

# Ковариантные производные тензорного поля, заданного в цилиндрических координатах.

ФИО группа

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## Условие:

Задано тензорное поле  $T(X^i)$ , где  $X^i$  - цилиндрические координаты. Найти:

- 1) ковариантные, контравариантные компоненты этого поля в базисах  $r_i, r^i$ , где  $r_i$  — ортогональный локальный базис цилиндрической системы координат;
- 2) ковариантные производные компонент тензорного поля в базисах  $r_i, r^i$

## Исходные данные:

Поле задано в виде:  $T = T^i j e_i \otimes e_j$ ;

$$T^{ij} = \begin{pmatrix} 0 & (X^2) + (X^1) & 0 \\ 2 \cdot (X^3) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

## Решение:

$r_i$  - ортогональный локальный базис цилиндрической системы координат:

$$\begin{cases} x^1 = X^1 \cdot \cos(X^2) \\ x^2 = X^1 \cdot \sin(X^2) \\ x^3 = X^3 \end{cases}$$

1) Для того, чтобы найти компоненты тензорного поля в базисах  $r_i, r^i$ , найдем сначала метрическую матрицу для цилиндрических координат  $X^i$  и локальные векторы базиса.

1.1) Найдем якобиеву матрицу для криволинейных координат  $X^i$ :

$$Q_k^i = \frac{\partial x^i}{\partial X^k}$$

$$\begin{array}{lll} Q_1^1 = \frac{\partial x^1}{\partial X^1} = \cos((X^2)) & Q_2^1 = \frac{\partial x^1}{\partial X^2} = -(X^1) \cdot \sin((X^2)) & Q_3^1 = \frac{\partial x^1}{\partial X^3} = 0 \\ Q_1^2 = \frac{\partial x^2}{\partial X^1} = \sin((X^2)) & Q_2^2 = \frac{\partial x^2}{\partial X^2} = (X^1) \cdot \cos((X^2)) & Q_3^2 = \frac{\partial x^2}{\partial X^3} = 0 \\ Q_1^3 = \frac{\partial x^3}{\partial X^1} = 0 & Q_2^3 = \frac{\partial x^3}{\partial X^2} = 0 & Q_3^3 = \frac{\partial x^3}{\partial X^3} = 1 \end{array}$$

Тогда якобиева матрица цилиндрической системы координат имеет вид:

$$Q_k^i = \begin{pmatrix} \cos((X^2)) & -(X^1) \cdot \sin((X^2)) & 0 \\ \sin((X^2)) & (X^1) \cdot \cos((X^2)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.2) Найдем локальные векторы базиса для цилиндрических координат  $X^i$ :

$$\mathbf{r}_k = \frac{\partial x^i}{\partial X^k} \overline{\mathbf{e}}_k = Q_k^i \overline{\mathbf{e}}_k$$

Матрица  $Q_k^i$  была найдена на предыдущем шаге.

1.3) Найдем метрическую матрицу для цилиндрических координат  $X^i$ :

$$g_{ij} = \mathbf{r}_i \cdot \mathbf{r}_j = Q_i^s Q_j^p \delta_{sp}$$

Найдем компоненты метрической матрицы для цилиндрической системы координат:

$$\begin{aligned} g_{11} &= Q_1^s \cdot Q_1^p \cdot \delta_{sp} = 1 \\ g_{12} &= Q_1^s \cdot Q_2^p \cdot \delta_{sp} = 0 \\ g_{13} &= Q_1^s \cdot Q_3^p \cdot \delta_{sp} = 0 \\ g_{21} &= Q_2^s \cdot Q_1^p \cdot \delta_{sp} = 0 \\ g_{22} &= Q_2^s \cdot Q_2^p \cdot \delta_{sp} = (X^1)^2 \\ g_{23} &= Q_2^s \cdot Q_3^p \cdot \delta_{sp} = 0 \\ g_{31} &= Q_3^s \cdot Q_1^p \cdot \delta_{sp} = 0 \\ g_{32} &= Q_3^s \cdot Q_2^p \cdot \delta_{sp} = 0 \\ g_{33} &= Q_3^s \cdot Q_3^p \cdot \delta_{sp} = 1 \end{aligned}$$

Запишем полученную метрическую матрицу для цилиндрической системы координат:

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (X^1)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Найдем обратную метрическую матрицу:

$$g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{(X^1)^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.4) Найдем векторы взаимного локального базиса для цилиндрических координат  $X^i$ :

$$\mathbf{r}^i = g^{ij} \mathbf{r}_j = g^{ij} Q_j^m \overline{\mathbf{e}}_m = Q^{im} \overline{\mathbf{e}}_m :$$

$$\begin{aligned}
Q^{11} &= g^{1j} Q_j^1 = g^{11} \cdot Q_1^1 + g^{12} \cdot Q_2^1 = g^{13} \cdot Q_3^1 = \cos((X^2)) + 0 + 0 = \cos((X^2)) \\
Q^{12} &= g^{1j} Q_j^2 = g^{11} \cdot Q_1^2 + g^{12} \cdot Q_2^2 = g^{13} \cdot Q_3^2 = \sin((X^2)) + 0 + 0 = \sin((X^2)) \\
Q^{13} &= g^{1j} Q_j^3 = g^{11} \cdot Q_1^3 + g^{12} \cdot Q_2^3 = g^{13} \cdot Q_3^3 = 0 + 0 + 0 = 0 \\
Q^{21} &= g^{2j} Q_j^1 = g^{21} \cdot Q_1^1 + g^{22} \cdot Q_2^1 = g^{23} \cdot Q_3^1 = 0 + -\sin((X^2))/(X^1) + 0 = -\sin((X^2))/(X^1) \\
Q^{22} &= g^{2j} Q_j^2 = g^{21} \cdot Q_1^2 + g^{22} \cdot Q_2^2 = g^{23} \cdot Q_3^2 = 0 + \cos((X^2))/(X^1) + 0 = \cos((X^2))/(X^1) \\
Q^{23} &= g^{2j} Q_j^3 = g^{21} \cdot Q_1^3 + g^{22} \cdot Q_2^3 = g^{23} \cdot Q_3^3 = 0 + 0 + 0 = 0 \\
Q^{31} &= g^{3j} Q_j^1 = g^{31} \cdot Q_1^1 + g^{32} \cdot Q_2^1 = g^{33} \cdot Q_3^1 = 0 + 0 + 0 = 0 \\
Q^{32} &= g^{3j} Q_j^2 = g^{31} \cdot Q_1^2 + g^{32} \cdot Q_2^2 = g^{33} \cdot Q_3^2 = 0 + 0 + 0 = 0 \\
Q^{33} &= g^{3j} Q_j^3 = g^{31} \cdot Q_1^3 + g^{32} \cdot Q_2^3 = g^{33} \cdot Q_3^3 = 0 + 0 + 1 = 1
\end{aligned}$$

Запишем полученную матрицу:

$$Q^{im} = \begin{pmatrix} \cos((X^2)) & \sin((X^2)) & 0 \\ -\sin((X^2))/(X^1) & \cos((X^2))/(X^1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.5) Найдем теперь компоненты тензорного поля.

