Advanced Fluids: Wind Tunnel Project

ENME 5910

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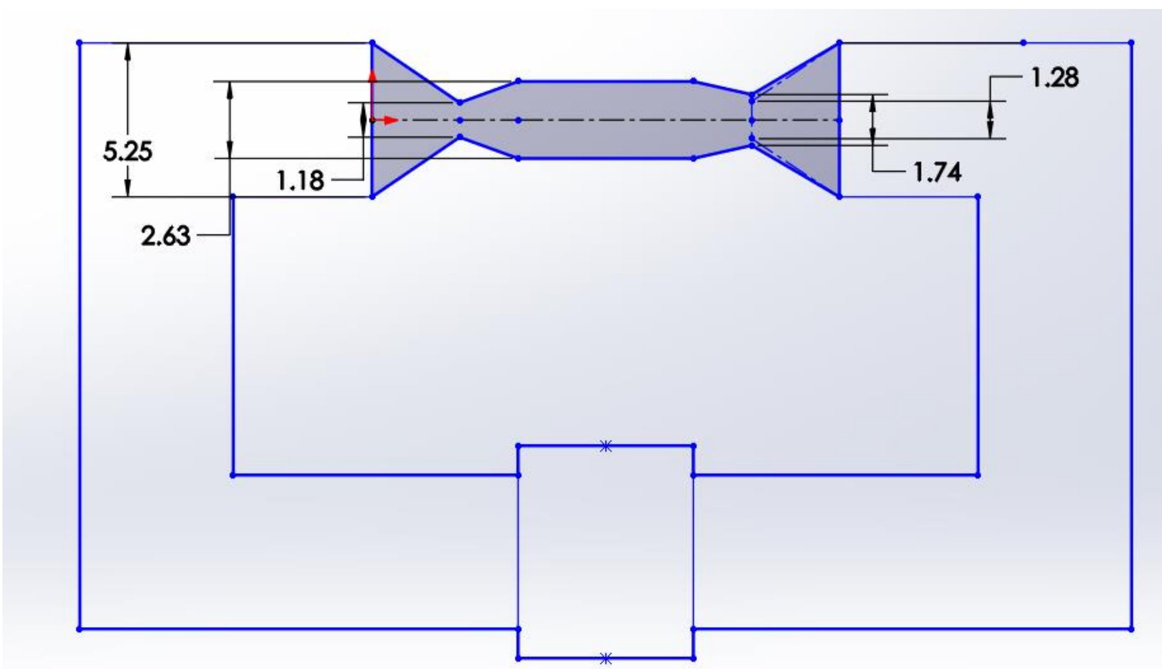

Process:

- Solved for Mass flow rate using Static Pressure and Temperature at a given at altitude.

$$m = \rho v a$$

- Solved for total pressure and total temperature in test section

$$T_o = T * \left(1 + \frac{\gamma - 1}{2} * Mach^2\right)$$

$$P_o = P * (1 + \frac{\gamma - 1}{2} * Mach^2)^{\frac{\gamma}{\gamma - 1}}$$

- Solved for the diameter of the nozzle

$$d = \sqrt{\frac{4 * m * \sqrt{T_o}}{\pi * P_o * \sqrt{\frac{\gamma}{R} * (\frac{2}{\gamma + 1})^{\frac{\gamma+1}{\gamma-1}}}}}$$

- Solved for Pressure ratio across shock

$$\frac{P_{o3}}{P_{o1}} = (\frac{((\gamma + 1) * Mach^2)}{((\gamma - 1) * Mach^2 + 2)})^{\frac{\gamma}{\gamma - 1}} * (\frac{(\gamma + 1)}{(2 * \gamma * Mach^2 - (\gamma - 1))})^{\frac{1}{\gamma - 1}}$$

