Chapter 1 – Introduction

logic = 'the word' / 'what is spoken' (Ancient Greek)
↳ 'thought' / 'reason' (today)

Chapter 2 – Logic

logical deduction:
premises — "All humans are mortal."
— "Socrates is human."
conclusion — "Socrates is mortal."

proposition – a statement that has a truth value

ex. "$\underbrace{\text{Delft}}_{\text{subject}}$ $\underbrace{\text{is a city.}}_{\text{predicate}}$" – either true or false

quantifiers – 'all', 'some', 'none'

1. Propositional Logic

1.1. Propositions – statement that is either true or false

p, q, r – propositional variable

mathematical generality

1.2 Logical operators / connectives

'and'	'or'	'not'
$\wedge$	$\vee$	$\neg$
conjunction	disjunction	negation

Def $p \wedge q = \text{true}$ only when $p = \text{true}$ and $q = \text{true}$
$p \vee q = \text{false}$ only when $p = \text{false}$ and $q = \text{false}$
$\neg p = \text{true}$ onl when $p = \text{false}$

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

1.3. Precedence rules

compound proposition – made up of simpler

I. $\neg$ II. $\wedge$ III. $\vee$

$\wedge$ – associative

main connective

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Consider the following sentence:

I do not drink tea or I am Spartacus, or I like cats.

Using the following propositions, what does this read in logic?

p = I like cats

q = I drink tea

r = I am Spartacus

$(\neg q \vee r) \vee p$

How many connectives are there in the following statement:

$(\neg s \vee t) \vee s$

3

Consider the following statement:

p ∧ q ∨ ¬ p

I like strawberries and I like melons, or I don't like strawberries.

What is the minimum number of propositional atoms ("variables") we need to translate this statement

2 → p and q

Consider the following sentence:

I do not like cats and I drink tea, and I am not Spartacus.

Using the following propositions, what does this read in logic?

p = I drink tea

q = I like cats

r = I am Spartacus

$(\neg q \wedge p) \wedge \neg r$

Consider the following sentence:

I like dogs or I am Spartacus, and I am a madman with a box.

Using the following propositions, what does this read in logic?

p = I am a madman with a box

q = I like dogs

r = I am Spartacus

$(q \vee r) \wedge p$

1.4. Logical equivalence

truth table

situation – individual combination

logically equivalent

Consider the following proposition:

$((q \wedge p) \vee p) \equiv p$

p	q	resulting truth value
0	0	0
0	1	0
1	0	1
1	1	1

Consider a statement which is comprised of 4 unique atoms and 12 connectives.

If we want to show **all** intermediate steps, how many *rows* should our truth table have?

16 $= 2^4$

Consider a statement which is comprised of 4 unique atoms and 13 connectives.

If we want to show **all** intermediate steps, how many *columns* should our truth table have?

17 $= 4 + 13$

1.5. More logical operaters

conditional operator $\rightarrow$

biconditional operator $\leftrightarrow$

exclusive or operator $\oplus$

p	q	$p \rightarrow q$	$p \leftrightarrow q$	$p \oplus q$
0	0	1	1	0
0	1	1	0	1
1	0	0	0	1
1	1	1	1	0

1.6. Implications in English

$p \rightarrow q$ – implication / conditional

p implies q

p – hypothesis / antecedent

q – conclusion / consequent

if true

p is sufficient for q / q is necessary for p

1.7 More forms of implication

$$\neg q \to \neg p \ \underset{\text{logically equivalent}}{\overset{\text{contrapositive of}}{\overline{\qquad\qquad}}} \ p \to q$$

$$q \to p \ \overset{\text{converse of}}{\overline{\qquad\qquad}} \ p \to q$$

$$\neg p \to \neg q \ \overset{\text{inverse of}}{\overline{\qquad\qquad}} \ p \to q$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

Consider the following sentence:

I do not like dogs if and only if I do not drink tea, if and only if I like lorek the bear.

Using the following propositions, what does this read in logic?

p = I like dogs

q = I like lorek the bear

r = I drink tea

$(\neg p \leftrightarrow \neg r) \leftrightarrow q$

Consider the following sentence:

I like dogs and I drink tea, and I am not a lawyer.

