

Content of Pure Core (Mandatory papers Y540 and Y541)

When this course is being co-taught with OCR's AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR Reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...	DfE Ref.
4.01 Proof				
4.01a 4.01b	Mathematical induction	<p>a) Be able to construct proofs using mathematical induction.</p> <p><i>This topic may be tested using any relevant content including divisibility, powers of matrices and results on powers, exponentials and factorials.</i></p> <p>e.g. Prove that $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ for $n \in \mathbb{Z}^+$.</p> <p>Prove that $7^n - 3^n$ is divisible by 4 for $n \in \mathbb{Z}^+$.</p> <p>Prove that $2^n > 2n$ for $n \geq 3, n \in \mathbb{Z}$.</p>	<p>b) Be able to construct proofs of a more demanding nature, including conjecture followed by proof.</p> <p><i>This topic may be tested using any relevant content including sums of series.</i></p> <p>e.g. Prove that $\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$ for all $n \in \mathbb{Z}$.</p> <p>Prove that $3^n > n^3$ for $n \geq 4, n \in \mathbb{Z}$.</p> <p>Prove that $n! < n^n$ for $n \geq 1, n \in \mathbb{Z}$.</p> <p>Prove that $(1+x)^n \geq 1+nx$ for any real number $x > -1$ and $n \in \mathbb{Z}^+$.</p> <p>Prove that the nth derivative of $x^2 e^x$ is $(x^2 + 2nx + n(n-1))e^x$.</p>	A1

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4.02 Complex Numbers				
4.02a	The language of complex numbers	a) Understand the language of complex numbers. <i>Know the meaning of “real part”, “imaginary part”, “conjugate”, “modulus” and “argument” of a complex number.</i>	d) Understand and be able to use the exponential form, $re^{i\theta}$, of a complex number.	B2 B5 B3
4.02b 4.02d		b) Be able to express a complex number z in either cartesian form $z = x + iy$, where $i^2 = -1$, or modulus-argument form $z = r(\cos \theta + i \sin \theta) = [r, \theta] = rcis\theta$, where $r \geq 0$ is the modulus of z and θ , measured in radians is the argument of z .		B9
4.02c		c) Understand and be able to use the notation: $z, z^*, \operatorname{Re}(z), \operatorname{Im}(z), \arg(z), z $. <i>Includes knowing that a complex number is zero if and only if both the real and imaginary parts are zero.</i> <i>The principal argument of a complex number, for uniqueness, will be taken to lie in either of the intervals $[0, 2\pi)$ or $(-\pi, \pi]$. Learners may use either as appropriate unless the interval is specified.</i> <i>In stage 1 knowledge of radians is assumed: see H240 section 1.05d.</i>		

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4.02e	Basic operations	<p>e) Be able to carry out basic arithmetic operations $(+, -, \times, \div)$ on complex numbers in both cartesian and modulus-argument forms.</p> <p><i>In stage 1 knowledge of radians and compound angle formulae is assumed: see H240 sections 1.05d and 1.05l.</i></p> <p><i>Learners may use the results</i> $z_1 z_2 = [r_1 r_2, \theta_1 + \theta_2]$ and $\frac{z_1}{z_2} = \left[\frac{r_1}{r_2}, \theta_1 - \theta_2 \right]$.</p>		B2 B5 B6
4.02f		<p>f) Convert between cartesian and modulus-argument forms.</p>		
4.02g	Solution of equations	<p>g) Know that, for a polynomial equation with real coefficients, complex roots occur in conjugate pairs.</p>		B1 B3
4.02h		<p>h) Be able to find algebraically the two square roots of a complex number.</p> <p><i>e.g. By squaring and comparing real and imaginary parts.</i></p>		
4.02i		<p>i) Be able to solve quadratic equations with real coefficients and complex roots.</p>		
4.02j		<p>j) Be able to use conjugate pairs, and the factor theorem, to solve or factorise cubic or quartic equations with real coefficients.</p> <p><i>Where necessary, sufficient information will be given to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics.</i></p>		

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4.02k 4.02m	Argand diagrams	k) Be able to use and interpret Argand diagrams. <i>e.g. To represent and interpret complex numbers geometrically.</i> <i>Understand and use the terms “real axis” and “imaginary axis”.</i>	m) Understand the geometrical effects of multiplying and dividing two complex numbers. <i>Includes raising complex numbers to positive integer powers.</i>	B4 B6
4.02l		l) Understand the geometrical effects of taking the conjugate of a complex number, and adding and subtracting two complex numbers.		
4.02n	Euler’s formula		n) Know and be able to use Euler’s formula $e^{i\theta} = \cos \theta + i \sin \theta$. <i>e.g. To express, and work with, complex numbers in the forms $r(\cos \theta + i \sin \theta) = re^{i\theta} = r\text{cis } \theta = [r, \theta]$.</i>	B6 B9

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4.02o	Loci	<p>o) Be able to illustrate equations and inequalities involving complex numbers by means of loci in an Argand diagram.</p> <p><i>i.e. Circle of the form $z - a = k$, half-lines of the form $\arg(z - a) = b$, lines of the form $\operatorname{Re}(z) = k$, $\operatorname{Im}(z) = k$ and $z - a = z - b$, and regions defined by inequalities in these forms.</i></p> <p><i>To include the convention of dashed and solid lines to show exclusion and inclusion respectively.</i></p> <p><i>No shading convention will be assumed. If not directed, learners should indicate clearly which regions are included.</i></p> <p><i>In stage 1 knowledge of radians is assumed: see H240 section 1.05d.</i></p>		B7
4.02p		<p>p) Understand and be able to use set notation in the context of loci.</p> <p><i>e.g. The region $z - a > k$ where $z = x + iy$, $a = x_a + iy_a$ and $k > 0$ may be represented by the set $\{x + iy : (x - x_a)^2 + (y - y_a)^2 > k^2\}$.</i></p> <p><i>In stage 1 knowledge of radians is assumed: see H240 section 1.05d.</i></p>		

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4.02q	De Moivre's theorem		<p>q) Understand de Moivre's theorem and use it to find multiple-angle formulae and sums of series involving trigonometric and/or exponential terms.</p> <p><i>Express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle.</i></p> <p><i>e.g. $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$.</i></p> <p><i>Use expressions for $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$ or equivalent relationships.</i></p> <p><i>e.g. $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$.</i></p> <p><i>Express powers of $\sin\theta$ and $\cos\theta$ in terms of series of trigonometric ratios of multiples of the fundamental angle.</i></p> <p><i>e.g. $\sin^5\theta = \frac{1}{16}(10\sin\theta - 5\sin(3\theta) + \sin(5\theta))$.</i></p>	B8
4.02r	n th roots		<p>r) Be able to find the n distinct nth roots of $re^{i\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon on an Argand diagram.</p> <p><i>Answers may be asked for in either cartesian or modulus-argument form.</i></p>	B10, B4
4.02s	Roots of unity		<p>s) Be able to use complex roots of unity to solve geometric problems.</p> <p><i>e.g. To locate the roots of unity on an Argand diagram or to prove results about sums of roots of unity.</i></p>	B11

