Chapter 2 - Miscellaneous Math

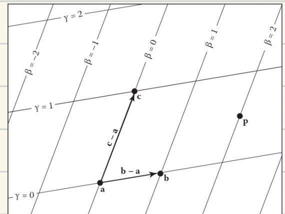
7. Triangles

information tagged onto vertices - interpolated across triangle

interpolation - barycentric coordinates

7.1. 2D Triangles

$$\begin{aligned}\text{area} &= \frac{1}{2}\begin{vmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{vmatrix} \\ &= \frac{1}{2}(x_a y_b + x_b y_c + x_c y_a - x_a y_c - x_b y_a - x_c y_b)\end{aligned}$$

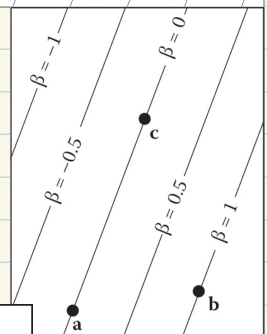


p(α, β, γ) = αa + βb + γc, where α + β + γ = 1

0 < a, b, c < 1 => p lies inside the triangle

one coordinate = 0; other two (0, 1) => point lies on edge

two coordinates = 0; third one (0, 1) => point is a vertex

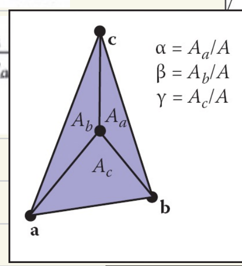


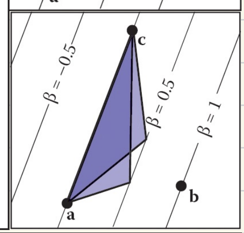
$$\begin{bmatrix} x_b - x_a & x_c - x_a \\ y_b - y_a & y_c - y_a \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} x_p - x_a \\ y_p - y_a \end{bmatrix}$$

$$\gamma = \frac{(y_a - y_b)x + (x_b - x_a)y + x_a y_b - x_b y_a}{(y_a - y_b)x_c + (x_b - x_a)y_c + x_a y_b - x_b y_a}$$

$$\beta = \frac{(y_a - y_c)x + (x_c - x_a)y + x_a y_c - x_c y_a}{(y_a - y_c)x_b + (x_c - x_a)y_b + x_a y_c - x_c y_a}$$

$$\alpha = 1 - \beta - \gamma$$





7.2. 3D Triangles

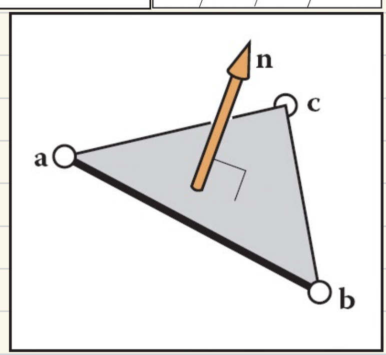
n = (b - a) x (c - a)

area = ll (b - a) x (c - a) ll / 2

α = n.na / llnll^2, where na = (c - b) x (p - b)

β = n.nb / llnll^2, where nb = (a - c) x (p - c)

γ = n.nc / llnll^2, where nc = (b - a) x (p - a)



Chapter 7 - Viewing

5. Field-of-View

window (l, r, b, t)

through center => l = -r; b = -t

constraint: square pixels

nx / ny = r / t = l / b

tan θ/2 = t / lnl