- Solve for 10 percent loss of total pressure across test section

$$P_{o2} = .9 * P_{01}$$

- Solved for area diffuser at start up

$$A_{du,su} = (\frac{P_{o1}}{P_{03}})_{shock} * Area_{nozzle}$$

- Solved for diameter diffuser at start up

$$d_{du,su} = \sqrt{\frac{4}{\pi} * A_{du,su}}$$

- Found available diameter that meets .3 ft travel requirements

$$(\frac{d}{.0914m})_{round} * .0914$$

- Solved for area at diffuser at steady state

$$A_{du,ss} = (\frac{P_{o1}}{P_{02}})_{loss} * Area_{nozzle}$$

- Found available diameter that meets .3 ft travel requirements

$$(\frac{d}{.0914m})_{round} * .0914$$

- Found the area ratio at diffuser throat between available and ideal

$$\frac{A_{diffuser\_ideal}}{A_{diffuser\_avail}}$$

- Solved for Mach number at $Area_{ratio}$

$$(Area_{ratio})^2 = \left(\frac{1}{Mach^2}\right) * (\left(\frac{2}{\gamma + 1}\right) * \left(1 + \frac{\gamma - 1}{2} * Mach^2\right))^{\frac{\gamma+1}{\gamma-1}}$$

- Solve for the Mach number after shock in the throat

$$Mach\ after\ throat_2^2 = \frac{1 + \frac{\gamma - 1}{2} * Mach_1^2}{\gamma * Mach_1^2 - \frac{\gamma - 1}{2}}$$

- Solved for total pressure across shock in throat

$$\frac{P_{oas}}{P_{obs}} = (\frac{\left((\gamma + 1) * Mach^2\right)}{\left((\gamma - 1) * Mach^2 + 2\right)})^{\frac{\gamma}{\gamma - 1}} * (\frac{(\gamma + 1)}{(2 * \gamma * Mach^2 - (\gamma - 1))})^{\frac{1}{\gamma - 1}}$$

- Found total pressure after shock

$$\frac{P_{oas}}{P_{obs}} = (\frac{\left((\gamma + 1) * Mach^2\right)}{\left((\gamma - 1) * Mach^2 + 2\right)})^{\frac{\gamma}{\gamma - 1}} * (\frac{(\gamma + 1)}{(2 * \gamma * Mach^2 - (\gamma - 1))})^{\frac{1}{\gamma - 1}} * P_{02}$$

- Found change in total pressure

$$\Delta P_o = P_{o2} - P_{as}$$

- Using area ratio between the available throat area and given exit area solved for Mach number at exit of the diffuser

$$(Area_{ratio,du,exit})^2 = \left(\frac{1}{Mach^2}\right) * (\left(\frac{2}{\gamma + 1}\right) * \left(1 + \frac{\gamma - 1}{2} * Mach^2\right))^{\frac{\gamma + 1}{\gamma - 1}}$$

- Found Mach number at the exit of the diffuser to determine temperature at exit of the diffuser

$$T_{du,ss,exit} = \left(\frac{T_0}{1 + \frac{\gamma - 1}{2} * Mach^2}\right)$$

- Solved for the velocity at the exit of the diffuser

$$V = Mach_{du,exit,ss} * \sqrt{\gamma * R * T_{du,ss,exit}}$$

- Using area ratio determined Mach number entering at opening before nozzle

$$(Area_{ratio,n,enter})^2 = \left(\frac{1}{Mach^2}\right) * (\left(\frac{2}{\gamma + 1}\right) * \left(1 + \frac{\gamma - 1}{2} * Mach^2\right))^{\frac{\gamma + 1}{\gamma - 1}}$$

- Solved for temperature at nozzle entrance

$$T_{N,ss,enter} = \left(\frac{T_0}{1 + \frac{\gamma - 1}{2} * Mach^2}\right)$$

- Solved for work of compressor

$$W = m * (\frac{\gamma R}{(\gamma - 1)} \frac{* \left(\frac{T_{N,ss,enter} + T_{du,ss,exit}}{2}\right)}{1000})$$

- Solved for total cost to run wind tunnel

$$Cost = W * \frac{cost}{Kwh} * H$$

Givens:
Mach number = 2.4
Gamma = 1.4
Gas Constant =287.05 M^2/S^2*K
Diameter= 1.8288 M
Static Temperature = 228.8 K
Static Pressure= 30150.17 Pa

Questions	S.I units	English units
The cost to run an isothermal compressor?	$64,888	$64,888
The loss in total pressure in the diffuser at steady state?	7117.1 Pa	1.0322 PSI
The Mach number and velocity at the exit of the diffuser at steady state?	.1435 Mach number	.1435 Mach number
	63.7142 meter/second	209.0360 feet/second
The diameter of the nozzle throat?	1.1797 meter	3.87 feet
The mass flow rate through the system?	877.5675 kg/s	1935.0 lbm/s
The minimum diameter of the diffuser throat at start up	1.7374 meters	5.7 feet
The area of the diffuser throat during steady state?	1.2870 meter^2	13.8544 feet^2

