Due September 26

Homework 3

Math 167 / CS 142: Complexity Theory

1. Problem 7.4.4 of the text.

> Let C be a *class of functions* from nonnegative integers to nonnegative integers. We say that C is closed under *left polynomial composition* of $f(n) \in C$ implies $p(f(n)) = \mathcal{O}(g(n))$ for some $g(n) \in C$, for all polynomials $p(n)$. We say that C is closed under *right polynomial composition* if $f(n) \in C$ implies $f(p(n)) = \mathcal{O}(g(n))$ for some $g(n) \in C$, for all polynomials $p(n)$.
>
> Intuitively, the first closure property implies that the corresponding complexity class is "computational model-independent", that is, it is robust under reasonable changes in the underlying model of computation (from RAM's to Turing machines, to multistring Turing machines, etc.) while closure under right polynomial composition suggests closure under *reductions* (see the next chapter).
>
> Which of the following classes of functions are closed under left polynomial composition, and which under right polynomial composition?
>
> (a) $\{n^k : k > 0\}$.
>
> (b) $\{k \cdot n : k > 0\}$.
>
> (c) $\{k^n : k > 0\}$.
>
> (d) $\{2^{n^k} : k > 0\}$.
>
> (e) $\{\log^k n : k > 0\}$.
>
> (f) $\{\log n : k > 0\}$.

(a)

(b)

(c)

(d)

(e)

(f)

2. Problem 7.4.6 of the text.

> Define the *Kleene star* of a language L to be $L^* = \{x_1 \ldots x_k : k \geq 0; x_1, \ldots, x_k \in L\}$ notice that our notation $\Sigma*$ is compatible with this definition). Show that $\mathbf{NP}$ is closed under Kleene star. Repeat for $\mathbf{P}$. (This last one is a little less obvious.)

3. Problem 7.4.7 of the text.

> Show that $\mathbf{NP} \neq \mathbf{SPACE}(n)$. (We have no idea if one includes the other, but we know they are different! Obviously, closure under some operation must be used.)