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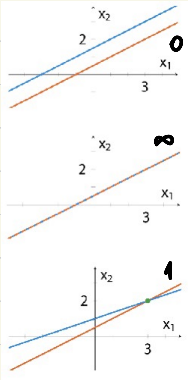
Lecture 1 – Linear systems and echelon forms

Number of solutions of a linear system



$$\begin{cases} x_1 + \cancel{5x_2} + \cancel{3x_3} = 7 \\ -3x_2 + \cancel{3x_3} = -2 \\ 6x_3 = 2 \end{cases} \qquad \begin{aligned} x_3 &= \tfrac{1}{3} \\ x_2 &= 1 \\ x_1 &= 1 \end{aligned}$$

$$\begin{cases} x_1 + 5x_2 + 3x_3 = 1 \\ 2x_1 + x_2 + 15x_3 = 8 \\ -x_1 + x_2 + 3x_3 = 1 \end{cases} \longrightarrow \left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 2 & 1 & 15 & 8 \\ -1 & 1 & 3 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 2 & 1 & 15 & 8 \\ -1 & 1 & 3 & 1 \end{array}\right] \xrightarrow{-2} \sim \left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ 0 & -9 & 9 & 6 \\ -1 & 1 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 5 & 3 & 1 \\ -1 & 1 & 3 & 1 \\ 0 & -9 & 9 & 6 \end{array}\right] \times -1/3$$

$$\left[\begin{array}{cc|c} 0 & -2 & 5 \\ 1 & 3 & -5 \\ 3 & -1 & 6 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 3 & -5 \\ 0 & -2 & 5 \\ 3 & -1 & 6 \end{array}\right] \xrightarrow{-3} \left[\begin{array}{cc|c} 1 & 3 & -5 \\ 0 & -2 & 5 \\ 0 & -10 & 21 \end{array}\right]$$

Def. System:
- consistent – has at least one solution
- inconsistent – has no solutions at all $[0\ 0 \dots 0 \mid c]$ $c \neq 0$

1.1./ 9. $\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$

$x_1 = 4$ $x_2 = 8$ $x_3 = 5$ $x_4 = 2$

13. $\begin{cases} x_1 - 3x_3 = 8 \\ 2x_1 + 2x_2 + 9x_3 = 7 \\ x_2 + 5x_3 = -2 \end{cases}$ $\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix} -2$

$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 0 & -1 & 11 \\ 0 & 1 & 5 & -2 \end{bmatrix}$ $x_1 = 5$ $x_2 = 3$ $x_3 = -1$

23. a) T b) F c) T d) T

rows × columns

Echleron form

a) All nonzero rows are above any row of all zeros.

b) Each leading entry (pivot) is in a column to the right of the leading entry in the previous row.

c) All entries below a leading entry are zero.

$$\left[\begin{array}{ccc|c} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccccccccc|c} 0 & \blacksquare & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 5 & 7 & 8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 4 & 0 \end{array}\right]$$

echelon form

Reduced echelon form

a) in echelon form

b) the pivot of each nonzero row is 1.

c) each leading 1 is the only nonzero entry in its column

$$\left[\begin{array}{ccc|c} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccccccccc|c} 0 & 1 & * & 0 & 0 & * & * & * & 0 & * \\ 0 & 0 & 0 & 1 & 0 & * & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} \blacksquare & * & * & * & * & * \\ 0 & \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare & 0 \end{array}\right]$$

not reduced echelon form

Solving Linear Systems

Def. Variable:

- basic – column contains pivot position (x_1, x_2, x_3, x_5)
- free – column doesn't contain pivot position (x_4)

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \end{array}$$
$$\left[\begin{array}{ccccc|c} \blacksquare & * & * & * & * & * \\ 0 & \blacksquare & * & * & * & * \\ 0 & 0 & \blacksquare & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare & 0 \end{array}\right]$$

consistent, ∞ solutions

1.2.1 3.
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \xrightarrow{\cdot(-4),\ \cdot(-6)} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix} \begin{matrix} \cdot(-\frac{1}{3}) \\ \cdot(-\frac{1}{5}) \end{matrix} \xrightarrow{-2,\ -1} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

13.
$$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 3x_2 - x_4 = -2$ $\quad$ $x_1 = 3x_5 + 5$ $\quad$ $x_4 = 4 - 9x_5$

$x_2 - 4x_5 = 1$ $\quad$ $x_2 = 4x_5 + 1$ $\quad$ x_5 - free variable

$x_4 + 9x_5 = 4$ $\quad$ x_3 - free variable

15. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & 0 \end{bmatrix}$ → consistent
1 solution

$\begin{bmatrix} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 & \blacksquare \end{bmatrix}$ → inconsistent

23. 3×5 matrix, 3 pivot columns } consistent, 3^{rd} row contains pivot

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Lecture 2 – Spans, vector equations and matrix equations

