Ковариантные производные тензорного поля, заданного в цилиндрических координатах.

Очкин Никита Валерьевич ФН11-42Б

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Условие:

Задано тензорное поле $\mathrm{T}(X^i)$, где X^i - цилиндрические координаты. Найти:
1) ковариантные, контравариантные компоненты этого поля в базисах $\mathrm{r_i}$, $\mathrm{r^i}$, где $\mathrm{r_i}$ — ортогональный локальный базис цилиндрической системы координат;
2) ковариантные производные компонент тензорного поля в базисах $\mathrm{r_i}$, $\mathrm{r^i}$

Исходные данные:

Поле задано в виде: $\mathrm{T} = T^i j \mathrm{e_i} \otimes \mathrm{e_j}$;

$$T^{ij} = \begin{pmatrix} 0 & -(X^2) + (X^1) & 0 \\ 0 & 0 & 0 \\ 2 \cdot (X^3) & 0 & 0 \end{pmatrix}$$

Решение:

$\mathrm{r_i}$ - ортогональный локальный базис цилиндрической системы координат:

$$\begin{cases} x^1 = X^1 \cdot \cos(X^2) \\ x^2 = X^1 \cdot \sin(X^2) \\ x^3 = X^3 \end{cases}$$

1) Для того, чтобы найти компоненты тензорного поля в базисах $\mathrm{r_i}$, $\mathrm{r^i}$, найдем сначала метрическую матрицу для цилиндрических координат X^i и локальные векторы базиса.
1.1) Найдем якобиеву матрицу для криволинейных координат X^i:

$$Q^i_k = \frac{\partial x^i}{\partial X^k}$$

$$
\begin{aligned}
&Q^1_1 = \frac{\partial x^1}{\partial X^1} = \cos((X^2)) \quad && Q^1_2 = \frac{\partial x^1}{\partial X^2} = -(X^1)\cdot\sin((X^2)) \quad && Q^1_3 = \frac{\partial x^1}{\partial X^3} = 0\\
&Q^2_1 = \frac{\partial x^2}{\partial X^1} = \sin((X^2)) \quad && Q^2_2 = \frac{\partial x^2}{\partial X^2} = (X^1)\cdot\cos((X^2)) \quad && Q^2_3 = \frac{\partial x^2}{\partial X^3} = 0\\
&Q^3_1 = \frac{\partial x^3}{\partial X^1} = 0 \quad && Q^3_2 = \frac{\partial x^3}{\partial X^2} = 0 \quad && Q^3_3 = \frac{\partial x^3}{\partial X^3} = 1
\end{aligned}
$$

Тогда якобиева матрица цилиндрической системы координат имеет вид:

$$
Q^i_k = \begin{pmatrix} \cos((X^2)) & -(X^1)\cdot\sin((X^2)) & 0\\ \sin((X^2)) & (X^1)\cdot\cos((X^2)) & 0\\ 0 & 0 & 1 \end{pmatrix}
$$

1.2) Найдем локальные векторы базиса для цилиндрических координат X^i:

$$
\mathrm{r_k} = \frac{\partial x^i}{\partial X^k}\overline{\mathrm{e_k}} = Q^i_k\overline{\mathrm{e_k}}
$$

Матрица Q^i_k была найдена на предыдущем шаге.
1.3) Найдем метрическую матрицу для цилиндрических координат X^i:

$$
\mathrm{g_{ij}} = \mathrm{r_i}\cdot\mathrm{r_j} = Q^s_i Q^p_j \delta_{sp}
$$

Найдем компоненты метрической матрицы для цилиндрической системы координат:

$$
\begin{aligned}
\mathrm{g_{11}} &= Q^s_1\cdot Q^p_1\cdot\delta_{sp} = 1\\
\mathrm{g_{12}} &= Q^s_1\cdot Q^p_2\cdot\delta_{sp} = 0\\
\mathrm{g_{13}} &= Q^s_1\cdot Q^p_3\cdot\delta_{sp} = 0\\
\mathrm{g_{21}} &= Q^s_2\cdot Q^p_1\cdot\delta_{sp} = 0\\
\mathrm{g_{22}} &= Q^s_2\cdot Q^p_2\cdot\delta_{sp} = (X^1)^2\\
\mathrm{g_{23}} &= Q^s_2\cdot Q^p_3\cdot\delta_{sp} = 0\\
\mathrm{g_{31}} &= Q^s_3\cdot Q^p_1\cdot\delta_{sp} = 0\\
\mathrm{g_{32}} &= Q^s_3\cdot Q^p_2\cdot\delta_{sp} = 0\\
\mathrm{g_{33}} &= Q^s_3\cdot Q^p_3\cdot\delta_{sp} = 1
\end{aligned}
$$

Запишем полученную метрическую матрицу для цилиндрической системы координат:

$$
g_{ij} = \begin{pmatrix} 1 & 0 & 0\\ 0 & (X^1)^2 & 0\\ 0 & 0 & 1 \end{pmatrix}
$$

Найдем обратную метрическую матрицу:

$$g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{(X^1)^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.4) Найдем векторы взаимного локального базиса для цилиндрических координат X^i:

$$\mathrm{r^i} = \mathrm{g^{ij}} \mathrm{r_j} = \mathrm{g^{ij}} Q^m_j \overline{\mathrm{e_m}} = Q^{im} \overline{\mathrm{e_m}} :$$

$Q^{11} = \mathrm{g^{1j}} Q^1_j = g^{11} \cdot Q^1_1 + g^{12} \cdot Q^1_2 = g^{13} \cdot Q^1_3 = \cos((X^2)) + 0 + 0 = \cos((X^2))$
$Q^{12} = \mathrm{g^{1j}} Q^2_j = g^{11} \cdot Q^2_1 + g^{12} \cdot Q^2_2 = g^{13} \cdot Q^2_3 = \sin((X^2)) + 0 + 0 = \sin((X^2))$
$Q^{13} = \mathrm{g^{1j}} Q^3_j = g^{11} \cdot Q^3_1 + g^{12} \cdot Q^3_2 = g^{13} \cdot Q^3_3 = 0 + 0 + 0 = 0$
$Q^{21} = \mathrm{g^{2j}} Q^1_j = g^{21} \cdot Q^1_1 + g^{22} \cdot Q^1_2 = g^{23} \cdot Q^1_3 = 0 + - \sin((X^2))/(X^1) + 0 = - \sin((X^2))/(X^1)$
$Q^{22} = \mathrm{g^{2j}} Q^2_j = g^{21} \cdot Q^2_1 + g^{22} \cdot Q^2_2 = g^{23} \cdot Q^2_3 = 0 + \cos((X^2))/(X^1) + 0 = \cos((X^2))/(X^1)$
$Q^{23} = \mathrm{g^{2j}} Q^3_j = g^{21} \cdot Q^3_1 + g^{22} \cdot Q^3_2 = g^{23} \cdot Q^3_3 = 0 + 0 + 0 = 0$
$Q^{31} = \mathrm{g^{3j}} Q^1_j = g^{31} \cdot Q^1_1 + g^{32} \cdot Q^1_2 = g^{33} \cdot Q^1_3 = 0 + 0 + 0 = 0$
$Q^{32} = \mathrm{g^{3j}} Q^2_j = g^{31} \cdot Q^2_1 + g^{32} \cdot Q^2_2 = g^{33} \cdot Q^2_3 = 0 + 0 + 0 = 0$
$Q^{33} = \mathrm{g^{3j}} Q^3_j = g^{31} \cdot Q^3_1 + g^{32} \cdot Q^3_2 = g^{33} \cdot Q^3_3 = 0 + 0 + 1 = 1$