По условию

$$\begin{aligned}
T^{11} &= 0 & T^{12} &= (X^2) + (X^1) & T^{13} &= 0 \\
T^{21} &= 2 \cdot (X^3) & T^{22} &= 0 & T^{23} &= 0 \\
T^{31} &= 0 & T^{32} &= 0 & T^{33} &= 0
\end{aligned}$$

Вычислим ковариантные компоненты по формуле:

$$T_{ij} = T^{kl} g_{ki} g_{lj};$$

$$\begin{aligned}
T_{11} &= T^{kl} g_{k1} g_{l1} = 0 \\
T_{12} &= T^{kl} g_{k1} g_{l2} = (X^1)^2 \cdot ((X^2) + (X^1)) \\
T_{13} &= T^{kl} g_{k1} g_{l3} = 0 \\
T_{21} &= T^{kl} g_{k2} g_{l1} = 2 \cdot (X^3) \cdot (X^1)^2 \\
T_{22} &= T^{kl} g_{k2} g_{l2} = 0 \\
T_{23} &= T^{kl} g_{k2} g_{l3} = 0 \\
T_{31} &= T^{kl} g_{k3} g_{l1} = 0 \\
T_{32} &= T^{kl} g_{k3} g_{l2} = 0 \\
T_{33} &= T^{kl} g_{k3} g_{l3} = 0
\end{aligned}$$

Запишем полученную матрицу:

$$T_{ij} = \begin{pmatrix} 0 & (X^1)^2 \cdot ((X^2) + (X^1)) & 0 \\ 2 \cdot (X^3) \cdot (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_j^i = T^{ik} g_{kj};$$

$$\begin{aligned} T_1^1 &= T^{13} g_{k1} = 0 \\ T_2^1 &= T^{13} g_{k2} = (X^1)^2 \cdot ((X^2) + (X^1)) \\ T_3^1 &= T^{13} g_{k3} = 0 \\ T_1^2 &= T^{23} g_{k1} = 2 \cdot (X^3) \\ T_2^2 &= T^{23} g_{k2} = 0 \\ T_3^2 &= T^{23} g_{k3} = 0 \\ T_1^3 &= T^{33} g_{k1} = 0 \\ T_2^3 &= T^{33} g_{k2} = 0 \\ T_3^3 &= T^{33} g_{k3} = 0 \end{aligned}$$

Запишем полученную матрицу:

$$T_j^i = \begin{pmatrix} 0 & (X^1)^2 \cdot ((X^2) + (X^1)) & 0 \\ 2 \cdot (X^3) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_i^j = T^{kj} g_{ki};$$

$$\begin{aligned} T_1^1 &= T^{31} g_{k1} = 0 \\ T_1^2 &= T^{32} g_{k1} = (X^2) + (X^1) \\ T_1^3 &= T^{33} g_{k1} = 0 \\ T_2^1 &= T^{31} g_{k2} = 2 \cdot (X^3) \cdot (X^1)^2 \\ T_2^2 &= T^{32} g_{k2} = 0 \\ T_2^3 &= T^{33} g_{k2} = 0 \\ T_3^1 &= T^{31} g_{k3} = 0 \\ T_3^2 &= T^{32} g_{k3} = 0 \\ T_3^3 &= T^{33} g_{k3} = 0 \end{aligned}$$

Запишем полученную матрицу:

$$T_i^j = \begin{pmatrix} 0 & (X^2) + (X^1) & 0 \\ 2 \cdot (X^3) \cdot (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Найдем символы Кристоффеля по формуле:

$$\Gamma_{ij}^m = \frac{1}{2} g^{km} \left( \frac{\partial g_{kj}}{\partial X^i} + \frac{\partial g_{ik}}{\partial X^j} - \frac{\partial g_{ij}}{\partial X^k} \right) :$$

При  $m = 1$ :

$$\begin{aligned} \Gamma_{11}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k1}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^1} - \frac{\partial g_{11}}{\partial X^k} \right) = 0 \\ \Gamma_{12}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k2}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^2} - \frac{\partial g_{12}}{\partial X^k} \right) = 0 \\ \Gamma_{13}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k3}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^3} - \frac{\partial g_{13}}{\partial X^k} \right) = 0 \\ \Gamma_{21}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k1}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^1} - \frac{\partial g_{21}}{\partial X^k} \right) = 0 \\ \Gamma_{22}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k2}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^2} - \frac{\partial g_{22}}{\partial X^k} \right) = -1.0 \cdot (X^1) \\ \Gamma_{23}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k3}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^3} - \frac{\partial g_{23}}{\partial X^k} \right) = 0 \\ \Gamma_{31}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k1}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^1} - \frac{\partial g_{31}}{\partial X^k} \right) = 0 \\ \Gamma_{32}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k2}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^2} - \frac{\partial g_{32}}{\partial X^k} \right) = 0 \\ \Gamma_{33}^1 &= \frac{1}{2} g^{k1} \left( \frac{\partial g_{k3}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^3} - \frac{\partial g_{33}}{\partial X^k} \right) = 0 \end{aligned}$$

При  $m = 2$ :

$$\begin{aligned}
\Gamma_{11}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k1}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^1} - \frac{\partial g_{11}}{\partial X^k}\right) = 0 \\
\Gamma_{12}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k2}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^2} - \frac{\partial g_{12}}{\partial X^k}\right) = 1.0/(X^1) \\
\Gamma_{13}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k3}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^3} - \frac{\partial g_{13}}{\partial X^k}\right) = 0 \\
\Gamma_{21}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k1}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^1} - \frac{\partial g_{21}}{\partial X^k}\right) = 1.0/(X^1) \\
\Gamma_{22}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k2}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^2} - \frac{\partial g_{22}}{\partial X^k}\right) = 0 \\
\Gamma_{23}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k3}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^3} - \frac{\partial g_{23}}{\partial X^k}\right) = 0 \\
\Gamma_{31}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k1}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^1} - \frac{\partial g_{31}}{\partial X^k}\right) = 0 \\
\Gamma_{32}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k2}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^2} - \frac{\partial g_{32}}{\partial X^k}\right) = 0 \\
\Gamma_{33}^2 &= \frac{1}{2}g^{k2}\left(\frac{\partial g_{k3}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^3} - \frac{\partial g_{33}}{\partial X^k}\right) = 0
\end{aligned}$$