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{array}\right] \longrightarrow x_1 \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

$$b = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix} \quad a_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad a_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$b = x_1 a_1 + x_2 a_2$
$b = 3a_1 - 2a_2$

Linear combinations and spans

Def. vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$ scalars $c_1, c_2, \ldots, c_p$
$y = c_1 v_1 + \ldots + c_p v_p$ – linear combination of $v_1 \ldots v_p$ with weights $c_1 \ldots c_n$

$$b = \begin{bmatrix} 6 \\ 4 \\ 34 \end{bmatrix} \quad a_1 = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} \quad a_2 = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix} \quad b = -2a_1 + 4a_2$$

Th $x_1 a_1 + x_2 a_2 + \ldots + x_p a_p = b \leftrightarrow [a_1 \ a_2 \ \ldots \ a_p \mid b]$

Def. vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$
Span$\{v_1, v_2, \ldots, v_p\}$ → set of all linear combinations of $v_1, v_2, \ldots, v_p$
$x_1 v_1 + x_2 v_2 + \ldots + x_p v_p$

1.3./ 11. $a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ $a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ $a_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

$b = 2a_1 + 3a_2$
$b = a_3 - 3a_1 - a_2$

13. $A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$ $\left| \begin{array}{l} A_1 - 4A_2 + 2A_3 = 3 \\ 3A_2 + 5A_3 = -7 \\ -2A_1 + 8A_2 - 4A_3 = -3 \end{array} \right.$ no solution

17. $a_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ $a_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$ $\left| \begin{array}{l} x_1 - 2x_2 = 4 \\ 4x_1 - 3x_2 = 1 \\ -2x_1 + 7x_2 = h \end{array} \right.$ $\left| \begin{array}{l} x_1 = 4 + 2x_2 \\ 16 + 5x_2 = 1 \\ -2x_1 + 7x_2 = h \end{array} \right.$ $\left| \begin{array}{l} x_1 = -2 \\ x_2 = -3 \\ h = -17 \end{array} \right.$

Matrix-vector product

Def. $Ax = [a_1\ a_2\ \dots\ a_n] \cdot \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = x_1 a_1 + x_2 a_2 + \dots + x_n a_n$

matrix A - n columns

vector x - n entries

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \dots & \dots & \dots & \dots \\ a_{m,1} & a_{m,2} & \dots & a_{m,n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} a_{1,1} \\ a_{2,1} \\ \dots \\ a_{m,1} \end{bmatrix} + x_2 \begin{bmatrix} a_{1,2} \\ a_{2,2} \\ \dots \\ a_{m,2} \end{bmatrix} + \dots + x_n \begin{bmatrix} a_{1,n} \\ a_{2,n} \\ \dots \\ a_{m,n} \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = DNE$$

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ 5 \\ 2 \end{bmatrix}$$

Th A - $m \times n$ matrix u, v - vectors $\in \mathbb{R}^n$ c - scalar

$A(u+v) = Au + Av$ $A(cu) = c(Au)$

Matrix equations

Th A - $m \times n$ matrix with columns $a_1 \dots a_n$ $\quad b \in \mathbb{R}^m$

matrix equation: $Ax = b$

vector equation: $x_1 a_1 + x_2 a_2 + \dots + x_n a_n = b$

linear system: $[a_1 \; a_2 \; \dots \; a_n \mid b]$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad Ax = 0 \Rightarrow \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \end{array}\right] \neq x_1 \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} + x_2 \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Th A - $m \times n$ matrix

$\forall b, b \in \mathbb{R}^m$, $Ax = b$ has a solution.

b - linear combination of the columns of A $\Big\}$ equivalent

Columns of A span $\mathbb{R}^m$.

A - pivot position in every row.

1.4./ 6. $\begin{bmatrix} 7 & -3 \\ 2 & 1 \\ 9 & -6 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix} \rightarrow -2 \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} - 5 \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$

15. $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad Ax = b \rightarrow x_1 \begin{bmatrix} 2 \\ -6 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ $\Bigg|$ $b_1 = 2x_1 - x_2$, $b_2 = -6x_1 + 3x_2 = -3b_1$

$\frac{b_2}{b_1} \neq -3 \rightarrow$ no solution

21. $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ $\quad \{v_1, v_2, v_3\}$ - no pivot in each row $\Rightarrow$ doesn't span $\mathbb{R}^4$

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Lecture 3 – Solution sets and linear independence

$\begin{cases} x_1 = -x_3 \\ x_2 = 2x_3 \\ x_3 - \text{free} \end{cases}$ $x = x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$ $x = x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\begin{array}{l} x_1 - x_3 = 0 \\ x_2 + x_3 = 0 \\ x_3 - \text{free} \end{array}$ $\begin{array}{l} x_1 = x_3 \\ x_2 = -x_3 \end{array}$

