Chapter 3 - Raster Images

raster display - images shown as rectangular array of pixels

pixel - short for “picture element”

vector image - descriptions of shapes; areas of colour bounded by lines or curves

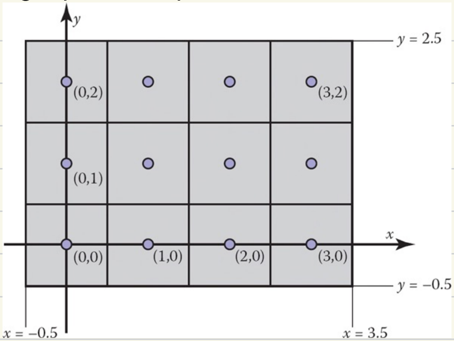
2. Images, Pixels and Geometry

I(x,y): R -> V; R - rectangular area (R^2); V - set of possible pixel values (R+^3)

display pixel - red, green and blue subpixels

pixel value - local average of the colour of the image; point sample

R = [-0.5, nx - 0.5] x [-0.5, ny - 0.5]



2.1. Pixel Values

floating-point numbers (32-bit)

-> high dynamic range (HDR)

8 bits -> [0,255]/255 = [0,1]

-> fixed range/ low dynamic range (LDR)

clipping - maximum value limit

banding - rounding values causes noticeable differences in intensity or colours

2.2. Monitor Intensities and Gamma

display intensity = (maximum intensity) a^γ; a - input pixel

γ = 2; input: 0, 0.5, 1; output: 0, 0.25, 1

Chapter 6 - Transformation Matrices

geometric transformations - rotation, translation, scaling and projection

1. 2D Linear Transformations

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix}$$

1.1. Scaling

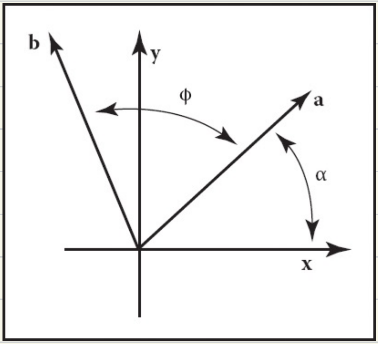
$$\text{scale}(s_x,\ s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$

1.2. Shearing

$$\text{shear-x}(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \quad \text{shear-y}(s) = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

1.3. Rotation



r = sqrt(xa^2 + ya^2)

=> xa = r cos(α); ya = r sin(α)

xb = r cos(α+ϕ) = r cos(α)cos(ϕ) - r sin(α)sin(ϕ)

yb = r sin(α+ϕ) = r sin(α)cos(ϕ) + r cos(α)sin(ϕ)

=> xb = xa cos(ϕ) - ya sin(ϕ)

yb = ya cos(ϕ) + xa sin(ϕ)

$$\text{rotate}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

1.4. Reflection

$$\text{reflect-y} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{reflect-x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

2. 3D Linear Transformations

$$\text{scale}(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

$$\text{shear-x}(d_y, d_z) = \begin{bmatrix} 1 & d_y & d_z \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{rotate-x}(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\text{rotate-y}(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\text{rotate-z}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Translation and Affine Transformations

translation: point (x,y) represented by a 3D vector [x y 1]T

$$\begin{aligned} x' &= x + x_t \\ y' &= y + y_t \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & x_t \\ m_{21} & m_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11}x + m_{12}y + x_t \\ m_{21}x + m_{22}y + y_t \\ 1 \end{bmatrix}$$

affine transformation - linear transformation + translation

adding extra dimension - homogeneous coordinates

directions or offsets - third coordinate should be 0

[x y 1]T - point; [x y 0]T - displacement or direction

$$\begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + x_t \\ y + y_t \\ z + z_t \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{x'_h - x'_l}{x_h - x_l} & 0 & 0 & \frac{x'_l x_h - x'_h x_l}{x_h - x_l} \\ 0 & \frac{y'_h - y'_l}{y_h - y_l} & 0 & \frac{y'_l y_h - y'_h y_l}{y_h - y_l} \\ 0 & 0 & \frac{z'_h, z'_l}{z_h - z_l} & \frac{z'_l z_h - z'_h z_l}{z_h - z_l} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{x'_h - x'_l}{x_h - x_l} & 0 & \frac{x'_l x_h - x'_h x_l}{x_h - x_l} \\ 0 & \frac{y'_h - y'_l}{y_h - y_l} & \frac{y'_l y_h - y'_h y_l}{y_h - y_l} \\ 0 & 0 & 1 \end{bmatrix}$$