Tsinghua University Combinatorics Final Exam –Fall 2019 (25 Points)

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.
[Total time: 1.5 hours]

Name : Student ID:

Write the formula with proper explanation. The exact number value is not necessary.

1. How many permutations are between “12764538” and “18263745” in lexicographic order? ( ) (2 points)

2. 1）Grandma invited six kids to have dinner with her and she prepared 6 pairs of chopsticks with different colors. Two sticks for each pair are the same. Bonnie helped Grandma to hand out the chopsticks to all the other 5 kids and herself. But unfortunately, there is no kid who got a matching pair of chopsticks but Bonnie got a pair. Please figure out how many different ways for Bonnie to distribute the chopsticks?（3 points）

   2）There are 52 students in Bonnie’s class. One day in the math course, the teacher asked the students to write down an integer number on cards and handed in. The teacher asked Bonnie to find two cards from them so that the numbers on two cards should be paired that the sum or the subtraction of those two numbers should be divisible by 100. Do you think Bonnie can always be successful to find two paired cards？（3 points）

3. Here we have 6 red beads and 6 blue beads. We want to embed all the corners of a cube with provided beads. How many in-equivalent ways to embed the beads? (5 points)

4. How many ways to compose a binary strings (made up of 0s and 1s) of length n that do not contain “010” or “101”? (7 points)

5. Transform the following problems into augmented form and solve it by simplex

Method and show at least 2 tables. (5 points)

$$Min \quad z = x_1 - 4x_2 + x_3$$

$$\begin{cases} 2x_1 + x_2 + 3x_3 \le 6 \\ -x_1 + x_2 - 6x_3 \ge -6 \\ -x_1 + 3x_2 - x_3 \le 9 \\ x_1 \le 1, x_2 \ge 3 \end{cases}$$