

1. A *tally language* is a language over a one-letter alphabet (that is, a subset of $\{0\}^*$). Show that $L \in \mathbf{P}^A$ for some sparse oracle A if and only if $L \in \mathbf{P}^B$ for some tally language B .
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2. A *tree* is a connected and acyclic undirected graph. Define the *tree isomorphism* problem $\text{TI} = \{\langle T, T' \rangle : T \text{ and } T' \text{ are isomorphic trees}\}$. Show that $\text{TI} \in \mathbf{P}$.
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3. Consider fully-parenthesized Boolean formulas with logical connectives \neg , \wedge , \vee , and \oplus (NOT, AND, OR, and EXCLUSIVE OR, for example $((x_1 \vee x) \wedge (x_1 \oplus (\neg x_2)))$). Define

$$\text{FVP} = \{\langle \Phi(x_1, \dots, x_m), c_1, \dots, c_m \rangle : \Phi(x_1, \dots, x_m) \text{ is a fully-parenthesized formula, and } \Phi(c_1, \dots, c_m) = 1\}.$$

Show that $\text{FVP} \in \mathbf{L}$.
