Classifying propositions

Def. tautology – a proposition is always true
contradiction – a proposition is always false
contingency – neither tautology nor contradiction

Ex. 2 a) $(p \land (p \to q)) \to q$ tautology

p	q	$p \to q$	
0	0	1	1
0	1	1	1
1	0	0	1
1	1	1	1

b) $((p \to q) \land (q \to r)) \to (p \to r)$ tautology

p	q	r	$p \to q$	$\land$	$q \to r$	$p \to r$	
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
1	0	0	0	0	1	0	1
0	1	1	1	1	1	1	1
1	0	1	0	0	1	0	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	1	1

c) $p \land \neg p$ contradiction

e) $p \lor \neg p$ tautology

d) $(p \vee q) \to (p \wedge q)$ contingency

p	q	$p \vee q$	$p \wedge q$	$\vee \to \wedge$	$\wedge \to \vee$
0	0	0	0	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	1	1	1

f) $(p \wedge q) \to (p \vee q)$ tautology

Substitution laws

Theorem (First Substitution Law)

Theorem (Second Substitution Law)

Simplifications

$p \to q \equiv \neg p \vee q$

Ex. 9 a) $p \wedge (q \wedge p) \equiv p \wedge q$

b) $(\neg p) \to q \equiv p \vee q$

c) $(p \vee q) \wedge \neg q \equiv p \wedge \neg q$

d) $p \to (q \to r) \equiv \neg p \vee (\neg q \vee r) \equiv (\neg p \vee \neg q) \vee r \equiv (p \wedge q) \to r$

e) $(p \to r) \wedge (q \to r) \equiv (\neg p \vee r) \wedge (\neg q \vee r) \equiv (\neg p \wedge \neg q) \vee r$
$\equiv (p \vee q) \to r$

f) $p \to (p \wedge q) \equiv \underbrace{\cancel{(p \to p)}}_{T} \wedge (p \to q) \equiv p \to q$

Ex. 10

a) $(p \wedge \overbrace{q) \vee \neg}^{T} q \equiv p \vee \neg q$

p	q	$p \wedge q$	p	$\vee \neg q$
0	0	0	1	1
0	1	0	p	0
1	0	0	1	1
1	1	1	1	1

b) $\neg(p \vee q) \wedge p \equiv \neg p \wedge \neg q \wedge p \equiv F$

p	q	$p \vee q$	$\neg$	$\wedge p$
0	0	0	1	0
0	1	1	0	0
1	0	1	0	0
1	1	1	0	0

c) $p \to \neg p \equiv \neg p \vee \neg p \equiv \neg p$

p	$\neg p$	$\to$
0	1	1
1	0	0

d) $\neg p \wedge (p \vee q) \equiv \neg p \wedge q$

$\neg p$	p	q	$p \vee q$	$\neg p$	$\wedge q$
1	0	0	0	0	0
1	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	0	0

e) $(q \wedge p) \rightarrow q \equiv \neg q \vee \neg p \vee q \equiv T$

p	q	$q \wedge p$	$\rightarrow$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

Ex. 4 a) It is sunny and cold.
It's neither sunny nor cold.

b) I won't have stroopwafel and I won't have appeltaart.
c) If today is Tuesday, this is Belgium.
$T \rightarrow B$ If today is Tuesday, this isn't Belgium.

T	B
0	0
0	1
1	1
1	0

d) If you pass the final exam, you don't pass the course.

Ex. 12 a) $s \wedge c$ $c \wedge s$
It is cold and sunny.

b) $s \vee a$ $a \vee s$

c) $T \rightarrow B$ $\neg T \vee B$
It's either not Tuesday or I'm in Belgium
d) $E \rightarrow C$ $\neg E \vee C$
You either don't pass the final or you pass the course.

Disjunctive Normal Form

Def. DNF – disjunction of conjunctions of simple terms

$(p \to q) \wedge r$ $\qquad$ $p \to q$

$(\neg p \vee q) \wedge r$ $\qquad$ $\neg p \vee q$

$(\neg p \wedge r) \vee (q \wedge r)$

$\neg((p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)) \vee r$

$(\neg p \oplus \neg q) \vee (p \oplus \neg q) \vee r$

$\neg(\neg q \to p) \wedge \neg r$

$\neg(q \vee p) \wedge \neg r$

$\neg q \wedge \neg p \wedge \neg r$

$\neg(\neg(\neg q \vee \neg r) \wedge \neg p)$

$\neg(q \wedge r \wedge \neg p)$

Deduction

Arguments

$$\begin{array}{l} p \to q \\ \underline{p \qquad\quad} \\ \therefore q \end{array}$$

modus ponens

$$\begin{array}{l} p \to q \\ \underline{\neg q \qquad\quad} \\ \therefore \neg p \end{array}$$

modus tollens

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

Law of Syllogism

Ex.1

$$\begin{array}{l} p \to q \\ \underline{\neg q \qquad\quad} \\ \therefore \neg p \end{array}$$

p	q	p → q
0	0	1
~~0~~	~~1~~	~~1~~
~~1~~	~~0~~	~~0~~
~~1~~	~~1~~	~~1~~

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

p	q	r	p → q	q → r	p → r
0	0	0	1	1	1
0	0	1	1	1	1
~~0~~	~~1~~	~~0~~	~~1~~	~~0~~	~~1~~
~~1~~	~~0~~	~~0~~	~~0~~	~~1~~	~~0~~
0	1	1	1	1	1
~~1~~	~~0~~	~~1~~	~~0~~	~~1~~	~~1~~
~~1~~	~~1~~	~~0~~	~~1~~	~~0~~	~~0~~
1	1	1	1	1	1

Ex.3

$$\begin{array}{l} p \to q \\ \underline{q \qquad\quad} \\ \therefore p \end{array}$$

$$\begin{array}{l} p \to q \\ \underline{\neg p \qquad\quad} \\ \therefore \neg q \end{array}$$

p	q	p → q
~~0~~	~~0~~	~~1~~
0	1	1
~~1~~	~~0~~	~~0~~
1	1	1

p	q	p → q
0	0	1
0	1	1
~~1~~	~~0~~	~~0~~
~~1~~	~~1~~	~~1~~

Ex. 4

a)
$p \to q$
$q \to s$
s
$\therefore p$

not valid

b)
$p \wedge q \Rightarrow p = q = T$
$q \to (r \vee s) \Rightarrow r \vee s = T \Rightarrow s = T$
$\neg r \Rightarrow r = F$
$\therefore s$

valid

c)
$p \vee q \Rightarrow q = T$
$q \to (r \wedge s) \Rightarrow r \wedge s = T$
$\neg p \Rightarrow p = F$
$\therefore s$

valid

d)
$(\neg p) \to t \Rightarrow p = T$
$q \to s$
$r \to q$
$\neg (q \vee t) \Rightarrow q \wedge t = F$
$\therefore p$

valid

e)
$p \Rightarrow p = T$
$s \to r \Rightarrow s = T$
$q \vee r \Rightarrow r = T$
$q \to \neg p \Rightarrow q = F$
$\therefore \neg s$

invalid

f)
$q \to t$
$p \to (t \to s) \Rightarrow t \to s = T$
$p \Rightarrow p = T$
$\therefore q \to s$

valid