Assignment 2

Instructions:

- There are 3 questions in this assignment, complete them all.
- There are 10 datasets, your lecturer will assign one dataset to you. Marks will be deducted for the wrong dataset used.
- Present your solutions in a Word document. Indicate the question number.
- State the dataset you are using at the beginning of the report. e.g. "2. Condo Tamp Paris Ris H1 price"
- Save the file as **x_name_admin.doc,** where x is your dataset number, name is your name, and admin is your admission number.
- Submit your report, code and video to Brightspace. The report should not exceed 13 pages.

- For the Python code, paste it into the report as instructed in the questions. In additional, place the code for Q1, Q2 and Q3 in one Jupyter notebook and submit it as well. Indicate clearly which question number the code is for.
- An oral presentation may be required at your tutor's discretion.
- Submit the *Declaration of Academic Integrity* before submitting your assignment.
- Use the following template to acknowledge the use of any AI Tools.

Name of AI tool	< For example, ChatGPT >
Input prompt	< Insert the question that you asked ChatGPT >
Date generated	< Insert the date that ChatGPT response was generated, since ChatGPT is an evolving technology >
Output generated	< Insert the response verbatim from ChatGPT >
Impact on submission	< Briefly explain which part of your submitted work was ChatGPT's response applied >

Introduction

Given a set of data points with at least one predictor and one continuous response variable, we want to construct a linear model to predict the response. This is the aim of **Linear Regression**, which is a supervised learning technique.

In the context of this assignment, the salary of NUS graduates and the GPA of students enrolled to NUS are extracted from following 2 sources:

https://www.moe.gov.sg/-/media/files/post-secondary/ges-2022/web-publication-nus-ges-2022.ashx
https://www.nus.edu.sg/oam/docs/default-source/undergradute-programmes/nus-igp.pdf?sfvrsn=f80385b1_22

This article provides some context for this assignment.

The data file is zipped together with this document. The following table lists two of the variables in the file and their descriptions:

Variable	**Description**
GPA (10th percentile)	10th percentile entry point in 2023 for the degree course in 2023
Basic Monthly Salary (Median)	Median Basic Monthly Salary of NUS Graduates by Bachelor Degree

The response variable is *Basic Monthly Salary (Median),* and the predictor is *GPA (10th percentile).*

Simple Linear Regression (SLR)

We will first build a SLR model using *GPA (10th percentile)* as the predictor to predict *Basic Monthly Salary (Median).*

In SLR notations, let:

x_i = predictor value of the *i*-th data point

y_i = actual response value of the *i*-th data point

$\hat{y}_i$ = predicted response value of the *i*-th data point based on model

Thus, $\hat{y}_i = a + bx_i$, where values of *a* (intercept) and *b* (slope) are to be determined.

The squared-error of the *i*th prediction is $e_i^2 = (y_i - \hat{y}_i)^2$. Errors (also known as residuals) are squared to remove the signs, so that errors of opposite signs do not cancel out each other, giving the false impression of small aggregated errors.

Then, we define **Error function** as the mean of squared-error (of the whole data set):

$$E(a, b) = \frac{1}{n}\sum_{i=1}^{n} e_i^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$= \frac{1}{n}[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + \cdots + (y_n - \hat{y}_n)^2] \ldots (1)$$

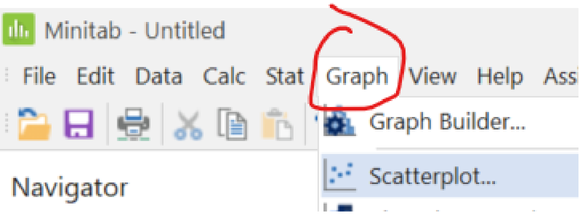
We want to find the values of *a* and *b* such that the Error function is **minimised**.
The resultant equation $\hat{y} = a + bx$ will give the best-fit line that passes through the data points.

<u>MODEL 1: SLR with intercept *a* fixed $\Rightarrow \hat{y}_i = bx_i$</u> (30 marks – 10 marks for each sub-qn)

Revision on SLR: https://youtu.be/HoqXask9cN8

We will first build a SLR model to predict *Basic Monthly Salary (Median)* (y) using *GPA (10th percentile)* (x) as the predictor. To get things started, we will simplify the regression line of $\hat{y} = a + bx$ to $\hat{y} = bx$ by setting $a = 0$.
Suppose it is believed that *Basic Monthly Salary (Median)* is directly proportional to *GPA (10th percentile)*. This means that $\hat{y}$ is a constant multiple of x and $a = 0$. Hence, in this SLR model 1, we will only need to determine the slope, b, that is $\hat{y} = bx$.