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4.03 Matrices				
4.03a	The language of matrices	<p>a) Understand the language of matrices.</p> <p><i>Understand the meaning of “conformable”, “equal”, “square”, “rectangular”, “m by n”, “determinant”, “zero” and “null”, “transpose” and “identity” when applied to matrices.</i></p> <p><i>Learners should be familiar with real matrices and complex matrices.</i></p>		C2
4.03b	Matrix addition and multiplication	<p>b) Be able to add, subtract and multiply conformable matrices; multiply a matrix by a scalar.</p> <p><i>Learners may perform any operations involving entirely numerical matrices by calculator.</i></p> <p><i>Includes raising square matrices to positive integer powers.</i></p> <p><i>Learners should understand the effects on a matrix of adding the zero matrix to it, multiplying it by the zero matrix and multiplying it by the identity matrix.</i></p>		C1
4.03c		<p>c) Understand that matrix multiplication is associative but not commutative.</p> <p><i>Understand the terms “associative” and “commutative”.</i></p>		

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4.03d	Linear transformations	<p>d) Be able to find and use matrices to represent linear transformations in 2-D.</p> <p><i>Includes:</i></p> <ul style="list-style-type: none"> <i>reflection in either coordinate axis and in the lines $y = \pm x$</i> <i>rotation about the origin (defined by the angle of rotation θ, where the direction of positive rotation is taken to be anticlockwise)</i> <i>enlargement centre the origin (defined by the the scale factor)</i> <i>stretch parallel to either coordinate axis (defined by the invariant axis and scale factor)</i> <i>shear parallel to either coordinate axis (defined by the invariant axis and the image of a transformed point).</i> <p><i>Includes the terms “object” and “image”.</i></p>		C3
4.03e		<p>e) Be able to find and use matrices to represent successive transformations.</p> <p><i>Includes understanding and being able to use the result that the matrix product \mathbf{AB} represents the transformation that results from the transformation represented by \mathbf{B} followed by the transformation represented by \mathbf{A}.</i></p>		

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4.03f		<p>f) Be able to use matrices to represent single linear transformations in 3-D.</p> <p><i>3-D transformations will be confined to reflection in one of the planes $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.</i></p> <p><i>Includes the terms “plane of reflection” and “axis of rotation”.</i></p> <p><i>In stage 1 knowledge of 3-D vectors is assumed: see H240 section 1.10b.</i></p>		C3
4.03g	Invariance	<p>g) Be able to find invariant points and lines for a linear transformation.</p> <p><i>Includes the distinction between invariant lines and lines of invariant points.</i></p> <p><i>[The 3-D transformations in section 4.03f are excluded.]</i></p>		C4
4.03h 4.03i	Determinants	<p>h) Be able to find the determinant of a 2×2 matrix with and without a calculator.</p> <p><i>Use and understand the notation $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ or \mathbf{M} or $\det \mathbf{M}$.</i></p> <p>i) Know that the determinant of a 2×2 matrix is the area scale factor of the transformation defined by that matrix, including the effect on the orientation of the image.</p> <p><i>Learners should know that a transformation preserves the orientation of the object if the determinant of the matrix which represents it is positive and that the transformation reverses orientation if the determinant is negative, and be able to interpret this geometrically.</i></p>		C5

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4.03j		j) Be able to calculate the determinant of a 3×3 matrix with and without a calculator.		C5
4.03k		k) Know that the determinant of a 3×3 matrix is the volume scale factor of the transformation defined by that matrix, including the effect on the preservation of the orientation of the image. <i>Learners should know that the sign of the determinant determines whether or not the corresponding transformation preserves orientation, but do not need to understand the geometric interpretation of this in 3-D.</i>		
4.03l		l) Understand and be able to use singular and non-singular matrices. <i>Includes understanding the significance of a zero determinant.</i>		
4.03m		m) Know and be able to use the result that $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$.		
4.03n	Inverses	n) Be able to find and use the inverse of a non-singular 2×2 matrix with and without a calculator.		C6
4.03o		o) Be able to find and use the inverse of a non-singular 3×3 matrix with and without a calculator.		
4.03p		p) Understand and be able to use simple properties of inverse matrices. <i>e.g. The result that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.</i>		
4.03q		q) Understand and be able to use the connection between inverse matrices and inverse transformations.		

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4.03r 4.03s	Solution of simultaneous equations	r) Be able to solve two or three linear simultaneous equations in two or three variables by the use of an inverse matrix, where a unique solution exists.	s) Be able to determine, for two or three linear simultaneous equations where no unique solution exists, whether the equations have an infinite set of solutions (the equations are consistent) or no solutions (the equations are inconsistent). <i>[Finding the solution set in the infinite case is excluded.]</i>	C7
4.03t	Intersection of planes		t) Be able to interpret the solution or failure of solution of three simultaneous linear equations in terms of the geometrical arrangement of three planes. <i>Learners should know and be able to identify the different ways in which two or three distinct planes can intersect in 3-D space, including cases where two or three of the planes are parallel.</i> <i>Learners should understand and be able to apply the geometric significance of the singularity of a matrix in relation to the solution(s) or non-existence of them.</i> <i>[Finding the line of intersection of two or more planes is excluded.]</i>	C8

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4.04 Further Vectors				
4.04a	Equation of a straight line	<p>a) Understand and be able to use the equation of a straight line, in 2-D and 3-D, in cartesian and vector form.</p> <p><i>Learners should know and be able to use the forms:</i> $y = mx + c$, $ax + by = c$ and $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ in 2-D, and $\frac{x - a_1}{u_1} = \frac{y - a_2}{u_2} = \frac{z - a_3}{u_3} (= \lambda)$ and $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ in 3-D. <i>Includes being able to convert from one form to another.</i></p>		F1
4.04b	Equation of a plane		<p>b) Understand and be able to use the equation of a plane in cartesian and vector form.</p> <p><i>Learners should know and be able to use the forms:</i> $ax + by + cz = d$, $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$, $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$ and $\mathbf{r} \cdot \mathbf{n} = p$. <i>Includes being able to convert from one form to another.</i></p>	F2
4.04c 4.04d	Scalar product	<p>c) Be able to calculate the scalar product and use it both to calculate the angles between vectors and/or lines, and also as a test for perpendicularity.</p> <p><i>Includes the notation $\mathbf{a} \cdot \mathbf{b}$</i></p>	<p>d) Be able to find the angle between two planes and the angle between a line and a plane.</p>	F3 F4
4.04e 4.04f	Intersections	<p>e) Be able to find, where it exists, the point of intersection between two lines.</p> <p><i>Includes determining whether or not lines intersect, are parallel or are skew.</i></p>	<p>f) Be able to find the intersection of a line and a plane.</p>	F5