Homogeneous systems

Def. System $Ax=b$:
- homogeneous if $b=0$ → always consistent
- nonhomogeneous if $b \neq 0$ → free variable → ∞ solutions

Solutions of (non)homogeneous systems

homogeneous system

$$\left[\begin{array}{ccc|c} 3 & 4 & -5 & 0 \\ -3 & -2 & 1 & 0 \\ 6 & 1 & 4 & 0 \end{array}\right] \to \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

non-homogeneous system

$$\left[\begin{array}{ccc|c} 3 & 4 & -5 & 1 \\ -3 & -2 & 1 & 1 \\ 6 & 1 & 4 & -5 \end{array}\right] \to \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Th nonhomogeneous $Ax=b$, $x = x_p + x_n$
x_p - particular solution
x_n - general solution of homogeneous $Ax=0$

$$A = \begin{bmatrix} 3 & 5 & 2 & 6 \\ 2 & 4 & 2 & 2 \\ 1 & 1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$Ax=0$
⇒ solution set: span of two vectors in $\mathbb{R}^4$

1.5./ 5. $\begin{cases} x_1 + 3x_2 + x_3 = 0 \\ -4x_1 - 9x_2 + 2x_3 = 0 \\ -3x_2 - 6x_3 = 0 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

11. $\left[\begin{array}{cccccc} 1 & -4 & \overset{0}{\cancel{-2}} & 0 & \overset{0}{\cancel{3}} & \overset{5}{\cancel{-5}} \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $\left|\begin{array}{l} x_1 - 4x_2 + 5x_6 = 0 \\ x_3 - x_6 = 0 \\ x_5 - 4x_6 = 0 \end{array}\right.$

35. $A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$ or $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$

14. $\left|\begin{array}{l} x_1 = 3x_4 \\ x_2 = 8 + x_4 \\ x_3 = 2 - 5x_4 \\ x_4 - \text{free} \end{array}\right\} x = \begin{bmatrix} 0 \\ 8 \\ 2 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 8 \\ -5 \\ 1 \end{bmatrix}$

Linear independence

Def. Set of vectors $\{v_1, v_2, \dots, v_p\} \in \mathbb{R}^n$:

- linearly independent if $x_1v_1 + x_2v_2 + \dots + x_pv_p = 0$ only trivial solution | set $\{v\}$ only if $v = 0$
- linearly dependent if otherwise

$\left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \end{bmatrix}\right\} \to$ linearly dependent

$\left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix}\right\} \to$ linearly dependent

$\left\{\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}\right\} \to$ linearly independent

$\left\{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}\right\}$ $\left|\begin{array}{l} 2x_1 - 3x_2 + x_3 = 0 \to 2x_1 + 4x_3 = 0 \quad x_1 = -2x_3 \\ \left.\begin{array}{l} x_1 + 4x_2 + 6x_3 = 0 \\ x_1 + 3x_2 + 5x_3 = 0 \end{array}\right\} x_2 + x_3 = 0 \quad x_2 = -x_3 \end{array}\right.$ linearly dependent

Th Set $\{v_1, v_2, \dots, v_p\}$:

- linearly dependent if ≥ 1 vector is linear combination of other
- linearly independent otherwise

Th $\{v_1, v_2, \dots, v_p\}$ in $\mathbb{R}^n$ if $p > n$, linearly dependent

$\{v_1, v_2, \dots, v_p\}$ in $\mathbb{R}^n$ if containing 0 vector, linearly dependent

$v_1, v_2, v_3, v_4 \in \mathbb{R}^4$

$\{v_1, v_2, v_3\} \rightarrow$ linearly dependent

$\Rightarrow \{v_1, v_2, v_3, v_4\} \rightarrow$ linearly dependent

$\{v_1, v_2, v_3, v_4\} \rightarrow$ linearly dependent

$\{v_1, v_2, v_3\} \rightarrow$ cannot determine

1.7./ 5. $\left\{ \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -8 \\ -7 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ -4 \\ 2 \end{bmatrix} \right\} \rightarrow$ linearly independent

21. a) T b) F c) F d) F

11. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ h \end{bmatrix} \right\}$

$$\begin{vmatrix} x_1 + 3x_2 - x_3 = 0 \\ -x_1 - 5x_2 + 5x_3 = 0 \\ 4x_1 + 7x_2 + hx_3 = 0 \end{vmatrix}$$

$$\begin{vmatrix} x_3 = x_1 + 3x_2 \\ 4x_1 + 10x_2 = 0 \\ (4+h)x_1 + (7+3h)x_2 = 0 \end{vmatrix}$$

$4(7+3h) = 10(4+h)$

$28 + 12h = 40 + 10h$

$2h = 12$ $h = 6$