Lemma

Rankine-Hugoniot condition for anisotropic plasmas is[1]

$$\begin{aligned}\Lambda_a(\epsilon_2,\theta_1,M_{n2}^2)\cdot\epsilon_1^2 M_{n1}^4 + 2\Lambda_b(\epsilon_1,\epsilon_2,\theta_1,\beta_1,M_{n2}^2)\cdot\epsilon_1 M_{n1}^2 \\ + \Lambda_c(\epsilon_1,\epsilon_2,\theta_1,\beta_1,M_{n2}^2) = 0\end{aligned}$$

where

$$\begin{aligned}\Lambda_a &= \frac{\gamma-1}{\gamma}\cdot\frac{\xi_2}{\cos^2\theta_1} - \xi_1 M_{n2}^2\tan^2\theta_1, \\ \Lambda_b &= \xi_2\left[\frac{\gamma-1}{\gamma}\frac{2(1-\epsilon_1)}{3\cos^2\theta_1} + \frac{\epsilon_1\beta_1}{2\cos^2\theta_1} - \epsilon_2 M_{n2}^2\right] + \epsilon_1\xi_1 M_{n2}^2\tan^2\theta_1, \\ \Lambda_c &= M_{n2}^2\left\{\epsilon_2^2\xi_2\left[\frac{\gamma+1}{\gamma}M_{n2}^2 - \frac{\epsilon_1\beta_1}{\epsilon_2\cos^2\theta_1} + \left(\frac{\epsilon_1}{\epsilon_2}-1\right)\right.\right. \\ &\quad \left.\left. +\frac{2}{3}\left(1-\frac{1}{\epsilon_2}\right)\left(\frac{2\gamma-2}{\gamma}-\tan^2\theta_1\right)\right] - \epsilon_1^2\xi_1\tan^2\theta_1\right\}\end{aligned}$$

and

$$\begin{aligned}\xi_1 &= \frac{\gamma-1}{\gamma}\left(M_{n2}^2 - 2 + \frac{1}{\epsilon_2}\right) - \frac{1}{3\gamma}\left(2+\frac{1}{\epsilon_2}\right), \\ \xi_2 &= \left(M_{n2}^2-1\right)^2\end{aligned}$$

proof

RH relations are as follows[2]

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$$[\rho V_n]_2^1 = 0, \tag{1}$$

$$\left[\rho V_n^2 + \bar{p} + \frac{1}{3}\left(\epsilon + \frac{1}{2}\right)\frac{\left|\boldsymbol{B}^2\right|}{4\pi} - \epsilon\frac{B_n^2}{4\pi}\right]_2^1 = 0, \tag{2}$$

$$\left[\rho V_n V_t - \epsilon\frac{B_n B_t}{4\pi}\right]_2^1 = 0, \tag{3}$$

$$\left[\rho V_n\left(V^2 + \frac{\gamma}{\gamma - 1}\frac{\bar{p}}{\rho}\right) + \frac{\epsilon + 2}{3}V_n\frac{\left|\boldsymbol{B}^2\right|}{4\pi} - \epsilon V_n\frac{B_n^2}{4\pi} - \epsilon V_t\frac{B_n B_t}{4\pi}\right]_2^1 = 0, \tag{4}$$

where $\bar{p} = (p_{\|} + 2p_{\perp})/3$.

□

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1. Higashimori ↩
2. Karimabadi ↩