Exercise Sheet 5

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1. **Motion dynamics of an adiabatic parcel.** Consider an adiabatic air parcel moving through still air. The parcel's motion is governed by two forces: net buoyancy and hydrodynamic drag. The equation of motion for such parcel may be cast as

$$\frac{d^2z}{dt^2} = g\frac{\theta_v - \theta_{v,0}}{\theta_{v,0}} + f_d, \tag{1}$$

where $f_d = F_d/m$ is the drag force divided by the parcel's mass. The hydrodynamic drag force may be obtained from the following relation:

$$\mathbf{F_d} = -\frac{1}{2}\rho_0 C_d A \mathbf{u}|\mathbf{u}|, \tag{2}$$

where ρ_0 is the density of the fluid around the object, $\mathbf{u}$ is the relative velocity between the object and the fluid, C_d is the drag coefficient and A is the reference area.

Show that for a spherical air parcel the drag term in equation 1 is given by:

$$f_d = -\frac{3}{8}\frac{\rho_0 C_d}{K\rho^{2/3}} w|w|, \quad K = (\rho_i R_i^3)^{1/3} \tag{3}$$

where ρ_i, R_i, ρ and w are respectively the initial density, initial radius, density and velocity of the parcel. Use $A = \pi R^2$ as the reference area.

2. **Computational Task.** Use equations 1 and 3 to simulate the motion dynamics of an adiabatic air parcel. Use $C_d = 0.47$ and the following initial conditions:

$$\begin{cases} z = 0 \\ w = 0 \\ \theta = \theta_0 + \Delta\theta \\ r_v = r_{v,0} \\ R_i = 1\text{m} \end{cases}, \tag{4}$$

where $\Delta\theta$ is a small increment in the parcel's potential temperature in order to propel it upwards. Use also the environmental profile below:

$$\theta_0(z) = 1 \begin{cases} 289 & \text{if } z \leq z_i \\ 289 + (z - z_i)^{1/3} & \text{if } z > z_i \end{cases} \tag{5a}$$

$$r_{v,0}(z) = 9\mathbf{g}/\mathbf{kg} \tag{5b}$$

with $z_i = 840\text{m}$

(a) Plot vertical profiles of potential temperature for the parcel and the environmental values.

(b) Plot the parcel's velocity versus its height. How does this graph relate to the previous one? Explain physically each stage of the motion.