Design and Analysis of Algorithms

Final Examination (Volume B)

January 06, 2012

Notice: This exam is closed book, no books, papers or recording devices permitted. You may use theorems from class, or the book provided you can recall them correctly. Add some annotation to your algorithm and pseudo code when essential.

Name________________No.______________Score________________

No.	1	2	3	4	5	6	7	8	Total
Percent									
Score									

Problem 1. Fundamentals (2*10=20 points)

1. If we merger two sorted tables including *n* elements respectively into an sorted table, we need compare __n__times at least.
2. In a min-heap containing *n* elements, the biggest elements may be stored in_____.

   A. $\lfloor n/2 \rfloor$ B. $\lfloor n/2 \rfloor$-1 C. 1 D. $\lfloor n/2 \rfloor$+2

3. $T(n)=9T(n/4)+n^2$, the tight asymptotic bounds for the recurrence is____________ O(n)
4. For an unsorted array, constructing a max-heap on it is asymptotically faster than constructing a red-black tree on it. (n lgn) __________(True/False)
5. Which is **not** prefix codes? _____.

   A. {00, 01, 10, 11} B. {01, 1, 00, 11}

   C. {0, 10, 110, 111} D. {1, 01, 000, 001}

6. Use depth–first search algorithm DFS on undirected graph $G=(V, E)$, and V={a, b, c, d, e ,f}, E={(a, b), (a, e), (a, c), (b, e), (c, f), (f, d), (e, d)}, the correct vertex sequence is _____.

   A. a, b, e, c, d, f B. a, c, f, e, b, d C. a, e, b, c, f, d D. a, e, d, f, c, b

7. What's the prerequisite for a directed graph can be topological sorted.

   ____

8. What's the run time of Dijkstra algorithm described in class used to find single source shortest path in weighted directed graph $G=(V, E)$. ____________

Problem 2. (5 points)
The following recursive algorithm sorts a sequence of n numbers. Write down a recurrence describing the running time of the algorithm as a function of n. You do not need to solve your recurrence.

```
triplesort(seq)
     if n <= 1 return
     if n == 2
           replace seq by [min(seq),max(seq)]
           return
     triplesort(first 2n/3 positions in seq)
     triplesort(last 2n/3 positions in seq)
     triplesort(first 2n/3 positions in seq)
```

Problem 3. (13 points)
Considering Dutch national flag problem, the flag of the Netherlands consists of three colors: red, white and blue. Given balls of these three colors arranged randomly in a line (the number of balls is n), the task is to arrange them such that all balls of the same color are together and their collective color groups are in red, white and blue order. For full credit, your algorithm must run in time $O(n)$.

Problem 4. (17 points)

1. We have introduced both Greedy Algorithms and Dynamic Programming during the course. Could you describe the major differences of the manner in which they solve the optimization problem? (4 points)

2. Given n integers (which can be negative) $a_1, a_2, \ldots, a_n$, please give an algorithm based on dynamic programming to seek for the maximum value of $\sum_{k=i}^{j} a_k$, where $1 \le i \le j \le n$. This is the Maximal-Subsegment-Summation Problem. We define the value of MSS to be 0 when all the integers are negative. Your algorithm should be bounded in $O(n)$ time. For example, for the case $(a_1, a_2, a_3, a_4, a_5, a_6) = (-2, 11, -4, 13, -5, -2)$, the answer for MSS is $\sum_{k=2}^{4} a_k = 20$. (13 points).

Problem 5. (10 points)
A sequence of n operations is performed on a data structure. The ith operation costs i if i is an exact power of 2, and 1 otherwise. Use aggregate analysis and accounting method to determine the amortized cost per operation.

Problem 6. (15 points)
Suppose you have an undirected graph with weighted edges, and perform a depth first search, such that the edges going out of each vertex are always explored in order by weight, smallest first. Is the depth first search tree resulting from this process guaranteed to be a minimum spanning tree? Explain why, if it is, or, if it isn't, provide a counterexample (specifying the start vertex for the DFS and showing both trees).

Problem 7. (10 points)

Consider the flow network $G = (V, E)$ in Fig. 1, with source s and sink t, and the given capacities and flow f. What is the value of the flow f ? Show that this flow is a maximum flow by exhibiting a suitable cut and the residual network contains no augmenting paths. (You may either copy the figure, or simply write down the cut and show the residual network. Also explain why the cut implies that the flow is maximum.)

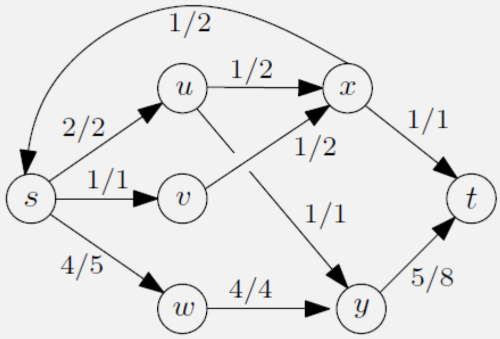

Figure 1: A flow network. Flows and capacities are indicated using the notation *flow/capacity*.