Maulana Abul Kalam Azad University of Technology

CA-3 Examination 2022-2023

Semester: 1st
Subject Name: **Discrete Structures**
Stream/ Discipline: **MCA**

Paper Code: **MCAC103**
Full marks: **25**
Time: **1 Hour**

Group A (Answer **all** questions.) — 5 × 1 = 5 marks

1. Let S be the set of all distinct numbers of the form $\frac{p}{q}$, where $p, q \in \{1,2,3\}$. The cardinality of the set S is
   a) 6 b) 7 c) 8 d) 9

2. If $\{1,2,3,4,5,6,7\}$ be the universal set and $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$. Then $(A \cup B)^c$ is equal to
   a) $\{5,6\}$ b) $\{5,6,7\}$ c) $\{7\}$ d) null set.

3. The cardinality of the power set of the set $S = \{a, b, c, d\}$ is
   a) 1 b) 4 c) 12 d) 16

4. Let S = $\{2,3,4,5\}$ and T = $\{-2, -1, 0, 1, 2\}$. The relation R between S and T is defined by s of S is related to t of T if and only if s + t is even. Then which of the following ordered pair is not in the relation R?
   a) $(2, -2)$ b) $(5, -1)$ c) $(4, 2)$ d) $(4, -1)$

5. Let $S = \{1, 2, 3, 4\}$, and $T = \{a, b, c, d\}$. Then which of the following relation between S and T is a mapping
   $f_1 = \{(1, a), (2, b), (3, c), (3, d), (4, d)\}$,
   $f_2 = \{(1, a), (3, b), (4, c)\}$,
   $f_3 = \{(1, a), (2, b), (3, d), (4, d)\}$.
   a) f_1 b) f_2 c) f_3 d) None of these.

6. Let $f: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = |x|$. Then f is
   a) injective, not surjective. b) surjective, not injective.
   c) bijective. d) neither injective nor surjective.

7. Which of the following element is the additive inverse of $\bar{4}$ in $(\mathbb{Z}_6, +)$?
   a) $\bar{0}$ b) $\bar{1}$ c) $\bar{2}$ d) inverse does not exists.

8. Which of the following is not a monoid?
   a) $(\mathbb{Z}, +)$ b) $(\mathbb{Z}, -)$ c) $(\mathbb{Q}, +)$ d) $(\mathbb{Q}, \cdot)$

9. The number of generators of the group $(S, \cdot)$, where $S = \{1, i, -1, -i\}$ are
   a) 0 b) 1 c) 2 d) 4

10. Let G be a group and $a \in G$. If $o(a) = 30$, then $o(a^{15})$ is
    a) 1 b) 2 c) 5 d) 30

Group B (Answer **all** questions.)

11. Let a relation R on the set $\mathbb{Z}$ is defined by "aRb if and only if $ab > 0$" for $a, b \in \mathbb{Z}$. Check whether R is a) reflexive, b) symmetric, c) transitive, d) equivalence relation? **5 marks**

12. a) Let $\circ$ on $\mathbb{Q}$ be the binary composition defined by $a \circ b = a + 2b$, $a, b \in \mathbb{Q}$. Justify whether ($\mathbb{Q}$, $\circ$) forms a group?
    b) Find the generators of the cyclic group $(\mathbb{Z}_{24}, +)$.
    c) Show that the group $(\mathbb{Q}, +)$ is not cyclic. **4+4+2 = 10 marks**

