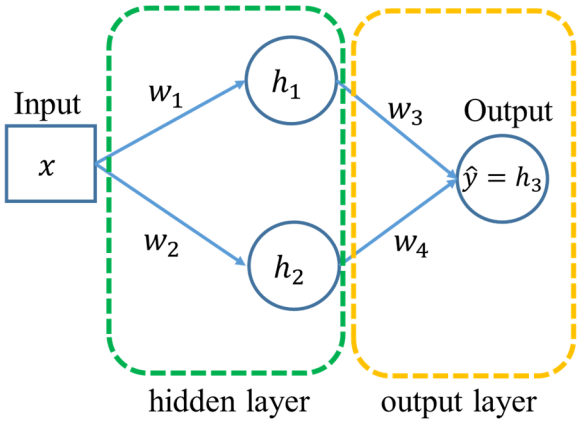
Homework Assignment 3

Part-1: Math

1. Backpropagation in a Neural network



$$h_1 = f_1(w_1 x + b_1)$$

$$h_2 = f_2(w_2 x + b_2)$$

$$\hat{y} = f_3(w_3 h_1 + w_4 h_2 + b_3)$$

$$f_n' = \frac{\partial f_n(v)}{\partial v},\ \mathrm{n} = 1,2,3$$

$x, w_1, w_2, w_3, w_4, b_1, b_2, b_3, h_1, h_2, h_3$ are scalars

(1) Compute the derivatives of the loss L with respect to parameters and input

$\frac{\partial L}{\partial w_1}$

$\frac{\partial L}{\partial w_2}$

$\frac{\partial L}{\partial w_3}$

$\frac{\partial L}{\partial w_4}$

$\frac{\partial L}{\partial b_1}$

$\frac{\partial L}{\partial b_2}$

$\frac{\partial L}{\partial b_3}$

$\frac{\partial L}{\partial x}$

2. Computational Graph

$h = 2x + 1$

$z = x^2$

$y = \frac{1}{1 + e^{-h}}$

(1) Draw the computational graph based on the above three equations

(2) What is $\frac{\partial y}{\partial z}$ from the graph?

3. Classification using Neural Networks

Assume in an application, there are two classes. We can use binary-cross-entropy loss or cross-entropy loss.

Assume we have two networks, Net-1 and Net-2.

Net-1 is trained with binary-cross-entropy loss

Net-2 is trained with cross-entropy loss

The structure of Net-1 is almost the same as that of Net-2, except the output layer.

What is the difference in the output layers of Net-1 and Net-2?

4. Input and Output Normalization for a Neural Network

Usually, we need to apply normalization/standardization to the inputs for classification and regression tasks. For example, if the input is an image, then every pixel is dived by 255, so that the pixel values of the normalized image are in the range of 0 to 1. Input normalization facilitates convergence of training algorithms.

We may also need to apply normalization to the output. Assume the input is an image of a person, the output vector has two components, $\hat{y}_{(1)}$ and $\hat{y}_{(2)}$: $\hat{y}_{(1)}$ is the monthly income (in the range of 0 to 10,000), and $\hat{y}_{(2)}$ is the age (in the range of 0 to 100). The MSE loss for a single data sample is

$$L = (\hat{y}_{(1)} - y_{(1)})^2 + (\hat{y}_{(2)} - y_{(2)})^2$$

where $y_{(1)}$ and $y_{(2)}$ are ground truth.

Question: Is output normalization necessary or not for this regression task? Why?

If it is necessary, what normalization can be applied (to both the network output and ground truth)?

5. Activation Functions for Regression (for Graduate Students)

Neural networks can be used for regression. Usually, we do not use nonlinear activation functions in the output layer. But, sometimes, there are requirements for outputs. For example, if the output is price, then the output should be nonnegative.

Assume $\boldsymbol{z}$ is the scalar output of a network, and the network does not have nonlinear activation function in the output layer. Now, you find out that there is some requirement for output, and you decide to add a nonlinear activation function.

You need to design nonlinear activation functions for three different requirements:

(1) the final output y should be nonnegative ($y \geq 0$), then what is the activation function $y = f(z)$?

(2) the final output y should be nonpositive ($y \leq 0$), then what is the activation function $y = f(z)$?

(3) the final output y should be $a \leq y \leq b$, then what is the activation function $y = f(z)$?

(4) Could we add the nonlinear activation function to the network only after the network is trained without the nonlinear activation function?

Grading

The number of points for each question/task

	Undergraduate Student	Graduate Student
1. Backpropagation	10	10
2. Computational Graph	10	10
3. Classification	10	10
4. Normalization	10	10
5. Activations for Regression	N.A. (bonus 5 points)	10
H3P2T1 (classification)	30	25
H3P2T2 (regression)	30	25

For programming tasks, H3P2T1 and H3P2T2, you can use either Keras or Pytorch.

In the programming task H3P2T2, you have the chance to get additional 5 points.

If you use both Keras and Pytorch, you will get additional 5 points (if there are no bugs in your code).

I highly suggest that you use both, so that you can put this line in your resume for a job:

“familiar with deep learning packages (Keras and Pytorch)”