Запишем полученную матрицу:

$$Q^{im} = \begin{pmatrix} \cos((X^2)) & \sin((X^2)) & 0 \\ -\sin((X^2))/(X^1) & \cos((X^2))/(X^1) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.5)Найдем теперь компоненты тензорного поля.
По условию

$$\begin{array}{lll} T^{11} = 0 & T^{12} = -(X^2) + (X^1) & T^{13} = 0 \\ T^{21} = 0 & T^{22} = 0 & T^{23} = 0 \\ T^{31} = 2 \cdot (X^3) & T^{32} = 0 & T^{33} = 0 \end{array}$$

Вычислим ковариантные компоненты по формуле:

$$T_{ij} = T^{kl} g_{ki} g_{lj};$$

$$
\begin{aligned}
T_{11} &= T^{kl}\mathrm{g_{k1}g_{l1}} = 0\\
T_{12} &= T^{kl}\mathrm{g_{k1}g_{l2}} = (X^1)^2 \cdot (-(X^2) + (X^1))\\
T_{13} &= T^{kl}\mathrm{g_{k1}g_{l3}} = 0\\
T_{21} &= T^{kl}\mathrm{g_{k2}g_{l1}} = 0\\
T_{22} &= T^{kl}\mathrm{g_{k2}g_{l2}} = 0\\
T_{23} &= T^{kl}\mathrm{g_{k2}g_{l3}} = 0\\
T_{31} &= T^{kl}\mathrm{g_{k3}g_{l1}} = 2 \cdot (X^3)\\
T_{32} &= T^{kl}\mathrm{g_{k3}g_{l2}} = 0\\
T_{33} &= T^{kl}\mathrm{g_{k3}g_{l3}} = 0
\end{aligned}
$$

Запишем полученную матрицу:

$$
T_{ij} = \begin{pmatrix} 0 & (X^1)^2 \cdot (-(X^2) + (X^1)) & 0 \\ 0 & 0 & 0 \\ 2 \cdot (X^3) & 0 & 0 \end{pmatrix}
$$

$$
T^i_j = T^{ik} g_{kj};
$$

$$
\begin{aligned}
T^1_1 &= T^{13}\mathrm{g_{k1}} = 0\\
T^1_2 &= T^{13}\mathrm{g_{k2}} = (X^1)^2 \cdot (-(X^2) + (X^1))\\
T^1_3 &= T^{13}\mathrm{g_{k3}} = 0\\
T^2_1 &= T^{23}\mathrm{g_{k1}} = 0\\
T^2_2 &= T^{23}\mathrm{g_{k2}} = 0\\
T^2_3 &= T^{23}\mathrm{g_{k3}} = 0\\
T^3_1 &= T^{33}\mathrm{g_{k1}} = 2 \cdot (X^3)\\
T^3_2 &= T^{33}\mathrm{g_{k2}} = 0\\
T^3_3 &= T^{33}\mathrm{g_{k3}} = 0
\end{aligned}
$$

Запишем полученную матрицу:

$$
T^i_j = \begin{pmatrix} 0 & (X^1)^2 \cdot (-(X^2) + (X^1)) & 0 \\ 0 & 0 & 0 \\ 2 \cdot (X^3) & 0 & 0 \end{pmatrix}
$$

$$
T^j_i = T^{kj} g_{ki};
$$

$$T_1^1 = T^{31}\mathrm{g_{k1}} = 0$$
$$T_1^2 = T^{32}\mathrm{g_{k1}} = -(X^2) + (X^1)$$
$$T_1^3 = T^{33}\mathrm{g_{k1}} = 0$$
$$T_2^1 = T^{31}\mathrm{g_{k2}} = 0$$
$$T_2^2 = T^{32}\mathrm{g_{k2}} = 0$$
$$T_2^3 = T^{33}\mathrm{g_{k2}} = 0$$
$$T_3^1 = T^{31}\mathrm{g_{k3}} = 2 \cdot (X^3)$$
$$T_3^2 = T^{32}\mathrm{g_{k3}} = 0$$
$$T_3^3 = T^{33}\mathrm{g_{k3}} = 0$$

Запишем полученную матрицу:

$$T_i^j = \begin{pmatrix} 0 & -(X^2) + (X^1) & 0 \\ 0 & 0 & 0 \\ 2 \cdot (X^3) & 0 & 0 \end{pmatrix}$$

Найдем символы Кристоффеля по формуле:

$$\Gamma_{ij}^m = \frac{1}{2} g^{km} (\frac{\partial g_{kj}}{\partial X^i} + \frac{\partial g_{ik}}{\partial X^j} - \frac{\partial g_{ij}}{\partial X^k}) :$$

При m = 1:

$$\Gamma_{11}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k1}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^1} - \frac{\partial g_{11}}{\partial X^k}) = 0$$
$$\Gamma_{12}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k2}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^2} - \frac{\partial g_{12}}{\partial X^k}) = 0$$
$$\Gamma_{13}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k3}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^3} - \frac{\partial g_{13}}{\partial X^k}) = 0$$
$$\Gamma_{21}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k1}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^1} - \frac{\partial g_{21}}{\partial X^k}) = 0$$
$$\Gamma_{22}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k2}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^2} - \frac{\partial g_{22}}{\partial X^k}) = -1.0 \cdot (X^1)$$
$$\Gamma_{23}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k3}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^3} - \frac{\partial g_{23}}{\partial X^k}) = 0$$
$$\Gamma_{31}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k1}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^1} - \frac{\partial g_{31}}{\partial X^k}) = 0$$
$$\Gamma_{32}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k2}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^2} - \frac{\partial g_{32}}{\partial X^k}) = 0$$
$$\Gamma_{33}^1 = \frac{1}{2} g^{k1} (\frac{\partial g_{k3}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^3} - \frac{\partial g_{33}}{\partial X^k}) = 0$$

При m = 2:

$$
\begin{aligned}
\Gamma^2_{11} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k1}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^1} - \frac{\partial g_{11}}{\partial X^k}) = 0\\
\Gamma^2_{12} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k2}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^2} - \frac{\partial g_{12}}{\partial X^k}) = 1.0/(X^1)\\
\Gamma^2_{13} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k3}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^3} - \frac{\partial g_{13}}{\partial X^k}) = 0\\
\Gamma^2_{21} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k1}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^1} - \frac{\partial g_{21}}{\partial X^k}) = 1.0/(X^1)\\
\Gamma^2_{22} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k2}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^2} - \frac{\partial g_{22}}{\partial X^k}) = 0\\
\Gamma^2_{23} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k3}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^3} - \frac{\partial g_{23}}{\partial X^k}) = 0\\
\Gamma^2_{31} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k1}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^1} - \frac{\partial g_{31}}{\partial X^k}) = 0\\
\Gamma^2_{32} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k2}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^2} - \frac{\partial g_{32}}{\partial X^k}) = 0\\
\Gamma^2_{33} &= \frac{1}{2}g^{k2}(\frac{\partial g_{k3}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^3} - \frac{\partial g_{33}}{\partial X^k}) = 0
\end{aligned}
$$

