Extra exercises

Week 9 material

The following exercises are practice material for Week 9. **You are not required to solve or review them** in this self-study, since we will only be teaching the material in the following week. **They are however part of the exam material.**

3 Amdahl's Law

Amdahl's law helps us determine the time it will take to execute a program with parallelization. It goes as follows:

$$T_p = T_s \cdot (f_s + \frac{f_p}{p})$$

Where:

- T_p = the time it takes to execute the parallelized program
- T_s = the time it takes to execute the program sequentially
- f_s = the fraction of the code that must be run sequentially
- f_p = the fraction of the code that can be run in parallel
- p = the number of cores which can be used for parallelization

1. (8 mins) Chris has been addicted to a game for the past three weeks and now only takes 16 seconds to win. Sára wanted to see whether Chris's hard work has paid off or whether she could beat him by quickly writing a program which would win faster. She was met with disappointment, since her program took 32 seconds. She was about to give up and admit to defeat when Ruben pointed out that 87% of her code is parallelizible. How many cores does she need to beat Chris's record?

1. ______________

$T_s = 32s$

$T_p < 16s$

$f_p = 87\%$

$f_s = 13\%$

$p = ?$

$$s = \frac{T_s}{T_p} \geq 2$$

$$\Rightarrow \frac{1}{f_s + \frac{f_p}{p}} \geq 2$$

$$\Rightarrow f_s + \frac{f_p}{p} \leq \frac{1}{2}$$

$$0.13 + \frac{0.87}{p} \leq 0.5$$

$$\frac{0.87}{p} \leq 0.37$$

$$p \geq \frac{0.87}{0.37} \sim 2.4$$

$\Rightarrow$ at least 3.

$p = 3$

$$T_p = T_s\left(f_s + \frac{f_p}{p}\right) = 32\left(0.13 + \frac{\overset{0.29}{\cancel{0.87}}}{\underset{1}{\cancel{3}}}\right) = 32(0.42) =$$

$$= 13.44s < 16s \checkmark$$

2. (8 mins) Aad created a program to draw an image of a Mandelbrot set in a small terminal. He wants to speed up his program, as it takes his original version about 42 seconds to draw the image on a single-core CPU. After making his program massively parallel, he runs it again on a GPU with 252 cores. His program now completes in 28 seconds. What is the value of the fraction f_p of the program he managed to parallellise? Assume his GPU has the same speed as his CPU and assume changing from a CPU to GPU introduces no overhead.

2. ______________

$T_s = 42s$

$p = 252$

$T_p = 28s$

$f_p = ?$

$f_s = 1 - f_p$

$$T_p = T_s\left(f_s + \frac{f_p}{p}\right)$$

$$\frac{\cancel{28}^{2}\,\cancel{s}}{\cancel{42}_{3}\,\cancel{s}} = 1 - f_p + \frac{f_p}{252}$$

$$\frac{2}{3} = 1 - \frac{251}{252} f_p$$

$$f_p = -\frac{1}{\cancel{3}} \cdot - \frac{\cancel{252}^{84}}{251}$$

$$f_p = \frac{84}{251} \sim 0.33$$

$f_p = 33\% = 0.33$

$f_s = 0.67 \quad p = 252$

$T_s = 42s \quad T_p = 28s$

$$T_p = T_s\left(f_s + \frac{f_p}{p}\right)$$

$$= 42\left(0.67 + \frac{0.33}{252}\right)$$

$$= 28(.195) \checkmark$$

4 Virtual memory

1. (3 mins) Find the total addressable virtual memory given that the size of a page is 24 KiB and there are in total $42 \cdot 2^{20}$ entries.

1. ____________

page $= 24$ KiB $= 3 \times 2^{13}$ B

42×2^{20} entries $= 21 \times 2^{21}$

total virtual memory:

$3 \times 2^{13} \times 21 \times 2^{21}$ B $=$

$= 63 \times 2^{34}$ B $=$

$= 63 \times 16$ GiB $=$

$= 1008$ GiB ~ 1 TB

2. (5 mins) Given a computer that uses 25 % of the total memory as the page table, with the total memory of 64 GiB. This computer also uses 24 bits to figure out the physical page number and 6 control bits, that together should be byte aligned. You also know that the size of a page is 2 KiB, find the total addressable virtual memory.

2. ____________

memory 64 GiB $= 2^{36}$ B

page table $= 2^{34}$ B $\xleftarrow{:4}$ $(\times 25\%)$ $\Big\}$ $\dfrac{2^{34}\text{B}}{32\text{ b}} = \dfrac{2^{34}\text{B}}{2^2\text{B}} = 2^{32}$ pages

(16 GiB) $\qquad$ $:4 = 4$B

24 bits for physical page number $\Big\}$ 30 bits for entry

6 control bits $\qquad$ byte-aligned $\downarrow$ 32 bits

page $= 2$ KiB $= 2^{11}$ B $\Big\}$ $2^{32} \times 2^{11}$ B $= 2^{43}$ B $=$ 8 TiB

2^{32} entries