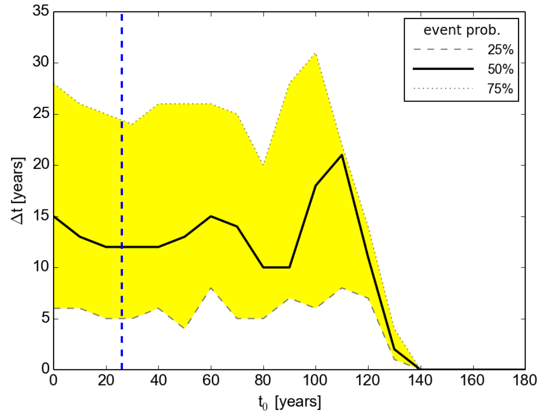

Fig. 5 The waiting times for the next magnitude ≥ 7.0 earthquake on the northern California faults, as a function of time since the last earthquake, t_0. The dark line indicates the median waiting time (50% probability), and the upper and lower edges of the yellow band represent the waiting times with 75% and 25% probability respectively. The vertical dashed line denotes $t_0 = 26$ years, the time elapsed since the M=7.0 Loma Prieta earthquake in 1989.

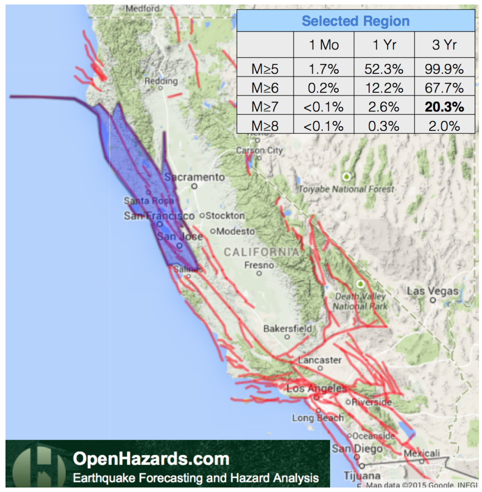

Fig. 6 The Northern California forecast as computed by OpenHazards (openhazards.com/viewer) for the region selected in blue; VC faults are in red. This forecast states there is a 20.3% chance of a $M \geq 7.0$ earthquake within 3 years, compared to the VC computed probability of 14.2%. OpenHazards data last accessed Jan. 7, 2015.

is described in detail in Rundle et al. [2012] and Holliday et al. [2014].

Figure 6 shows the Northern California region selected for comparison to the Virtual California forecast, with the same fault model used in the VC simulation shown in red. The forecast generated from their Hazards Viewer — www.openhazards.com/viewer — is completely independent from the physics governing the Virtual California simulator and completely independent from how VC computes probabilities. Taking the VC conditional probabilities from Figure 4, we find there is a 14.2% chance of a $M \geq 7.0$ earthquake occurring on the faults shown in Figure 3 in the next 3 years. OpenHazards computes a 20.3% probability for a $M \geq 7.0$ earthquake occurring anywhere within the blue region shown in Figure 6 within the next 3 years.

5 Conclusions and Discussion

Here we have introduced and highlighted a few applications of Virtual Quake. We have shown that Virtual Quake can produce earthquake probabilities that are fairly consistent with a more sophisticated time-dependent forecasting method. We have also illustrated how Virtual Quake can be used to generate catalogs of thousands of maps of geophysical observables like coseismic gravity changes and InSAR interferograms.

Tullis et al. [2012b] provided a concise overview of today's earthquake simulators, including Virtual Quake. Tullis et al. [2012a] compared the output of Virtual Quake to observed seismicity and to other earthquake simulators. These papers provide very general validation of the Virtual Quake method and statistics, but there are still many questions that Virtual Quake can answer. How does the inclusion of an Epidemic Type Aftershock (ETAS) model affect fault interactions and earthquake statistics? How would analytically-derived values of the stress drop for faults affect earthquake statistics compared to those used in the Tullis et al. [2012a] study (derived by tuning the stress drops to match observed recurrence intervals using one particular simulator)? How sensitive are simulation-derived conditional probabilities to the model parameterization? What percentage of simulated earthquakes have slip that jumps between adjacent faults, and what are the statistics of these jumps? These are but a few examples of the research pathways afforded by the Virtual Quake simulator.

Acknowledgements This research was supported by National Aeronautics and Space Administration (NASA) Earth and Space Science fellowship number NNX11AL92H. The release version of Virtual California and the Users' Manual are hosted by the Computational Infrastructure for Geodynamics which is supported by NSF Grant EAR-0949446.

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