При  $m = 3$ :

$$\begin{aligned}
\Gamma_{11}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k1}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^1} - \frac{\partial g_{11}}{\partial X^k}\right) = 0 \\
\Gamma_{12}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k2}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^2} - \frac{\partial g_{12}}{\partial X^k}\right) = 0 \\
\Gamma_{13}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k3}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^3} - \frac{\partial g_{13}}{\partial X^k}\right) = 0 \\
\Gamma_{21}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k1}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^1} - \frac{\partial g_{21}}{\partial X^k}\right) = 0 \\
\Gamma_{22}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k2}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^2} - \frac{\partial g_{22}}{\partial X^k}\right) = 0 \\
\Gamma_{23}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k3}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^3} - \frac{\partial g_{23}}{\partial X^k}\right) = 0 \\
\Gamma_{31}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k1}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^1} - \frac{\partial g_{31}}{\partial X^k}\right) = 0 \\
\Gamma_{32}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k2}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^2} - \frac{\partial g_{32}}{\partial X^k}\right) = 0 \\
\Gamma_{33}^3 &= \frac{1}{2}g^{k3}\left(\frac{\partial g_{k3}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^3} - \frac{\partial g_{33}}{\partial X^k}\right) = 0
\end{aligned}$$

Запишем результат:

$$\Gamma_{ij}^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1.0 \cdot (X^1) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma_{ij}^2 = \begin{pmatrix} 0 & 1.0/(X^1) & 0 \\ 1.0/(X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma_{ij}^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.1) Вычислим ковариантную производную контравариантных компонент поля по формуле:

$$\nabla_k T^{ij} = \frac{\partial T^{ij}}{\partial X^k} + T^{mj} \Gamma_{mk}^i + T^{im} \Gamma_{mk}^j;$$

При  $k = 1$ :

$$\begin{aligned} \nabla_1 T^{11} &= \frac{\partial T^{11}}{\partial X^1} + T^{m1} \Gamma_{m1}^1 + T^{1m} \Gamma_{m1}^1 = 0 \\ \nabla_1 T^{12} &= \frac{\partial T^{12}}{\partial X^1} + T^{m2} \Gamma_{m1}^1 + T^{1m} \Gamma_{m1}^2 = 1.0 \cdot (X^2)/(X^1) + 2.0 \\ \nabla_1 T^{13} &= \frac{\partial T^{13}}{\partial X^1} + T^{m3} \Gamma_{m1}^1 + T^{1m} \Gamma_{m1}^3 = 0 \\ \nabla_1 T^{21} &= \frac{\partial T^{21}}{\partial X^1} + T^{m1} \Gamma_{m1}^2 + T^{2m} \Gamma_{m1}^1 = 2.0 \cdot (X^3)/(X^1) \\ \nabla_1 T^{22} &= \frac{\partial T^{22}}{\partial X^1} + T^{m2} \Gamma_{m1}^2 + T^{2m} \Gamma_{m1}^2 = 0 \\ \nabla_1 T^{23} &= \frac{\partial T^{23}}{\partial X^1} + T^{m3} \Gamma_{m1}^2 + T^{2m} \Gamma_{m1}^3 = 0 \\ \nabla_1 T^{31} &= \frac{\partial T^{31}}{\partial X^1} + T^{m1} \Gamma_{m1}^3 + T^{3m} \Gamma_{m1}^1 = 0 \\ \nabla_1 T^{32} &= \frac{\partial T^{32}}{\partial X^1} + T^{m2} \Gamma_{m1}^3 + T^{3m} \Gamma_{m1}^2 = 0 \\ \nabla_1 T^{33} &= \frac{\partial T^{33}}{\partial X^1} + T^{m3} \Gamma_{m1}^3 + T^{3m} \Gamma_{m1}^3 = 0 \end{aligned}$$

При  $k = 2$ :

$$\begin{aligned}
\nabla_2 T^{11} &= \frac{\partial T^{11}}{\partial X^2} + T^{m1} \Gamma_{m2}^1 + T^{1m} \Gamma_{m2}^1 = (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) \\
\nabla_2 T^{12} &= \frac{\partial T^{12}}{\partial X^2} + T^{m2} \Gamma_{m2}^1 + T^{1m} \Gamma_{m2}^2 = 1 \\
\nabla_2 T^{13} &= \frac{\partial T^{13}}{\partial X^2} + T^{m3} \Gamma_{m2}^1 + T^{1m} \Gamma_{m2}^3 = 0 \\
\nabla_2 T^{21} &= \frac{\partial T^{21}}{\partial X^2} + T^{m1} \Gamma_{m2}^2 + T^{2m} \Gamma_{m2}^1 = 0 \\
\nabla_2 T^{22} &= \frac{\partial T^{22}}{\partial X^2} + T^{m2} \Gamma_{m2}^2 + T^{2m} \Gamma_{m2}^2 = (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) / (X^1) \\
\nabla_2 T^{23} &= \frac{\partial T^{23}}{\partial X^2} + T^{m3} \Gamma_{m2}^2 + T^{2m} \Gamma_{m2}^3 = 0 \\
\nabla_2 T^{31} &= \frac{\partial T^{31}}{\partial X^2} + T^{m1} \Gamma_{m2}^3 + T^{3m} \Gamma_{m2}^1 = 0 \\
\nabla_2 T^{32} &= \frac{\partial T^{32}}{\partial X^2} + T^{m2} \Gamma_{m2}^3 + T^{3m} \Gamma_{m2}^2 = 0 \\
\nabla_2 T^{33} &= \frac{\partial T^{33}}{\partial X^2} + T^{m3} \Gamma_{m2}^3 + T^{3m} \Gamma_{m2}^3 = 0
\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla_3 T^{11} &= \frac{\partial T^{11}}{\partial X^3} + T^{m1} \Gamma_{m3}^1 + T^{1m} \Gamma_{m3}^1 = 0 \\
\nabla_3 T^{12} &= \frac{\partial T^{12}}{\partial X^3} + T^{m2} \Gamma_{m3}^1 + T^{1m} \Gamma_{m3}^2 = 0 \\
\nabla_3 T^{13} &= \frac{\partial T^{13}}{\partial X^3} + T^{m3} \Gamma_{m3}^1 + T^{1m} \Gamma_{m3}^3 = 0 \\
\nabla_3 T^{21} &= \frac{\partial T^{21}}{\partial X^3} + T^{m1} \Gamma_{m3}^2 + T^{2m} \Gamma_{m3}^1 = 2 \\
\nabla_3 T^{22} &= \frac{\partial T^{22}}{\partial X^3} + T^{m2} \Gamma_{m3}^2 + T^{2m} \Gamma_{m3}^2 = 0 \\
\nabla_3 T^{23} &= \frac{\partial T^{23}}{\partial X^3} + T^{m3} \Gamma_{m3}^2 + T^{2m} \Gamma_{m3}^3 = 0 \\
\nabla_3 T^{31} &= \frac{\partial T^{31}}{\partial X^3} + T^{m1} \Gamma_{m3}^3 + T^{3m} \Gamma_{m3}^1 = 0 \\
\nabla_3 T^{32} &= \frac{\partial T^{32}}{\partial X^3} + T^{m2} \Gamma_{m3}^3 + T^{3m} \Gamma_{m3}^2 = 0 \\
\nabla_3 T^{33} &= \frac{\partial T^{33}}{\partial X^3} + T^{m3} \Gamma_{m3}^3 + T^{3m} \Gamma_{m3}^3 = 0
\end{aligned}$$