1(a) Use Minitab to plot the scatter plot of *Basic Monthly Salary (Median)* and *GPA (10th percentile)* with the regression line in it, paste the scatter plot to your report. Write down the equation of the regression line obtained from Minitab, in the context of your dataset, e.g. $sale\ of\ \widehat{ice} - cream = 2.31\ temperature$.



(Hint: This question we want a regression line of $\hat{y} = a + bx$ with a = 0, to do that in Minitab, look for “Fit Intercept” in “Data View” in the Scatterplot options, then uncheck it)
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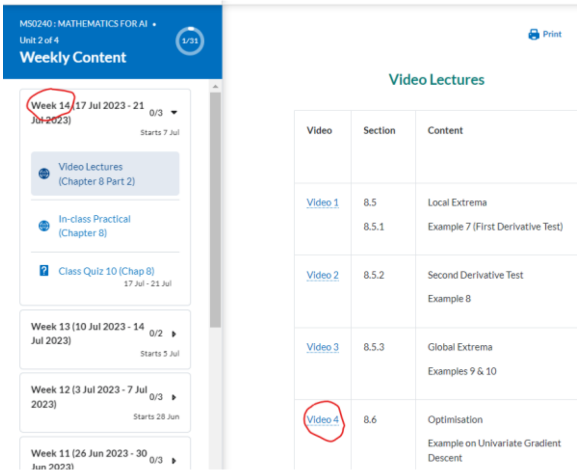
In this assignment, we are going to learn how to obtain the regression line from scratch by using the gradient descent algorithm, which will involve differentiation.

We are going to use the univariate gradient descent algorithm to obtain the value of the slope, b, which is reported by Minitab in Q1(a).

The video in Brightspace: Week 14 – Video 4 (see pic) explains the gradient descent algorithm.


MS0240 : MATHEMATICS FOR AI •
Unit 2 of 4
Weekly Content
1/31
Week 14 (17 Jul 2023 - 21 Jul 2023) 0/3
Starts 7 Jul
Video Lectures (Chapter 8 Part 2)
In-class Practical (Chapter 8)
Class Quiz 10 (Chap 8)
17 Jul - 21 Jul
Week 13 (10 Jul 2023 - 14 Jul 2023) 0/2
Starts 5 Jul
Week 12 (3 Jul 2023 - 7 Jul 2023) 0/3
Starts 28 Jun
Week 11 (26 Jun 2023 - 30 Jun 2023) 0/3
Print

Video Lectures

Video	Section	Content
Video 1	8.5 8.5.1	Local Extrema Example 7 (First Derivative Test)
Video 2	8.5.2	Second Derivative Test Example 8
Video 3	8.5.3	Global Extrema Examples 9 & 10
Video 4	8.6	Optimisation Example on Univariate Gradient Descent

This YouTube video explains the error function (Equation 1) in page 2 and its derivative. Do watch until 8:55 min only: https://youtu.be/sDv4f4s2SB8

Note:

(i) There is a slight difference in the loss function in the YouTube video, and in page 2, Equation 1, there is a division by n in Equation 1. In all your calculations for this assignment, stick to Equation 1:

$$\frac{1}{n}[(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + \cdots + (y_n - \hat{y}_n),^2]$$

(ii) In the video, it explains the concept of obtaining the value of the intercept. That can be applied to calculate the value of the slope in Q1(b)

1(b) Express Error function $E(b)$ in terms of b only since *a=0*, indicate the value of n in your Error function. Hence, derive $E'(b)$.

1(c) Use univariate gradient descent algorithm to find the value of b for which $E(b)$ is at its minimum.

1c(i) Write your Python code in a single cell and copy-paste (do NOT take screenshots) your **code** and **output** into your report. Refer to the sample output below as an example.

Sample output:

```
Number of iterations is xx

The local minimum occurs when b is xx

Minimum error is xx
```

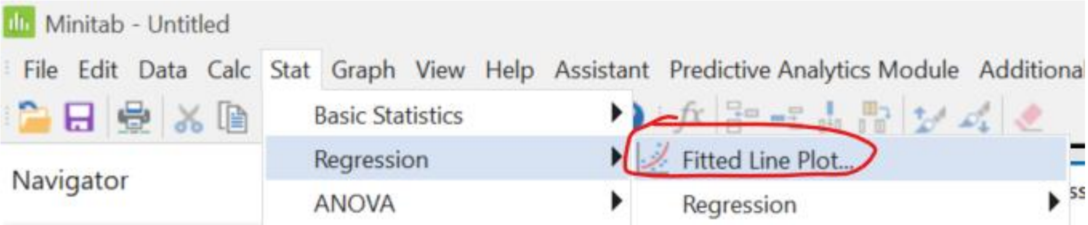
1c(ii) Write down the equation of the regression line you have obtained from gradient descent in the context of your dataset, e.g. $sale\ of\ \widehat{ice} - cream = 2.31\ temperature.$

