Homework 9

Math 167 / CS 142: Complexity Theory

1. Problem 14.5.10 of the text. (Hint: Do part (b) first.)

   > Define a *robust* oracle machine $M^?$ deciding language L to be one such that $L(M^A) = L$ for all oracles A. That is, the answers are always correct, independently of the oracle (although the number of steps may vary from oracle to oracle). If furthermore M^A works in polynomial time, we say that oracle A *helps* the robust machine $M^?$. Let $\mathbf{P}_\mathrm{h}$ be the class of languages decidable in polynomial time by deterministic robust oracle machines that can be helped; and $\mathbf{NP}_\mathrm{h}$ for nondeterministic machines.
   >
   > (a) Show that $\mathbf{P}_\mathrm{h} = \mathbf{NP} \cap \mathbf{coNP}$.
   >
   > (b) Show that $\mathbf{NP}_\mathrm{h} = \mathbf{NP}$.

   (a)

   (b)

2. Reduce the computation of the parity $x_1 \oplus \cdots \oplus x_n$ of n Boolean variables to the multiplication of two n^2-bit factors (represented in binary) to form a $2n^2$-bit product, by setting some of the bits of the factors to constants in such a way that the parity of the remaining bits emerges as one of the bits of the product.

3. Show how the n Boolean functions $y_1 = x_1, y_2 = x_1 \vee x_2, \ldots, y_n = x_1 \vee x_2 \vee \cdots \vee x_n$ of the n Boolean variables $x_1, x_2, \ldots, x_n$ can be computed by an unbounded fan-in circuit of NOT-, AND- and OR-gates of size $\mathcal{O}(n)$ and depth $\mathcal{O}(1)$. (Hint: Partition the n inputs into about $\sqrt{n}$ blocks of about $\sqrt{n}$ inputs each.)