Запишем результат:

$$\nabla_1 T^{ij} = \begin{pmatrix} 0 & 1.0 \cdot (X^2) / (X^1) + 2.0 & 0 \\ 2.0 \cdot (X^3) / (X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\nabla_2 T^{ij} = \begin{pmatrix} (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) & 1 & 0 \\ 0 & (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1))/(X^1) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_3 T^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.2) Вычислим ковариантную производную ковариантных компонент поля по формуле:

$$\nabla_k T_{ij} = \frac{\partial T_{ij}}{\partial X^k} - T_{mj} \Gamma_{ik}^m - T_{im} \Gamma_{jk}^m;$$

При k = 1:

$$\begin{aligned} \nabla_1 T_{11} &= \frac{\partial T_{11}}{\partial X^1} - T_{m1} \Gamma_{11}^m - T_{1m} \Gamma_{11}^m = 0 \\ \nabla_1 T_{12} &= \frac{\partial T_{12}}{\partial X^1} - T_{m2} \Gamma_{11}^m - T_{1m} \Gamma_{21}^m = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) \\ \nabla_1 T_{13} &= \frac{\partial T_{13}}{\partial X^1} - T_{m3} \Gamma_{11}^m - T_{1m} \Gamma_{31}^m = 0 \\ \nabla_1 T_{21} &= \frac{\partial T_{21}}{\partial X^1} - T_{m1} \Gamma_{21}^m - T_{2m} \Gamma_{11}^m = 2.0 \cdot (X^3) \cdot (X^1) \\ \nabla_1 T_{22} &= \frac{\partial T_{22}}{\partial X^1} - T_{m2} \Gamma_{21}^m - T_{2m} \Gamma_{21}^m = 0 \\ \nabla_1 T_{23} &= \frac{\partial T_{23}}{\partial X^1} - T_{m3} \Gamma_{21}^m - T_{2m} \Gamma_{31}^m = 0 \\ \nabla_1 T_{31} &= \frac{\partial T_{31}}{\partial X^1} - T_{m1} \Gamma_{31}^m - T_{3m} \Gamma_{11}^m = 0 \\ \nabla_1 T_{32} &= \frac{\partial T_{32}}{\partial X^1} - T_{m2} \Gamma_{31}^m - T_{3m} \Gamma_{21}^m = 0 \\ \nabla_1 T_{33} &= \frac{\partial T_{33}}{\partial X^1} - T_{m3} \Gamma_{31}^m - T_{3m} \Gamma_{31}^m = 0 \end{aligned}$$

При k = 2:

$$\begin{aligned}
\nabla_2 T_{11} &= \frac{\partial T_{11}}{\partial X^2} - T_{m1} \Gamma_{12}^m - T_{1m} \Gamma_{12}^m = (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) \\
\nabla_2 T_{12} &= \frac{\partial T_{12}}{\partial X^2} - T_{m2} \Gamma_{12}^m - T_{1m} \Gamma_{22}^m = (X^1)^2 \\
\nabla_2 T_{13} &= \frac{\partial T_{13}}{\partial X^2} - T_{m3} \Gamma_{12}^m - T_{1m} \Gamma_{32}^m = 0 \\
\nabla_2 T_{21} &= \frac{\partial T_{21}}{\partial X^2} - T_{m1} \Gamma_{22}^m - T_{2m} \Gamma_{12}^m = 0 \\
\nabla_2 T_{22} &= \frac{\partial T_{22}}{\partial X^2} - T_{m2} \Gamma_{22}^m - T_{2m} \Gamma_{22}^m = (X^1)^3 \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) \\
\nabla_2 T_{23} &= \frac{\partial T_{23}}{\partial X^2} - T_{m3} \Gamma_{22}^m - T_{2m} \Gamma_{32}^m = 0 \\
\nabla_2 T_{31} &= \frac{\partial T_{31}}{\partial X^2} - T_{m1} \Gamma_{32}^m - T_{3m} \Gamma_{12}^m = 0 \\
\nabla_2 T_{32} &= \frac{\partial T_{32}}{\partial X^2} - T_{m2} \Gamma_{32}^m - T_{3m} \Gamma_{22}^m = 0 \\
\nabla_2 T_{33} &= \frac{\partial T_{33}}{\partial X^2} - T_{m3} \Gamma_{32}^m - T_{3m} \Gamma_{32}^m = 0
\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla_3 T_{11} &= \frac{\partial T_{11}}{\partial X^3} - T_{m1} \Gamma_{13}^m - T_{1m} \Gamma_{13}^m = 0 \\
\nabla_3 T_{12} &= \frac{\partial T_{12}}{\partial X^3} - T_{m2} \Gamma_{13}^m - T_{1m} \Gamma_{23}^m = 0 \\
\nabla_3 T_{13} &= \frac{\partial T_{13}}{\partial X^3} - T_{m3} \Gamma_{13}^m - T_{1m} \Gamma_{33}^m = 0 \\
\nabla_3 T_{21} &= \frac{\partial T_{21}}{\partial X^3} - T_{m1} \Gamma_{23}^m - T_{2m} \Gamma_{13}^m = 2 \cdot (X^1)^2 \\
\nabla_3 T_{22} &= \frac{\partial T_{22}}{\partial X^3} - T_{m2} \Gamma_{23}^m - T_{2m} \Gamma_{23}^m = 0 \\
\nabla_3 T_{23} &= \frac{\partial T_{23}}{\partial X^3} - T_{m3} \Gamma_{23}^m - T_{2m} \Gamma_{33}^m = 0 \\
\nabla_3 T_{31} &= \frac{\partial T_{31}}{\partial X^3} - T_{m1} \Gamma_{33}^m - T_{3m} \Gamma_{13}^m = 0 \\
\nabla_3 T_{32} &= \frac{\partial T_{32}}{\partial X^3} - T_{m2} \Gamma_{33}^m - T_{3m} \Gamma_{23}^m = 0 \\
\nabla_3 T_{33} &= \frac{\partial T_{33}}{\partial X^3} - T_{m3} \Gamma_{33}^m - T_{3m} \Gamma_{33}^m = 0
\end{aligned}$$