При m = 3:

$$
\begin{aligned}
\Gamma^3_{11} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k1}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^1} - \frac{\partial g_{11}}{\partial X^k}) = 0\\
\Gamma^3_{12} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k2}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^2} - \frac{\partial g_{12}}{\partial X^k}) = 0\\
\Gamma^3_{13} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k3}}{\partial X^1} + \frac{\partial g_{1k}}{\partial X^3} - \frac{\partial g_{13}}{\partial X^k}) = 0\\
\Gamma^3_{21} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k1}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^1} - \frac{\partial g_{21}}{\partial X^k}) = 0\\
\Gamma^3_{22} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k2}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^2} - \frac{\partial g_{22}}{\partial X^k}) = 0\\
\Gamma^3_{23} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k3}}{\partial X^2} + \frac{\partial g_{2k}}{\partial X^3} - \frac{\partial g_{23}}{\partial X^k}) = 0\\
\Gamma^3_{31} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k1}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^1} - \frac{\partial g_{31}}{\partial X^k}) = 0\\
\Gamma^3_{32} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k2}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^2} - \frac{\partial g_{32}}{\partial X^k}) = 0\\
\Gamma^3_{33} &= \frac{1}{2}g^{k3}(\frac{\partial g_{k3}}{\partial X^3} + \frac{\partial g_{3k}}{\partial X^3} - \frac{\partial g_{33}}{\partial X^k}) = 0
\end{aligned}
$$

Запишем результат:

$$
\Gamma^1_{ij} = \begin{pmatrix} 0 & 0 & 0\\ 0 & -1.0\cdot(X^1) & 0\\ 0 & 0 & 0 \end{pmatrix}
$$

$$\Gamma^2_{ij} = \begin{pmatrix} 0 & 1.0/(X^1) & 0 \\ 1.0/(X^1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma^3_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2.1)Вычислим ковариантную производную контравариантных компонент поля по формуле:

$$\nabla_k T^{ij} = \frac{\partial T^{ij}}{\partial X^k} + T^{mj}\Gamma^i_{mk} + T^{im}\Gamma^j_{mk};$$

При k = 1:

$$\nabla_1 T^{11} = \frac{\partial T^{11}}{\partial X^1} + T^{m1}\Gamma^1_{m1} + T^{1m}\Gamma^1_{m1} = 0$$
$$\nabla_1 T^{12} = \frac{\partial T^{12}}{\partial X^1} + T^{m2}\Gamma^1_{m1} + T^{1m}\Gamma^2_{m1} = -1.0 \cdot (X^2)/(X^1) + 2.0$$
$$\nabla_1 T^{13} = \frac{\partial T^{13}}{\partial X^1} + T^{m3}\Gamma^1_{m1} + T^{1m}\Gamma^3_{m1} = 0$$
$$\nabla_1 T^{21} = \frac{\partial T^{21}}{\partial X^1} + T^{m1}\Gamma^2_{m1} + T^{2m}\Gamma^1_{m1} = 0$$
$$\nabla_1 T^{22} = \frac{\partial T^{22}}{\partial X^1} + T^{m2}\Gamma^2_{m1} + T^{2m}\Gamma^2_{m1} = 0$$
$$\nabla_1 T^{23} = \frac{\partial T^{23}}{\partial X^1} + T^{m3}\Gamma^2_{m1} + T^{2m}\Gamma^3_{m1} = 0$$
$$\nabla_1 T^{31} = \frac{\partial T^{31}}{\partial X^1} + T^{m1}\Gamma^3_{m1} + T^{3m}\Gamma^1_{m1} = 0$$
$$\nabla_1 T^{32} = \frac{\partial T^{32}}{\partial X^1} + T^{m2}\Gamma^3_{m1} + T^{3m}\Gamma^2_{m1} = 0$$
$$\nabla_1 T^{33} = \frac{\partial T^{33}}{\partial X^1} + T^{m3}\Gamma^3_{m1} + T^{3m}\Gamma^3_{m1} = 0$$

При k = 2:

$$\nabla_2 T^{11} = \frac{\partial T^{11}}{\partial X^2} + T^{m1}\Gamma^1_{m2} + T^{1m}\Gamma^1_{m2} = 1.0 \cdot (X^1) \cdot ((X^2) - (X^1))$$
$$\nabla_2 T^{12} = \frac{\partial T^{12}}{\partial X^2} + T^{m2}\Gamma^1_{m2} + T^{1m}\Gamma^2_{m2} = -1$$
$$\nabla_2 T^{13} = \frac{\partial T^{13}}{\partial X^2} + T^{m3}\Gamma^1_{m2} + T^{1m}\Gamma^3_{m2} = 0$$
$$\nabla_2 T^{21} = \frac{\partial T^{21}}{\partial X^2} + T^{m1}\Gamma^2_{m2} + T^{2m}\Gamma^1_{m2} = 0$$
$$\nabla_2 T^{22} = \frac{\partial T^{22}}{\partial X^2} + T^{m2}\Gamma^2_{m2} + T^{2m}\Gamma^2_{m2} = -1.0 \cdot (X^2)/(X^1) + 1.0$$
$$\nabla_2 T^{23} = \frac{\partial T^{23}}{\partial X^2} + T^{m3}\Gamma^2_{m2} + T^{2m}\Gamma^3_{m2} = 0$$
$$\nabla_2 T^{31} = \frac{\partial T^{31}}{\partial X^2} + T^{m1}\Gamma^3_{m2} + T^{3m}\Gamma^1_{m2} = 0$$
$$\nabla_2 T^{32} = \frac{\partial T^{32}}{\partial X^2} + T^{m2}\Gamma^3_{m2} + T^{3m}\Gamma^2_{m2} = 2.0 \cdot (X^3)/(X^1)$$
$$\nabla_2 T^{33} = \frac{\partial T^{33}}{\partial X^2} + T^{m3}\Gamma^3_{m2} + T^{3m}\Gamma^3_{m2} = 0$$

При k = 3:

$$\nabla_3 T^{11} = \frac{\partial T^{11}}{\partial X^3} + T^{m1}\Gamma^1_{m3} + T^{1m}\Gamma^1_{m3} = 0$$
$$\nabla_3 T^{12} = \frac{\partial T^{12}}{\partial X^3} + T^{m2}\Gamma^1_{m3} + T^{1m}\Gamma^2_{m3} = 0$$
$$\nabla_3 T^{13} = \frac{\partial T^{13}}{\partial X^3} + T^{m3}\Gamma^1_{m3} + T^{1m}\Gamma^3_{m3} = 0$$
$$\nabla_3 T^{21} = \frac{\partial T^{21}}{\partial X^3} + T^{m1}\Gamma^2_{m3} + T^{2m}\Gamma^1_{m3} = 0$$
$$\nabla_3 T^{22} = \frac{\partial T^{22}}{\partial X^3} + T^{m2}\Gamma^2_{m3} + T^{2m}\Gamma^2_{m3} = 0$$
$$\nabla_3 T^{23} = \frac{\partial T^{23}}{\partial X^3} + T^{m3}\Gamma^2_{m3} + T^{2m}\Gamma^3_{m3} = 0$$
$$\nabla_3 T^{31} = \frac{\partial T^{31}}{\partial X^3} + T^{m1}\Gamma^3_{m3} + T^{3m}\Gamma^1_{m3} = 2$$
$$\nabla_3 T^{32} = \frac{\partial T^{32}}{\partial X^3} + T^{m2}\Gamma^3_{m3} + T^{3m}\Gamma^2_{m3} = 0$$
$$\nabla_3 T^{33} = \frac{\partial T^{33}}{\partial X^3} + T^{m3}\Gamma^3_{m3} + T^{3m}\Gamma^3_{m3} = 0$$