Using the following propositions, what does this read in logic?

p = I am a lawyer

q = I like dogs

r = I drink tea

$(q \wedge r) \wedge \neg p$

Ex. 3

p	0	0	1	1
q	0	1	0	1
$p \leftrightarrow q$	1	0	0	1

a) $(p \to q) \wedge (q \to p)$

$(p \to q) \to r$, $(q \to p) \to s$

p	q	r	s	$r \wedge s$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

b) $\neg p \leftrightarrow \neg q$

$(\neg p \leftrightarrow \neg q) \to r$

$\neg p$	$\neg q$	r
1	1	1
1	0	0
0	1	0
0	0	1

c) $(p \to q) \wedge (\neg p \to \neg q)$

$(p \to q) \to r$, $(\neg p \to \neg q) \to s$

p	q	r	s	$r \wedge s$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

d) $\neg(p \oplus q)$

p	q	$p \oplus q \equiv s$	$\neg s$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

Ex.4 $\to$ associative?

$$(p \to q) \to r \overset{?}{\equiv} p \to (q \to r)$$

p	q	r	s	t	$s \to r$	$p \to t$
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
1	0	0	0	1	1	1
0	1	1	1	1	1	1
1	0	1	0	1	1	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

not associative

$\leftrightarrow$ associative?

$$(p \leftrightarrow q) \leftrightarrow r \overset{?}{\equiv} p \leftrightarrow (q \leftrightarrow r)$$

associative

Ex.5 a) $p \vee q$ b) $p \oplus q$

Ex.6 a) Galileo wasn't accused and the Earth is the centre of the universe.

b) The Earth moves, therefore the Earth is not the centre of the universe.

c) The Earth moves if and only if the Earth is not the centre of the universe

d) The Earth moves so Galileo was accused, or the Earth is the centre of the universe so Galileo wasn't accused.

Ex.7 a) If you are good, Sinterklaas brings you toys
converse: If Sinterklaas brings you toys, you are good.
contrapositive: If Sinterklaas doesn't bring you toys, you are not good.

b) If the package weighs more than one kilo, then you need extra postage.
converse: If you need extra postage, then the package weighs more than one kilo.
contrapositive: If you don't need extra postage, then the package doesn't weigh more than one kilo.

c) If I have a choice, I don't eat courgette.
If I don't eat courgette, I have a choice.
If I eat courgette, I don't have a choice.

Ex. 8 a) 1 b) 7 c) 1 d) 1

TA 1/1

1. **Formulating precisely**

Consider the following claims and arguments written in English. Formulate a logically precise claim in propositional logic using logical operators to represent the claim. For instance for "I like puzzles and I like tea", one might use $p \wedge q$ where p is "I like puzzles" and r is "I like tea.".

(a) (2 min.) If I do this homework, then I have a greater chance at passing the course.

(b) (2 min.) Only if I do this homework will I get feedback from TA's.

(c) (2 min.) I can choose to do the MC-test in week 3 or I can choose not to.

(d) (4 min.)
If I pass the course, then I have practiced well.
If I pass the course, then I have passed the endterm.
I have practiced well and passed the endterm.
Therefore, I have passed the course.

(e) (4 min.)
It is not true that: I do the homework and I do not get feedback from TA's.
I do the homework or I do not have a greater chance of passing the course.
I do not get feedback from the TA's
Therefore, I do not have a greater chance of passing the course.

a) $p \rightarrow q$

b) $p \leftrightarrow r$

c) $s \vee \neg s$

$Q(p) \vee Q(\neg p)$

d)
$p \rightarrow q$
$p \rightarrow r$
q, r
---
p

e)
$\neg(p \wedge q)$
$p \vee r$
q
---
r

2. Boolean Algebra

Double negation	$\neg(\neg p) \equiv p$
Excluded middle	$p \vee \neg p \equiv \mathbb{T}$
Contradiction	$p \wedge \neg p \equiv \mathbb{F}$
Identity laws	$\mathbb{T} \wedge p \equiv p$ $\mathbb{F} \vee p \equiv p$
Idempotent laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Commutative laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
DeMorgan's laws	$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

TA1 / 3

a) $\left(p \vee \overset{s}{\cancel{(q \wedge r)}}\right) \overset{?}{\equiv} \left(\neg p \longrightarrow \overset{s}{\cancel{(q \wedge r)}}\right)$

$p = 1$ holds
$p = 0$ $s = 0$ holds
$s = 1$ holds

logically equivalent

b) $(p \vee q) \longrightarrow (r \vee q) \overset{?}{\equiv} (\neg r \vee \neg q) \longrightarrow (\neg p \vee \neg q)$

1 0

$q = p = 1$
$r = 0$

c) $(q \vee (p \rightarrow q)) \overset{?}{\equiv} \neg(\neg q \wedge (p \wedge \neg q))$

$q \vee (\neg p \vee q)$

p	q	$p \rightarrow q$	$\neg p \vee q$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

logically equivalent

d) $((\neg s \leftrightarrow p) \wedge (r \rightarrow p)) \overset{?}{\equiv} ((\neg p \wedge \neg r \wedge \neg s) \vee (p \wedge s))$

0 1

$p = s = 1$
$r = 0$