06/09 George Boole (1815 - 1864)

— Boolean Algebra

not (coffee or tea) $\neg(p \lor q)$

⇔ not coffee and not tea $\neg p \land \neg q$

To pass the course R82, it is necessary that you pass the exam.

pass R82 ⟶ pass exam

cannot be $1 \to 0$

$p = 1$

$p \lor q = 1$ $\quad q \lor \neg q = 1$ $\quad p \to q$ $\quad q \to p = 1$

$p \lor \neg q = 1$

only true if $q = 1$

If there are puzzles, Stefan is happy.

contrapositive → If Stefan isn't happy, there are no puzzles.

equivalence $p \equiv q$ $(p \leftrightarrow q)$

when $p \to q$ and $q \to p$

$$(\neg p \to (q \lor \neg r)) \overset{?}{\equiv} ((r \to p) \lor \neg q)$$

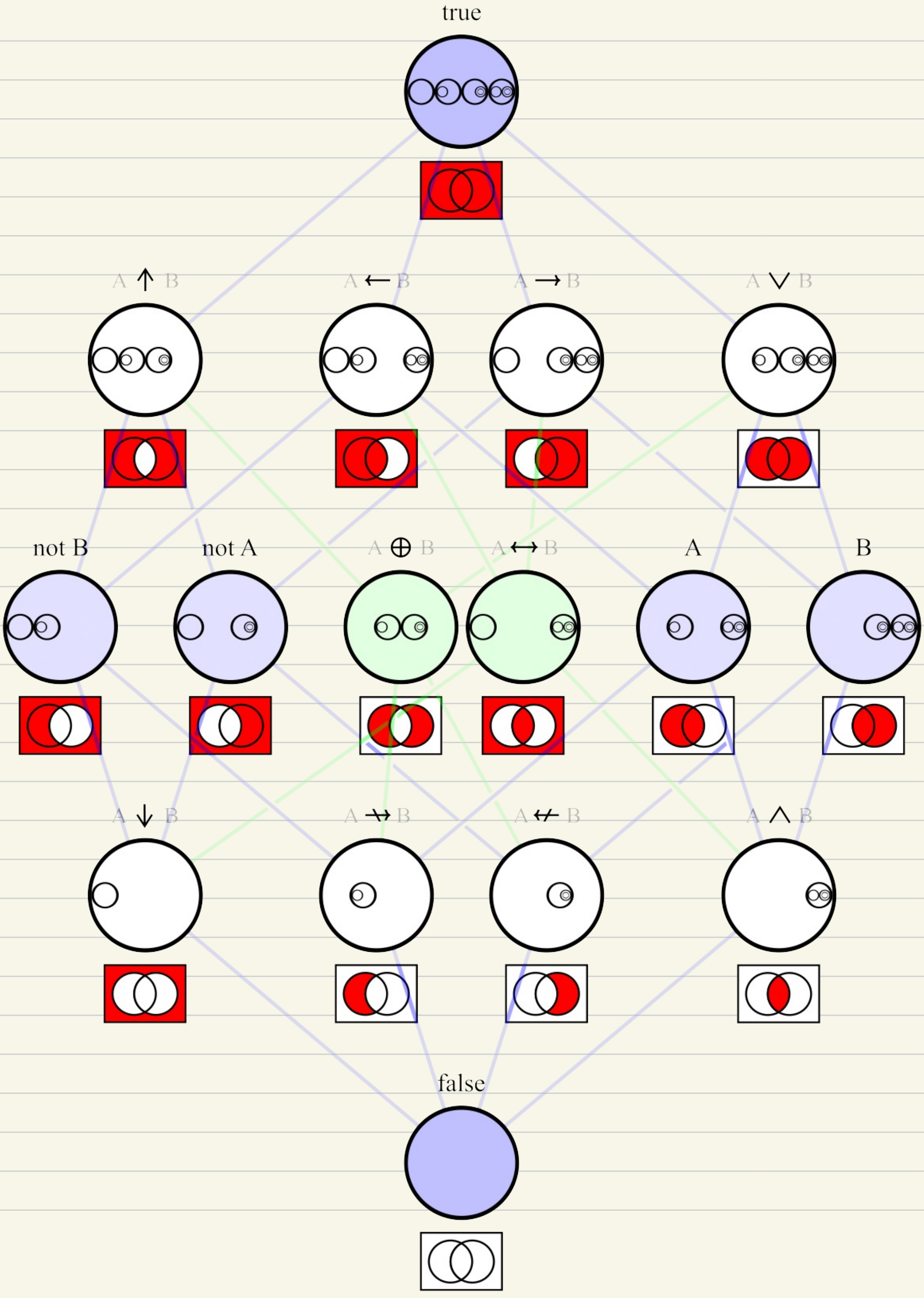
0 1

$r = 1$

$p = q = 0$ not equivalent

binary connectives $\to 2^4 = 16$

p	q	b.c.
0	0	.
0	1	.
1	0	.
1	1	.



