Due 15 November

Homework 8

Math 167 / CS 142: Complexity Theory

1. A *tally language* is a language over a one-letter alphabet (that is, a subset of $\{0\}^*$). Show that $L \in \mathbf{P}^A$ for some sparse oracle A if and only if $L \in \mathbf{P}^B$ for some tally language B.

2. A *tree* is a connected and acyclic undirected graph. Define the *tree isomorphism* problem $\text{TI} = \{\langle T, T' \rangle : T \text{ and } T' \text{ are isomorphic trees}\}$. Show that $\text{TI} \in \mathbf{P}$.

3. Consider fully-parenthesized Boolean formulas with logical connectives $\neg$, $\wedge$, $\vee$, and $\oplus$ (NOT, AND, OR, and EXCLUSIVE OR, for example $((x_1 \vee x) \wedge (x_1 \oplus (\neg x_2)))$. Define

   $$\text{FVP} = \{\langle \Phi(x_1, \ldots, x_m), c_1, \ldots, c_m \rangle : \Phi(x_1, \ldots, x_m) \text{ is a fully-parenthesized formula, and } \Phi(c_1, \ldots, c_m) = 1\}.$$

   Show that $\text{FVP} \in \mathbf{L}$.