Запишем результат:

$$\nabla_1 T^{ij} = \begin{pmatrix} 0 & -1.0 \cdot (X^2)/(X^1) + 2.0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T^{ij} = \begin{pmatrix} 1.0 \cdot (X^1) \cdot ((X^2) - (X^1)) & -1 & 0 \\ 0 & -1.0 \cdot (X^2)/(X^1) + 1.0 & 0 \\ 0 & 2.0 \cdot (X^3)/(X^1) & 0 \end{pmatrix}$$

$$\nabla_3 T^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

2.2)Вычислим ковариантную производную ковариантных компонент поля по формуле:

$$\nabla_k T_{ij} = \frac{\partial T_{ij}}{\partial X^k} - T_{mj}\Gamma^m_{ik} - T_{im}\Gamma^m_{jk};$$

При k = 1:

$$\begin{aligned}
\nabla_1 T_{11} &= \frac{\partial T_{11}}{\partial X^1} - T_{m1}\Gamma^m_{11} - T_{1m}\Gamma^m_{11} = 0 \\
\nabla_1 T_{12} &= \frac{\partial T_{12}}{\partial X^1} - T_{m2}\Gamma^m_{11} - T_{1m}\Gamma^m_{21} = (X^1) \cdot (-1.0 \cdot (X^2) + 2.0 \cdot (X^1)) \\
\nabla_1 T_{13} &= \frac{\partial T_{13}}{\partial X^1} - T_{m3}\Gamma^m_{11} - T_{1m}\Gamma^m_{31} = 0 \\
\nabla_1 T_{21} &= \frac{\partial T_{21}}{\partial X^1} - T_{m1}\Gamma^m_{21} - T_{2m}\Gamma^m_{11} = 0 \\
\nabla_1 T_{22} &= \frac{\partial T_{22}}{\partial X^1} - T_{m2}\Gamma^m_{21} - T_{2m}\Gamma^m_{21} = 0 \\
\nabla_1 T_{23} &= \frac{\partial T_{23}}{\partial X^1} - T_{m3}\Gamma^m_{21} - T_{2m}\Gamma^m_{31} = 0 \\
\nabla_1 T_{31} &= \frac{\partial T_{31}}{\partial X^1} - T_{m1}\Gamma^m_{31} - T_{3m}\Gamma^m_{11} = 0 \\
\nabla_1 T_{32} &= \frac{\partial T_{32}}{\partial X^1} - T_{m2}\Gamma^m_{31} - T_{3m}\Gamma^m_{21} = 0 \\
\nabla_1 T_{33} &= \frac{\partial T_{33}}{\partial X^1} - T_{m3}\Gamma^m_{31} - T_{3m}\Gamma^m_{31} = 0
\end{aligned}$$

При k = 2:

$$\nabla_2 T_{11} = \frac{\partial T_{11}}{\partial X^2} - T_{m1}\Gamma^m_{12} - T_{1m}\Gamma^m_{12} = 1.0 \cdot (X^1) \cdot ((X^2) - (X^1))$$
$$\nabla_2 T_{12} = \frac{\partial T_{12}}{\partial X^2} - T_{m2}\Gamma^m_{12} - T_{1m}\Gamma^m_{22} = -(X^1)^2$$
$$\nabla_2 T_{13} = \frac{\partial T_{13}}{\partial X^2} - T_{m3}\Gamma^m_{12} - T_{1m}\Gamma^m_{32} = 0$$
$$\nabla_2 T_{21} = \frac{\partial T_{21}}{\partial X^2} - T_{m1}\Gamma^m_{22} - T_{2m}\Gamma^m_{12} = 0$$
$$\nabla_2 T_{22} = \frac{\partial T_{22}}{\partial X^2} - T_{m2}\Gamma^m_{22} - T_{2m}\Gamma^m_{22} = 1.0 \cdot (X^1)^3 \cdot (-(X^2) + (X^1))$$
$$\nabla_2 T_{23} = \frac{\partial T_{23}}{\partial X^2} - T_{m3}\Gamma^m_{22} - T_{2m}\Gamma^m_{32} = 0$$
$$\nabla_2 T_{31} = \frac{\partial T_{31}}{\partial X^2} - T_{m1}\Gamma^m_{32} - T_{3m}\Gamma^m_{12} = 0$$
$$\nabla_2 T_{32} = \frac{\partial T_{32}}{\partial X^2} - T_{m2}\Gamma^m_{32} - T_{3m}\Gamma^m_{22} = 2.0 \cdot (X^3) \cdot (X^1)$$
$$\nabla_2 T_{33} = \frac{\partial T_{33}}{\partial X^2} - T_{m3}\Gamma^m_{32} - T_{3m}\Gamma^m_{32} = 0$$

При k = 3:

$$\nabla_3 T_{11} = \frac{\partial T_{11}}{\partial X^3} - T_{m1}\Gamma^m_{13} - T_{1m}\Gamma^m_{13} = 0$$
$$\nabla_3 T_{12} = \frac{\partial T_{12}}{\partial X^3} - T_{m2}\Gamma^m_{13} - T_{1m}\Gamma^m_{23} = 0$$
$$\nabla_3 T_{13} = \frac{\partial T_{13}}{\partial X^3} - T_{m3}\Gamma^m_{13} - T_{1m}\Gamma^m_{33} = 0$$
$$\nabla_3 T_{21} = \frac{\partial T_{21}}{\partial X^3} - T_{m1}\Gamma^m_{23} - T_{2m}\Gamma^m_{13} = 0$$
$$\nabla_3 T_{22} = \frac{\partial T_{22}}{\partial X^3} - T_{m2}\Gamma^m_{23} - T_{2m}\Gamma^m_{23} = 0$$
$$\nabla_3 T_{23} = \frac{\partial T_{23}}{\partial X^3} - T_{m3}\Gamma^m_{23} - T_{2m}\Gamma^m_{33} = 0$$
$$\nabla_3 T_{31} = \frac{\partial T_{31}}{\partial X^3} - T_{m1}\Gamma^m_{33} - T_{3m}\Gamma^m_{13} = 2$$
$$\nabla_3 T_{32} = \frac{\partial T_{32}}{\partial X^3} - T_{m2}\Gamma^m_{33} - T_{3m}\Gamma^m_{23} = 0$$
$$\nabla_3 T_{33} = \frac{\partial T_{33}}{\partial X^3} - T_{m3}\Gamma^m_{33} - T_{3m}\Gamma^m_{33} = 0$$

