01/04

Academic Reasoning 2

① $A^2 = I \Rightarrow \lambda = 1$ is the only eigenvalue of A

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$, but A has characteristic equation:
$\lambda^2 - 1 = 0 \Rightarrow \lambda_1 = 1 \quad \lambda_2 = -1 \Rightarrow$ False

② A, B - $n \times n$ matrices with eigenvector $v \Rightarrow v$ - eigenvector of AB

v - eigenvector of $A, B \Rightarrow Av = \lambda_a v$ and $Bv = \lambda_b v$
$\Rightarrow ABv = A\lambda_b v = \lambda_b Av = \lambda_b \lambda_a v \Rightarrow v$ - eigenvector of AB

③ U - (real) orthogonal matrix $\Rightarrow \det(U) = -1$ or $\det(U) = 1$

U - orthogonal matrix $\Rightarrow UU^T = I$ and $U^T U = I$
$\Rightarrow \det(U) \cdot \det(U^T) = \det(I)$, but $\det(I) = 1$ and $\det(U^T) = \det(U)$
$\Rightarrow (\det(U))^2 = 1 \Rightarrow \det(U) = -1$ or $\det(U) = 1$

④ U - (real) orthogonal matrix $\Rightarrow \lambda = 1$ is the only real eigenvalue of U

Let $U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ - orthonormal $\Rightarrow U$ - orthogonal matrix. But U has characteristic eq.: $\lambda^2 - 1 = 0 \Rightarrow \lambda_1 = 1 \quad \lambda_2 = -1 \Rightarrow$ False