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4.04g	Vector product	<p>g) Be able to use the vector product to find a vector perpendicular to two given vectors.</p> <p><i>Includes the notation $\mathbf{a} \times \mathbf{b}$.</i></p> <p><i>When the vector product is required, either a calculator or a formula may be used. The formula below will be given:</i></p> $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}.$ <p><i>[The magnitude of the vector product is excluded.]</i></p>		Essential content for F2, F3, F5, C6
4.04h 4.04i	Shortest distances		<p>h) Be able to find the distance between two parallel lines and the shortest distance between two skew lines.</p> <p><i>For skew lines, the formula $D = \frac{ (\mathbf{b} - \mathbf{a}) \cdot \mathbf{n} }{ \mathbf{n} }$, where \mathbf{a} and \mathbf{b} are position vectors of points on each line and \mathbf{n} is a mutual perpendicular to both lines, will be given.</i></p> <p><i>Either \mathbf{n} will be given, or it must be established from given information including by use of the vector product.</i></p> <p>i) Be able to find the shortest distance between a point and a line.</p> <p><i>The formula $D = \frac{ ax_1 + by_1 - c }{\sqrt{a^2 + b^2}}$ where the coordinates of the point are (x_1, y_1) and the equation of the line is given by $ax + by = c$, will be given.</i></p>	F5

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4.04j			<p>j) Be able to find the shortest distance between a point and a plane.</p> <p>The formula $D = \frac{ \mathbf{b} \cdot \mathbf{n} - p }{ \mathbf{n} }$ where \mathbf{b} is the position vector of the point and the equation of the plane is given by $\mathbf{r} \cdot \mathbf{n} = p$, will be given.</p>	F5
4.05 Further Algebra				
4.05a	Roots of equations	<p>a) Understand and be able to use the relationships between the symmetric functions of the roots of polynomial equations and the coefficients.</p> <p><i>Up to, and including, quartic equations.</i></p> <p><i>e.g. For the quadratic equation $ax^2 + bx + c = 0$ with roots α and β, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.</i></p>		D1
4.05b	Transformation of equations	<p>b) Be able to use a substitution to obtain an equation whose roots are related to those of the original equation.</p> <p><i>Equations will be of at least cubic degree.</i></p>		D2
4.05c	Partial fractions		<p>c) Extend their knowledge of partial fractions up to rational functions in which the denominator may include quadratic factors of the form $ax^2 + c$ for $c > 0$, and in which the degree of the numerator may be equal to, or exceed, the degree of the denominator.</p> <p><i>See H240 section 1.02y.</i></p>	D4 E4

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4.06 Series				
4.06a	Summation of series		<p>a) Understand and be able to use formulae for the sums of integers, squares and cubes and use these to sum related series.</p> <p><i>Formulae for $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$ will be given, but learners may be asked to prove them.</i></p>	D3
4.06b	Method of differences		<p>b) Understand and be able to use the method of differences to find the sum of a (finite or infinite) series.</p> <p><i>Including the use of partial fractions.</i></p> <p><i>e.g. Find $\sum_{r=1}^n \frac{1}{r(r+2)}$.</i></p>	D4
4.07 Hyperbolic Functions				
4.07a	Definitions		<p>a) Understand and be able to use the definitions of the hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, in terms of exponentials.</p> <p><i>Including the domain and range of each function.</i></p>	H1
4.07b			<p>b) Know and be able to sketch the graphs of the hyperbolic functions.</p>	
4.07c			<p>c) Know and be able to use the identity $\cosh^2 x - \sinh^2 x \equiv 1$.</p> <p><i>Learners may be asked to derive or use other identities, but no prior knowledge of them is assumed.</i></p> <p><i>[Prior knowledge of other identities is excluded.]</i></p>	

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4.07d	Differentiation and integration		d) Be able to differentiate and integrate hyperbolic functions.	H2
4.07e	Inverse hyperbolic functions		e) Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.	H3 H4
4.07f			f) Be able to derive and use expressions in terms of logarithms for the inverse hyperbolic functions. <i>Includes the notation: $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$ and $\operatorname{arsinh} x$, $\operatorname{arcosh} x$, $\operatorname{artanh} x$.</i>	
4.08 Further Calculus				
4.08a	Maclaurin series		a) Be able to find the Maclaurin series of a function, including the general term.	D5 D6
4.08b			b) Recognise and be able to use the Maclaurin series for e^x , $\sin x$, $\cos x$, $\ln(1+x)$ and $(1+x)^n$, and functions based on these. <i>The interval of validity should be understood.</i> <i>[Proof of the interval of validity and the use of non-real values of x are excluded.]</i>	
4.08c	Improper integrals		c) Be able to evaluate improper integrals where either the integrand is undefined at a value in the range of integration or where the range of integration is infinite. <i>e.g. $\int_0^2 \frac{1}{\sqrt{x}} dx$ or $\int_1^\infty \frac{1}{x^2} dx$.</i>	E1

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4.08d	Volumes of solids of revolution		<p>d) Be able to derive formulae for and calculate volumes of solids of revolution.</p> <p><i>To include solids generated using either coordinate axis as the axis of rotation, and the volume of a solid formed by rotation of a region between two curves.</i></p> <p><i>This includes curves defined parametrically.</i></p>	E2
4.08e	Mean values		<p>e) Understand and be able to evaluate the mean value of a function.</p> <p><i>Includes the use of: mean value = $\frac{1}{b-a} \int_a^b f(x) dx$.</i></p>	E3
4.08f	Partial fractions		<p>f) Be able to integrate using partial fractions.</p> <p><i>See Further Algebra section 4.05c and H240 section 1.02y for permitted forms.</i></p>	E4
4.08g	Inverse trigonometric and hyperbolic functions		<p>g) Be able to derive and use the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\sinh^{-1} x$, $\cosh^{-1} x$ and $\tanh^{-1} x$.</p>	E5
4.08h	Further integration		<p>h) Be able to integrate functions of the form: $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{a^2 + x^2}$, $\frac{1}{\sqrt{x^2 - a^2}}$ and $\frac{1}{\sqrt{x^2 + a^2}}$ and use an appropriate inverse trigonometric or hyperbolic substitution for the evaluation of associated definite or indefinite integrals.</p>	E6 H5