Запишем результат:

$$\nabla_1 T_{ij} = \begin{pmatrix} 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) & 0 \\ 2.0 \cdot (X^3) \cdot (X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T_{ij} = \begin{pmatrix} (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) & (X^1)^2 & 0 \\ 0 & (X^1)^3 \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_3 T_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 2 \cdot (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.3) Вычислим ковариантную производную смешанных компонент поля по формуле:

$$\nabla_k T_j^i = \frac{\partial T_j^i}{\partial X^k} + T_j^m \Gamma_{mk}^i - T_m^i \Gamma_{jk}^m,$$

При k = 1:

$$\begin{aligned} \nabla_1 T_1^1 &= \frac{\partial T_1^1}{\partial X^1} + T_1^m \Gamma_{m1}^1 - T_m^1 \Gamma_{11}^m = 0 \\ \nabla_1 T_2^1 &= \frac{\partial T_2^1}{\partial X^1} + T_2^m \Gamma_{m1}^1 - T_m^1 \Gamma_{21}^m = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) \\ \nabla_1 T_3^1 &= \frac{\partial T_3^1}{\partial X^1} + T_3^m \Gamma_{m1}^1 - T_m^1 \Gamma_{31}^m = 0 \\ \nabla_1 T_1^2 &= \frac{\partial T_1^2}{\partial X^1} + T_1^m \Gamma_{m1}^2 - T_m^2 \Gamma_{11}^m = 2.0 \cdot (X^3)/(X^1) \\ \nabla_1 T_2^2 &= \frac{\partial T_2^2}{\partial X^1} + T_2^m \Gamma_{m1}^2 - T_m^2 \Gamma_{21}^m = 0 \\ \nabla_1 T_3^2 &= \frac{\partial T_3^2}{\partial X^1} + T_3^m \Gamma_{m1}^2 - T_m^2 \Gamma_{31}^m = 0 \\ \nabla_1 T_1^3 &= \frac{\partial T_1^3}{\partial X^1} + T_1^m \Gamma_{m1}^3 - T_m^3 \Gamma_{11}^m = 0 \\ \nabla_1 T_2^3 &= \frac{\partial T_2^3}{\partial X^1} + T_2^m \Gamma_{m1}^3 - T_m^3 \Gamma_{21}^m = 0 \\ \nabla_1 T_3^3 &= \frac{\partial T_3^3}{\partial X^1} + T_3^m \Gamma_{m1}^3 - T_m^3 \Gamma_{31}^m = 0 \end{aligned}$$

При k = 2:

$$\begin{aligned}
\nabla_2 T_1^1 &= \frac{\partial T_1^1}{\partial X^2} + T_1^m \Gamma_{m2}^1 - T_m^1 \Gamma_{12}^m = (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) \\
\nabla_2 T_2^1 &= \frac{\partial T_2^1}{\partial X^2} + T_2^m \Gamma_{m2}^1 - T_m^1 \Gamma_{22}^m = (X^1)^2 \\
\nabla_2 T_3^1 &= \frac{\partial T_3^1}{\partial X^2} + T_3^m \Gamma_{m2}^1 - T_m^1 \Gamma_{32}^m = 0 \\
\nabla_2 T_1^2 &= \frac{\partial T_1^2}{\partial X^2} + T_1^m \Gamma_{m2}^2 - T_m^2 \Gamma_{12}^m = 0 \\
\nabla_2 T_2^2 &= \frac{\partial T_2^2}{\partial X^2} + T_2^m \Gamma_{m2}^2 - T_m^2 \Gamma_{22}^m = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) \\
\nabla_2 T_3^2 &= \frac{\partial T_3^2}{\partial X^2} + T_3^m \Gamma_{m2}^2 - T_m^2 \Gamma_{32}^m = 0 \\
\nabla_2 T_1^3 &= \frac{\partial T_1^3}{\partial X^2} + T_1^m \Gamma_{m2}^3 - T_m^3 \Gamma_{12}^m = 0 \\
\nabla_2 T_2^3 &= \frac{\partial T_2^3}{\partial X^2} + T_2^m \Gamma_{m2}^3 - T_m^3 \Gamma_{22}^m = 0 \\
\nabla_2 T_3^3 &= \frac{\partial T_3^3}{\partial X^2} + T_3^m \Gamma_{m2}^3 - T_m^3 \Gamma_{32}^m = 0
\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla_3 T_1^1 &= \frac{\partial T_1^1}{\partial X^3} + T_1^m \Gamma_{m3}^1 - T_m^1 \Gamma_{13}^m = 0 \\
\nabla_3 T_2^1 &= \frac{\partial T_2^1}{\partial X^3} + T_2^m \Gamma_{m3}^1 - T_m^1 \Gamma_{23}^m = 0 \\
\nabla_3 T_3^1 &= \frac{\partial T_3^1}{\partial X^3} + T_3^m \Gamma_{m3}^1 - T_m^1 \Gamma_{33}^m = 0 \\
\nabla_3 T_1^2 &= \frac{\partial T_1^2}{\partial X^3} + T_1^m \Gamma_{m3}^2 - T_m^2 \Gamma_{13}^m = 2 \\
\nabla_3 T_2^2 &= \frac{\partial T_2^2}{\partial X^3} + T_2^m \Gamma_{m3}^2 - T_m^2 \Gamma_{23}^m = 0 \\
\nabla_3 T_3^2 &= \frac{\partial T_3^2}{\partial X^3} + T_3^m \Gamma_{m3}^2 - T_m^2 \Gamma_{33}^m = 0 \\
\nabla_3 T_1^3 &= \frac{\partial T_1^3}{\partial X^3} + T_1^m \Gamma_{m3}^3 - T_m^3 \Gamma_{13}^m = 0 \\
\nabla_3 T_2^3 &= \frac{\partial T_2^3}{\partial X^3} + T_2^m \Gamma_{m3}^3 - T_m^3 \Gamma_{23}^m = 0 \\
\nabla_3 T_3^3 &= \frac{\partial T_3^3}{\partial X^3} + T_3^m \Gamma_{m3}^3 - T_m^3 \Gamma_{33}^m = 0
\end{aligned}$$

Запишем результат:

$$\nabla_1 T_j^i = \begin{pmatrix} 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) & 0 \\ 2.0 \cdot (X^3)/(X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T_j^i = \begin{pmatrix} (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) & (X^1)^2 & 0 \\ 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_3 T_j^i = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.4) Вычислим ковариантную производную смешанных компонент поля по формуле:

$$\nabla_k T_i^j = \frac{\partial T_i^j}{\partial X^k} - T_m^j \Gamma_{ik}^m + T_i^m \Gamma_{mk}^j;$$

