SDSE Homework 1

Due date: February 8th, 2022

Problem 1 (15 points)

Prove whether the following are valid PDFs.

(a) $p(x) = \frac{1}{2}\cos(x) \quad , \quad \Omega = [-\pi/2, \pi/2]$

(b) $p(x) = \dfrac{1}{1+e^{-x}} \quad , \quad \Omega = \mathbb{R}$

(c) $p(x) = \frac{1}{3}\,x^3 \quad , \quad \Omega = [0, 1]$

(d) $p(x) = 1/x \quad , \quad \Omega = \mathbb{N}$

(e) $p(x) = 2^{-x} \quad , \quad \Omega = \mathbb{N}$

Problem 2 (4+4+4+1 points)

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x_1, x_2) = \frac{x_1^3}{3} - 4x_1 + \frac{x_2^3}{3} - 16x_2$$

(a) Find the set of stationary points of f.

(b) Determine which of these stationary points are local maxima and local minima.

(c) Plot the function using matplolib. Indicate the stationary points and extrema of the function.

(d) What is the solution of the problem: maximize $f(x_1, x_2)$ over $\mathbb{R}^2$?

Problem 3 (2+2+3+3 points)

A study of washing machines found their lifetime to follow an exponential distribution:

$$p_T(t) = 0.1e^{-0.1t}$$

with t measured in years.

(a) What is the mean lifetime of washing machines?

(b) What is the standard deviation?

(c) What percentage of washing machine are expected to fail in 10 years?

(d) What is the median life of washing machines?

Problem 4 (2+3+3 points)

Measurements are made on the length L and width W of a rectangular component. Assume that $\Omega_L = [9.95, 10.05]$ and $p_L(l) = 10$, and $\Omega_W = [4.9, 5.1]$ and $p_W(w) = 5$. Assume that L and W are independent.

(a) Find $Prob(L < 9.98)$

(b) Draw the region in the LW plane represented by $(L \in [9.96, 9.98] \ \wedge \ W \in [5.0, 5.05])$ and compute its probability.

(c) Draw the region in the LW plane represented by $(L \in [9.96, 9.98] \ \vee \ W \in [5.0, 5.05])$ and compute its probability.

Problem 5 (3 points)

Certain pipes are specified to have a roughness coefficient of between 0.2 and 0.3. The manufacturing process is known to produce pipes with normally distributed roughnesses with mean 0.25 and standard deviation of 0.03. What percentage of pipes is expected to meet the specification? Hint: `stats.norm`.