Validity of Arguments (2.5)

premise → proposition known to be true
argument → claim that conclusion follows from premises

<u>Def.</u>| $P \Rightarrow Q$ $P \to Q$ is tautology logically implies/deduced

valid argument → conclusion follows from premises

$$\begin{array}{c} p \to q \\ \underline{p} \\ \therefore q \end{array}$$
modus ponens

$$\begin{array}{c} p \to q \\ \underline{\neg q} \\ \therefore \neg p \end{array}$$
modus tollens

$$\begin{array}{c} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$
Law of Syllogism

$$\begin{array}{c} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$$

$$\begin{array}{c} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$$

$$\begin{array}{c} \underline{p \wedge q} \\ \therefore p \end{array}$$

$$\begin{array}{c} \underline{p} \\ \therefore p \vee q \end{array}$$

<u>Def.</u>| formal proof → sequence of propositions (premises or deduced)

Number Sets and Divisibility (3.2.2)

natural numbers $(\mathbb{N}) \to 0, 1, 2 \ldots$ $+ \times$
integers $(\mathbb{Z}) \to 0, -1, 1 \ldots$ $+ - \times$
rational numbers $(\mathbb{Q}) \to \frac{m}{n}$ $+ - \times /$
real numbers $(\mathbb{R}) \to$ decimal form $+ - \times /$
irrational numbers $\to \sqrt{3}, \pi$

n, m - integers
$m | n$ if $n = mk$ for integer k
$2 | n \Rightarrow n$ - even $2 \nmid n \Rightarrow n$ - odd
$n > 1$ - prime if $1 | n$ $n | n$ and no other

3.2./ Ex. 1 n^2+n+41 is not prime

↳ $n(n+1)+41$

$n=40$ $\quad 40(41)+41=41^2$

$n=41$ $\quad 41(42)+41=41\times 43$

Introduction to Proofs (3. – 3.2.1.)

hypothesis – assumption in a theorem

$\forall x\, P(x)$ → proof by generalization

$\exists x\, P(x)$ → existence proof

$\neg(\forall x\, P(x)) \equiv \exists x(\neg P(x))$ → counterexample