Запишем результат:

$$\nabla_1 T_{ij} = \begin{pmatrix} 0 & (X^1) \cdot (-1.0 \cdot (X^2) + 2.0 \cdot (X^1)) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T_{ij} = \begin{pmatrix} 1.0 \cdot (X^1) \cdot ((X^2) - (X^1)) & -(X^1)^2 & 0 \\ 0 & 1.0 \cdot (X^1)^3 \cdot (-(X^2) + (X^1)) & 0 \\ 0 & 2.0 \cdot (X^3) \cdot (X^1) & 0 \end{pmatrix}$$

$$\nabla_3 T_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

2.3)Вычислим ковариантную производную смешанных компонент поля по формуле:

$$\nabla_k T^i_j = \frac{\partial T^i_j}{\partial X^k} + T^m_j \Gamma^i_{mk} - T^i_m \Gamma^m_{jk};$$

При k = 1:

$$\begin{aligned}
\nabla_1 T^1_1 &= \frac{\partial T^1_1}{\partial X^1} + T^m_1 \Gamma^1_{m1} - T^1_m \Gamma^m_{11} = 0 \\
\nabla_1 T^1_2 &= \frac{\partial T^1_2}{\partial X^1} + T^m_2 \Gamma^1_{m1} - T^1_m \Gamma^m_{21} = (X^1) \cdot (-1.0 \cdot (X^2) + 2.0 \cdot (X^1)) \\
\nabla_1 T^1_3 &= \frac{\partial T^1_3}{\partial X^1} + T^m_3 \Gamma^1_{m1} - T^1_m \Gamma^m_{31} = 0 \\
\nabla_1 T^2_1 &= \frac{\partial T^2_1}{\partial X^1} + T^m_1 \Gamma^2_{m1} - T^2_m \Gamma^m_{11} = 0 \\
\nabla_1 T^2_2 &= \frac{\partial T^2_2}{\partial X^1} + T^m_2 \Gamma^2_{m1} - T^2_m \Gamma^m_{21} = 0 \\
\nabla_1 T^2_3 &= \frac{\partial T^2_3}{\partial X^1} + T^m_3 \Gamma^2_{m1} - T^2_m \Gamma^m_{31} = 0 \\
\nabla_1 T^3_1 &= \frac{\partial T^3_1}{\partial X^1} + T^m_1 \Gamma^3_{m1} - T^3_m \Gamma^m_{11} = 0 \\
\nabla_1 T^3_2 &= \frac{\partial T^3_2}{\partial X^1} + T^m_2 \Gamma^3_{m1} - T^3_m \Gamma^m_{21} = 0 \\
\nabla_1 T^3_3 &= \frac{\partial T^3_3}{\partial X^1} + T^m_3 \Gamma^3_{m1} - T^3_m \Gamma^m_{31} = 0
\end{aligned}$$

При k = 2:

$$
\begin{aligned}
\nabla_2 T_1^1 &= \frac{\partial T_1^1}{\partial X^2} + T_1^m \Gamma_{m2}^1 - T_m^1 \Gamma_{12}^m = 1.0 \cdot (X^1) \cdot ((X^2) - (X^1)) \\
\nabla_2 T_2^1 &= \frac{\partial T_2^1}{\partial X^2} + T_2^m \Gamma_{m2}^1 - T_m^1 \Gamma_{22}^m = -(X^1)^2 \\
\nabla_2 T_3^1 &= \frac{\partial T_3^1}{\partial X^2} + T_3^m \Gamma_{m2}^1 - T_m^1 \Gamma_{32}^m = 0 \\
\nabla_2 T_1^2 &= \frac{\partial T_1^2}{\partial X^2} + T_1^m \Gamma_{m2}^2 - T_m^2 \Gamma_{12}^m = 0 \\
\nabla_2 T_2^2 &= \frac{\partial T_2^2}{\partial X^2} + T_2^m \Gamma_{m2}^2 - T_m^2 \Gamma_{22}^m = 1.0 \cdot (X^1) \cdot (-(X^2) + (X^1)) \\
\nabla_2 T_3^2 &= \frac{\partial T_3^2}{\partial X^2} + T_3^m \Gamma_{m2}^2 - T_m^2 \Gamma_{32}^m = 0 \\
\nabla_2 T_1^3 &= \frac{\partial T_1^3}{\partial X^2} + T_1^m \Gamma_{m2}^3 - T_m^3 \Gamma_{12}^m = 0 \\
\nabla_2 T_2^3 &= \frac{\partial T_2^3}{\partial X^2} + T_2^m \Gamma_{m2}^3 - T_m^3 \Gamma_{22}^m = 2.0 \cdot (X^3) \cdot (X^1) \\
\nabla_2 T_3^3 &= \frac{\partial T_3^3}{\partial X^2} + T_3^m \Gamma_{m2}^3 - T_m^3 \Gamma_{32}^m = 0
\end{aligned}
$$

При k = 3:

$$
\begin{aligned}
\nabla_3 T_1^1 &= \frac{\partial T_1^1}{\partial X^3} + T_1^m \Gamma_{m3}^1 - T_m^1 \Gamma_{13}^m = 0 \\
\nabla_3 T_2^1 &= \frac{\partial T_2^1}{\partial X^3} + T_2^m \Gamma_{m3}^1 - T_m^1 \Gamma_{23}^m = 0 \\
\nabla_3 T_3^1 &= \frac{\partial T_3^1}{\partial X^3} + T_3^m \Gamma_{m3}^1 - T_m^1 \Gamma_{33}^m = 0 \\
\nabla_3 T_1^2 &= \frac{\partial T_1^2}{\partial X^3} + T_1^m \Gamma_{m3}^2 - T_m^2 \Gamma_{13}^m = 0 \\
\nabla_3 T_2^2 &= \frac{\partial T_2^2}{\partial X^3} + T_2^m \Gamma_{m3}^2 - T_m^2 \Gamma_{23}^m = 0 \\
\nabla_3 T_3^2 &= \frac{\partial T_3^2}{\partial X^3} + T_3^m \Gamma_{m3}^2 - T_m^2 \Gamma_{33}^m = 0 \\
\nabla_3 T_1^3 &= \frac{\partial T_1^3}{\partial X^3} + T_1^m \Gamma_{m3}^3 - T_m^3 \Gamma_{13}^m = 2 \\
\nabla_3 T_2^3 &= \frac{\partial T_2^3}{\partial X^3} + T_2^m \Gamma_{m3}^3 - T_m^3 \Gamma_{23}^m = 0 \\
\nabla_3 T_3^3 &= \frac{\partial T_3^3}{\partial X^3} + T_3^m \Gamma_{m3}^3 - T_m^3 \Gamma_{33}^m = 0
\end{aligned}
$$

Запишем результат:

$$
\nabla_1 T_j^i = \begin{pmatrix} 0 & (X^1) \cdot (-1.0 \cdot (X^2) + 2.0 \cdot (X^1)) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$\nabla_2 T^i_j = \begin{pmatrix} 1.0 \cdot (X^1) \cdot ((X^2) - (X^1)) & -(X^1)^2 & 0 \\ 0 & 1.0 \cdot (X^1) \cdot (-(X^2) + (X^1)) & 0 \\ 0 & 2.0 \cdot (X^3) \cdot (X^1) & 0 \end{pmatrix}$$

$$\nabla_3 T^i_j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

2.4)Вычислим ковариантную производную смешанных компонент поля по формуле:

$$\nabla_k T^j_i = \frac{\partial T^j_i}{\partial X^k} - T^j_m \Gamma^m_{ik} + T^m_i \Gamma^j_{mk};$$

