Notice When you submit this assignment, you are certifying therewith that you understand and accept the following policy, which applies to all assignments.

Collaboration Policy The writeup that you submit must be your own work. You are encouraged to get help from the professor and grutors. You may discuss the problems with classmates, but if you do so, it should be in groups of no more than three. You are not allowed to copy or transcribe solutions from other sources, including the work of other students, the internet, previous solution sets, and images photographed from a whiteboard or blackboard. There is to be no "partnering" where two or more students submit the same writeup. If you get help on a problem, you should say who provided the help on a per-problem basis. Blanket statements such as "worked with John and Mary" are not allowed. Detected infractions may impact your academic career.

Formatting Policy All work must be typeset in electronic media and submitted as a single pdf file, one problem on each page as shown in the following pages. Retain this header page. Handwritten and photographed or scanned work is not allowed.

Do not use inverse video (light typography on dark background). Do not rotate images. You will not get credit for difficult-to-read submissions.

For JAPE proofs take a screenshot of your proof and paste it into the document. For written out proofs, just type into a copy of the Google doc master (not the pdf), or use some other method such as LaTex if you must. You may scrape formulas and symbols directly from the doc master, or from my page full of symbols that I use throughout the course. Once your document is complete, make a pdf and submitted to Gradescope.

[5 points] 1. Show that a constructive proof of $\neg A \to A \vdash A$ is not possible by deriving LEM ($\vdash B \vee \neg B$) from it, using only constructive rules in addition.

[10 points] 2. Use the tableau method to establish whether or not the following is valid. If not valid, derive a counterexample from the tableau.

$$A \to (B \to \neg C), C \vDash \neg A \vee \neg B$$

[10 points] 3. Use the tableau method to establish whether or not the following is valid. If not valid, derive a counterexample from the tableau.

$$(A \to B) \to C \models A \to (B \to C)$$

[10 points] 4. Determine whether the following is valid using a tableau. If not, extract a counterexample from the tableau.

$$\vDash A \vee \neg B \vee (\neg A \wedge \neg C) \vee (B \wedge \neg C) \vee (C \wedge D)$$

In 5–7, use the method described in lecture (and approximately slides 343–363 of "A Multi-Threaded Approach to Logic (Propositional Logic)"), to prove

$$(A \to C) \land (B \to C) \vdash (A \lor B) \to C.$$

Note that this is intended to exercise a specific method.

[10 points] 5. First construct a tableau showing

$$(A \to C) \land (B \to C) \vDash (A \lor B) \to C.$$

[25 points] 6. Having constructed the closed tableau in the previous problem, convert it using the prescribed method to a natural deduction proof of

$$(A \to C) \land (B \to C) \vdash (A \lor B) \to C.$$

(Using JAPE, you will need to prove some of the five lemmas in order to carry out your proof, some of which you did on the previous assignment). Note that in JAPE, if you want to get a pair of $\land$ eliminations to stack left to right for aesthetic reasons, you should choose "$\land$ elimination preserving right" first. Also, you may need to select the conclusion inside a box, in addition to the antecedent, to get a hypothesis to be inside the box.)

Include proofs of any lemmas or derived rules you used. There should be an evident correspondence between the tableau of problem 5 and the natural deduction proof of problem 6.

[10 points]7. Show that there is a constructive proof of

$$(A \to C) \land (B \to C) \vdash (A \lor B) \to C.$$

[20 points] 8. The Sequent Calculus is a method similar to the tableau method, but which does not explicitly negate the conclusion. It constructs a proof by working backward from the desired sequent, constructing a tree. One difference is that, in the sequent calculus, there can be multiple statements on both sides of the turnstile: On the left-hand side, statements have their usual conjunctive meaning.

On the right-hand side, statements have a disjunctive meaning. For example, $A \to B \vdash \neg A, B$ means that from $A \to B$ prove either $\neg A$ or B. In the sequent calculus, one sequent is replaced with another, until we get to a point where the same statement exists on both sides of $\vdash$ (a closed path) or until we can make no further replacements (an open path). In some cases, branching takes place. The original sequent is proved if all paths can be made to close. The rules for sequent calculus deal with each connective as it occurs on the left and on the right. Here Γ and Δ represent arbitrary sets of statements.

Closure Rule $\Gamma \cup \{A\} \vdash \Delta \cup \{A\}$ closes

$\wedge$ **Left Rule** $\Gamma \cup \{A \wedge B\} \vdash \Delta$ is replaced with $\Gamma \cup \{A, B\} \vdash \Delta$

$\wedge$ **Right Rule** $\Gamma \vdash \Delta \cup \{A \wedge B\}$ splits into $\Gamma \vdash \Delta \cup \{A\}$ and $\Gamma \vdash \Delta \cup \{B\}$

$\vee$ **Left Rule** $\Gamma \cup \{A \vee B\} \vdash \Delta$ splits into $\Gamma \cup \{A\} \vdash \Delta$ and $\Gamma \cup \{B\} \vdash \Delta$

$\vee$ **Right Rule** $\Gamma \vdash \Delta \cup \{A \vee B\}$ is replaced with $\Gamma \vdash \Delta \cup \{A, B\}$

$\to$ **Left Rule** $\Gamma \cup \{A \to B\} \vdash \Delta$ splits into $\Gamma \vdash \Delta \cup \{A\}$ and $\Gamma \cup \{B\} \vdash \Delta$

$\to$ **Right Rule** $\Gamma \vdash \Delta \cup \{A \to B\}$ is replaced with $\Gamma \cup \{A\} \vdash \Delta \cup \{B\}$

$\neg$ **Left Rule** $\Gamma \cup \{\neg A\} \vdash \Delta$ is replaced $\Gamma \vdash \Delta \cup \{A\}$

$\neg$ **Right Rule** $\Gamma \vdash \Delta \cup \{\neg A\}$ is replaced with $\Gamma \cup \{A\} \vdash \Delta$

Here is an example of a sequent calculus derivation:

1.	$\neg(A \vee B) \vdash (\neg A \wedge \neg B)$		Goal
2.	$\vdash (A \vee B), (\neg A \wedge \neg B)$		$\neg$ Left Rule 1
3.	$\vdash A, B, (\neg A \wedge \neg B)$		$\vee$ Right Rule 2
4.	$\vdash A, B, \neg A$	$\vdash A, B \neg B$	$\wedge$ Right Rule 3
5.	$A \vdash A, B$	$B \vdash A, B$	$\neg$ Right 4
	$\otimes$	$\otimes$	

The problem is to construct a sequent calculus proof of

$$(A \to C) \wedge (B \to C) \vdash (A \wedge B) \to C.$$