true
A ↑ B
A ← B
A → B
A ∨ B
not B
not A
A ⊕ B
A ↔ B
A
B
A ↓ B
A ↛ B
A ↚ B
A ∧ B
false

4 suspects → 3 lying, 1 truthful

Mia: not Larry T
Payne: not Sahwit F
Larry: not me and not Mia F
Sahwit: Larry F

Sahwit

08/09 $(p \to q) \leftrightarrow (\neg p \to q) \equiv (\neg p \vee q) \leftrightarrow (p \vee q) \equiv$

p	q	$p \to q$	$\neg p \to q$	$\leftrightarrow$
0	0	1	0	0
0	1	1	1	1
1	0	0	1	0
1	1	1	1	1

$(\neg p \vee p) \leftrightarrow q \equiv T \leftrightarrow q \equiv q$

logically valid → if in all situations where the premises are true, the conclusion also is

$c \vee \neg t \quad t \to c \quad \neg c \to \neg t$

$\neg w \to \neg p \quad w \vee \neg p \quad p \to w$

knaves – always lie
knights – always true
J&B – either knaves or knights

✓ J → We are the same
✗ B → We are different

$\neg((p \wedge q) \to \neg(\underbrace{r \to q}_{\neg r \vee q})) \equiv$

$\neg((p \wedge q) \to (r \wedge \neg q)) \equiv$

$\neg(\neg p \vee \neg q \vee (r \wedge \neg q)) \equiv$

$p \wedge q \wedge (\neg r \vee q) \equiv$

$p \wedge q \wedge \neg r$

$(\neg p \to q) \equiv (p \vee q) \equiv (\neg q \to p)$

$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$

$(p \wedge (q \vee r)) \equiv ((p \wedge q) \vee (p \wedge r))$

$(p \to (q \wedge r)) \equiv ((p \to q) \wedge (p \to r))$

$p \vee \neg q \to$ contingency

$(\neg p \to (p \to q)) \equiv p \vee (\neg p \vee q) \to$ tautology

$((p \to q) \leftrightarrow (\neg p \vee q)) \to$ tautology

$\neg p \vee q$

$((p \vee q) \to ((p \wedge r) \vee (q \wedge r))) \equiv [(\overset{s}{p \vee q}) \to ((\overset{s}{p \vee q}) \wedge r)]$ } contingency

$(p \vee q) \wedge r$

$s \to (s \wedge r)$ $\neg s \vee s \wedge r$

$\neg(p \to (q \to p)) \equiv \neg(\neg p \vee \neg q \vee p) \equiv p \wedge q \wedge \neg p \to$ contradiction

$A \equiv B$

$\neg(A \vee B) \to$ contingency (not guaranteed)

$A \to B \to$ tautology

$B \wedge \neg A \to$ contradiction

$(B \to A) \to B \equiv (\neg B \vee A) \to B \equiv (B \wedge \neg A) \vee B$

If $x > 10$, then $x > 5$.

$P(x) = x > 10$ $\quad Q(x) = x > 5$

$\forall x (P(x) \to Q(x))$

All students from Delft have a bicycle.

$\llcorner$ negation

$\rightarrow$ There exists a student from Delft without a bike

All students from Delft have a bike or a car.

$\llcorner$ negation

$\rightarrow$ Every student from Delft has no bike and no car.

If all students in Delft have a bike, then all students in Delft have no car.

