1. Prove constructively: $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$

2. Prove constructively: $B \to A \vdash (A \to C) \to (B \to C)$

3. Prove constructively: $A \to \neg B \vdash B \to \neg A$

4. Prove constructively: $\neg\neg\neg A \vdash \neg A$

5. Prove constructively: $A \to \neg A \vdash \neg A$

6. Prove classically: $\neg A \to A \vdash A$

7. Prove classically: $\neg(\neg A \wedge \neg B) \vdash A \vee B$

8. Prove constructively: $A \vee B \vdash \neg(\neg A \wedge \neg B)$

9. Note that the problems 7 and 8 are converses of each other. How does this tell us that $\vee$Introduction cannot be used as the last step of the proof of $\neg(\neg A \wedge \neg B) \vdash A \vee B$?

10. Use propositional logic to prove this set-theoretic inclusion: If $A \subseteq C$ and $B \subseteq C$ then $A \cup B \subseteq C$?

11. Use propositional logic to prove this set-theoretic identity:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

12. State, in your own words, the difference in meanings of $\Gamma \vdash A$ vs. $\Gamma \vDash A$, where Γ is a set of statements and A is a statement.

13. Using a truth table, determine whether or not

$$(A \to B) \to C \vDash (A \wedge B) \to C.$$

14. Show that, in classical logic,

$$\Gamma \vdash A \text{ if, and only if, } \Gamma \cup \{\neg A\} \vdash \bot$$

15. Show that, in classical logic,

$$\Gamma \vDash A \text{ if, and only if, } \Gamma \cup \{\neg A\} \vDash \bot$$