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

SCHOOL OF INFORMATION SCIENCE AND TECHNOLOGY

END SEMESTER EXAMINATION

SIST/*MCA /SEM-01 /MCAC103*/2022-23

FEBRUARY – 2023

PAPER NAME: Discrete Structures | *PAPER CODE:MCAC103*

SEMESTER: 1st

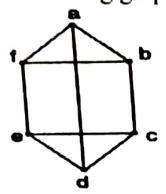
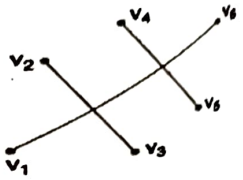
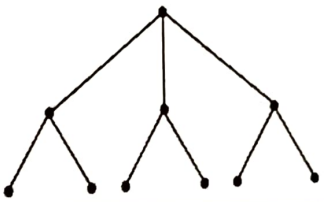
Time : 3 Hours] [Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1	**Choose the correct alternatives of the following: Any *ten***	**10 □1 = 10**			
		MARKS	CO	PO	BL
i)	If U is universal and $A \subset U$. Then which of the following is not correct?	1	CO1	PO1	
a.	$A \cap A^c = \emptyset$				
b.	$A \cup A^c = U$				
c.	$A - U = U - A$				
d.	All of the above are correct				
ii)	Let $S = \{1, 2, 3, 4\}$, and $T = \{a, b, c, d\}$. Then which of the following relation between S and T is a mapping $f_1 = \{(1, a), (2, b), (3, c), (3, d), (4, d)\}$, $f_2 = \{(1, a), (3, b), (4, c)\}$, $f_3 = \{(1, a), (2, b), (3, d), (4, d)\}$.	1	CO1 CO4	PO1 PO2	
a.	f_1				
b.	f_2				
c.	f_3				
d.	None of these				
iii)	Which of the following element is not invertible in $(\mathbb{Q}, .)$?		CO1	PO1 PO2 PO4	
a.	0				
b.	1				
c.	½				
d.	None of these				
iv)	Let $(G, \circ)$ be a group. Define a mapping $f: G \to G$ by $f(x) = x^{-1}, x \in G$. Then f is	1	CO1	PO1 PO2 PO12	
a.	injective not surjective				
b.	surjective not injective				

c.	Bijective			
d.	None of these			
v)	A graph with all vertices having equal degree is known as a	1	CO1 CO4	PO1 PO3 PO12
a.	Multi-graph			
b.	Regular graph			
c.	Simple graph			
d.	Complete graph			
vi)	Which of the following graph(s) with 6 vertices is/are connected?  (A)  (B)	1	CO1	PO2 PO3 PO4 PO12
a.	(A) only			
b.	(B) only			
c.	Both (A) and (B)			
d.	Neither (A) nor (B)			
vii)	The following tree is 	1	CO4	PO1 PO2 PO4 PO6
a.	full binary tree, but not a complete binary tree			
b.	complete binary tree			
c.	binary tree but not a full binary tree			
d.	hot a binary tree			
viii)	Let p: be the proposition 'A is intelligent' and q be a proposition 'A is tall'. Then $\sim q \wedge \sim p$ states that	1	CO3	PO2 PO4 PO5 PO6 PO7
a.	A is either intelligent or tall.			
b.	A is neither tall nor intelligent			
c.	A is not intelligent or A is not tall			
d.	A is not intelligent			
ix)	Let $P(x)$ be the statement "$x > 8$", Then which one of these has truth value true?	1	CO2	PO2 PO4 PO6
a.	$P(0)$			

b.	$P(8)$			PO7 PO10	
c.	$P(9)$				
d.	None of these				
x)	The number of distinct permutations from all the letters of the word INDIAN is	1	CO5	PO4 PO9 PO10	
a.	1				
b.	120				
c.	180				
d.	720				
xi)	The recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$ is	1	CO1 CO5	PO1 PO4 PO12	
a.	linear and homogeneous				
b.	linear and inhomogeneous				
c.	nonlinear and homogeneous				
d.	nonlinear and inhomogeneous				

GROUP – B

(Short Answer Type Questions)

Answer the following. **3 × 5 = 15**

		MARKS	CO	PO	BL
2.a.	Let a relation R on the set $\mathbb{Z}$ is defined by "aRb if and only if a is a divisor of b" for $a, b \in \mathbb{Z}$. Check whether R is (i) reflexive, (ii) symmetric, (iii) transitive, (iv) equivalence relation, (v) poset?	5	CO1 CO4	PO1 PO2 PO4	
	OR				
2.b.	Let the mapping $f: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 3x$. Check whether the mapping f is (i) injective, (ii) surjective, (iii) bijective?	5	CO1 CO4	PO1 PO2 PO4	
3.a.	Let $A = \{2, 3, 5, 6\}$, $B = \{8, 10, 13, 20\}$, $C = \{a, b, c, d\}$ and the relations R_1 and R_2 are defined by $R_1 = \{(2,8), (2,20), (3,10), (5,10), (6,20)\}$ and $R_2 = \{(8, b), (8, c), (10, a), (13, d), (20, c)\}$. Find $R_2 \circ R_1$. Also find R_1^{-1}.	5	CO1	PO1 PO4 PO12	
	OR				
3.b.	Prove that the set of rational numbers $\mathbb{Q}$ is countable.	5	CO1	PO1 PO12	
4.a.	Show that $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.	5	CO2 CO3	PO3 PO4 PO5 PO6	