$\llcorner$ negation

Student Eddie from Delft has no car, and no student from Delft has no bike

$E(x) \rightarrow x$ is even

$P(x) \rightarrow x$ is prime number

All prime nubers are even. $\forall x\,(P(x) \rightarrow E(x))$

There exists an even prime number. $\exists x\,(P(x) \land E(x))$

There exists exactly one prime number.

$$\exists x\,(P(x) \rightarrow E(x)) \land \forall y\,[(P(y) \rightarrow E(y)) \rightarrow (y = x)]$$

Mario is a plumber and Jenny is a lawyer.

m $\quad P(x) \quad j \quad L(x)$ $\quad P(m) \land L(j)$

13/09

p	$\to q$
0	0
0	1
1	1
1	0

$1100111_2 = 103_{10}$

F → Every student from Delft has no bicycle and no car.
X → All students from Delft have a bicycle or a car.

$$\frac{F}{\therefore \neg X}$$

invalid

F — 'vacuously' true: might have no students in Delft

$\exists x\,(P(x) \land \forall y(\neg Q(y) \lor \neg R(x,y)))$

→ negation of $\forall x\,(P(x) \to \exists y(Q(y) \land R(x,y)))$

Pierce's arrow ($\downarrow$) NOR

p	q	$p \downarrow q$
0	0	1
0	1	0
1	0	0
1	1	0

$D = \{a, b, c, d, e\}$

$R = \{(a,a), (b,b), (c,c), (a,d), (e,e)\}$

15/09 skipped

20/09 n – even iff n^2 – even

$n = 2k \quad 2 \mid n \qquad n = 2k+1 \quad 2 \nmid n$

$n^2 = 4k^2 \quad 2 \mid n^2 \qquad n^2 = 4k^2 + 4k + 1 \quad 2 \nmid n^2$

$f: \mathbb{Z} \to \mathbb{N}$ sum of digits

S: $n \to$ triple iff $3 \mid f(n)$ theorem

II: $n \to$ triple iff $3 \mid n$ definition

$5 \nmid n \to P(n)$ holds

$5 \nmid k$

$k = 5m+1$ prove $P(k)$ holds

$k = 5m+3$ prove $P(k)$ holds

$k = 2m$ prove $P(k)$ holds $\Leftarrow$ $5 \mid m$ might be true

QED $5 \nmid n \to P(n)$ holds $\quad 5m+2 \mid 5m+4$ not covered $m \to$ odd

$3 \mid n^3 - n$

arbitrary $k \in \mathbb{N}$

$k^3 - k = \boxed{k}(k^2-1) = k(k-1)(k+1) \to$ multiplication of 3 conseq. numbers $\Rightarrow$ divisible by 3

$a, b \to$ irrational $\quad a^b \to$ rational

$e^{\ln 2} = 2 \qquad a = e \quad b = \ln 2$

21/09 Study Session

$s \to t$
$q \to p$
$p \wedge s$
$t \to z$
$\therefore z$

s
$s \to t$
$t \to z$
$\therefore z$

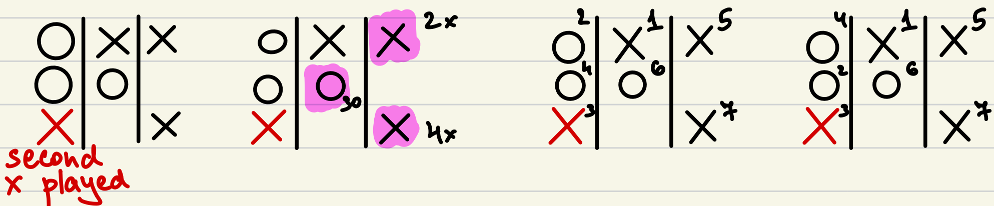
Pete has only two friends w/o a hat.
$p \to$ Pete
$F(x,y) \to x$ and y are friends
$H(x) \to x$ has a hat
$\exists x \exists y (F(p,x) \wedge \neg H(x) \wedge F(p,y) \wedge \neg H(y)) \to \forall z (x \neq z \wedge y \neq z \wedge \neg F(p,z))$

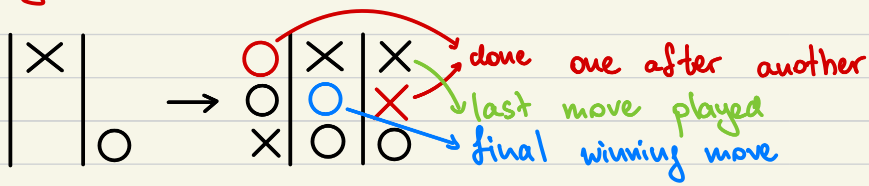
$p \Leftrightarrow q \to$ might be true or not
$p \equiv q$ / $p \Longleftrightarrow q \to$ statement / $p \Leftrightarrow q$ is tautology

2 quantifiers $(\exists, \forall)$
2 unary predicates $(P(x))$
1 binary predicate $(P(x,y))$

$\exists x (\text{Lawyer}(x) \wedge \forall y (\text{Prisoner}(y) \to \neg \text{Met}(x,y)))$
$\forall x (\neg \text{Lawyer}(x) \vee \exists y (\text{Prisoner}(y) \wedge \text{Met}(x,y)))$

All natural numbers containing the digit 5 are divisible by 3.
Proof by contradiction:
Assume there exists a natural number containing the digit 5 that is not divisible by 3.