Solution:

Source Code

```
clear, clc

%global values
global gamma;
global Ae;
global a_star_ratio_diffuser
global A_nozzle;
global A_Diffuser_ss_avail

%givens
Mach_testsection=input("please enter a mach number in test
section");
Temp_testsection=input("please enter a static tempature in
kelvin that corolates height at atmospheric conditions");
Pressure_testsection=input("please enter a static pressure
in pa that corolates height at atmospheric conditions");
gas_constant=input("enter gas constant value in
m^2/s^2*K");
gamma=input("enter desired specific heat ratio");
D=input("please enter a value for the Diameter of the test
section in meters");
price=.15;
h=1;

% known Areas
Aenter=((pi/4)*(1.8288)^2);
Ae=2*((pi/4)*(1.8288)^2);
disp("")
disp("")

%solving for total pressure ratio in test section
Total_pressure_ratio_testsection=(((gamma+1)*Mach_testsecti
on^2)/((gamma-1)*Mach_testsection^2+2))^((gamma)/(gamma-
1))*(((gamma+1)/(2*gamma*Mach_testsection^2-(gamma-
1))))^((1)/(gamma-1));
disp("this is the ratio of total pressures")
disp(Total_pressure_ratio_testsection)
disp("");
disp("")
```

```
%solving for total tempature value throughout wind tunnel
in kelvin
To=Temp_testsection*(1+(((gamma-1)/2)*Mach_testsection^2));
disp("this is the total temp in the test section in
kelvin")
disp(To)
disp("")
disp("")

%solving for total pressure valvue in test section in pa
Po1=Pressure_testsection*(1+(((gamma-
1)/2)*Mach_testsection^2))^((gamma/(gamma-1)));
disp("this is the total pressure in the test section")
disp(Po1)
disp("")
disp("")

%solving for the mass flow rate in kg/s
m_dot=((Pressure_testsection)/(gas_constant*Temp_testsectio
n))*Mach_testsection*sqrt(gamma*gas_constant*Temp_testsecti
on)*(pi/4)*(D)^2;
disp("this is the mass flow rate in kg/s")
disp(m_dot)
disp("")
disp("")

%solving for the mass flow rate in lbm/s the value of 2.205
is a standard conversion factor between kg/s and lbm/s
m_dot_english=m_dot*2.205;
disp("this is the mass flow rate in lbm/s")
disp(m_dot_english)
disp("")
disp("")

%solving initially for the A_nozzle in meters and then
converting to feet
%and inches
A_nozzle=(m_dot*sqrt(To))/(Po1*sqrt((gamma/gas_constant)*((
2/(gamma+1))^((gamma+1)/(gamma-1)))));
A_nozzle_english_in=A_nozzle*3.281*3.281*144;
A_nozzle_english_ft=A_nozzle*3.281*3.281;


%Solving for Diamter of nozzle in meters
```

```
D_nozzle=sqrt(A_nozzle*(4/pi));
disp("this is the nozzle diameter in meters")
disp(D_nozzle)
disp("")
disp("")

%Solving for Diamter of nozzle in ft
D_nozzle_english=D_nozzle*3.281;
disp("this is the diameter of the nozzle in feet")
disp(D_nozzle_english)
disp("")
disp("")


%change in pressure caused by oblique shocks in test
section
Po2_ss=.90*Po1;
disp("at Steady state the 10% decrease in total pressure is
equal to")
disp(Po2_ss)
disp("")
disp("")

%solving for Po after shock at steady state
disp("this is total pressure ratio across shock in throat
at steady state")
Po_after_shock=Po2_ss*Total_pressure_ratio_testsection;
disp(Po_after_shock)

%solving for area of diffuser at start up conditions in m^2
A_Diffuser_su=(Po1/Po_after_shock)*A_nozzle;
disp("this is the diffuser area in meters for startup")
disp(A_Diffuser_su)
disp("")
disp("")

%solving for diameter of diffuser at start up conditions in
meters
disp("this is the diffuser diamter in meters for startup")
D_Diffuser_su=sqrt(A_Diffuser_su*(4/pi));
disp(D_Diffuser_su)
disp("")
disp("")
```

```
%solving for diamter of diffuser at start up conditions in
ft
disp("this is the diffuser diamter in feet for startup")
D_Diffuser_english_su=D_Diffuser_su*3.281;
disp(D_Diffuser_english_su)
disp("")
disp("")

%solving for availiable diameter with travel in feet