При k = 1:

$$\begin{aligned}
\nabla_1 T^1_1 &= \frac{\partial T^1_1}{\partial X^1} - T^1_m \Gamma^m_{11} + T^m_1 \Gamma^1_{m1} = 0 \\
\nabla_1 T^2_1 &= \frac{\partial T^2_1}{\partial X^1} - T^2_m \Gamma^m_{11} + T^m_1 \Gamma^2_{m1} = -1.0 \cdot (X^2)/(X^1) + 2.0 \\
\nabla_1 T^3_1 &= \frac{\partial T^3_1}{\partial X^1} - T^3_m \Gamma^m_{11} + T^m_1 \Gamma^3_{m1} = 0 \\
\nabla_1 T^1_2 &= \frac{\partial T^1_2}{\partial X^1} - T^1_m \Gamma^m_{21} + T^m_2 \Gamma^1_{m1} = 0 \\
\nabla_1 T^2_2 &= \frac{\partial T^2_2}{\partial X^1} - T^2_m \Gamma^m_{21} + T^m_2 \Gamma^2_{m1} = 0 \\
\nabla_1 T^3_2 &= \frac{\partial T^3_2}{\partial X^1} - T^3_m \Gamma^m_{21} + T^m_2 \Gamma^3_{m1} = 0 \\
\nabla_1 T^1_3 &= \frac{\partial T^1_3}{\partial X^1} - T^1_m \Gamma^m_{31} + T^m_3 \Gamma^1_{m1} = 0 \\
\nabla_1 T^2_3 &= \frac{\partial T^2_3}{\partial X^1} - T^2_m \Gamma^m_{31} + T^m_3 \Gamma^2_{m1} = 0 \\
\nabla_1 T^3_3 &= \frac{\partial T^3_3}{\partial X^1} - T^3_m \Gamma^m_{31} + T^m_3 \Gamma^3_{m1} = 0
\end{aligned}$$

При k = 2:

$$\nabla_2 T_1^1 = \frac{\partial T_1^1}{\partial X^2} - T_m^1 \Gamma_{12}^m + T_1^m \Gamma_{m2}^1 = 1.0 \cdot (X^1) \cdot ((X^2) - (X^1))$$
$$\nabla_2 T_1^2 = \frac{\partial T_1^2}{\partial X^2} - T_m^2 \Gamma_{12}^m + T_1^m \Gamma_{m2}^2 = -1$$
$$\nabla_2 T_1^3 = \frac{\partial T_1^3}{\partial X^2} - T_m^3 \Gamma_{12}^m + T_1^m \Gamma_{m2}^3 = 0$$
$$\nabla_2 T_2^1 = \frac{\partial T_2^1}{\partial X^2} - T_m^1 \Gamma_{22}^m + T_2^m \Gamma_{m2}^1 = 0$$
$$\nabla_2 T_2^2 = \frac{\partial T_2^2}{\partial X^2} - T_m^2 \Gamma_{22}^m + T_2^m \Gamma_{m2}^2 = 1.0 \cdot (X^1) \cdot (-(X^2) + (X^1))$$
$$\nabla_2 T_2^3 = \frac{\partial T_2^3}{\partial X^2} - T_m^3 \Gamma_{22}^m + T_2^m \Gamma_{m2}^3 = 0$$
$$\nabla_2 T_3^1 = \frac{\partial T_3^1}{\partial X^2} - T_m^1 \Gamma_{32}^m + T_3^m \Gamma_{m2}^1 = 0$$
$$\nabla_2 T_3^2 = \frac{\partial T_3^2}{\partial X^2} - T_m^2 \Gamma_{32}^m + T_3^m \Gamma_{m2}^2 = 2.0 \cdot (X^3)/(X^1)$$
$$\nabla_2 T_3^3 = \frac{\partial T_3^3}{\partial X^2} - T_m^3 \Gamma_{32}^m + T_3^m \Gamma_{m2}^3 = 0$$

При k = 3:

$$\nabla_3 T_1^1 = \frac{\partial T_1^1}{\partial X^3} - T_m^1 \Gamma_{13}^m + T_1^m \Gamma_{m3}^1 = 0$$
$$\nabla_3 T_1^2 = \frac{\partial T_1^2}{\partial X^3} - T_m^2 \Gamma_{13}^m + T_1^m \Gamma_{m3}^2 = 0$$
$$\nabla_3 T_1^3 = \frac{\partial T_1^3}{\partial X^3} - T_m^3 \Gamma_{13}^m + T_1^m \Gamma_{m3}^3 = 0$$
$$\nabla_3 T_2^1 = \frac{\partial T_2^1}{\partial X^3} - T_m^1 \Gamma_{23}^m + T_2^m \Gamma_{m3}^1 = 0$$
$$\nabla_3 T_2^2 = \frac{\partial T_2^2}{\partial X^3} - T_m^2 \Gamma_{23}^m + T_2^m \Gamma_{m3}^2 = 0$$
$$\nabla_3 T_2^3 = \frac{\partial T_2^3}{\partial X^3} - T_m^3 \Gamma_{23}^m + T_2^m \Gamma_{m3}^3 = 0$$
$$\nabla_3 T_3^1 = \frac{\partial T_3^1}{\partial X^3} - T_m^1 \Gamma_{33}^m + T_3^m \Gamma_{m3}^1 = 2$$
$$\nabla_3 T_3^2 = \frac{\partial T_3^2}{\partial X^3} - T_m^2 \Gamma_{33}^m + T_3^m \Gamma_{m3}^2 = 0$$
$$\nabla_3 T_3^3 = \frac{\partial T_3^3}{\partial X^3} - T_m^3 \Gamma_{33}^m + T_3^m \Gamma_{m3}^3 = 0$$

Запишем результат:

$$\nabla_1 T_i^j = \begin{pmatrix} 0 & -1.0 \cdot (X^2)/(X^1) + 2.0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla_2 T_i^j = \begin{pmatrix} 1.0 \cdot (X^1) \cdot ((X^2) - (X^1)) & -1 & 0 \\ 0 & 1.0 \cdot (X^1) \cdot (-(X^2) + (X^1)) & 0 \\ 0 & 2.0 \cdot (X^3)/(X^1) & 0 \end{pmatrix}$$

$$\nabla_3 T_i^j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

Вычислим контравариантные производные.
2.5)Вычислим контравариантную производную от контравариантных компонент по формуле:

$$\nabla^m T^{ij} = g^{mk} \nabla_k T^{ij};$$

При k = 1:

$$\begin{aligned}
\nabla^1 T^{11} &= g^{1k} \nabla_k T^{11} = 0 \\
\nabla^1 T^{12} &= g^{1k} \nabla_k T^{12} = -1.0 \cdot (X^2)/(X^1) + 2.0 \\
\nabla^1 T^{13} &= g^{1k} \nabla_k T^{13} = 0 \\
\nabla^1 T^{21} &= g^{1k} \nabla_k T^{21} = 0 \\
\nabla^1 T^{22} &= g^{1k} \nabla_k T^{22} = 0 \\
\nabla^1 T^{23} &= g^{1k} \nabla_k T^{23} = 0 \\
\nabla^1 T^{31} &= g^{1k} \nabla_k T^{31} = 0 \\
\nabla^1 T^{32} &= g^{1k} \nabla_k T^{32} = 0 \\
\nabla^1 T^{33} &= g^{1k} \nabla_k T^{33} = 0
\end{aligned}$$