22/09

$P(n) \equiv \sum_{i=0}^{n} \frac{1}{2^i} \stackrel{?}{=} \frac{2^{n+1}-1}{2^n}$ base step: $P(0) \stackrel{?}{=}$ true

$P(2) \equiv \underbrace{1+\frac{1}{2}}_{}+\frac{1}{4} \stackrel{?}{=} \frac{8-1}{4} = \frac{7}{4}$ inductive step: $P(n) \stackrel{?}{\rightarrow} P(n+1)$ $n \geq 0$

$\sum_{i=1}^{n} i.2^i = (n-1).2^{n+1}+2$ $n \geq 1$

Base:
$n=1$ $1.2^1 = (1-1).2^2+2 = 2$ ✓

$n=k$ $\sum_{i=1}^{k} i.2^i = (k-1).2^{k+1}+2$

inductive
$n=k+1$

$$\sum_{i=1}^{k+1} i.2^i = (k+1-1).2^{k+1+1}+2$$

$$\underbrace{\sum_{i=1}^{k} i.2^i + (k+1)2^{k+1}}_{} = k.2^{k+2}+2$$

$(k-1).2^{k+1}+2+(k+1)2^{k+1} =$

$(k\cancel{-1}+k\cancel{+1}).2^{k+1}+2 =$

$= 2k.2^{k+1}+2 = k.2^{k+2}+2$ ✓

$$\neg(\underbrace{(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)}_{F}) \vee r \Rightarrow \neg(F) \vee r \equiv T \vee r \equiv T$$

$mb \rightarrow fa$
$fa \rightarrow na$
$\therefore mb \rightarrow na$
$\downarrow$ suf. $\downarrow$ nes.

$\forall x(Puzzle(x) \rightarrow \exists y(Answer(y) \wedge Solves(y,x)))$

There is no answer that solves a particular puzzle.

27/09 $\forall n \in \mathbb{N},\ n \geq 1,\ 6 \mid (n^3 - n)$

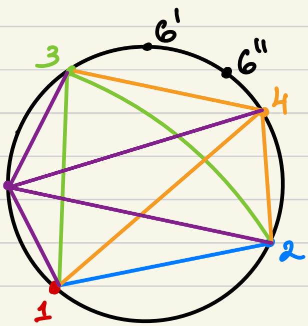
inductive step: $\exists a \in \mathbb{N}$ s.t. $k^3 - k = 6a$, arbitrary $k \in \mathbb{N}$
$\Rightarrow \exists b \in \mathbb{N}$ s.t. $(k+1)^3 - (k+1) = 6b$

base case: $n = 1 \quad 6 \mid 0$
ind. ass.: $6 \mid k^3 - k$
ind. case: $(k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1$

$$= k^3 + 3k^2 + 2k =$$
$$= (k^3 - k) + 3k^2 + 3k$$
$$= (k^3 - k) + 3k(k+1)$$

$6 \mid k^3 - k$
$3 \mid 3k(k+1)$ } $6 \mid 3k(k+1)$ $\Rightarrow \checkmark$
$2 \mid 3k(k+1)$
↳ multiplication of conseq. numbers

n	S_n
1	1 (2^0)
2	2 (2^1)
3	4 (2^2)
4	8 (2^3)
5	16 (2^4)
n	$\Rightarrow 2^{n-1}$



n	a_n	b_n
1	1	1
2	3	4
3	5	9
4	7	16
5	9	25
6	11	36
n	$2n-1$	n^2

$\forall n\, (b_n = n^2)$ $\qquad b_k = k^2 = \sum_{i=1}^{k} a_i$

$\forall n \left(\sum_{i=1}^{n} a_i = n^2 \right)$ $\qquad b_{k+1} = \sum_{i=1}^{k+1} a_i = \sum_{i=1}^{k} a_i + a_k + 2 = b_k + a_k + 2$

