The topic of my thesis is *puzzle generation*. When we talk about complexity problems & algorithms, we mostly focus on *solving* problems (i.e., "given so-and-so setup, find so-and-so satisfying some conditions"). But what does it take to *make* a good puzzle? How hard is it to generate a good Sudoku puzzle? What does *good* even mean? Are there general ways to generate arbitrary, NP-complete/NP-hard "flavored" puzzles?

Annotated bibliography

- Laura A. Sanchis and Mark A. Fulk. "On the Efficient Generation of Language Instances". In: *SIAM Journal on Computing* 19.2 (1990), pp. 281–296. DOI: `10.1137/0219019`. URL: `https://epubs.siam.org/doi/abs/10.1137/0219019`

    This paper introduces the simplest formal model for efficient puzzle generators. A *polynomial time constructor* (PTC) for a language L is a deterministic program that, on input 1^n, runs in polynomial-time and returns a string in L with length n iff one exists. *Polynomial time generators* (PTGs) is the nondeterministic analog of PTCs, with the additional requirement that every string in L of length n must be reachable. PTCs and PTGs could be thought of as programs that produce solvable puzzles of given sizes (e.g., given n, generate a solvable $n^2 \times n^2$ Sudoku board).

    The main question explored here is: which classes of languages (puzzles/problems) have PTCs and PTGs? Relevant results include:

    - Every language that has a PTG is in NP.
    - For any language L in NP, L has a PTC iff it has a PTG.
    - Every P language has a PTG iff every NP language has a PTG.

    Surprisingly enough, that last result indicates that we don't know whether every P language has a PTG. This paper goes on to define various special types of PTGs (e.g., categorical, lexicographical, etc.) and establishes various connections between PTG-existence questions and polynomial-hierarchy relations.

- Laura A. Sanchis. "On the complexity of test case generation for NP-hard problems". In: *Information Processing Letters* 36 (3 Nov. 1, 1990), pp. 135–140. DOI: `10.1016/0020-0190(90)90082-9`. URL: `https://www.sciencedirect.com/science/article/pii/0020019090900829`

    In this paper, Sanchis generalizes the notion of puzzle generators introduced in Sanchis and Fulk ("On the Efficient Generation of Language Instances"). Define a *Test Instance Construction Method* (TICM) with respect to some fixed problem Π as a non-deterministic, polynomial-time program that, given a *desired* answer α (along with some desired parameters on the input, e.g., length), attempts to return an instance/input of Π that has answer α and which meets the target parameters.

    The key result from this paper is that, unless NP = co-NP, most NP-hard problems do *not* have efficient TICMs that can generate all input instances (with given known answers). This establishes theoretical bounds on how comprehensive we can reasonably expect a puzzle generator to be in its coverage of available/possible inputs.

- Laura A. Sanchis. "Generating hard and diverse test sets for NP-hard graph problems". In: *Discrete Applied Mathematics* 58 (1 Mar. 10, 1995), pp. 35–66. DOI: `10.1016/0166-218X(93)E0140-T`. URL: `https://www.sciencedirect.com/science/article/pii/0166218X93E0140T`

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