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4.09 Polar Coordinates				
4.09a	Polar coordinates		a) Understand and be able to use polar coordinates (using the convention $r \geq 0$) and be able to convert between polar and cartesian coordinates.	G1
4.09b	Sketching curves		b) Be able to sketch polar curves, with r given as a function of θ . <i>Identify significant features of polar curves such as symmetry, and least and greatest values of r.</i> <i>Includes use of trigonometric functions.</i>	G2
4.09c	Area		c) Be able to find the area enclosed by a polar curve. <i>Be able to use the formula $\frac{1}{2} \int r^2 d\theta$.</i>	G3

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4.10 Differential Equations				
4.10a	General and particular solutions		<p>a) Understand the difference between, and be able to find, general and particular solutions to differential equations.</p> <p><i>Includes understanding that the general solution will include arbitrary constant(s) and that the particular solution may be found from initial or boundary conditions.</i></p>	I2
4.10b	Modelling		<p>b) Be able to use differential equations in modelling in kinematics and in other contexts.</p> <p><i>Includes use of Newton's second law of motion and the language of kinematics including the notation $v = \dot{x} = \frac{dx}{dt}$ and $a = \dot{v} = \ddot{x} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$.</i></p> <p><i>Includes problems involving variable force.</i></p> <p><i>Problems may include formulating differential equations which learners cannot solve analytically.</i></p> <p><i>[Problems involving either variable mass or the form $a = v \frac{dv}{dx}$ are excluded.]</i></p>	I3, I2
4.10c	Integrating factor method for first order differential equations		<p>c) Be able to find and use an integrating factor $e^{\int P(x)dx}$ to solve differential equations of the form $\frac{dy}{dx} + P(x)y = Q(x)$.</p> <p><i>Includes recognising when it is appropriate to do so, rearranging into the given form when necessary.</i></p>	I1

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4.10d	Second order homogeneous differential equations		<p>d) Be able to solve differential equations of the form $y'' + ay' + by = 0$, where a and b are constants, by using the auxiliary equation.</p> <p><i>Includes rearranging into the given form when necessary.</i></p> <p><i>Learners should be able to interpret the sign of the discriminant of the auxiliary equation and how it determines the form of the complementary function.</i></p> <p><i>Including the cases when the roots of the auxiliary equation are:</i></p> <p>(i) <i>distinct and real,</i></p> <p>(ii) <i>repeated,</i></p> <p>(iii) <i>complex.</i></p>	14 16 12
4.10e	Second order non-homogeneous differential equations		<p>e) Be able to solve differential equations of the form $y'' + ay' + by = f(x)$, where a and b are constants, by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential or trigonometric function).</p> <p><i>Includes cases where the form of the complementary function affects the choice of trial integral for the particular integral.</i></p> <p><i>Includes cases where the form of the particular integral is given.</i></p>	15

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...	DfE Ref.
4.10f	Simple harmonic motion		<p>f) Be able to solve the equation for simple harmonic motion (SHM) $\ddot{x} = -\omega^2 x$ and relate the solution to the motion.</p> <p><i>Includes use of the formulae</i></p> <p>$x = A \cos(\omega t) + B \sin(\omega t)$ and $x = R \sin(\omega t + \phi)$ in modelling situations.</p> <p><i>Learners may quote these formulae without proof when not asked to derive it or to solve the SHM equation.</i></p>	17
4.10g	Damped oscillations		<p>g) Be able to model damped oscillations using second order differential equations and interpret their solutions.</p> <p><i>The terms “underdamping”, “overdamping” and “critical damping” should be known and understood informally.</i></p>	18
4.10h	Linear systems		<p>h) Be able to analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled, simultaneous, first order differential equations, and be able to solve them.</p> <p><i>e.g. Predator-prey models, continuous population models, industrial processes.</i></p> <p><i>Includes solution by eliminating one variable to produce a single second order differential equation.</i></p> <p><i>Systems will be of the form</i></p> $\frac{dx}{dt} = ax + by + f(t), \quad \frac{dy}{dt} = cx + dy + g(t)$ <p><i>or easily reducible to this form.</i></p>	19

2d. Content of Statistics (Optional paper Y542)

Introduction to Statistics.

In **Statistics** learners will explore the theory which underlies the statistics content in A Level Mathematics, as well as extending their tool box of statistical concepts and techniques. This area covers probability involving combinatorics, probability distributions for discrete and continuous random variables, hypothesis tests and confidence intervals for a population mean, chi-squared tests, non-parametric tests, correlation and regression.

5.01 Probability

The work on probability in A Level Mathematics is extended to include problems involving arrangements and selections.

5.02 Discrete Random Variables

The general concept of a discrete random variable introduced in A Level Mathematics is further developed, along with the calculation of expectation and variance. The discrete uniform, binomial, geometric and Poisson distributions are studied.

5.03 Continuous Random Variables

The general concept of a continuous random variable introduced in A Level Mathematics is further developed, including probability density functions and cumulative distribution functions. Calculus is used to find expectation, median and quartiles.

5.04 Linear Combinations of Random Variables

Formulae are introduced and applied for linear combinations.

5.05 Hypothesis Tests and Confidence Intervals

The study of hypothesis tests is formalised and developed further, including the central limit theorem. The concept of a confidence interval is introduced, applied in context.

5.06 Chi-squared Tests

The use of a Chi-squared test to test for independence and goodness of fit is explored, including the interpretation of the results.

5.07 Non-parametric Tests

The concept of a non-parametric test is introduced and explored through the Wilcoxon signed-rank tests and the Wilcoxon rank-sum test (or Mann-Whitney U test), and applied in hypothesis tests concerning population median and identity of populations.

5.08 Correlation

The concept of correlation introduced in A Level Mathematics is formalised and explored further, including the study of rank correlation.

5.09 Linear Regression

Regressions lines are calculated and used in context for estimation.

Assumed knowledge

Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. They are also assumed to know the content of Pure Core (Y540 and Y541). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Learners should use spreadsheets or statistical software to generate tables and diagrams, and to perform standard statistical calculations.
2. Hypothesis tests: Learners should use spreadsheets or statistical software to carry out hypothesis tests using the techniques in this paper.
3. Probability: Learners should use random number generators, including spreadsheets, to simulate tossing coins, rolling dice etc, and to investigate probability distributions.
4. Central limit theorem: Learners should use simulations to investigate the central limit theorem, including sampling from a variety of distributions.

- 5 Use of data: Learners are expected to have explored different data sets, using appropriate technology, during the course. No particular data set is expected to be studied, and there will not be any pre-release data.

Hypothesis Tests

Hypotheses should be stated in terms of parameter values (where relevant) and the meanings of symbols should be stated. For example,

" $H_0: p = 0.7$, $H_1: p < 0.7$, where p is the population proportion in favour of the resolution".