disp("this is the diameter in feet with .3ft travel in ft")
D_Diffuser_english_su_avail=ceil(D_Diffuser_english_su/.3)*
.3;
disp(D_Diffuser_english_su_avail)
disp("")
disp("")

%solving for availiable diameter with travel in meter
disp("this is the diameter in meters with .3ft travel in
meters")
D_Diffuser_su_avail=ceil(D_Diffuser_su/.09144)*.09144;
disp(D_Diffuser_su_avail)
disp("")
disp("")

%solving for the availiable area in ft^2
disp("this is the area availiable to meet travel .3
requierments in feet")
A_Diffuser_su_english_avail=(pi/4)*(D_Diffuser_english_su_a
vail)^2;
disp(A_Diffuser_su_english_avail)
disp("")
disp("")

%solving for the availible area in m^2
disp("this is the area availiable to meet travel .3
requierments in meters")
A_Diffuser_su_avail=A_Diffuser_su_english_avail/(3.281*3.28
1);
disp(A_Diffuser_su_avail)
disp("")
disp("")

%solving for area of Diffuser at steady state in m^2
A_Diffuser_ss=(A_nozzle)*(Po1/Po2_ss);
```

```
disp("this is the area of the diffuser in m^2")
disp(A_Diffuser_ss);
disp("")
disp("")

%solving for area of Diffuser at steady state in in^2
disp("this is the area of the diffuser in in^2")
A_Diffuser_english_ss_in=A_Diffuser_ss*3.281*3.281*144;
disp(A_Diffuser_english_ss_in)
disp("")
disp("")

%solving for area of Diffuser at steady state in ft^2
disp("this is the area of the diffuser in ft^2")
A_Diffuser_english_ss=A_Diffuser_ss*3.281*3.281;
disp(A_Diffuser_english_ss)
disp("")
disp("")

%solving for diffuser diameter at steady state in meters
D_Diffuser_ss=sqrt(A_Diffuser_ss*(4/pi));
disp("The Diameter Diffuser in meters at steady state is")
disp(D_Diffuser_ss)
disp("")
disp("")

%solving for diffuser diameter at steady state in feet
D_Diffuser_english_ss=D_Diffuser_ss*3.281;
disp("The Diffuser Diameter at steady state in ft")
disp(D_Diffuser_english_ss)
disp("")
disp("")

%solving for diffuser diameter at steady state in inches
D_Diffuser_english_ss_inches=D_Diffuser_english_ss*12;
disp("the Diffuser Diameter at steady state in inches")
disp(D_Diffuser_english_ss_inches)
disp("")
disp("")

%solving for diameter with availiable travel in feet
disp("value of diameter that can be reached with .3ft of
travel in feet")
```

```
D_Diffuser_english_ss_avail=ceil(D_Diffuser_english_ss/.3)*
.3;
disp(D_Diffuser_english_ss_avail)
disp("")
disp("")


%solving for the diameter availiable with .3 ft travel in
meter
D_Diffuser_ss_avail= (D_Diffuser_english_ss_avail)/3.281;
disp("Availiable travel in diffuser in meters")
disp(D_Diffuser_ss_avail)
disp("")
disp("")


%solving for the diameter availiable with .3 ft travel in
inches
D_Diffuser_english_ss_avail_inches=(D_Diffuser_english_ss_a
vail)*(12);
disp("Availiable diamter travel in diffuser in inches")
disp(D_Diffuser_english_ss_avail_inches)
disp("")
disp("")

%solving for the area available with .3ft travel in feet
disp("Availiable area travel in diffuser in feet")
A_Diffuser_english_ss_avail=(pi/4)*(D_Diffuser_english_ss_a
vail)^2 ;
disp(A_Diffuser_english_ss_avail)
disp("")
disp("")

%solving for the area avilable with .3ft travel in meters
disp("Availiable area travel in diffuser in meters")
A_Diffuser_ss_avail=A_Diffuser_english_ss_avail/(3.281*3.28
1);
disp(A_Diffuser_ss_avail)
disp("")
disp("")

%solving area ratio between ideal and availiable
disp("this is the area ratio between optimal and what is
availiable with given travel")
```

```
a_star_ratio_diffuser=A_Diffuser_english_ss_avail/A_Diffuse
r_english_ss;
disp(a_star_ratio_diffuser)
disp("")
disp("")

%solving for mach number at throat
k=input("select value slightly larger then
A_star_ratio_diffuser in command window  to get supersonic
