06/03 **Exercises**

① A - 5×8 matrix
$\#$pivots $=\dim \mathrm{Col}\, A = 0 \div 5$
$\Rightarrow \#$free $= \dim \mathrm{Nul}\, A = 8 - \;\; = 3 \div 8$

② $A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$ a) $\rightarrow \begin{bmatrix} 1 & 0 & -5 & -5 & 0 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

b) rank $= \dim \mathrm{Col}\, A = 2$
c) $\dim \mathrm{Nul}\, A = 3$
d) $\mathrm{Col}\, A = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 3 \\ 8 \end{bmatrix} \right\}$

③ G, H - subspaces $\mathbb{R}^n$
i) intersection $G \cap H \rightarrow$ subspace
ii) union $G \cup H \rightarrow$ cannot determine
iii) sum $G + H = \{ g + h \mid g \in G,\ h \in H \} \rightarrow$ subspace

④ A, B - $n\times n$ matrices
i) $\mathrm{Nul}(AB) \subset \mathrm{Nul}(A)$ ✗
ii) $\mathrm{Nul}(AB) \subset \mathrm{Nul}(B)$ ✓
iii) $\mathrm{Nul}(A) \subset \mathrm{Nul}(AB)$ ✓
iv) $\mathrm{Nul}(B) \subset \mathrm{Nul}(AB)$ ✗
v) $\mathrm{Col}(AB) \subset \mathrm{Col}(A)$ ✓
vi) $\mathrm{Col}(AB) \subset \mathrm{Col}(B)$ ✗
vii) $\mathrm{Col}(A) \subset \mathrm{Col}(AB)$ ✗
viii) $\mathrm{Col}(B) \subset \mathrm{Col}(AB)$ ✓

⑤ A - $m\times n$ matrix $\Big\}$ $m = n$
$\mathrm{Col}\, A$ in $\mathbb{R}^n$ square

⑥ $A = [\mathbf{a}_1\, \mathbf{a}_2\, \mathbf{a}_3\, \mathbf{a}_4\, \mathbf{a}_5] = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 1 & 4 & 1 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix} U = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\beta_1 = \{a_1, a_2, a_3\}$
$\beta_2 = \{a_1, a_3, a_5\} \rightarrow$ basis for $\mathrm{Col}\, A$
$\beta_3 = \{a_1, a_4, a_5\}$

⑦ $A = \begin{bmatrix} \| & \| & \| & \| & \| \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 & \mathbf{a}_5 \\ \| & \| & \| & \| & \| \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

1) $\{e_1, e_2, e_3\}$
4) $\{a_2, a_3, a_5\}$
$\rightarrow$ both can be basis for $\mathrm{Col}\, A$