При k = 2:

$$\begin{aligned}
\nabla^2 T^{11} &= g^{2k} \nabla_k T^{11} = 1.0 \cdot (X^2)/(X^1) - 1.0 \\
\nabla^2 T^{12} &= g^{2k} \nabla_k T^{12} = -1/(X^1)^2 \\
\nabla^2 T^{13} &= g^{2k} \nabla_k T^{13} = 0 \\
\nabla^2 T^{21} &= g^{2k} \nabla_k T^{21} = 0 \\
\nabla^2 T^{22} &= g^{2k} \nabla_k T^{22} = 1.0 \cdot (-(X^2) + (X^1))/(X^1)^3 \\
\nabla^2 T^{23} &= g^{2k} \nabla_k T^{23} = 0 \\
\nabla^2 T^{31} &= g^{2k} \nabla_k T^{31} = 0 \\
\nabla^2 T^{32} &= g^{2k} \nabla_k T^{32} = 2.0 \cdot (X^3)/(X^1)^3 \\
\nabla^2 T^{33} &= g^{2k} \nabla_k T^{33} = 0
\end{aligned}$$

При k = 3:

$$
\begin{aligned}
\nabla^3 T^{11} &= g^{3k}\nabla_k T^{11} = 0\\
\nabla^3 T^{12} &= g^{3k}\nabla_k T^{12} = 0\\
\nabla^3 T^{13} &= g^{3k}\nabla_k T^{13} = 0\\
\nabla^3 T^{21} &= g^{3k}\nabla_k T^{21} = 0\\
\nabla^3 T^{22} &= g^{3k}\nabla_k T^{22} = 0\\
\nabla^3 T^{23} &= g^{3k}\nabla_k T^{23} = 0\\
\nabla^3 T^{31} &= g^{3k}\nabla_k T^{31} = 2\\
\nabla^3 T^{32} &= g^{3k}\nabla_k T^{32} = 0\\
\nabla^3 T^{33} &= g^{3k}\nabla_k T^{33} = 0
\end{aligned}
$$

Запишем результат:

$$
\nabla^1 T^{ij} = \begin{pmatrix} 0 & -1.0\cdot(X^2)/(X^1)+2.0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}
$$

$$
\nabla^2 T^{ij} = \begin{pmatrix} 1.0\cdot(X^2)/(X^1)-1.0 & -1/(X^1)^2 & 0\\ 0 & 1.0\cdot(-(X^2)+(X^1))/(X^1)^3 & 0\\ 0 & 2.0\cdot(X^3)/(X^1)^3 & 0 \end{pmatrix}
$$

$$
\nabla^3 T^{ij} = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 2 & 0 & 0 \end{pmatrix}
$$

2.6)Вычислим контравариантную производную от ковариантных компонент по формуле:

$$
\nabla^m T_{ij} = g^{mk}\nabla_k T_{ij};
$$

При k = 1:

$$
\begin{aligned}
\nabla^1 T_{11} &= g^{1k}\nabla_k T_{11} = 0\\
\nabla^1 T_{12} &= g^{1k}\nabla_k T_{12} = (X^1)\cdot(-1.0\cdot(X^2)+2.0\cdot(X^1))\\
\nabla^1 T_{13} &= g^{1k}\nabla_k T_{13} = 0\\
\nabla^1 T_{21} &= g^{1k}\nabla_k T_{21} = 0\\
\nabla^1 T_{22} &= g^{1k}\nabla_k T_{22} = 0\\
\nabla^1 T_{23} &= g^{1k}\nabla_k T_{23} = 0\\
\nabla^1 T_{31} &= g^{1k}\nabla_k T_{31} = 0\\
\nabla^1 T_{32} &= g^{1k}\nabla_k T_{32} = 0\\
\nabla^1 T_{33} &= g^{1k}\nabla_k T_{33} = 0
\end{aligned}
$$

При k = 2:

$$\begin{aligned}
\nabla^2 T_{11} &= g^{2k}\nabla_k T_{11} = 1.0\cdot(X^2)/(X^1) - 1.0\\
\nabla^2 T_{12} &= g^{2k}\nabla_k T_{12} = -1\\
\nabla^2 T_{13} &= g^{2k}\nabla_k T_{13} = 0\\
\nabla^2 T_{21} &= g^{2k}\nabla_k T_{21} = 0\\
\nabla^2 T_{22} &= g^{2k}\nabla_k T_{22} = 1.0\cdot(X^1)\cdot(-(X^2)+(X^1))\\
\nabla^2 T_{23} &= g^{2k}\nabla_k T_{23} = 0\\
\nabla^2 T_{31} &= g^{2k}\nabla_k T_{31} = 0\\
\nabla^2 T_{32} &= g^{2k}\nabla_k T_{32} = 2.0\cdot(X^3)/(X^1)\\
\nabla^2 T_{33} &= g^{2k}\nabla_k T_{33} = 0
\end{aligned}$$

При k = 3:

$$\begin{aligned}
\nabla^3 T_{11} &= g^{3k}\nabla_k T_{11} = 0\\
\nabla^3 T_{12} &= g^{3k}\nabla_k T_{12} = 0\\
\nabla^3 T_{13} &= g^{3k}\nabla_k T_{13} = 0\\
\nabla^3 T_{21} &= g^{3k}\nabla_k T_{21} = 0\\
\nabla^3 T_{22} &= g^{3k}\nabla_k T_{22} = 0\\
\nabla^3 T_{23} &= g^{3k}\nabla_k T_{23} = 0\\
\nabla^3 T_{31} &= g^{3k}\nabla_k T_{31} = 2\\
\nabla^3 T_{32} &= g^{3k}\nabla_k T_{32} = 0\\
\nabla^3 T_{33} &= g^{3k}\nabla_k T_{33} = 0
\end{aligned}$$

Запишем результат:

$$\nabla^1 T_{ij} = \begin{pmatrix} 0 & (X^1)\cdot(-1.0\cdot(X^2)+2.0\cdot(X^1)) & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T_{ij} = \begin{pmatrix} 1.0\cdot(X^2)/(X^1) - 1.0 & -1 & 0\\ 0 & 1.0\cdot(X^1)\cdot(-(X^2)+(X^1)) & 0\\ 0 & 2.0\cdot(X^3)/(X^1) & 0 \end{pmatrix}$$

$$\nabla^3 T_{ij} = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 2 & 0 & 0 \end{pmatrix}$$