Conclusions should be stated in such a way as to reflect the fact that they are not certain. For example, "There is evidence at the 5% level to reject H_0 . It is likely that the mean mass is less than 500g." "There is no evidence at the 2% level to reject H_0 . There is no reason to suppose that the mean journey time has changed."

Some examples of incorrect conclusion are as follows:

" H_0 is rejected. Waiting times have increased."

"Accept H_0 . Plants in this area have the same height as plants in other areas."

2

Content of Statistics (Optional paper Y542)

When this course is being co-taught with AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.01 Probability			
5.01a	Probability	<p>a) Be able to evaluate probabilities by calculation using permutations and combinations.</p> <p><i>Includes the terms "permutation" and "combination".</i></p> <p><i>Includes the notation ${}_nP_r = {}^nP_r$ and ${}_nC_r = {}^nC_r$.</i></p> <p><i>For underlying content on probability see H240 section 2.03.</i></p>	
5.01b		<p>b) Be able to evaluate probabilities by calculation in contexts involving selections and arrangements.</p> <p><i>Selection problems include, for example, finding the probability that 3 vowels and 2 consonants are chosen when 5 letters are chosen at random from the word 'CALCULATOR'.</i></p> <p><i>Arrangement problems only involve arrangement of objects in a line and include:</i></p> <ol style="list-style-type: none"> <i>repetition, e.g. the probability that the word 'ARTIST' is formed when the letters of the word 'STRAIT' are chosen at random.</i> <i>restriction, e.g. the probability that two consonants are (or are not) next to each other when the letters of the word 'TRAITS' are placed in a random order.</i> 	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.02 Discrete Random Variables			
5.02a	Probability distributions for general discrete random variables	<p>a) Understand and be able to use discrete probability distributions.</p> <p><i>Includes using and constructing probability distribution tables and functions relating to a given situation involving a discrete random variable.</i></p> <p><i>Any defined non-standard distribution will be finite.</i></p>	
5.02b		<p>b) Understand and be able to calculate the expectation and variance of a discrete random variable.</p> <p><i>Includes knowing and being able to use the formulae</i> $\mu = E(X) = \sum x_i p_i$ $\sigma^2 = \text{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2.$ <i>[Proof of these results is excluded.]</i></p>	
5.02c		<p>c) Know and be able to use the effects of linear coding on the mean and variance of a random variable.</p>	
5.02d	The binomial distribution	<p>d) Know and be able to use the formulae $\mu = np$ and $\sigma^2 = np(1 - p)$ for a binomial distribution.</p> <p><i>[Proof of these results is excluded.]</i></p> <p><i>For the underlying content on binomial distributions, see H240 sections 2.04b and 2.04c.</i></p>	
5.02e	The discrete uniform distribution	<p>e) Know and be able to use the conditions under which a random variable will have a discrete uniform distribution, and be able to calculate probabilities and the mean and variance for a given discrete uniform distribution.</p> <p><i>Includes use of the notation $X \sim U(n)$ for the uniform distribution over the interval $[1, n]$.</i></p>	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.02f	The geometric distribution	<p>f) Know and be able to use the conditions under which a random variable will have a geometric distribution.</p> <p><i>Includes use of the notation $X \sim \text{Geo}(p)$, where X is the number of trials up to and including the first success.</i></p>	
5.02g		<p>g) Be able to calculate probabilities using the geometric distribution.</p> <p><i>Learners may use the formulae $P(X = x) = (1 - p)^{x-1}p$ and $P(X > x) = (1 - p)^x$.</i></p>	
5.02h		<p>h) Know and be able to use the formulae $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{1-p}{p^2}$ for a geometric distribution.</p> <p><i>[Proof of these results is excluded.]</i></p>	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.02i	The Poisson distribution	<p>i) Understand informally the relevance of the Poisson distribution to the distribution of random events, and be able to use the Poisson distribution as a model.</p> <p><i>Includes use of the notation $X \sim \text{Po}(\lambda)$, where X is the number of events in a given interval.</i></p>	
5.02j		<p>j) Understand and be able to use the formula</p> $P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}.$	
5.02k		<p>k) Be able to calculate probabilities using the Poisson distribution, using appropriate calculator functions.</p> <p><i>Learners are expected to have a calculator with the ability to access probabilities from the Poisson distribution.</i></p> <p><i>[Use of the Poisson distribution to calculate numerical approximations for a binomial distribution is excluded.]</i></p>	
5.02l		<p>l) Know and be able to use the conditions under which a random variable will have a Poisson distribution.</p> <p><i>Learners will be expected to identify which of the modelling conditions [assumptions] is/are relevant to a given scenario and to explain them in context.</i></p>	
5.02m		<p>m) Be able to use the result that if $X \sim \text{Po}(\lambda)$ then the mean and variance of X are each equal to λ.</p>	
5.02n		<p>n) Know and be able to use the result that the sum of independent Poisson variables has a Poisson distribution.</p>	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.03 Continuous Random Variables			
5.03a	Continuous random variables		<p>a) Understand and be able to use the concept of a continuous random variable, a probability density function (p.d.f.) and a cumulative distribution function (c.d.f).</p> <p><i>Includes the normal, continuous uniform and exponential distributions.</i></p> <p><i>Includes understanding informally the link between the exponential and Poisson distributions.</i></p> <p><i>Includes knowing and being able to use the formula for the mean and variance of the continuous uniform and exponential distributions.</i></p> <p><i>For the underlying content on normal distributions, see H240 sections 2.04e, 2.04f and 2.04g.</i></p>
5.03b	Probability density functions		<p>b) Be able to use a probability density function (including where defined piecewise) to solve problems involving probabilities.</p> <p><i>Includes knowing and being able to use $\int_{-\infty}^{\infty} f(x) dx = 1$.</i></p>
5.03c			<p>c) Be able to calculate the mean and/or variance of a distribution using the formulae $\mu = E(X) = \int xf(x) dx$ and</p> $\sigma^2 = \text{Var}(X) = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2.$
5.03d			<p>d) Be able to use the general result $E(g(X)) = \int g(x)f(x) dx$, where $f(x)$ is the probability density function of the continuous random variable X and $g(x)$ is a function of X.</p>

