Due September 19

Homework 2

Math 167 / CS 142: Complexity Theory

1. Suppose that a Boolean circuit with n inputs has a AND- and OR-gates and b NOT-gates. Show that the same Boolean function can be computed by a circuit with $2a$ AND- and OR-gates and n NOT-gates.

2. Construct a Boolean circuit that has three inputs x, y, and z, and three outputs $\mathrm{NOT}(x)$, $\mathrm{NOT}(y)$, and $\mathrm{NOT}(z)$. You may use any number of AND- and OR-gates but only *two* NOT-gates.

3. Problem 4.4.13 of the text.

> A *monotone* Boolean function F is one that has the following property: If one of the inputs changes from **false** to **true**, the value of the function cannot change from **true** to **false**. Show that F is monotone if and only if it can be expressed as a circuit with only AND and OR gates.