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1.

y	$P(y)$
1	1/42
2	2/42
3	6/42
4	4/42
6	6/42
7	7/42
8	16/42

$$E(Y) = 1\times\frac{1}{42} + 2\times\frac{1}{42} + 3\times\frac{6}{42} + 4\times\frac{4}{42} + 6\times\frac{6}{42} + 7\times\frac{7}{42} + 8\times\frac{16}{42}$$
$$= \boxed{5.952}$$

2.

$$E(XY) = \iint xy\, f(x,y)\, dx dy$$
$$= \int_0^1\int_0^x xy\, 12y^2\, dx\, dy = \int_0^1\int_0^x 12xy^3\, dx dy$$
$$= \int_0^1 3xy^4\Big|_0^x dx$$
$$= \int_0^1 3x^5\, dx = \frac{x^6}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

3.

$$E[(X_1-2X_2+X_3)^2] = E(X_1^2+4X_2^2+X_3^2-4X_1X_2+2X_1X_3-4X_2X_3)$$
$$= E(X_1^2)+4E(X_2^2)+E(X_3^2)-4E(X_1X_2)+2E(X_1X_3)-4(X_2X_3)$$
$$= E(X_1^2)+4E(X_2^2)+E(X_3^2)-4E(X_1)E(X_2)+2E(X_1)E(X_3)-4E(X_2)E(X_3)$$

$$E(X_i) = \frac{1}{2}$$
$$E(X_i^2) = \int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\therefore E[(X_1-2X_2+X_3)^2] = \frac{1}{3}+4\times\frac{1}{3}+\frac{1}{3}-4\times\frac{1}{2}\times\frac{1}{2}+2\times\frac{1}{2}\times\frac{1}{2}-4\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{3}+\frac{4}{3}+\frac{1}{3}-1+\frac{1}{2}-1$$
$$= 2-1+\frac{1}{2}-1 = \boxed{\frac{1}{2}}$$

4) $f(x)=e^{-x},\ x>0$

$Y=e^{\frac{3}{4}x}$

$F(Y)=P(Y\le y)=P(e^{\frac{3}{4}x}\le y)=P(\frac{3}{4}x\le \log(y))=P(x\le\frac{4}{3}\log(y))$

$=\int_0^{\frac{4}{3}\log y} e^{-x}\,dx$

$=-e^{-x}\Big|_0^{\frac{4}{3}\log y}$

$=-e^{-\frac{4}{3}\log y}-1=-y^{-\frac{4}{3}}-1$

$f(y)=\frac{4}{3}y^{-\frac{7}{3}}$

$E(Y)=\int_1^{\infty} Y\cdot\frac{4}{3}y^{-\frac{7}{3}}\,dy$

$=\int_1^{\infty}\frac{4}{3}y^{-\frac{4}{3}}\,dy=-\frac{4}{3}\times3\times y^{-\frac{1}{3}}\Big|_1^{\infty}$

$\boxed{=4}$

5)

X	1	2	3	4	5	6
P	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Y	3	9	19	33	51	73
P	$\frac{1}{6}$	......				

$E(Y)=3\times\frac{1}{6}+9\times\frac{1}{6}+19\times\frac{1}{6}+33\times\frac{1}{6}+51\times\frac{1}{6}+73\times\frac{1}{6}$

$\boxed{=\frac{89}{3}}$

6) $f(x)=2(1-x),\ 0<x<1$

$Y=(2x+1)$

$F(x)=\int_0^1 2-2x\,dx=2x-x^2$

$P(Y<y)=P(2x+1<y)=P(x<\frac{y-1}{2})=\int_0^{\frac{y-1}{2}}2-2x\,dx=2x-x^2\Big|_0^{\frac{y-1}{2}}$

$=y-1-(\frac{y-1}{2})^2=y-1-\frac{(y-1)^2}{4}$

$=\frac{1}{4}(6y-y^2-5)$

$f(y)=\frac{3}{2}-\frac{1}{2}y$

$E(y)=\int_1^3 y(\frac{3}{2}-\frac{1}{2}y)dy=\boxed{\frac{5}{3}}$

7) $E[(ax+b)^n]=E(\sum_{i=1}^{n}\binom{n}{i}(ax)^{n-i}\cdot b^i)=\sum_{i=0}^{n}\binom{n}{i}E((ax)^{n-i}b^i)=\sum_{i=0}^{n}\binom{n}{i}a^{n-i}b^iE(X^{n-i})$

8) $E(x-Y)=E(x)-E(Y)=np-((1-p)n)=2np-n$

$n=20,\ p=0.05$

$E(X-Y)=2\times20\times0.05-20=-18.$

The expected number of defective part are less than good part.