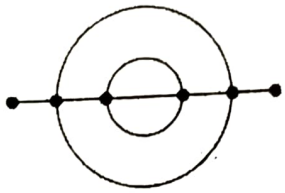
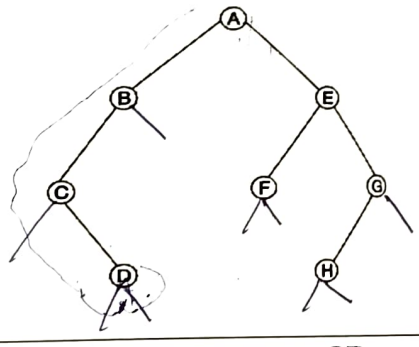
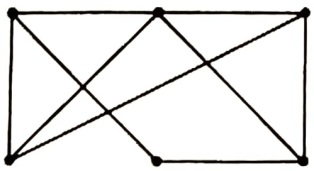
	OR			
4.b.	Using truth table show that $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$	**5**	CO2 CO3	PO3 PO4 PO5 PO6

GROUP – C

(Long Answer Type Questions)

Answer the following. **3 ×15 = 45**

		MARKS	CO	PO	BL
5.a.i.	Let $\circ$ on $\mathbb{Q}$ be the binary composition defined by $a \circ b = ab + 1, a, b \in \mathbb{Q}$. Justify whether $(\mathbb{Q}, \circ)$ forms a group?	**5**	CO1 CO3 CO4	PO1 PO2 PO3 PO4 PO5 PO6 PO12	
ii.	Find the generators of the cyclic group $(\mathbb{Z}_{36}, +)$.	**3**			
iii.	Show that the group $(\mathbb{Q}, +)$ is not cyclic.	**3**			
iv.	What are conditional and bi-conditional propositions. Write down the truth table for conditional and bi-conditional propositions.	**4**			
	OR				
5.b.i.	Examine which of the following forms a monoid? (A) $(\mathbb{Z}, -)$ (B) $(\mathbb{Q}, \cdot)$ (C) $(\mathbb{Z}_6, \cdot)$	**5**	CO1 CO3 CO4	PO1 PO2 PO3 PO4 PO12	
ii.	In a group $(G, \circ)$, a is an element of order 40. Find the order of a^{24}.	**3**			
iii.	Find the inverse permutation of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$.	**3**			
iv.	Define the divisor of zero of a ring. What are the divisors of zero in the ring $(\mathbb{Z}_6, +, \cdot)$.	**4**			
6.a.i.	Out of 12 employees a group of 4 trainees is to be sent for a software training. In how many ways these can be selected if there are two employees who refuse to go together?	**5**	CO5	PO2 PO3 PO4 PO6 PO9 PO10 PO12	
ii.	A store sells only red, blue and black color pens. How many different selections are possible if a man wants to buy 7 pens?	**4**			
iii.	Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$ by characteristic root method subject to the initial conditions $a_n = 1$ for $n = 0$; $a_n = 1$ for $n = 1$. Hence find a_7.	**6**			
	OR				
6.b.i.	Use principle of mathematical induction to prove that $2^{3n+1} + 3.5^{2n+1}$ is	**5**	CO5	PO2	

	divisible by 17.			PO3 PO4 PO6 PO9 PO10 PO12	
ii.	Find the number of integers between 1 to 500 that are divisible by 7 or 11.	4			
iii.	Define generating function. Find the generating function for the sequence $\{5 + 3n\}$.	6			
7.a.i.	State the handshaking theorem. Whether it is valid for multi-graphs and pseudo-graphs.	4	CO1 CO4	PO1 PO2 PO4 PO5 PO6 PO7 PO8 PO12	
ii.	Define Euler graph. Check whether the following graph has an Euler trail or Euler circuit. If it exists, then write the trial or circuit. 	5			
iii.	Find the A) preorder, B) inorder, and C) postorder traversal of the following tree: 	6			
	OR				
7.b.i.	Find the number of edges in a complete graph with n vertices.	4	CO1 CO4	PO1 PO2 PO4 PO5 PO6 PO7 PO8 PO12	
ii.	Define chromatic number of a graph. Find the chromatic number of the following graph. 	5			
iii.	Draw the unique binary search tree whose pre-order traversal is given below: Pre-order: 6, 1, 5, 11, 3, 4, 8, 7, 2	6			