case in diffuser throat between A* theoretical and A*
Possiable");
Mach_in_diffuser_throat_ss=fzero(@l,k);
disp("This is the Mach number exiting the throat of the
diffuser the Diffuser")
disp(Mach_in_diffuser_throat_ss)
disp("")
disp("")

%solving for mach number after the shock occurs
Mach_after_shock_diffuser=sqrt((1+((gamma-
1)/(2))*Mach_in_diffuser_throat_ss^2)/(gamma*Mach_in_diffus
er_throat_ss^2-((gamma-1)/(2))));
disp("this is the mach number after the shock in the
diffuser")
disp(Mach_after_shock_diffuser)
disp("")
disp("")

%solving for the change in total pressure ratio across
shock
Total_pressure_ratio_diffuser_exit=(((gamma+1)*Mach_in_diff
user_throat_ss^2)/((gamma-
1)*Mach_in_diffuser_throat_ss^2+2))^((gamma)/(gamma-
1))*(((gamma+1)/(2*gamma*Mach_in_diffuser_throat_ss^2-
(gamma-1))))^((1)/(gamma-1));
Po_Diffuser_exit=Total_pressure_ratio_diffuser_exit*Po2_ss;
Delta_Po=Po2_ss-Po_Diffuser_exit;
disp("this is the change in total pressure across the
diffuser at steady state in pa")
disp(Delta_Po)
disp("")
disp("")

%converting pascals to psi
```

```
disp("this is the change in total pressure across the
diffuser at steady state in psi")
Delta_Po_english=Delta_Po*0.0001450377;
disp(Delta_Po_english)
disp("")
disp("")

%solving for Exit Mach number
disp("this is the mach number exiting the diffuser at
steady state")
Mach_leaving_Diffuser=fzero(@b,k);
disp(Mach_leaving_Diffuser)

%solving for the Tempature after the shock in the throat of
the diffuser
disp("this is the Tempature of the fluid leaving the
Diffuser in kelvin")
T_Diffuser_exit=To/((1+(((gamma-
1)/2)*Mach_leaving_Diffuser^2)));
disp(T_Diffuser_exit)
disp("")
disp("")

%solving for the velocity in m/s after the shock in the
throat of the
%diffuser
disp("This is the velocity in diffuser in meters per
second" )
Velocity_Diffuser=Mach_leaving_Diffuser*sqrt(gas_constant*g
amma*T_Diffuser_exit);
disp(Velocity_Diffuser)
disp("")
disp("")

%solving for the velocity in ft/s after the shock in the
throat of the
%difusser
%3.28084 is a conversion factor between meters and feet
Velocity_Diffuser_english=Velocity_Diffuser*(3.28084);
disp("this is the velocity of diffuser in ft/s")
disp(Velocity_Diffuser_english)
disp("")
disp("")
```

```
%solving for the mach number entering the wind tunnel
x=input("if subsonic case press .1 if supersonic case 5");
Mach_enterance=fzero(@e,x);
disp("this is Mach number at enterance to nozzle")
disp(Mach_enterance)
disp("")
disp("")

%solving for the tempature entering the wind tunnel
disp("This is the Tempature of the fluid entering the
nozzle in kelvin")
T_nozzle_i=To/(1+(((gamma-1)/(2))*Mach_enterance^2));
disp(T_nozzle_i)
disp("")
disp("")

%solving for the work done with tempature average
W=((m_dot*(((gamma*gas_constant)/(gamma-
1))*((T_Diffuser_exit+T_nozzle_i)/2))))/1000;
disp("this is the work being  in kw")
disp(W);
disp("")
disp("")

%solving total cost
cost=W*h*price;
disp("this is the cost to run the wind tunnel for one
hour")
disp(cost)
disp("")
disp("")

function y=l(Mach_in_Diffuser_throat_ss)
global gamma
global a_star_ratio_diffuser
y=(1/Mach_in_Diffuser_throat_ss^2)*(((2/(gamma+1)))*(1+(((g
amma-
1)/(2))*Mach_in_Diffuser_throat_ss^2)))^((gamma+1)/(gamma-
1))-(a_star_ratio_diffuser)^2;
end

function y=b(Mach_leaving_Diffuser)
global gamma
global Ae
```

```
global A_Diffuser_ss_avail
y=(1/Mach_leaving_Diffuser^2)*(((2/(gamma+1)))*(1+(((gamma-
1)/(2))*Mach_leaving_Diffuser^2)))^((gamma+1)/(gamma-1))-
(Ae/A_Diffuser_ss_avail)^2;
end

function y=e(Mach_enterance)
global gamma
global Ae
global A_nozzle
y=(1/Mach_enterance^2)*(((2/(gamma+1)))*(1+(((gamma-
1)/(2))*Mach_enterance^2)))^((gamma+1)/(gamma-1))-
(Ae/A_nozzle)^2;
end
```

