Homework 1

Physics 116: Quantum Mechanics — Spring 2020

Exercise (Townsend 1.3, 5 pts)**.** *In Problem 3.2 we will see that the state of a spin-$\frac{1}{2}$ particle that is spin up along the axis whose direction is specified by the unit vector in spherical coordinates*

$$\boldsymbol{n} = \sin\theta\cos\phi\,\boldsymbol{i} + \sin\theta\sin\phi\,\boldsymbol{j} + \cos\theta\,\boldsymbol{k}$$

with θ and ϕ being the familiar spherical coordinates shown in Fig. 1.11, is given by

$$|{+\boldsymbol{n}}\rangle = \cos\frac{\theta}{2}\,|{+\boldsymbol{z}}\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|{-\boldsymbol{z}}\rangle.$$

(a) Verify that the state $|{+\boldsymbol{n}}\rangle$ reduces to the states $|{+\boldsymbol{x}}\rangle$ and $|{+\boldsymbol{y}}\rangle$ given in the chapter for the appropriate choice of the angles θ and ϕ.

(b) Suppose that a measurement of S_z is carried out on a particle in the state $|{+\boldsymbol{n}}\rangle$. What is the probability that the measuremnet yields (i) $\hbar/2$? (ii) $-\hbar/2$?

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Exercise (Townsend 1.6+, 5 pts). *Show that the state*

$$|-\boldsymbol{n}\rangle = \sin\frac{\theta}{2}\,|+\boldsymbol{z}\rangle - e^{i\phi}\cos\frac{\theta}{2}\,|-\boldsymbol{z}\rangle$$

(a) *really does correspond to* $|+\boldsymbol{n}\rangle$ *with* $\boldsymbol{n}$ *pointing in the opposite direction;*

(b) *is normalized such that* $\langle -\boldsymbol{n}| - \boldsymbol{n}| = 1$*;*

(c) *is orthogonal to* $|+\boldsymbol{n}\rangle$*.*

(d) *Show that as* 3*-vectors,* $(-\boldsymbol{n}) \cdot \boldsymbol{n} = -1$ *.*

Note that (d) is consistent with having the quantum states be orthogonal: $\langle -n| + n| = 0$*. The vector* $\boldsymbol{n}$ *lives in physical 3D space whereas* $|\boldsymbol{n}\rangle$ *lives in a 2D abstract "Hilbert" space. The states* $|\boldsymbol{n}\rangle$ *and* $|-\boldsymbol{n}\rangle$ *are orthogonal because they give two different, definite, and mutually exclusive results when their spin is measured along the* $\boldsymbol{n}$ *direction.*

Hint for part (a): How shsould you change the polar angle to get a vector pointing in the opposite direction?