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.03e	Cumulative distribution functions		e) Be able to find and use a cumulative distribution function (including where defined piecewise) to solve problems involving probabilities. <i>Includes being able to use $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$</i>
5.03f			f) Know and be able to use the relationship between the probability density function, $f(x)$, and the cumulative distribution function, $F(x)$, and use either to evaluate the median, quartiles and other percentiles.
5.03g			g) Be able to find and use the cumulative distribution functions of related variables. <i>e.g. Given the c.d.f. of X, find the c.d.f. of Y and hence the p.d.f. of Y where $Y = X^3$.</i>
5.04 Linear Combinations of Random Variables			
5.04a	Linear combinations of any random variables		a) Be able to use the following results, including the cases where $a = b = \pm 1$ and/or $c = 0$: 1. $E(aX + bY + c) = aE(X) + bE(Y) + c$, 2. if X and Y are independent then $\text{Var}(aX + bY + c) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$.
5.04b	Linear combinations of normal random variables		b) Be able to use the following results: 1. if X has a normal distribution then $aX + b$ has a normal distribution, 2. if X and Y have independent normal distributions then $aX + bY$ has a normal distribution.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.05 Hypothesis Tests and Confidence Intervals			
5.05a	The distribution of \bar{X} and the central limit theorem		<p>a) Know that for any randomly and independently selected sample, X_i, of size n taken from a population, then for the sample mean \bar{X}:</p> <ol style="list-style-type: none"> $E(\bar{X}) = \mu$, $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$ and \bar{X} is approximately normally distributed when n is large (approximately $n > 25$). <p><i>[Proof of these results is excluded.]</i></p>
5.05b	Unbiased estimates of population mean and variance		<p>b) Know that unbiased estimates of the population mean and variance are given by $\frac{\sum x}{n}$ and $\frac{n}{n-1} \left(\frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2 \right)$ respectively.</p> <p><i>[Proof of these results is excluded.]</i> <i>Only an informal understanding of “unbiased” is required.</i></p>
5.05c	Using the normal distribution in hypothesis tests		<p>c) Be able to use a normal distribution to carry out a hypothesis test for a population mean in the following cases:</p> <ol style="list-style-type: none"> a sample drawn from a normal population of known, given or assumed variance, a large sample drawn from any population with known, given or assumed variance, a large sample, drawn from any population with unknown variance.
5.05d	Confidence intervals		<p>d) Be able to use a normal distribution to find a confidence interval for a population mean in each of the above cases.</p>

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.06 Chi-squared Tests			
5.06a	Contingency tables	<p>a) Be able to use a chi-squared (χ^2) test with the appropriate number of degrees of freedom to test for independence in a contingency table and interpret the results of such a test.</p> <p><i>Rows or columns, as appropriate, should be combined so that each expected frequency is at least 5, and Yates' correction should be used in the special case of a 2×2 table.</i></p> <p><i>A table of critical values of the χ^2 distribution will be provided.</i></p> <p><i>Includes calculation of expected frequencies and contributions to the test statistic.</i></p> <p><i>Questions may require candidates to calculate some expected frequencies and contributions to the test statistic, but will not involve lengthy calculations.</i></p>	
5.06b 5.06c	Fitting a theoretical distribution	<p>b) Be able to fit a theoretical distribution, as prescribed by a given hypothesis involving a given ratio, proportion or discrete uniform distribution, to given data.</p> <p><i>Questions may require candidates to calculate some expected frequencies, but will not involve lengthy calculations.</i></p>	<p>c) Extend their knowledge of fitting distributions to other known or given discrete and continuous distributions.</p> <p><i>Questions may require candidates to calculate some expected frequencies, but will not involve lengthy calculations.</i></p>
5.06d	Goodness of fit test	<p>d) Be able to use a χ^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit test.</p> <p><i>Where necessary, adjacent classes should be combined so that each expected frequency is at least 5.</i></p> <p><i>A table of critical values of the χ^2 distribution will be provided.</i></p>	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.07 Non-parametric tests			
5.07a	Non-parametric tests		a) Understand what is meant by a non-parametric hypothesis test, appreciate situations where such tests are useful and be able to select an appropriate test.
5.07b	The basis of non-parametric tests		b) Understand the basis of sign tests, the Wilcoxon signed-rank test and the Wilcoxon rank-sum test (also known as the Mann-Whitney U test). <i>Tables of critical values of T and W will be provided. Learners should know the notation W_+ and W_-.</i>
5.07c	Single-sample hypothesis tests		c) Be able to test a hypothesis concerning a population median using a single-sample sign test and a single-sample Wilcoxon signed-rank test. <i>[Problems in which observations coincide with the hypothetical population median are excluded.]</i>
5.07d 5.07e	Paired-sample and two-sample hypothesis tests		d) Understand the difference between a paired-sample test and a two-sample test, and be able to select the appropriate form when solving problems. e) Be able to test for medians or identity of population as appropriate, using a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and (for unpaired samples) a Wilcoxon rank-sum test. <i>[Problems involving tied ranks are excluded.]</i>

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.07f	Normal approximations		<p>f) Be able to carry out tests using the Wilcoxon signed-rank test and the Wilcoxon rank-sum test for large samples using the approximations:</p> <p>Wilcoxon signed-rank test</p> $T \sim N\left(\frac{1}{4}n(n+1), \frac{1}{24}n(n+1)(2n+1)\right)$ <p>Wilcoxon rank-sum test (samples of sizes m and n, with $m \leq n$)</p> $W \sim N\left(\frac{1}{2}m(m+n+1), \frac{1}{12}mn(m+n+1)\right).$ <p><i>Includes the use of continuity corrections.</i></p>
5.08 Correlation			
5.08a	Pearson's product-moment correlation coefficient	<p>a) Be able to calculate the product-moment correlation coefficient (pmcc) for a set of bivariate data; raw data or summarised data may be given.</p> <p><i>Use of appropriate calculator functions is expected.</i></p> <p><i>Learners will not be required to enter large amounts of data into a calculator during the examination.</i></p>	
5.08b		<p>b) Understand that the value of a correlation coefficient is unaffected by linear coding of the variables.</p>	
5.08c		<p>c) Understand Pearson's product-moment correlation coefficient as a measure of how close data points lie to a straight line.</p>	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.08d	Hypothesis tests using Pearson's product-moment correlation coefficient	<p>d) Use and be able to interpret Pearson's product-moment correlation coefficient in hypothesis tests, using either a given critical value, or a p-value and a table of critical values.</p> <p><i>When using Pearson's coefficient in a hypothesis test, the data may be assumed to come from a bivariate normal distribution.</i></p> <p><i>A table of critical values of Pearson's coefficient will be provided.</i></p>	
5.08e	Spearman's rank correlation coefficient	<p>e) Be able to calculate Spearman's rank correlation coefficient for a maximum of 10 pairs of data values or ranks.</p> <p><i>Includes being able to draw basic conclusions about the meaning of a value of the coefficient in relation to the ranks before, or without, carrying out a hypothesis test.</i></p> <p><i>Includes understanding the conditions under which the use of rank correlation may be appropriate.</i></p> <p><i>[Tied ranks are excluded.]</i></p>	
5.08f	Hypothesis tests using Spearman's coefficient	<p>f) Be able to carry out a hypothesis test for association in a population.</p> <p><i>Includes understanding that this is a non-parametric test, as it makes no assumptions about the population.</i></p> <p><i>Tables of critical values of Spearman's coefficient will be provided.</i></p>	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.08g	Comparison of coefficients	g) Be able to choose between Pearson's product-moment correlation coefficient and Spearman's rank correlation coefficient for a given context. <i>Includes interpreting a scatter diagram and distinguishing between linear correlation and association.</i>	
5.09 Linear Regression			
5.09a	Dependent and independent variables	a) Understand the difference between an independent (or controlled) variable and a dependent (or response) variable. <i>Includes appreciating that, in a given situation, neither parameter may be independent.</i>	
5.09b 5.09c 5.09d	Calculation of the equation of the regression line	b) Understand the concepts of least squares and regression lines in the context of a scatter diagram. c) Be able to calculate, both from raw data and from summarised data, the equation of the regression line of y on x , where the independent variable (if any) is x . <i>[The regression line of x on y is excluded in the case when x is independent.]</i> d) Understand the effect on a regression line of linear coding on one or both variables.	
5.09e	Use of the regression line	e) Be able to use, in the context of a problem, the regression line of y on x to estimate a value of y , and be able to interpret in context the uncertainties of such an estimate.	