$\forall n \left(\sum_{i=1}^{n} 2i+1 = n^2 \right)$ $\qquad b_{k+1} = k^2 + 2k - 1 + 2 = k^2 + 2k + 1 = (k+1)^2$

Proof by mathematical induction:

Let $P(n)$ be $\left\{\sum_{i=1}^{n} 2i-1 = n^2\right\}$

Base case ($n=1$):

$\left\{ \qquad \right\}$

Inductive step: T.P. $\forall n\, (P(n) \rightarrow P(n+1))$

Take an arbitrary k, s.t. $P(k)$ holds. (IH)

T.P. $P(k+1)$

$\left\{ \qquad \right\}$

Since k was arbitrary chosen, it holds $\forall n\, (P(n) \rightarrow P(n+1))$.

So by the principle of induction, $\underset{\geq 1}{\forall n} \left(\sum_{i=1}^{n} 2i-1 = n^2\right)$ $\boxed{\text{QED}}$

Invariants:

$\frac{b}{x-a} = \text{const.} = y$

$x-a$ div. by y

$\frac{b}{y} = \text{const.} = x-a$

$b = (x-a)y$ ✓

$b \geq 0$ ✓

$0 \leq a \leq x$ ✓

$y \mid b$ ✓

$x, y = \text{const.}$ ✓

25w, 25b in a bag

take 2

- if same color, put black in
- if diff. color, put the white in

	black	white	tot
BB	-1	$=$	-1
WW	$+1$	-2	-1
WB	-1	$=$	-1

#W is always odd

1) beginning: $W = 25 \rightarrow$ odd ✓
2) after iteration: $W = / -2 \rightarrow$ odd ✓
3) end: total always $-1 \rightarrow$ will end after 49 iterations ✓
4) intend to work: only 1 left $\rightarrow$ white ✓

28/09

Study Session

$\neg r \vee (p \to q) \equiv \neg r \vee \neg p \vee q$

$(p \vee r) \wedge \neg q$

All elephants have green feet.

$\forall x\,(E(x) \to Feet(x, green))$

$\sqrt{2}$ is irrational

Proof by contradiction

Assume $\sqrt{2}$ is rational.

$\Rightarrow \sqrt{2} = \frac{k}{p}$ for some integers k, p that have no common factor.

$\Rightarrow \sqrt{2}\,p = k \quad |\, (\ )^2$

$2p^2 = k^2$

$2 \mid k^2 \Leftrightarrow 2 \mid k \Rightarrow k = 2n$

$2p^2 = (2n)^2$

$2p^2 = 4n^2$

$p^2 = 2n^2$

$2 \mid p^2 \Leftrightarrow 2 \mid p \Rightarrow p = 2q$

But p, k have no common factor.

$\Rightarrow \sqrt{2}$ is irrational

$(\exists x\,(P(x)) \wedge \exists y\,(Q(y))) \leftrightarrow \forall x\,(P(x) \wedge Q(x))$

$\left.\begin{array}{l} D = P = Q = \{a\} \\ D = \{a\} \quad P = Q = \varnothing \end{array}\right\}$ true

$D = P = Q = \{\} \to$ false

Base case $(n=1)$

The only person with green eyes leaves the first night.

truth tables — **Propositional Logic** — DeMorgan, equivalency, functionally complete set

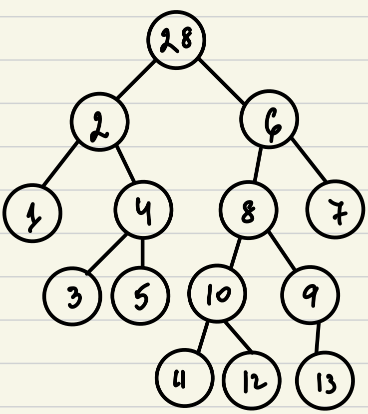
sufficient -> necessary — **Predicate Logic** — quantifiers, domain of discourse, vaguosly true

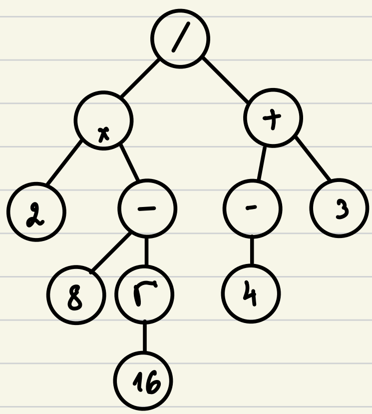
Proofs — contradiction, generalization, explosion

