Notice When you submit this assignment, you are certifying therewith that you understand and accept the following policy, which applies to all assignments.

Collaboration Policy The writeup that you submit must be your own work. You are encouraged to get help from the professor and grutors. You may discuss the problems with classmates, but if you do so, it should be in groups of no more than three. You are not allowed to copy or transcribe solutions from other sources, including the work of other students, the internet, previous solution sets, and images photographed from a whiteboard or blackboard. There is to be no "partnering" where two or more students submit the same writeup. If you get help on a problem, you should say who provided the help on a per-problem basis. Blanket statements such as "worked with John and Mary" are not allowed. Detected infractions may impact your academic career.

Formatting Policy All work must be typeset in electronic media and submitted as a single pdf file, one problem on each page as shown in the following pages. Retain this header page. Handwritten and photographed or scanned work is not allowed. Do not use inverse video (light typography on dark background). Do not rotate images. You will not get credit for difficult-to-read submissions.

For JAPE proofs take a screenshot of your proof and paste it into the document. For written out proofs, just type into a copy of the Google doc master (not the pdf), or use some other method such as LaTeX if you must. You may scrape formulas and symbols directly from the doc master, or from my page full of symbols that I use throughout the course. Once your document is complete, make a pdf and submit to Gradescope.

Please help us grade by placing your solutions in the space provided.

Translate the following English statements into predicate logic, assuming L is a binary predicate representing Likes on a domain of people, A is a unary predicate representing whether its argument is an Athlete, and M is a unary predicate representing whether its argument is a Musician. When we speak of first, second, third person, etc. it is possible that any or all could be the same person as well; they do not have to be distinct.

[3 points] 1. Everyone likes some athlete and some musician.

[3 points] 2. There is someone who likes a musician who likes every athlete.

[3 points] 3. For every pair of people, if the first likes the second, then the second likes the first. (This is called the symmetric property.)

[3 points] 4. For every pair of people, if the first likes the second, then the second does not like the first. (This is called the antisymmetric property.)

[3 points] 5. For any three people, if the first likes the second, and the second likes the third, then the first likes the third. (This is called the transitive property.)

Translate the following predicate logic statements into english, with the same assumptions about L, A, and M as previous. Try to make your statements simple and concise.

[3 points] 6. $\forall x\, \exists y\, (L(x, y) \land \neg L(y, x))$

[3 points] 7. $\exists x\, (M(x) \land \forall y\, (A(y) \to \neg L(x, y)))$

[3 points] 8. $\forall x\, \forall y\, (L(x, y) \to \forall z\, (L(x, z) \to L(y, z)))$

[3 points] 9. $\forall x\, \forall y\, (L(x, y) \to \exists z\, (L(x, z) \land L(y, z)))$

[3 points] 10. $\exists x\, (M(x) \to \forall y\, M(y))$

Present an informal argument that the following is valid (evaluates to true) for any interpretation.

[5 points] 11. $(\exists x\, A(x)) \rightarrow ((\forall x\, M(x)) \rightarrow (\exists x\, M(x)))$

Present an informal argument that the following is valid (evaluates to true) for any interpretation.

[5 points] 12. $(\forall x\, M(x)) \vee (\exists x\, \neg M(x))$

Give an interpretation for which the following is not valid. An interpretation with a small domain is preferred. In this counterexample, you should show the domain and interpretation of each predicate explicitly, then argue that the formula evaluates to false (0).

[5 points] 13. $(\forall x\, M(x)) \vee \neg\exists x\, M(x)$

Give an interpretation for which the following is not valid. An interpretation with a small domain is preferred. In this counterexample, you should show the domain and interpretation of each predicate explicitly, then argue that the formula evaluates to false (0).

[5 points] 14. $(\exists x\, A(x)) \to \forall y\, A(y)$

Give an interpretation for which the following is not valid. An interpretation with a small domain is preferred. In this counterexample, you should show the domain and interpretation of each predicate explicitly, then argue that the formula evaluates to false (0).

[10 points] 15. $((\forall x\, A(x)) \to (\forall x\, M(x))) \to \forall x\, (A(x) \to M(x))$

Using JAPE, provide a proof of the following sequent. JAPE requires periods after quantified variables. We include them so that you can scrape the formula into JAPE directly.

[15 points] 16. $(\forall x.(A(x) \to B(x))) \to (\exists x.A(x)) \to \exists x.B(x)$ [constructive proof required.]

Using JAPE, provide a proof of the following sequent. JAPE requires periods after quantified variables. We include them so that you can scrape the formula into JAPE directly.

[10 points] 17. $(\forall x.A(x)) \vee \exists x.\neg A(x)$

[This will be a classical proof, somewhat analogous to LEM.]

Using JAPE, provide a proof of the following sequent. JAPE requires periods after quantified variables. We include them so that you can scrape the formula into JAPE directly.

[15 points] 18. $(\exists x.A(x)) \rightarrow \exists x.(\neg A(x) \vee \forall y.A(y))$

[Classical and tricky. I recommend using problem 17 as a lemma.]