Hint: Adjust the learning rate to get the minimum error. You may continue to watch this video from 8:55 to 15:40 min to enhance your understanding of the algorithm of gradient descent: https://youtu.be/sDv4f4s2SB8?t=534

MODEL 2: SLR ⇒ $\hat{y}_i = a + bx_i$ (40 marks – 10 marks each sub-qn)

Now we apply the SLR model where **both intercept *a* and slope *b* are to be determined**, when predicting *Basic Monthly Salary (Median)* (*y*) using *GPA (10th percentile)* (*x*) as the predictor.

2(a) Use Minitab to plot the scatter plot of *Basic Monthly Salary (Median)* and *GPA (10th percentile)* with the regression line in it, paste the scatter plot to your report. Your plot should include the equation of the regression line obtained from Minitab, in the context of your dataset, e.g. $sale\ of\ \widehat{ice - cream} = 10.9 + \ 2.31\ temperature$
Use the regression line to estimate the value of the response variable with a suitable predictor value of your choice.


Minitab - Untitled
File Edit Data Calc Stat Graph View Help Assistant Predictive Analytics Module Additiona
Navigator
Basic Statistics
Regression
ANOVA
Fitted Line Plot...
Regression

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We are going to use gradient descent to obtain the regression line, $\hat{y} = a + bx$, from scratch. As there are 2 variables involved, we will need to apply partial derivative to the error function. You can start watching videos in week 16 folder in Brightspace to learn partial derivatives.

You may want to watch this YouTube video from 15:43 till the end, for the implementation of gradient descent involving two variables: https://youtu.be/sDv4f4s2SB8?t=943

2(b) Express Error function $E(a, b)$ in terms of *a* and *b*. Hence, derive $E_a(a, b)$ and $E_b(a, b)$.

2(c) Use gradient descent algorithm **to find the values of *a* and *b*** for which $E(a, b)$ is at its minimum.

2c(i) Write your Python code in a single cell and copy-paste (do NOT take screenshots) your **code** and **output** into your report, refer to the sample output below as an example.

Sample output: Number of iterations is xx

The local minimum occurs when b is xx

Minimum error is xx

2c(ii) Write down the equation of the regression line you have obtained from gradient descent in the context of your dataset, e.g. $sale\ of\ \widehat{ice - cream} = 10.9 +$ $2.31\ temperature.$

2(d) Describe the changes and decisions you made on the parameters for your solution to reach convergence.

MODEL 3: MLR $\Rightarrow$ $\hat{y}_i = a + bx_i + cw_i$ (30 marks – 10 marks each sub-qn)

We can extend the SLR model to include more predictors. A linear regression model with more than one predictor is called **Multiple Linear Regression** (MLR) model.
Apply the MLR model where intercept *a*, and slopes *b* and *c* are to be determined, when predicting *Basic Monthly Salary (Median)* (*y*) using *GPA (10th percentile)* (*x*) and ***w* as the predictors**.

You decide on a suitable variable *w* based on the context of your dataset.

Explain your Model 3 in a video. Your face must be visible for the entire duration of the recording.

Your video should include the following:

3(a) **Data Collection**

- Pick at least 10 records (rows) from your existing dataset, then insert data for the field *w*. The inserted data can come from the existing data given (if any), your own measurements, some randomly assigned values, etc.
- Explain your data collection procedure.
- Copy and paste the records you used to create Model 3 into your report.

3(b) **Implementation: Error Function**

- Obtain the expressions for the error function and any related functions needed for the gradient descent algorithm.
- Explain how the above expressions are derived mathematically.
- Explain how the above expressions are used in the gradient descent algorithm to find the values of *a*, *b* and *c* in the regression line, $\hat{y} = a + bx + cw$

3(c) **Implementation: Coding and Verification**

- Code the gradient descent algorithm in Python to find the values of *a*, *b* and *c*
- Explain your code.
- Run your code with the data inserted to demonstrate how you obtain the regression line.
- Explain and demonstrate how you verify that your regression line for Model 3 is correct.
- Copy-paste (do NOT take screenshots) your code and output into your report and include it in the .ipynp file.

Summary of files to be submitted:

1. Word document
2. Video file for Q3
3. One .ipynp file that contains the codes for Q1, Q2 and Q3. Label the question number clearly in your code.

Marks will be awarded for clear explanations. Your video should not exceed 4 minutes.
Marks will be deducted for video exceeding 4 minutes.

END OF ASSIGNMENT