20/02 **Quiz**

① $\left|\begin{array}{l} 2x_1 + x_2 + 2x_3 = 8 \\ x_1 + 2x_2 + 3x_3 = 5 \\ 2x_1 + 5x_2 + 8x_3 = 10 \end{array}\right. \rightarrow \begin{bmatrix} 2 & 1 & 2 & 8 \\ 1 & 2 & 3 & 5 \\ 2 & 5 & 8 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$

② $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 4 & h & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & h-2 & 1 \end{array}\right]$

consistent $\Rightarrow h \neq 2$

③ unique solution $\Rightarrow h \in \emptyset$

④ $\{b_1, b_2, b_3\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

I. $\text{Span}\{b_1, b_2, b_3\} = \text{Span}\{b_1, b_2\}$ T $\quad b_3 = \frac{1}{3}(b_1 + b_2)$

II. $\text{Span}\{b_1, b_2\} = \text{Span}\{b_2, b_3\}$ T $\quad b_1 = 3b_3 - b_2$

⑤ $A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$ s.t. $a_1 + a_4 = a_2 + a_3$

$Ax = 0$ solution: u

$u = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad w = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

Exercises

① $\left[\begin{array}{cc|c} 1 & w & -5 \\ 2 & -8 & 6 \end{array}\right] \forall w \qquad \left[\begin{array}{cc|c} -4 & 12 & w \\ 2 & -6 & -3 \end{array}\right] w = 6$

② $\left[\begin{array}{cc|c} 33 & 34 & 35 \\ 34 & 35 & 37 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 33 \\ 0 & 1 & -31 \end{array}\right] \qquad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

③ 2×3: 7

$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \end{bmatrix} \quad \begin{bmatrix} 1 & * & * \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & * & * \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix}$

(4) a. $\left[\begin{array}{cc|c} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \end{array}\right] \to$ inconsistent

b. $\left[\begin{array}{cccc|c} \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & * \end{array}\right]$ consistent, ∞ solutions, $\to$ free variable

(5) $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\frac{1}{4}(u+v) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\frac{1}{2}(v-u) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(6) $\left[\begin{array}{ccccc|c} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & -4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$

$x_1 = 4x_2 + 5$
$x_3 = -1 \Rightarrow x = \begin{pmatrix} 5 \\ 0 \\ -1 \\ 0 \\ -4 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
$x_5 = -4$
x_2, x_4 - free

(7) $\left.\begin{array}{l} Au_1 = b_1 \\ Au_2 = b_2 \\ Ax = b_1 - 2b_2 \end{array}\right\} x = u_1 - 2u_2$

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(8) $\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ 3 ind. $\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2 ind. $\begin{bmatrix} 1 & * & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2 ind. $\begin{bmatrix} 1 & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2 ind. $\begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(9) $\{a_1, a_2, a_3, a_4\}$ - independent

a. $\{b_1, b_2, b_3, b_4\} = \{a_1 - a_2, a_2 - a_3, a_3 - a_4, a_4 - a_1\}$ - dependent
$-b_1 - b_2 - b_3 = -a_1 + \cancel{a_2} - \cancel{a_2} + \cancel{a_3} - \cancel{a_3} + a_4 = a_4 - a_1 = b_4$

b. $\{c_1, c_2, c_3, c_4\} = \{a_1, a_2 + a_1, a_3 + a_2 + a_1, a_4 + a_3 + a_2 + a_1\}$ - independent

(10) $b_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ $b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ $b_3 = \begin{bmatrix} h_1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ $b_4 = \begin{bmatrix} 2 \\ h_2 \\ 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 3 & 1 & h_1 & 2 \\ 1 & 1 & 2 & h_2 \\ 2 & 1 & -1 & 1 \\ 1 & -1 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 3 & h_2 \\ 0 & 0 & h_1+1 & 0 \end{bmatrix} \Rightarrow$ $h_1 \neq -1$, $\forall h_2$

$\{b_1, b_2, b_3, b_4\}$ independent