20/03 **Exercises**

① $\det(B_2)=0 \quad \det(B_3) = 1\cdot\begin{pmatrix}1&1\\1&0\end{pmatrix} + 1\cdot\begin{pmatrix}1&1\\1&1\end{pmatrix} = -1+\det(B_2) = -1 \quad \Rightarrow \det(B_{100}) = -98$

$\det(B_4) = -1\cdot\begin{pmatrix}1&1&0\\1&0&1\\1&0&0\end{pmatrix} + 1\cdot\begin{pmatrix}1&1&1\\1&1&0\\1&0&1\end{pmatrix} = -1\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix} + \det(B_3) = -2 \qquad \det(B_{1000}) = -998$

② i) $D_2 = \det(A_2) = 0 \quad D_3 = \det(A_3) = 1\cdot\begin{pmatrix}1&1\\1&1\end{pmatrix} - 1\cdot\begin{pmatrix}1&1\\0&1\end{pmatrix} = \det(A_2) - 1 = -1$

$D_4 = \det(A_4) = 1\cdot\begin{pmatrix}1&1&0\\1&1&1\\0&1&1\end{pmatrix} - 1\cdot\begin{pmatrix}1&0&0\\1&1&1\\0&1&1\end{pmatrix} = \det(A_3) - \det(A_2) = -1$

ii) $D_{n+1} = D_n - D_{n-1} \qquad \Rightarrow D_{1000} = D_{1000 \bmod 6} = D_4 = -1$

iii) $D_5 = D_4 - D_3 = 0 \quad D_6 = D_5 - D_4 = 1 \quad D_7 = D_6 - D_5 = 1 \quad D_8 = D_7 - D_6 = 0 \quad D_9 = -1$

③ $A - 3\times 3 \quad A^3 = -2A$

i) $(\det(A))^3 = -8\det(A) \Rightarrow \det(A) = 0 \Rightarrow A$ - not invertible

ii) $\det(A) = 0 \Rightarrow \lambda = 0$

iii) $A^{100} = A\cdot(A^3)^{33} = (-2)^{33}\cdot A^{34} = (-2)^{33}\cdot A\cdot(A^3)^{11} = (-2)^{44}\cdot A^{12} = (-2)^{44}\cdot(A^3)^4 = (-2)^{48}\cdot A^4 = -2^{49}A^2$

④ $\det(A-\lambda I_4) = (1-\lambda)\cdot\begin{pmatrix}1-\lambda&0&0\\0&1-\lambda&-1\\0&1&-1-\lambda\end{pmatrix} - 1\cdot\begin{pmatrix}1&0&0\\0&1-\lambda&-1\\0&1&-1-\lambda\end{pmatrix} = (1-\lambda)^2\big((1-\lambda)(-1-\lambda)+1\big) - 1\big((1-\lambda)(-1-\lambda)+1\big)$

$= (\lambda^2-2\lambda)\lambda^2 = \lambda^3(\lambda-2) \qquad \lambda_1 = 0 \quad \lambda_2 = 2$

$\lambda=0 \quad A-\lambda I = \begin{bmatrix}1&1&0&0\\1&1&0&0\\0&0&1&-1\\0&0&1&-1\end{bmatrix} \Rightarrow x = \begin{pmatrix}-1\\1\\0\\0\end{pmatrix}x_2 + \begin{pmatrix}0\\0\\1\\1\end{pmatrix}x_4$

g.m.$(\lambda_0) = 2$ a.m.$(\lambda_0) = 3$
$\Rightarrow$ not diagonalizable

$\lambda=2 \quad A-\lambda I = \begin{bmatrix}-1&1&0&0\\1&-1&0&0\\0&0&-1&-1\\0&0&1&-3\end{bmatrix} \Rightarrow x = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}x_2$

$B = \begin{bmatrix}1&2&3&4\\1&2&3&4\\1&2&3&4\\1&2&3&4\end{bmatrix}$

⑤ a) F b) F c) F

$B-\lambda I = \begin{bmatrix}1-\lambda&2&3&4\\1&2-\lambda&3&4\\1&2&3-\lambda&4\\1&2&3&4-\lambda\end{bmatrix} \sim \begin{bmatrix}1&0&0&-1\\0&1&0&-1\\0&0&1&-1\\0&0&0&\lambda-10\end{bmatrix}$ factor: λ^3

⑥ a) T b) F c) T

⑦ $\det(B-\lambda I) = \lambda^3(\lambda-10) \quad \lambda_1 = 0 \quad \lambda_2 = 10 \quad$ g.m.$(\lambda_0) =$ a.m.$(\lambda_0) = 3 \quad$ g.m.$(\lambda_{10}) =$ a.m.$(\lambda_{10}) = 1$

⑧ a) C has column with only 4 -> $\begin{pmatrix}0\\0\\1\\1\end{pmatrix}$

b) $\det(C) = 0 \quad \lambda = 0$

c)