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Givens:

Mach # = 2.4

$P_{Test\,section} = 30150.17$ pa

$T_{Test} = 228.8$ K

$D_{Test} = 1.8288$ m

$R = 287.05 \frac{m^2}{s^2 \cdot K}$

$\gamma = 1.4$

cost = .15 kwh

hours = 1

Unknowns:

mass flow rate kg/s and lbm/s

Area/Diameter nozzle m^2, ft^2, ft, m

$(\text{Area/Diameter Diffuser})_{Su}$ m^2, ft^2, ft, m

$(\text{Area/Diameter Diffuser})_{SS}$ m^2, ft^2, ft, m

$(\text{Pressure loss in Diffuser})_{SS}$ PSI, kpa

$(\text{Mach \# and velocity})_{SS}$ at Diffuser Exit

Required work for compressor.

cost to run wind tunnel 1 hr.

Solving total pressure In test section

$$\frac{P_0}{P} = \left(1 + \left(\frac{\gamma-1}{2}\right) \cdot Mach_{Test}^2\right)^{\frac{\gamma}{\gamma-1}} = P_0 = P\left(1 + \frac{\gamma-1}{2} \cdot Mach_{test}^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_0 = 30150.17\,pa\left(1 + \left(\frac{.4}{2}\right) 2.4^2\right)^{\frac{1.4}{.4}} = P_0 = 440796.0557\ Pa$$

Solving total Temp in test section

$$\frac{T_0}{T} = \left(1 + \left(\frac{\gamma-1}{2}\right) \cdot Mach_{Test}^2\right) = T_0 = T \cdot \left(1 + \frac{\gamma-1}{2} \cdot mach_{Test}^2\right)$$

$$T_0 = 228.8K \cdot \left(1 + \frac{.4}{2} \cdot 2.4^2\right) = T_0 = 492.3776\,K$$

Solving for mass flow rate kg/s

$$\dot{m} = \rho v a = \frac{P}{RT} \cdot Mach_{Test} \cdot \sqrt{\gamma \cdot R \cdot T_{Test}} \cdot \frac{\pi}{4}(d_{Test})^2$$

$$\dot{m} = \frac{30150.17\,kg}{287.05 \frac{m^2}{s^2 K} \cdot 228.8\,k \cdot m \cdot s^2} \cdot 2.4 \cdot \sqrt{1.4 \cdot 287.05 \frac{m^2}{s^2 K}\ 228.8K} \cdot \frac{\pi}{4}(1.8288)^2$$

$$\boxed{\dot{m} = 877.56\ kg/s}$$

Converting to lbm/s

$$877.56 \frac{kg}{s}\ 2.205 \frac{lbm}{kg} = \boxed{1935.0198 \frac{lbm}{s}}$$

Finding Area of nozzle in m^2

$$\dot{M} = \frac{P_0 \cdot A_N}{\sqrt{T_0}} \cdot \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}} = A_N = \frac{\dot{m}\sqrt{T_0}}{P_0 \cdot \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}$$

$$A_N = \frac{877.56 \cdot \sqrt{492.37\,K}}{440796.0557\,Pa \cdot \sqrt{\frac{1.4}{287.05}\left(\frac{2}{2.4}\right)^{\frac{2.4}{.4}}}} = 1.093\,m^2$$