2e. Content of Mechanics (Optional paper Y543)

Introduction to Mechanics.

In **Mechanics** learners extend their knowledge of particles, kinematics and forces from A Level Mathematics, using their extended pure mathematical knowledge to explore more complex physical systems. The area covers dimensional analysis, work, energy, power, impulse, momentum, centres of mass, circular motion and variable force.

6.01 Dimensional Analysis

The relationships between physical quantities are analysed by considering their dimensions (length, mass and time), in order to construct or check models.

6.02 Work, Energy and Power

The fundamental concepts of work, energy and power are introduced, including kinetic energy, gravitational potential energy and elastic potential energy. The principle of conservation of mechanical energy is used to solve problems.

6.03 Impulse and Momentum

Problems involving collisions in 1-D and 2-D are studied, using the principle of conservation of linear momentum and Newton's experimental law.

6.04 Centre of Mass

Rigid bodies are modelled as particles at their centres of mass. Techniques for finding the centre of mass of a body or system of bodies are explored, including integration.

6.05 Motion in a circle

The motion of a particle in a horizontal or vertical circle is explored, including motion which is not restricted to a circular path.

6.06 Further Dynamics and Kinematics

The techniques of differential equations studied in Pure Core are extended to linear motion under a variable force.

Assumed knowledge

Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. They are also assumed to know the content of Pure Core (Y540 and Y541). All of this content is assumed, but

will only be explicitly assessed where it appears in this section.

Resolving forces

The technique of **resolving forces** is found in 'Stage 2' of the A Level mathematics content, and therefore 'Stage 1' learners may not have learned this technique yet. As it is a vital underlying skill in the more advanced mechanics topics met in this paper, it is taken as assumed knowledge for 'Stage 1'. This includes both being able to express a force as two mutually perpendicular components, and being able to find the resultant of two or more forces acting at a point. See sections 6.02b, 6.02l and 6.05c.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Spreadsheets: Learners should use spreadsheets to generate tables of values for functions and to investigate functions numerically.
2. Learners should use graphing software for modelling, including kinematics and projectiles, and in visualising physical systems.
3. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including derivatives and integrals, and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.
4. Complex problem solving: Learners could use CAS to perform computation when solving complex problems in mechanics, including those which lead to equations or systems that they cannot solve analytically.
5. Practical mechanics: Learners could use computers and/or mobile phones to enrich practical mechanics tasks, using them for data logging, to create videos of moving objects, or to share and analyse data.

Content of Mechanics (Optional paper Y543)

When this course is being co-taught with AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.01 Dimensional Analysis			
6.01a	Dimensional analysis	<p>a) Be able to find the dimensions of a quantity in terms of M, L and T, and understand that some quantities are dimensionless.</p> <p><i>Includes understanding and using the notation $[d]$ for the dimension of the quantity d.</i></p> <p><i>Learners are expected to know or be able to derive the dimensions of any quantity for which they know the units. Dimensions of other quantities will be given, or their derivation will be the focus of assessment.</i></p>	
6.01b		b) Understand and be able to use the relationship between the units of a quantity and its dimensions.	
6.01c		<p>c) Be able to use dimensional analysis as an error check.</p> <p><i>e.g. Verify the relationship that power is proportional to the product of the driving force and the velocity.</i></p>	
6.01d		<p>d) Be able to use dimensional analysis to determine unknown indices in a proposed formulation.</p> <p><i>e.g. Determine the period of oscillation of a simple pendulum in terms of its length, mass and the acceleration due to gravity, g.</i></p>	
6.01e		e) Be able to formulate models and derive equations of motion using a dimensional argument.	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.02 Work, Energy and Power			
6.02a 6.02c	Work	a) Understand the concept of work done by a force.	c) Be able to calculate the work done by a constant force in two dimensions using vectors ($\mathbf{F} \cdot \mathbf{x}$) or by a variable force $\left(\int F dx\right)$ in one dimension only.
6.02b		b) Be able to calculate the work done by a constant force. <i>The force may not act in the direction of motion of the body and so learners will be expected to resolve forces in two dimensions.</i>	
6.02d 6.02f	Energy	d) Understand the concept of the mechanical energy of a body. <i>i.e. The kinetic and potential energy.</i>	f) Be able to calculate the kinetic energy of a body using the scalar product $\frac{1}{2} m \mathbf{v} \cdot \mathbf{v}$ <i>Learners may be expected to use the formula $\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2 \mathbf{a} \cdot \mathbf{x}$, in solving a variety of problems, for example in calculating the kinetic energy of a body.</i>
6.02e		e) Be able to calculate the gravitational potential energy (mgh) and kinetic energy $\left(\frac{1}{2} mv^2\right)$ of a body.	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.02g 6.02h	Hooke's law		<p>g) Understand and be able to use Hooke's law, in the form $T = \frac{\lambda x}{l}$, for elastic strings and springs. <i>Includes an informal understanding of when Hooke's law does not apply.</i></p> <p>h) Be able to calculate the elastic potential energy $\left(E = \frac{\lambda x^2}{2l} \right)$ stored in a string or spring. <i>Learners will be expected to state the formula for the elastic potential energy stored in a string or spring unless they are explicitly asked to derive it.</i></p>
6.02i 6.02j	Conservation of energy	<p>i) Understand and be able to use the principle of the conservation of mechanical energy and the work-energy principle for dynamic systems, including consideration of energy loss.</p>	<p>j) Extend their knowledge of the principle of the conservation of mechanical energy and the work-energy principle to systems which include elastic strings or springs.</p>
6.02k 6.02l 6.02m	Power	<p>k) Understand and be able to use the definition of power (the rate at which a force does work). <i>Includes average power = $\frac{\text{work done}}{\text{time elapsed}}$.</i></p> <p>l) Be able to use the relationship between power, the tractive force and velocity ($P = Fv$) to solve problems. <i>e.g. Motion on an inclined plane.</i> <i>Includes maximum velocity and speed.</i> <i>Learners will be required to resolve forces in two dimensions.</i></p>	<p>m) Be able to calculate the power associated with a variable force in two dimensions using the scalar product $P = \mathbf{F} \cdot \mathbf{v}$</p>