При k = 1:

$$\begin{aligned} \nabla_1 T_1^1 &= \frac{\partial T_1^1}{\partial X^1} - T_m^1 \Gamma_{11}^m + T_1^m \Gamma_{m1}^1 = 0 \\ \nabla_1 T_1^2 &= \frac{\partial T_1^2}{\partial X^1} - T_m^2 \Gamma_{11}^m + T_1^m \Gamma_{m1}^2 = 1.0 \cdot (X^2)/(X^1) + 2.0 \\ \nabla_1 T_1^3 &= \frac{\partial T_1^3}{\partial X^1} - T_m^3 \Gamma_{11}^m + T_1^m \Gamma_{m1}^3 = 0 \\ \nabla_1 T_2^1 &= \frac{\partial T_2^1}{\partial X^1} - T_m^1 \Gamma_{21}^m + T_2^m \Gamma_{m1}^1 = 2.0 \cdot (X^3) \cdot (X^1) \\ \nabla_1 T_2^2 &= \frac{\partial T_2^2}{\partial X^1} - T_m^2 \Gamma_{21}^m + T_2^m \Gamma_{m1}^2 = 0 \\ \nabla_1 T_2^3 &= \frac{\partial T_2^3}{\partial X^1} - T_m^3 \Gamma_{21}^m + T_2^m \Gamma_{m1}^3 = 0 \\ \nabla_1 T_3^1 &= \frac{\partial T_3^1}{\partial X^1} - T_m^1 \Gamma_{31}^m + T_3^m \Gamma_{m1}^1 = 0 \\ \nabla_1 T_3^2 &= \frac{\partial T_3^2}{\partial X^1} - T_m^2 \Gamma_{31}^m + T_3^m \Gamma_{m1}^2 = 0 \\ \nabla_1 T_3^3 &= \frac{\partial T_3^3}{\partial X^1} - T_m^3 \Gamma_{31}^m + T_3^m \Gamma_{m1}^3 = 0 \end{aligned}$$

При k = 2:

$$\begin{aligned}
\nabla_2 T_1^1 &= \frac{\partial T_1^1}{\partial X^2} - T_m^1 \Gamma_{12}^m + T_1^m \Gamma_{m2}^1 = (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) \\
\nabla_2 T_1^2 &= \frac{\partial T_1^2}{\partial X^2} - T_m^2 \Gamma_{12}^m + T_1^m \Gamma_{m2}^2 = 1 \\
\nabla_2 T_1^3 &= \frac{\partial T_1^3}{\partial X^2} - T_m^3 \Gamma_{12}^m + T_1^m \Gamma_{m2}^3 = 0 \\
\nabla_2 T_2^1 &= \frac{\partial T_2^1}{\partial X^2} - T_m^1 \Gamma_{22}^m + T_2^m \Gamma_{m2}^1 = 0 \\
\nabla_2 T_2^2 &= \frac{\partial T_2^2}{\partial X^2} - T_m^2 \Gamma_{22}^m + T_2^m \Gamma_{m2}^2 = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) \\
\nabla_2 T_2^3 &= \frac{\partial T_2^3}{\partial X^2} - T_m^3 \Gamma_{22}^m + T_2^m \Gamma_{m2}^3 = 0 \\
\nabla_2 T_3^1 &= \frac{\partial T_3^1}{\partial X^2} - T_m^1 \Gamma_{32}^m + T_3^m \Gamma_{m2}^1 = 0 \\
\nabla_2 T_3^2 &= \frac{\partial T_3^2}{\partial X^2} - T_m^2 \Gamma_{32}^m + T_3^m \Gamma_{m2}^2 = 0 \\
\nabla_2 T_3^3 &= \frac{\partial T_3^3}{\partial X^2} - T_m^3 \Gamma_{32}^m + T_3^m \Gamma_{m2}^3 = 0
\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla_3 T_1^1 &= \frac{\partial T_1^1}{\partial X^3} - T_m^1 \Gamma_{13}^m + T_1^m \Gamma_{m3}^1 = 0 \\
\nabla_3 T_1^2 &= \frac{\partial T_1^2}{\partial X^3} - T_m^2 \Gamma_{13}^m + T_1^m \Gamma_{m3}^2 = 0 \\
\nabla_3 T_1^3 &= \frac{\partial T_1^3}{\partial X^3} - T_m^3 \Gamma_{13}^m + T_1^m \Gamma_{m3}^3 = 0 \\
\nabla_3 T_2^1 &= \frac{\partial T_2^1}{\partial X^3} - T_m^1 \Gamma_{23}^m + T_2^m \Gamma_{m3}^1 = 2 \cdot (X^1)^2 \\
\nabla_3 T_2^2 &= \frac{\partial T_2^2}{\partial X^3} - T_m^2 \Gamma_{23}^m + T_2^m \Gamma_{m3}^2 = 0 \\
\nabla_3 T_2^3 &= \frac{\partial T_2^3}{\partial X^3} - T_m^3 \Gamma_{23}^m + T_2^m \Gamma_{m3}^3 = 0 \\
\nabla_3 T_3^1 &= \frac{\partial T_3^1}{\partial X^3} - T_m^1 \Gamma_{33}^m + T_3^m \Gamma_{m3}^1 = 0 \\
\nabla_3 T_3^2 &= \frac{\partial T_3^2}{\partial X^3} - T_m^2 \Gamma_{33}^m + T_3^m \Gamma_{m3}^2 = 0 \\
\nabla_3 T_3^3 &= \frac{\partial T_3^3}{\partial X^3} - T_m^3 \Gamma_{33}^m + T_3^m \Gamma_{m3}^3 = 0
\end{aligned}$$

Запишем результат:

$$\nabla_1 T_i^j = \begin{pmatrix} 0 & 1.0 \cdot (X^2)/(X^1) + 2.0 & 0 \\ 2.0 \cdot (X^3) \cdot (X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T_i^j = \begin{pmatrix} (X^1) \cdot (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) & 1 & 0 \\ 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_3 T_i^j = \begin{pmatrix} 0 & 0 & 0 \\ 2 \cdot (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Вычислим контравариантные производные.