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Find diameter of nozzle in m

$d_N = \sqrt{\frac{4}{\pi} \cdot A_N} = d_N = \sqrt{\frac{4}{\pi} \cdot 1.093 m^2} = 1.1796\ m$

Converting diameter of nozzle from m to feet

$1.1796\ m \cdot 3.281 \frac{ft}{m} = 3.8705\ ft$

Area of nozzle in ft^2

$A_N = \frac{\pi}{4}(3.8705\ ft)^2 = 11.765\ ft^2$

Total pressure ratio across shock at mach # 2.4

$P_R = \left(\frac{(\gamma+1) \cdot mach_{Test}^2}{(\gamma-1) mach_{Test}^2 + 2}\right)^{\frac{\gamma}{\gamma-1}} \cdot \left(\frac{\gamma+1}{2 \cdot \gamma \cdot mach_{Test}^2 - (\gamma-1)}\right)^{\frac{1}{\gamma-1}}$

$P_R = \left(\frac{(2.4)(2.4)^2}{(.4) \cdot 2.4^2 + 2}\right)^{\frac{1.4}{.4}} \cdot \left(\frac{2.4}{2 \cdot 1.4 \cdot 2.4^2 - (.4)}\right)^{\frac{1}{.4}}$

$P_r = .54$

10% loss in total pressure across Test section

$P_{O_2} = P_{O_1} \cdot .9 = P_{O_2} = 440796.0557\ pa \cdot .9 = 396716.4501\ pa$

Change in P_O across Shock

$\frac{P_{O_3}}{P_{O_2}} = .54 = P_{O_3} = 396716.4501 \cdot .54$

$P_{O_3} = 214226.8831\ pa$

Sizing diffuser at start up

$\left(\frac{A_{DT}}{A_{NT}}\right)_{su} = \frac{P_{O_1}}{P_{O_3}} = (A_{DT})_{su} = 1.093\ m^2 \left(\frac{440796.0557\ pa}{214226.8831\ pa}\right)$

$(A_{DT})_{su} = 2.2489\ m^2$

Sizing diameter for diffuser at startup

$D_{Du,su} = \sqrt{\frac{4}{\pi}(2.2489\ m^2)} = 1.329\ m$

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Sizing Diffuser with travel restrictions in m

$$\left(\frac{1.329}{.09144}\right)_{round} = 15$$

$$(15 \times .09144) = 1.3716\,m$$

Converting from meters to feet diameter

$$1.3716\,M \cdot 3.281\,\frac{ft}{m} = 4.5\text{ feet}$$

Solving for Area of diffuser at start up in ft^2 with travel

$$(A_{D,SU})_{ft} = \frac{\pi}{4}(4.5)^2_{ft} = \boxed{15.9\,ft^2}$$

Solving for Area of diffuser at SS m^2

$$\frac{A_{DT}}{A_{MT}}\left(\frac{P_{01}}{P_{02}}\right) = A_{DT} = 1.093\,m^2\left(\frac{4410796.0557}{3967116.4501}\right) = 1.2144\,m^2$$

Solving for diameter of diffuser as S.S. M

$$D_{Du,SS} = \sqrt{\frac{4}{\pi}(1.2144\,m^2)} = 1.24347\,M$$

Sizing diffuser with travel restrictions in m^2 at SS

$$\left(\frac{1.24347}{.09144}\right)_{round} = 14$$

$$(14).09144 = 1.28016\,m^2$$

Converting from meters to feet diameter

$$1.28016\,m \cdot 3.281\,\frac{ft}{m} = 4.2\,ft$$

Sizing diffuser with travel restriction in ft^2 at SS

$$A_{Du,SS} = \frac{\pi}{4}(4.2\,ft)^2 = 13.85\,ft^2$$

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Sizing diffuser with travel restrictions in m^2 at SS

$A_{Di,SS} = \frac{\pi}{4}(1.28016)^2 = 1.287\ m^2$

Finding ratio between ideal and available

$\left(\frac{1.287}{1.2144}\right) = 1.0597$

Finding Mach # at area ratio

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M_{Throat}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_{Throat}^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

$$1.0597^2 = \frac{1}{M_{Throat}}\left[\frac{2}{2.4}\left(1+\frac{.4}{2}\cdot M_{Throat}\right)\right]^{\frac{2.4}{.4}}$$ using computer