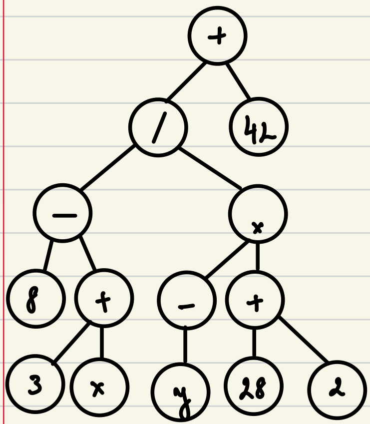
$P(k) \rightarrow P(k+1)$
$P(0) \wedge \dots \wedge P(k)$
$\rightarrow P(k+1)$

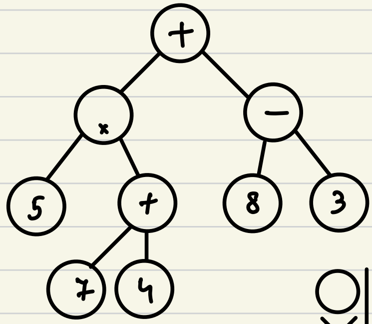
Induction & Recursion — inductive assumption, base case, weak / strong { mathematical induction, invariants

29/09

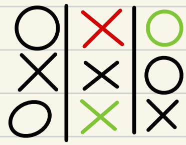








prefix: + − × 8 3 5 + 7 4
infix: (7+4)×5+8−3
postfix: 7 4 + 5 3 8 × − +



Binary tree
- → 2 children at most
- → 1 right child and 1 left

iterating { pre-order, in-order, post-order }

proper and full → every node has either 0 or 2 children

pre-order traversal:
1) value of current node
2) pre-order tranversal of the left child
3) pre-order tranversal of the right child

in-order traversal:
1) in-order traversal of the left child
2) value od current node
3) in-order traversal of the right child

post-order traversal:
1) post-order traversal of the left child
2) post-order traversal of the right child
3) value od current node

$f_1 = 2$ $\quad f_n = \begin{cases} n+1, & 1 \le n \le 3 \\ f_{n-1} f_{n-2} f_{n-3}, & n \ge 4 \end{cases}$

$f_2 = 3$

$f_3 = 4$

weak induction $\quad P(1)$

$P(n) \to P(n+1) \ \forall n \ge \underline{1}$

strong induction $\quad P(1)$

$P(1) \wedge \ldots \wedge P(n) \to P(n+1) \ \forall n \ge \underline{1}$

11/10

Set Theory

$|\mathbb{B}| \to$ how many elements the set $\mathbb{B}$ has $\qquad \mathbb{B}$ – binary set $\{0,1\}$

$\{0,1,0\} = \{1,0\} = \{0,1\}$ $\qquad \underset{\text{binary}}{\mathbb{B}} \to \overset{\text{natural}}{\mathbb{N}} \to \underset{\text{integer}}{\mathbb{Z}} \to \overset{\text{decimal}}{\mathbb{Q}} \to \underset{\text{real}}{\mathbb{R}} \to \overset{\text{complex}}{\mathbb{C}}$

$A = \{x^2 \mid \frac{x}{2} \in \mathbb{Z}\}$ $0, 4, 16, 36, 64, \ldots$

$A \subseteq B$

$B = \{x \in \mathbb{N} \mid \exists y \in \mathbb{Z} : x = 4y\}$ $0, 4, 8, 12, 16, \ldots$

$\cancel{\emptyset \in \emptyset}$ $\qquad x \in \emptyset$ is false for $\forall x$ $\qquad x \ne \{x\}$

$\emptyset \subseteq X$ is true for $\forall x$

$\emptyset \not\subset \emptyset$ $\quad X \not\subset X$ $\quad$ a set can never be a proper subset of itself

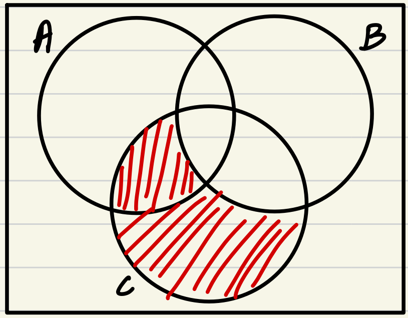
$A \subseteq B \quad \forall x (x \in A \to x \in B)$

