

1. Suppose that a Boolean circuit with  $n$  inputs has  $a$  AND- and OR-gates and  $b$  NOT-gates. Show that the same Boolean function can be computed by a circuit with  $2a$  AND- and OR-gates and  $n$  NOT-gates.

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2. Construct a Boolean circuit that has three inputs  $x$ ,  $y$ , and  $z$ , and three outputs  $\text{NOT}(x)$ ,  $\text{NOT}(y)$ , and  $\text{NOT}(z)$ . You may use any number of AND- and OR-gates but only *two* NOT-gates.

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3. Problem 4.4.13 of the text.

A *monotone* Boolean function  $F$  is one that has the following property: If one of the inputs changes from **false** to **true**, the value of the function cannot change from **true** to **false**. Show that  $F$  is monotone if and only if it can be expressed as a circuit with only AND and OR gates.

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