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.03 Impulse and Momentum			
6.03a 6.03c 6.03b 6.03d	Linear momentum	<p>a) Recall and be able to use the definition of linear momentum in one dimension.</p> <p>b) Understand and be able to apply the principle of conservation of linear momentum in one dimension applied to two particles.</p> <p><i>Includes using the formula $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$.</i></p>	<p>c) Recall and be able to use the definition of momentum in two dimensions including the vector form $m\mathbf{v}$.</p> <p>d) Understand and be able to apply the principle of conservation of linear momentum in two dimensions applied to two particles.</p> <p><i>Includes using the vector form $m_1 \mathbf{u}_1 + m_2 \mathbf{u}_2 = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$.</i></p>
6.03e 6.03f 6.03g	Impulse	<p>e) Understand and be able to use the concept of the impulse imparted by a force.</p> <p>f) Be able to use the relationship between the instantaneous impulse of a force and the change in momentum ($I = mv - mu$).</p> <p><i>The instantaneous impulse is the impulse associated with an instantaneous change in velocity.</i></p> <p><i>Learners will only be required to apply this to instantaneous events in one dimension.</i></p> <p><i>e.g.</i> <i>The direct impact of two smooth spheres.</i> <i>An impulsive force acting in the direction of an inelastic string.</i></p> <p><i>Questions involving collision(s) between particles may include multiple collisions and the conditions under which further collisions occur.</i></p>	<p>g) Understand and be able to apply the impulse – momentum principle in two dimensions including the vector form $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$.</p> <p><i>e.g.</i> <i>The oblique impact of two smooth spheres.</i> <i>A smooth sphere with a fixed plane surface.</i> <i>An impulsive force acting at an angle to an inelastic string.</i></p>
6.03h			<p>h) Understand and be able to apply the impulse – momentum principle for a constant force expressed as force \times time or for a variable force in one dimension only as $\int F dt$.</p>

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.03i	Restitution	i) Recall and be able to use the definition of the coefficient of restitution, including $0 \leq e \leq 1$. <i>[Superelastic collisions are excluded.]</i>	
6.03j		j) Understand and be able to use the terms “perfectly elastic” ($e = 1$) and “inelastic” ($e = 0$) for describing collisions. <i>Learners should know that for perfectly elastic collisions there will be no loss of kinetic energy and for inelastic collisions the bodies coalesce and there is maximum loss of kinetic energy.</i>	
6.03k		k) Recall and be able to use Newton’s experimental law in one dimension for problems of direct impact. <i>e.g. Between two smooth spheres ($v_1 - v_2 = -e(u_1 - u_2)$) and a smooth sphere with a fixed plane surface ($v = -eu$), where u and v are the velocities before and after impact.</i>	
6.03l			

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.04 Centre of Mass			
6.04a	Centre of mass		<p>a) Understand and be able to apply the principle that the effect of gravity is equivalent to a single force acting at the body's centre of mass.</p> <p><i>Includes understanding that, in terms of linear motion, a rigid body may be modelled by a particle of the same mass at its centre of mass.</i></p>
6.04b			<p>b) Be able to find the position of the centre of mass of a uniform rigid body using symmetry, for example a rectangular lamina.</p>
6.04c			<p>c) Be able to determine the centre of mass of a system of particles or the centre of mass of a composite rigid body.</p> <p><i>Questions may involve any of the rigid bodies listed in the Formulae Booklet, but will be limited to compound shapes such as a uniform L-shaped lamina or a hemisphere abutting a cylinder with a common axis.</i></p> <p><i>Includes composition by addition or subtraction, for example a rectangular lamina with a semicircle attached to one side, or a rectangular lamina with a semicircle removed.</i></p>
6.04d			<p>d) Be able to use integration to determine the position of the centre of mass of a uniform lamina or a uniform solid of revolution.</p>
6.04e	Rigid bodies		<p>e) Be able to solve problems involving the equilibrium of a single rigid body under the action of coplanar forces.</p> <p><i>e.g. Suspension of a rigid body from a given point or problems involving the toppling or sliding of a rigid body placed on an inclined plane.</i></p> <p><i>May include rigid bodies which are hinged to a surface.</i></p> <p><i>[Hinged bodies are excluded.]</i></p>

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.05 Motion in a Circle			
6.05a	Uniform motion in a circle	a) Understand and be able to use the definitions of angular velocity, velocity, speed and acceleration in relation to a particle moving in a circular path, or a point rotating in a circle, with constant speed. <i>Includes the use of both ω and $\dot{\theta}$.</i>	
6.05b		b) Be able to use and apply the relationships $v = r\dot{\theta}$ and $a = \frac{v^2}{r} = r\dot{\theta}^2 = v\dot{\theta}$ for motion in a circle with constant speed.	
6.05c		c) Be able to solve problems regarding motion in a horizontal circle. <i>e.g.</i> <i>Motion of a conical pendulum.</i> <i>Motion on a banked track.</i> <i>Problems will be restricted to those involving constant forces but learners will be required to resolve forces in two dimensions.</i>	
6.05d 6.05e	Motion in a vertical circle	d) Understand the motion of a particle in a circle with variable speed. <i>In 'Stage 1' Learners will be expected to use energy considerations to calculate the speed of a particle at a given point on a circular path.</i>	e) Extend their understanding of the motion of a particle in a circle with variable speed to include the radial and tangential components of the acceleration.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
6.05f			<p>f) Be able to solve problems involving motion round a vertical circle including motion which is not restricted to a circular path.</p> <p><i>This is restricted to a combination of motion in a circle and free fall.</i></p> <p><i>e.g. The subsequent motion of a particle moving on the outside of a smooth circular surface.</i></p> <p><i>The motion of a particle on a string moving in a vertical circle and then as a projectile.</i></p>
6.06 Further Dynamics and Kinematics			
6.06a	Linear motion under a variable force		<p>a) Be able to use $a = \frac{dv}{dt}$ or $a = v \frac{dv}{dx}$ to model the linear motion of a particle under the action of a variable force in one dimension only.</p> <p><i>Learners will be required to solve problems in which the corresponding differential equation can be solved by either the method of separation of variables or an integrating factor.</i></p>