$C \nsubseteq D \quad \exists x (x \in C \wedge x \notin D)$

$E \not\subset F \quad \exists x (x \in E) \wedge \forall x (x \in E \to x \notin F \vee x \in F \to x \notin E)$

$\exists x (x \in E \wedge x \notin F) \vee (E = F) = (E \subseteq F \wedge F \subseteq E)$

$\forall x\,(x \notin (A \cap B) \to x \notin B)$
$\exists x\,(x \in (A \cap B) \land x \notin B)$



$((A - B) \cap (C \cup B)) \cup (B^c \cap C)$

Structural Induction

$\mathcal{P}(A) = 2^A$ power set
$\mathcal{P}(\{a\}) = \{\{a\}, \emptyset\}$
$\mathcal{P}(\{a, b\}) = \{\{a\}, \{b\}, \{a, b\}, \emptyset\}$
$\mathcal{P}(\{\}) = \{\emptyset\}$
$\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

$\emptyset = \{\}$

26/10 Study Session

$A = \{a, b, c, z\}$

$B = \{a, c, b, f\}$

$((A \times B) \times B) = 4 \times 4 \times 4 = 64$

There is at least one TU Delft student who loves CO.
$\exists x\,(\text{Student}(x, \text{TU Delft}) \cap \text{Loves}(x, \text{CO}))$

$\neg(a \to (b \leftrightarrow c)) \land a$

$a = 0$	F	
$a = 1$	$b = c$	T
	$b \neq c$	F

$x \in \mathbb{Z},\ 30|x \to 3|x$

Proof by generalization

Take an arbitrary k s.t. $30|k$

$\Rightarrow k = 30m$ for some integer m.

$\Rightarrow k = 3(10m) \Rightarrow 3|k$

Arbitrary $k \Rightarrow$ holds for any integer.

$\sqrt{6} \to$ irrational

directed: (a, b) directed graph $E \subseteq V \times V$

undirected: $\{a, b\}$ undirected graph $E \not\subseteq \mathcal{P}(V)$

$\not\subseteq \mathcal{P}(V)^2$

$= \{\{a, b\} \mid a, b \in V\}$ $+ a \neq b$

$= \{\{a, b\}\}$

height = 1

$2^1 - 1 = 1$ ✓

height = k

$2^k - 1$ nodes

height = $k+1$

$2^{k+1} - 1$ nodes?

$2^k - 1 + \underbrace{2^k}_{\text{next level}} = 2^{k+1} - 1$ ✓

$f: \mathbb{N} \to \mathbb{N}$

$f(x) = \sqrt{x}$

$f = \{(0, 1), (1, 2), (2, 3)\} \to$ function?

$\to$ relation $\to$ set of duples

$A = \{0, 1, 2\}$

$f: A \to (A \cup \{3\})$ $f(0) \neq f(1) \neq f(2)$ $f(x) \neq 0$

function $\uparrow$ not

bijective $\to$ injective $\cap$ surjective

1. bijective

$f: \mathbb{Z} \to \{420\}$

$f(x) = 420$

surjective

2. injective xor surjective

$f: \mathbb{N} \to \mathbb{N}$

$f = \{(69, 420), (420, 69)\}$

not well defined

3. not well identified

$f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = x+1$

bijective

hotel $k: \quad 2^k$
lens $1: \quad 3^{k+1}$
$2: \quad 5^{k+1}$
$3: \quad 7^{k+1}$

	0	1	2	3	4	5 ...
hotel	0	2	5	9		
lens 1	1	4	8			
2	3	7				
3	6					
...						

01/11

$a \in S$
$x \in S \rightarrow xi \in S, \; axa \in S$
$x, y \in S \rightarrow ixiyixi \in S$

$w \in S \rightarrow$ odd number of a's
$x = a \rightarrow ai \rightarrow 1 ✓ \rightarrow$ next adds only i's
$\rightarrow aaa \rightarrow 3 ✓$ next adds even number of a's
$x = y = a \rightarrow iaiaiai \rightarrow 3 ✓$

$m \in S$
$x, y \in S \rightarrow xiy \in S \quad mia \quad \begin{matrix} x = m \\ y = a \end{matrix}$

$\frac{1}{2} \in T$

$t = \frac{2^k}{2^l} \quad k, l \in \mathbb{N}$

I) $\frac{1}{2} \in T$
II) $x, y \in T \rightarrow \frac{x}{y} \in T$
$x \in T \rightarrow 2x \in T, \; \frac{x}{2} \in T$
III) Nothing else in T.