2.5) Вычислим контравариантную производную от контравариантных компонент по формуле:

$$\nabla^m T^{ij} = g^{mk} \nabla_k T^{ij};$$

При  $k = 1$ :

$$\begin{aligned} \nabla^1 T^{11} &= g^{1k} \nabla_k T^{11} = 0 \\ \nabla^1 T^{12} &= g^{1k} \nabla_k T^{12} = 1.0 \cdot (X^2)/(X^1) + 2.0 \\ \nabla^1 T^{13} &= g^{1k} \nabla_k T^{13} = 0 \\ \nabla^1 T^{21} &= g^{1k} \nabla_k T^{21} = 2.0 \cdot (X^3)/(X^1) \\ \nabla^1 T^{22} &= g^{1k} \nabla_k T^{22} = 0 \\ \nabla^1 T^{23} &= g^{1k} \nabla_k T^{23} = 0 \\ \nabla^1 T^{31} &= g^{1k} \nabla_k T^{31} = 0 \\ \nabla^1 T^{32} &= g^{1k} \nabla_k T^{32} = 0 \\ \nabla^1 T^{33} &= g^{1k} \nabla_k T^{33} = 0 \end{aligned}$$

При  $k = 2$ :

$$\begin{aligned} \nabla^2 T^{11} &= g^{2k} \nabla_k T^{11} = (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1))/(X^1) \\ \nabla^2 T^{12} &= g^{2k} \nabla_k T^{12} = \frac{1}{(X^1)^2} \\ \nabla^2 T^{13} &= g^{2k} \nabla_k T^{13} = 0 \\ \nabla^2 T^{21} &= g^{2k} \nabla_k T^{21} = 0 \\ \nabla^2 T^{22} &= g^{2k} \nabla_k T^{22} = (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1))/(X^1)^3 \\ \nabla^2 T^{23} &= g^{2k} \nabla_k T^{23} = 0 \\ \nabla^2 T^{31} &= g^{2k} \nabla_k T^{31} = 0 \\ \nabla^2 T^{32} &= g^{2k} \nabla_k T^{32} = 0 \\ \nabla^2 T^{33} &= g^{2k} \nabla_k T^{33} = 0 \end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla^3 T^{11} &= g^{3k} \nabla_k T^{11} = 0 \\
\nabla^3 T^{12} &= g^{3k} \nabla_k T^{12} = 0 \\
\nabla^3 T^{13} &= g^{3k} \nabla_k T^{13} = 0 \\
\nabla^3 T^{21} &= g^{3k} \nabla_k T^{21} = 2 \\
\nabla^3 T^{22} &= g^{3k} \nabla_k T^{22} = 0 \\
\nabla^3 T^{23} &= g^{3k} \nabla_k T^{23} = 0 \\
\nabla^3 T^{31} &= g^{3k} \nabla_k T^{31} = 0 \\
\nabla^3 T^{32} &= g^{3k} \nabla_k T^{32} = 0 \\
\nabla^3 T^{33} &= g^{3k} \nabla_k T^{33} = 0
\end{aligned}$$

Запишем результат:

$$\nabla^1 T^{ij} = \begin{pmatrix} 0 & 1.0 \cdot (X^2)/(X^1) + 2.0 & 0 \\ 2.0 \cdot (X^3)/(X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T^{ij} = \begin{pmatrix} (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1))/(X^1) & \frac{1}{(X^1)^2} & 0 \\ 0 & (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1))/(X^1)^3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^3 T^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.6) Вычислим контравариантную производную от ковариантных компонент по формуле:

$$\nabla^m T_{ij} = g^{mk} \nabla_k T_{ij};$$

При  $k = 1$ :

$$\begin{aligned}
\nabla^1 T_{11} &= g^{1k} \nabla_k T_{11} = 0 \\
\nabla^1 T_{12} &= g^{1k} \nabla_k T_{12} = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) \\
\nabla^1 T_{13} &= g^{1k} \nabla_k T_{13} = 0 \\
\nabla^1 T_{21} &= g^{1k} \nabla_k T_{21} = 2.0 \cdot (X^3) \cdot (X^1) \\
\nabla^1 T_{22} &= g^{1k} \nabla_k T_{22} = 0 \\
\nabla^1 T_{23} &= g^{1k} \nabla_k T_{23} = 0 \\
\nabla^1 T_{31} &= g^{1k} \nabla_k T_{31} = 0 \\
\nabla^1 T_{32} &= g^{1k} \nabla_k T_{32} = 0 \\
\nabla^1 T_{33} &= g^{1k} \nabla_k T_{33} = 0
\end{aligned}$$



При  $k = 2$ :

$$\begin{aligned}
\nabla^2 T_{11} &= g^{2k} \nabla_k T_{11} = (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) / (X^1) \\
\nabla^2 T_{12} &= g^{2k} \nabla_k T_{12} = 1 \\
\nabla^2 T_{13} &= g^{2k} \nabla_k T_{13} = 0 \\
\nabla^2 T_{21} &= g^{2k} \nabla_k T_{21} = 0 \\
\nabla^2 T_{22} &= g^{2k} \nabla_k T_{22} = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) \\
\nabla^2 T_{23} &= g^{2k} \nabla_k T_{23} = 0 \\
\nabla^2 T_{31} &= g^{2k} \nabla_k T_{31} = 0 \\
\nabla^2 T_{32} &= g^{2k} \nabla_k T_{32} = 0 \\
\nabla^2 T_{33} &= g^{2k} \nabla_k T_{33} = 0
\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla^3 T_{11} &= g^{3k} \nabla_k T_{11} = 0 \\
\nabla^3 T_{12} &= g^{3k} \nabla_k T_{12} = 0 \\
\nabla^3 T_{13} &= g^{3k} \nabla_k T_{13} = 0 \\
\nabla^3 T_{21} &= g^{3k} \nabla_k T_{21} = 2 \cdot (X^1)^2 \\
\nabla^3 T_{22} &= g^{3k} \nabla_k T_{22} = 0 \\
\nabla^3 T_{23} &= g^{3k} \nabla_k T_{23} = 0 \\
\nabla^3 T_{31} &= g^{3k} \nabla_k T_{31} = 0 \\
\nabla^3 T_{32} &= g^{3k} \nabla_k T_{32} = 0 \\
\nabla^3 T_{33} &= g^{3k} \nabla_k T_{33} = 0
\end{aligned}$$

Запишем результат:

$$\nabla^1 T_{ij} = \begin{pmatrix} 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) & 0 \\ 2.0 \cdot (X^3) \cdot (X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T_{ij} = \begin{pmatrix} (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) / (X^1) & 1 & 0 \\ 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^3 T_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 2 \cdot (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.7) Аналогично вычислим контравариантную производную от смешанных компонент тензора по формуле:

$$\nabla^m T_j^i = g^{mk} \nabla_k T_j^i;$$

При  $k = 1$ :

$$\begin{aligned}\nabla^1 T_1^1 &= g^{1k} \nabla_k T_1^1 = 0 \\ \nabla^1 T_2^1 &= g^{1k} \nabla_k T_2^1 = (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) \\ \nabla^1 T_3^1 &= g^{1k} \nabla_k T_3^1 = 0 \\ \nabla^1 T_1^2 &= g^{1k} \nabla_k T_1^2 = 2.0 \cdot (X^3)/(X^1) \\ \nabla^1 T_2^2 &= g^{1k} \nabla_k T_2^2 = 0 \\ \nabla^1 T_3^2 &= g^{1k} \nabla_k T_3^2 = 0 \\ \nabla^1 T_1^3 &= g^{1k} \nabla_k T_1^3 = 0 \\ \nabla^1 T_2^3 &= g^{1k} \nabla_k T_2^3 = 0 \\ \nabla^1 T_3^3 &= g^{1k} \nabla_k T_3^3 = 0\end{aligned}$$

