13/02 # Lecture 1 – Linear systems and echleron forms

1.1 ~~6~~, ~~7~~, ~~15~~, ~~17~~, ~~21~~, ~~22~~, ~~23(a-c)~~, ~~25~~
1.2 ~~11~~, ~~19~~, ~~24~~, ~~25~~, ~~31~~

1.1/6. $\left[\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 1 & 6 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 62 \\ 0 & 1 & 0 & 0 & 25/2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$

7. $\left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

15. $\left| \begin{array}{l} x_1 + 3x_3 = 2 \\ x_2 - 3x_4 = 3 \\ -2x_2 + 3x_3 + 2x_4 = 1 \\ 3x_1 + 7x_4 = -5 \end{array} \right. \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$ consistent, unique non-trivial solution

17. $\left| \begin{array}{l} x_1 - 4x_2 = 1 \\ 2x_1 - x_2 = -3 \\ -x_1 - 3x_2 = 4 \end{array} \right. \longrightarrow \left[\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right]$ or $\left| \begin{array}{l} x - 4y = 1 \\ 2x - y = -3 \\ -x - 3y = 4 \end{array} \right\}$ intersection point $(-1.857, -0.714)$ $\longrightarrow$ unique non-trivial solution $\begin{pmatrix} -1.857 \\ -0.714 \end{pmatrix}$ $x_1 = -1.857$ $x_2 = -0.714$

21. $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & h+12 & 0 \end{bmatrix} \Rightarrow h \in \mathbb{R}$

22. $\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -3 & h \\ 0 & 0 & 5+3h \end{bmatrix} \Rightarrow$ consistent only when $h = \frac{-5}{3}$

25. $\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{array}\right] \Rightarrow k+h+2g=0$

1.2/11. $\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$3x_1 - 4x_2 + 2x_3 = 0$

$x_1 = \dfrac{4x_2 + 2x_3}{3} \Rightarrow x = \begin{pmatrix} \frac{4}{3} \\ 1 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} \frac{2}{3} \\ 0 \\ 1 \end{pmatrix} x_3$

19. $\left| \begin{array}{l} x_1 + hx_2 = 2 \\ 4x_1 + 8x_2 = k \end{array} \right. \rightarrow \begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix} \rightarrow \begin{bmatrix} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{bmatrix}$

a) no solution $\left| \begin{array}{l} k-8 \neq 0 \\ 8-4h = 0 \end{array} \right.$ $\left| \begin{array}{l} k \neq 8 \\ h = 2 \end{array} \right.$

b) unique solution
$8-4h \neq 0$ $h \neq 2$

c) ∞ solutions
$\left| \begin{array}{l} 8-4h = 0 \\ k-8 = 0 \end{array} \right.$ $\left| \begin{array}{l} h = 2 \\ k = 8 \end{array} \right.$

24. inconsistent

25. consistent

31. yes, $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

14/02 # Lecture 2 – Spans, vector equations and matrix equations

1.3	~~9~~, ~~15~~, ~~18~~, ~~**23**~~, ~~25~~
1.4	~~11~~, ~~17~~, ~~**23**~~, ~~24~~, 26, 31, 33, 36

1.3./9.
$$\begin{cases} x_2 + 5x_3 = 0 \\ 4x_1 + 6x_2 - x_3 = 0 \\ -x_1 + 3x_2 - 8x_3 = 0 \end{cases} \longrightarrow \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix} x_1 + \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix} x_2 + \begin{pmatrix} 5 \\ -1 \\ -8 \end{pmatrix} x_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

15. $v_1 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}$ $v_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$ $a_1, a_2, a_3, a_4, a_5 \in \text{Span}\{v_1, v_2\}$

$a_1 = v_1 + v_2 = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$ $a_2 = \frac{1}{2} a_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ $a_3 = 2a_1 = \begin{bmatrix} 4 \\ 8 \\ -12 \end{bmatrix}$ $a_4 = -a_1 = \begin{bmatrix} -2 \\ -4 \\ 6 \end{bmatrix}$ $a_5 = -a_2 = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$

18. $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ $v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$ $y = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$ $\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & 2h+7 \end{bmatrix}$ dependent

$\Rightarrow 2h + 7 = 0$

$h = -\frac{7}{2}$

23. a) F b) F c) T d) T e) T

25. $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$

$W = \text{Span}\{a_1, a_2, a_3\}$

a) $b \notin \{a_1, a_2, a_3\} \rightarrow 3$ vectors

b) $b \in W \rightarrow \infty$ vectors

c) $a_1 \in W$ $a_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} A$

1.4./11. $Ax = b$ $\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $x = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$

17. $A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 3 pivots

no solution

26. $u = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ $w = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$

$3u - 5v - w = 0 \Rightarrow w = 3u - 5v$

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$$

23. a) F b) T c) F d) F e) T f) T

24. a) T b) T c) T d) T e) T f) T

17/02 # Lecture 3 – Solution sets and Linear independence

1.5	~~13~~, ~~23~~, ~~26~~, ~~27~~, ~~29~~, ~~36~~
1.7	~~7~~, ~~13~~, ~~17~~, ~~23~~, ~~25~~, ~~27~~, ~~31~~, ~~35~~

1.5./13. $\begin{cases} x_1 = 5 + 4x_3 \\ x_2 = -2 - 7x_3 \\ x_3 - \text{free} \end{cases} \Rightarrow x = \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix} x_3$

23. a) T b) T c) F d) F e) T

27. $\forall x \in \mathbb{R}^3$

29. a) No b) Yes

36. A – 3×3 matrix s.t. $Ax = 0$

$x = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad A = \begin{bmatrix} a & a & a \\ b & b & b \\ c & c & c \end{bmatrix}$

1.7./7. $\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow$ linearly dependent

13. $\begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \forall h \in \mathbb{R}$

17. linearly dependent

23. $\begin{bmatrix} \blacksquare & \star & \star \\ 0 & \blacksquare & \star \\ 0 & 0 & \blacksquare \end{bmatrix}$

25. $\begin{bmatrix} \blacksquare & \star \\ 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

27. 5

31. $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

35. False, zero vector