$(A \subseteq B \land (B \cup C) \subseteq A) \rightarrow A = B$
$(\underline{A \subseteq B \land B \subseteq A} \land C \subseteq A) \rightarrow A = B$

$x = \sum_{i=0}^{n} i$

```
F(n)
x ← 1
c ← 0
  while c<n do
    c ← c+1
    x ← x+c
  end while
  return x
end function
```

03/11

Review

f: TREE $\to \mathbb{Z}$ / $\mathbb{N}$ → surjective

$$f(t) = \begin{cases} 0 & t = \emptyset \\ 0 & t = (x, \emptyset) \wedge 3 \nmid x \\ 1 & t = (x, \emptyset) \wedge 3 \mid x \\ 1 + \sum_{i=1}^{k} f(T_i) & t = (x, (T_1, \dots, T_k)) \wedge 3 \mid x \\ 0 + \sum_{i=1}^{k} f(T_i) & t = (x, (T_1, \dots, T_k)) \wedge 3 \nmid x \end{cases}$$

$f((x, \emptyset))$

injective $f(k) = f(m) \Rightarrow x$

surjective a, $f(b) = a$; no negative values $\Rightarrow x$

g: TREE$^3 \longrightarrow \{0, 1\}$

$$g(t_1, t_2, t_3) = \begin{cases} 1 & t_1 = t_2 = t_3 = \emptyset \\ 1 & t_1 = (x_1, \emptyset) \wedge t_2 = (x_2, \emptyset) \wedge t_3 = (x_3, \emptyset) \wedge |x_1 - x_2| = x_3 \\ 1 & t_1 = (x_1, (T_1, \dots, T_k)) \wedge t_2 = (x_2, (P_1, \dots, P_k)) \wedge t_3 = (x_3, (Q_1, \dots, Q_k)) \wedge \forall k (|T_k - P_k| = Q_k) \wedge |x_1 - x_2| = x_3 \\ 0 & \text{else} \end{cases}$$

If both Socrates and Plato are mortal, all living beings are mortal

If being human makes you mortal, then Socrates is mortal and Plato is a human.

$(B \in \mathcal{P}(A) \wedge C \in A) \to (C \in A \Delta B)$

$A = \{1, \{2, 4\}\}$

$B = \{1, \{2, 4\}\}$

$C = \{2, 4\}$

$A \Delta B = \emptyset$

$\mathcal{P}(A) \subseteq B \to A \in B$ ✓

$\emptyset \neq \{\emptyset\}$

$\mathcal{P}(\emptyset) \neq \mathcal{P}(\mathcal{P}(\emptyset))$

$|\mathcal{P}(A)| \in B \to |B| \geq 2^{|A|}$

$A = \{1, 2\}$ $\quad 1 \not\geq 2^2$

$B = \{4\}$ $\quad 1 \not\geq 2^1$

$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

$A = \{1\}$ $\quad \mathcal{P}(A) = \{\emptyset, \{1\}\}$

$B = \{2\}$

end up on the same (ground) floor $1^{st}\ 2^{nd}\ 3^{rd}\ 4^{th}$ floor

before start: taking 1 staircase: stairs[] : $\{a, b, c, d, \dots\}$

stairs[i] -> even all above

stairs[i ? 1] -> even all above

ground floor -> no below

below -> ground floor

have taken 1 staircase left odd

$f(i)$ = # of stairs used on floor i

$\forall i < k \quad 2 \nmid f(i)$

$\forall i \geq k \quad 2 \mid f(i)$

k: → $k+1$ $f(k)$++ $\forall i < k+1 \quad 2 \nmid f(i)$ → k: $f(k+1)$ -> even

$\forall i \geq k+1 \quad 2 \mid f(i)$ ✓ → $k+1$: $f(k+1)$ -> odd

k: → $k-1$ $f(k-1)$++ $\forall i < k-1 \quad 2 \nmid f(i)$ ✓ k: $f(k-1)$ -> odd

$\forall x \geq k-1 \quad 2 \mid f(i)$ → $k-1$: $f(k-1)$ -> even