При  $k = 2$ :

$$\begin{aligned}\nabla^2 T_1^1 &= g^{2k} \nabla_k T_1^1 = (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1))/(X^1) \\ \nabla^2 T_2^1 &= g^{2k} \nabla_k T_2^1 = 1 \\ \nabla^2 T_3^1 &= g^{2k} \nabla_k T_3^1 = 0 \\ \nabla^2 T_1^2 &= g^{2k} \nabla_k T_1^2 = 0 \\ \nabla^2 T_2^2 &= g^{2k} \nabla_k T_2^2 = (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1))/(X^1) \\ \nabla^2 T_3^2 &= g^{2k} \nabla_k T_3^2 = 0 \\ \nabla^2 T_1^3 &= g^{2k} \nabla_k T_1^3 = 0 \\ \nabla^2 T_2^3 &= g^{2k} \nabla_k T_2^3 = 0 \\ \nabla^2 T_3^3 &= g^{2k} \nabla_k T_3^3 = 0\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}\nabla^3 T_1^1 &= g^{3k} \nabla_k T_1^1 = 0 \\ \nabla^3 T_2^1 &= g^{3k} \nabla_k T_2^1 = 0 \\ \nabla^3 T_3^1 &= g^{3k} \nabla_k T_3^1 = 0 \\ \nabla^3 T_1^2 &= g^{3k} \nabla_k T_1^2 = 2 \\ \nabla^3 T_2^2 &= g^{3k} \nabla_k T_2^2 = 0 \\ \nabla^3 T_3^2 &= g^{3k} \nabla_k T_3^2 = 0 \\ \nabla^3 T_1^3 &= g^{3k} \nabla_k T_1^3 = 0 \\ \nabla^3 T_2^3 &= g^{3k} \nabla_k T_2^3 = 0 \\ \nabla^3 T_3^3 &= g^{3k} \nabla_k T_3^3 = 0\end{aligned}$$

Запишем результат:

$$\nabla^1 T_j^i = \begin{pmatrix} 0 & (X^1) \cdot (1.0 \cdot (X^2) + 2.0 \cdot (X^1)) & 0 \\ 2.0 \cdot (X^3)/(X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T_j^i = \begin{pmatrix} (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1))/(X^1) & 1 & 0 \\ 0 & (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1))/(X^1) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^3 T_j^i = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.8) Аналогично вычислим контравариантную производную от смешанных компонент тензора по формуле:

$$\nabla^m T_i^j = g^{mk} \nabla_k T_i^j;$$

При  $k = 1$ :

$$\begin{aligned} \nabla^1 T_1^1 &= g^{1k} \nabla_k T_1^1 = 0 \\ \nabla^1 T_1^2 &= g^{1k} \nabla_k T_1^2 = 1.0 \cdot (X^2)/(X^1) + 2.0 \\ \nabla^1 T_1^3 &= g^{1k} \nabla_k T_1^3 = 0 \\ \nabla^1 T_2^1 &= g^{1k} \nabla_k T_2^1 = 2.0 \cdot (X^3) \cdot (X^1) \\ \nabla^1 T_2^2 &= g^{1k} \nabla_k T_2^2 = 0 \\ \nabla^1 T_2^3 &= g^{1k} \nabla_k T_2^3 = 0 \\ \nabla^1 T_3^1 &= g^{1k} \nabla_k T_3^1 = 0 \\ \nabla^1 T_3^2 &= g^{1k} \nabla_k T_3^2 = 0 \\ \nabla^1 T_3^3 &= g^{1k} \nabla_k T_3^3 = 0 \end{aligned}$$

При  $k = 2$ :

$$\begin{aligned}
\nabla^2 T_1^1 &= g^{2k} \nabla_k T_1^1 = (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) / (X^1) \\
\nabla^2 T_1^2 &= g^{2k} \nabla_k T_1^2 = \frac{1}{(X^1)^2} \\
\nabla^2 T_1^3 &= g^{2k} \nabla_k T_1^3 = 0 \\
\nabla^2 T_2^1 &= g^{2k} \nabla_k T_2^1 = 0 \\
\nabla^2 T_2^2 &= g^{2k} \nabla_k T_2^2 = (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) / (X^1) \\
\nabla^2 T_2^3 &= g^{2k} \nabla_k T_2^3 = 0 \\
\nabla^2 T_3^1 &= g^{2k} \nabla_k T_3^1 = 0 \\
\nabla^2 T_3^2 &= g^{2k} \nabla_k T_3^2 = 0 \\
\nabla^2 T_3^3 &= g^{2k} \nabla_k T_3^3 = 0
\end{aligned}$$

При  $k = 3$ :

$$\begin{aligned}
\nabla^3 T_1^1 &= g^{3k} \nabla_k T_1^1 = 0 \\
\nabla^3 T_1^2 &= g^{3k} \nabla_k T_1^2 = 0 \\
\nabla^3 T_1^3 &= g^{3k} \nabla_k T_1^3 = 0 \\
\nabla^3 T_2^1 &= g^{3k} \nabla_k T_2^1 = 2 \cdot (X^1)^2 \\
\nabla^3 T_2^2 &= g^{3k} \nabla_k T_2^2 = 0 \\
\nabla^3 T_2^3 &= g^{3k} \nabla_k T_2^3 = 0 \\
\nabla^3 T_3^1 &= g^{3k} \nabla_k T_3^1 = 0 \\
\nabla^3 T_3^2 &= g^{3k} \nabla_k T_3^2 = 0 \\
\nabla^3 T_3^3 &= g^{3k} \nabla_k T_3^3 = 0
\end{aligned}$$

Запишем результат:

$$\nabla^1 T_i^j = \begin{pmatrix} 0 & 1.0 \cdot (X^2) / (X^1) + 2.0 & 0 \\ 2.0 \cdot (X^3) \cdot (X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T_i^j = \begin{pmatrix} (-1.0 \cdot (X^2) - 2.0 \cdot (X^3) - 1.0 \cdot (X^1)) / (X^1) & \frac{1}{(X^1)^2} & 0 \\ 0 & (1.0 \cdot (X^2) + 2.0 \cdot (X^3) + 1.0 \cdot (X^1)) / (X^1) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^3 T_i^j = \begin{pmatrix} 0 & 0 & 0 \\ 2 \cdot (X^1)^2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$