$M_{Throat} = 1.28$

Finding pressure ratio at shock in throat

$$PR_{Throat} = \left(\frac{(\gamma+1)\cdot Mach_{Throat}^2}{(\gamma-1)\cdot Mach_{Throat}^2+2}\right)^{\frac{\gamma}{\gamma-1}} \cdot \left(\frac{\gamma+1}{2\cdot\gamma\cdot Mach_{throat}^2-\gamma-1}\right)^{\frac{1}{\gamma-1}}$$

$$PR_{Throat} = \left(\frac{2.4\cdot(1.28)^2}{(.4)(1.28)^2+2}\right)^{\frac{1.4}{.4}} \cdot \left(\frac{2.4}{2\cdot1.4\cdot1.28^2-(.4)}\right)^{\frac{1}{.4}}$$

$PR_{Throat} = .9826$

Finding change in pressure across shock in Diffuser throat

$\frac{Po_3'}{Po_2} = .9826 = Po_3' = 396,716.4501(.9826) = 389846.1776$

$Po_2 - Po_3' = (\Delta po)_{shock} = 396,716.4501 - 389846.1776 = \boxed{6870.27\ Pa}$

$\boxed{\text{in PSI} = .996\ \frac{lbf}{in^2}}$

Finding Mach # at diffuser end

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{Mach_{DE}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}Mach_{DE}^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

$$(4.081)^2 = \frac{1}{Mach_{DE}}\left[\frac{2}{2.4}\left(1+\frac{.4}{2}Mach_{DE}^2\right)\right]^{\frac{2.4}{.4}}$$ using computer

$\boxed{Mach_{DE,SS} = .1435}$

Solving for velocity at diffuser end m/s

$$\frac{T_0}{T_{DE}} = \left(1+\left(\frac{\gamma-1}{2}\right) mach_{DE}^2\right) = T_{DE} = \frac{T_0}{1+\frac{\gamma-1}{2} mach_{DE}} = T_{DE} = \frac{492.3776}{1+\frac{.4}{2}\cdot(.1435}$$

$T_{DE} = 478.6406$ K

$$V = M \cdot a = mach_{DE} \cdot \sqrt{\gamma R T_{DE}} \quad .1435\sqrt{(1.4)(287.05)(478.6406)}$$

$V = 62.936$ m/s

Converting m/s to ft/s

$$62.936 \, m/s \cdot 3.8705 \frac{ft}{m} = 243.5937 \frac{ft}{s}$$

Solving for mach # at nozzle enterance

$$\left(\frac{A}{A^*}\right) = \frac{1}{Mach_{NE}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2} mach_{NE}^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}$$

$$\left(\frac{5.2535}{1.093}\right)^2 = \frac{1}{Mach_{NE}}\left[\frac{2}{2.4}\left(1+\frac{.4}{2} mach_{NE}^2\right)\right)^{\frac{2.4}{.4}}$$

using computer

$Mach_{NE} = .1205$

Solving for temp at nozzle enterance

$$\frac{T_N}{T_0} = \frac{T_0}{\left(1+\frac{\gamma-1}{2}\cdot M_{NE}\right)} = \frac{492.3776K}{\left(1+\frac{.4}{2}\cdot .1205\right)} \quad T_N = 480.79 \text{ K}$$

Solving for work at compressor

$$\dot{W} = \dot{m}(h_2-h_1) = \dot{m}\,c_p(T_2-T_1) = \dot{m}\,c_p\left(\frac{T_2+T_1}{2}\right) = \dot{m}\,\frac{\gamma R}{\gamma-1}\left(\frac{T_2+T_1}{2}\right)$$

$$\dot{W} = 877.56 \frac{kg}{s} \cdot \frac{(1.4)(287.05)\frac{m^2}{s^2 K}}{.4}\left(\frac{480.79+478.6406}{2}\right)$$

$\boxed{\dot{W} = 422947.0353 \text{ kw}}$ 1000

Solving for cost

$cost = \dot{W} \cdot t \cdot price$

$cost = 422947.0353 \, kw \cdot 1 \cdot .15 \frac{kwh}{hr} =$ $\boxed{63,412 \text{ dollars and 5 cents}}$