

# week6\_lec1

## Main Ideas

- Edit Distance

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Problem : Two strings  $A$  of length  $m$  and  $B$  of length  $n$ . Transform  $A$  into  $B$  with minimum number of operations. The operations are:

1. Delete a character from  $A$ .
2. Insert a character into  $A$ .
3. Change some character in  $A$  into a new character.

The Edit Distance between two strings is the minimal number of operations required to convert one string to the other.

## Dynamic Programming Solution

We can use dynamic programming to solve this.

**Input:** Two text strings  $A$  of length  $m$  and  $B$  of length  $n$ .

Before going to a solution, let us consider the possible operations for converting  $A$  into  $B$ .

1. If  $m > n$ , we need to remove some characters of  $A$ .
2. If  $m == n$ , we need to convert some characters of  $A$ .
3. If  $m < n$ , we need to insert some characters in  $A$ .

We generate the recursive formulation of the problem.

Let,  $T(i, j)$  represents the minimum cost required to transform first  $i$  characters of  $A$  to first  $j$  characters of  $B$ . That means,  $A[1...i] \text{ to } B[1...j]$   $T(i, j) = \min(1 + T(i - 1, j); 1 + T(i, j - 1); T(i - 1, j - 1) + \text{diff}(i, j))$

$\text{diff}(i, j)$  is 0 if the characters are equal and 1 otherwise

DP table is

			$j - 1$	$j$			$n$
$i - 1$							
$i$							
$m$							GOAL

We have the following cases

- If we insert  $i^{th}$  character in  $A$ , then convert these  $i$  characters of  $A$  to  $j$  characters of  $B$ .
- If we delete  $i^{th}$  character from  $A$ , then we have to convert remaining  $i - 1$  characters of  $A$  to  $j$  characters of  $B$ .
- If  $A[i] == B[j]$ , then we have to convert the remaining  $i-1$  characters of  $A$  to  $j - 1$  characters of  $B$ .
- If  $A[i] \neq B[j]$ , then we have to replace  $i^{th}$  character of  $A$  to  $j^{th}$  character of  $B$  and convert remaining  $i - 1$  characters of  $A$  to  $j - 1$  characters of  $B$ .

### Subproblems :

Range of  $i$  :  $1 \leq i \leq m$

Range of  $j$  :  $1 \leq j \leq n$

No. of subproblems :  $m*n$

Time Complexity for each subproblem :  $O(1)$

Total Time Complexity :  $O(mn)$

Space Complexity :  $O(mn)$  where  $m$  is number of rows and  $n$  is number of columns in the given matrix.