2.7)Аналогично вычислим контравариантную производную от смешанных компонент тензора по формуле:

$$\nabla^m T^i_j = g^{mk} \nabla_k T^i_j;$$

При k = 1:

$$\nabla^1 T^1_1 = g^{1k} \nabla_k T^1_1 = 0$$
$$\nabla^1 T^1_2 = g^{1k} \nabla_k T^1_2 = (X^1) \cdot (-1.0 \cdot (X^2) + 2.0 \cdot (X^1))$$
$$\nabla^1 T^1_3 = g^{1k} \nabla_k T^1_3 = 0$$
$$\nabla^1 T^2_1 = g^{1k} \nabla_k T^2_1 = 0$$
$$\nabla^1 T^2_2 = g^{1k} \nabla_k T^2_2 = 0$$
$$\nabla^1 T^2_3 = g^{1k} \nabla_k T^2_3 = 0$$
$$\nabla^1 T^3_1 = g^{1k} \nabla_k T^3_1 = 0$$
$$\nabla^1 T^3_2 = g^{1k} \nabla_k T^3_2 = 0$$
$$\nabla^1 T^3_3 = g^{1k} \nabla_k T^3_3 = 0$$

При k = 2:

$$\nabla^2 T^1_1 = g^{2k} \nabla_k T^1_1 = 1.0 \cdot (X^2)/(X^1) - 1.0$$
$$\nabla^2 T^1_2 = g^{2k} \nabla_k T^1_2 = -1$$
$$\nabla^2 T^1_3 = g^{2k} \nabla_k T^1_3 = 0$$
$$\nabla^2 T^2_1 = g^{2k} \nabla_k T^2_1 = 0$$
$$\nabla^2 T^2_2 = g^{2k} \nabla_k T^2_2 = -1.0 \cdot (X^2)/(X^1) + 1.0$$
$$\nabla^2 T^2_3 = g^{2k} \nabla_k T^2_3 = 0$$
$$\nabla^2 T^3_1 = g^{2k} \nabla_k T^3_1 = 0$$
$$\nabla^2 T^3_2 = g^{2k} \nabla_k T^3_2 = 2.0 \cdot (X^3)/(X^1)$$
$$\nabla^2 T^3_3 = g^{2k} \nabla_k T^3_3 = 0$$

При k = 3:

$$\nabla^3 T^1_1 = g^{3k} \nabla_k T^1_1 = 0$$
$$\nabla^3 T^1_2 = g^{3k} \nabla_k T^1_2 = 0$$
$$\nabla^3 T^1_3 = g^{3k} \nabla_k T^1_3 = 0$$
$$\nabla^3 T^2_1 = g^{3k} \nabla_k T^2_1 = 0$$
$$\nabla^3 T^2_2 = g^{3k} \nabla_k T^2_2 = 0$$
$$\nabla^3 T^2_3 = g^{3k} \nabla_k T^2_3 = 0$$
$$\nabla^3 T^3_1 = g^{3k} \nabla_k T^3_1 = 2$$
$$\nabla^3 T^3_2 = g^{3k} \nabla_k T^3_2 = 0$$
$$\nabla^3 T^3_3 = g^{3k} \nabla_k T^3_3 = 0$$

Запишем результат:

$$\nabla^1 T^i_j = \begin{pmatrix} 0 & (X^1)\cdot(-1.0\cdot(X^2)+2.0\cdot(X^1)) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T^i_j = \begin{pmatrix} 1.0\cdot(X^2)/(X^1)-1.0 & -1 & 0 \\ 0 & -1.0\cdot(X^2)/(X^1)+1.0 & 0 \\ 0 & 2.0\cdot(X^3)/(X^1) & 0 \end{pmatrix}$$

$$\nabla^3 T^i_j = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

2.8)Аналогично вычислим контравариантную производную от смешанных компонент тензора по формуле:

$$\nabla^m T^j_i = g^{mk}\nabla_k T^j_i;$$

При k = 1:

$$\begin{aligned}
\nabla^1 T^1_1 &= g^{1k}\nabla_k T^1_1 = 0\\
\nabla^1 T^2_1 &= g^{1k}\nabla_k T^2_1 = -1.0\cdot(X^2)/(X^1)+2.0\\
\nabla^1 T^3_1 &= g^{1k}\nabla_k T^3_1 = 0\\
\nabla^1 T^1_2 &= g^{1k}\nabla_k T^1_2 = 0\\
\nabla^1 T^2_2 &= g^{1k}\nabla_k T^2_2 = 0\\
\nabla^1 T^3_2 &= g^{1k}\nabla_k T^3_2 = 0\\
\nabla^1 T^1_3 &= g^{1k}\nabla_k T^1_3 = 0\\
\nabla^1 T^2_3 &= g^{1k}\nabla_k T^2_3 = 0\\
\nabla^1 T^3_3 &= g^{1k}\nabla_k T^3_3 = 0
\end{aligned}$$

При k = 2:

$$\begin{aligned}
\nabla^2 T_1^1 &= g^{2k}\nabla_k T_1^1 = 1.0\cdot(X^2)/(X^1) - 1.0\\
\nabla^2 T_1^2 &= g^{2k}\nabla_k T_1^2 = -1/(X^1)^2\\
\nabla^2 T_1^3 &= g^{2k}\nabla_k T_1^3 = 0\\
\nabla^2 T_2^1 &= g^{2k}\nabla_k T_2^1 = 0\\
\nabla^2 T_2^2 &= g^{2k}\nabla_k T_2^2 = -1.0\cdot(X^2)/(X^1) + 1.0\\
\nabla^2 T_2^3 &= g^{2k}\nabla_k T_2^3 = 0\\
\nabla^2 T_3^1 &= g^{2k}\nabla_k T_3^1 = 0\\
\nabla^2 T_3^2 &= g^{2k}\nabla_k T_3^2 = 2.0\cdot(X^3)/(X^1)^3\\
\nabla^2 T_3^3 &= g^{2k}\nabla_k T_3^3 = 0
\end{aligned}$$

При k = 3:

$$\begin{aligned}
\nabla^3 T_1^1 &= g^{3k}\nabla_k T_1^1 = 0\\
\nabla^3 T_1^2 &= g^{3k}\nabla_k T_1^2 = 0\\
\nabla^3 T_1^3 &= g^{3k}\nabla_k T_1^3 = 0\\
\nabla^3 T_2^1 &= g^{3k}\nabla_k T_2^1 = 0\\
\nabla^3 T_2^2 &= g^{3k}\nabla_k T_2^2 = 0\\
\nabla^3 T_2^3 &= g^{3k}\nabla_k T_2^3 = 0\\
\nabla^3 T_3^1 &= g^{3k}\nabla_k T_3^1 = 2\\
\nabla^3 T_3^2 &= g^{3k}\nabla_k T_3^2 = 0\\
\nabla^3 T_3^3 &= g^{3k}\nabla_k T_3^3 = 0
\end{aligned}$$

Запишем результат:

$$\nabla^1 T_i^j = \begin{pmatrix} 0 & -1.0\cdot(X^2)/(X^1) + 2.0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla^2 T_i^j = \begin{pmatrix} 1.0\cdot(X^2)/(X^1) - 1.0 & -1/(X^1)^2 & 0\\ 0 & -1.0\cdot(X^2)/(X^1) + 1.0 & 0\\ 0 & 2.0\cdot(X^3)/(X^1)^3 & 0 \end{pmatrix}$$

$$\nabla^3 T_i^j = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 2 & 0 & 0 \end{pmatrix}$$