CSE1400 - Computer Organisation

Self-Study: Week 3
Data Representation

Delft University of Technology

2021/2022 Q1

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Important information:

1. If any question is unclear please consult Answers. For more specific questions, you can use the Queue during lab hours.
2. The average time for solving this self study is **3** hours, and **1** hour is allocated to giving feedback. Timings are included for each exercise to give you a more clear overview of how much time you should be spending on them.
3. The maximum amount of points for this self study is 200 points. To get the points you should submit a serious attempt on Peer and **properly review** your peers' submissions (100 points per full cycle, including review evaluation).
4. Answers will be provided during the weekly tutorial sessions.

1 Integer Conversions

1. (5 mins) Convert the following numbers from radix-10 to radix-2:

   (a) 263

   (a) 100000111

   263 : 2 = 131 1
   131 : 2 = 65 1 ↑
   65 : 2 = 32 1
   32 : 2 = 16 0
   16 : 2 = 8 0
   8 : 2 = 4 0 ↑
   4 : 2 = 2 0
   2 : 2 = 1 0
   1 : 2 = 0 1 → MSB
   check: $2^9 = 512 > 263$
   $2^8 = 256 < 263$

   (b) 1759

   (b) 11011011111

   1759 : 2 = 879 1
   879 : 2 = 439 1 ↑
   439 : 2 = 219 1
   219 : 2 = 109 1
   109 : 2 = 54 1
   54 : 2 = 27 0 ↑
   27 : 2 = 13 1
   13 : 2 = 6 1
   6 : 2 = 3 0
   3 : 2 = 1 1 ↑
   1 : 2 = 0 1 → MSB
   check:
   $2^{10} = 1024 < 1759$
   $2^{11} = 2048 < 1759$

2. (7 mins) Convert 3645_8 to:

   (a) radix-2

   (a) 11110100101

   $3_8 \downarrow 011_2$ $6_8 \downarrow 110_2$ $4_8 \downarrow 100_2$ $5_8 \downarrow 101_2$

   (b) radix-10

   (b) 1957

   $= 5 \cdot 8^0 + 4 \cdot 8^1 + 6 \cdot 8^2 + 3 \cdot 8^3 = 5 + 32 + 6 \cdot 64 + 3 \cdot 512 =$
   $= 37 + 384 + 1536 =$
   $= 1957$

   (c) radix-16

   (c) 7A5

   binary: $\underbrace{1\ 1\ 1}_{7_{10} = 7_{12}}\ \underbrace{1\ 0\ 1\ 0}_{10_{10} = A_{12}}\ \underbrace{0\ 1\ 0\ 1}_{5_{10} = 5_{12}}$

3. (8 mins) Convert CE_{16}:

(a) radix-2

(a) 11001110

$C_{16} = 12_{10} \downarrow 1100_2$ $\quad$ $E_{16} = 14_{10} \downarrow 1110_2$

(b) radix-8

(b) 316

Binary: $\underbrace{1\ 1}_{3_{10}=3_8}\ \underbrace{0\ 0\ 1}_{1_{10}=1_8}\ \underbrace{1\ 1\ 0}_{6_{10}=6_8}$

4. (5 mins) What is 204335_6 in radix-36?

4. crw

$\underbrace{2\ 0}_{2\times6+0=12_{10}}\ \underbrace{4\ 3}_{4\times6+3=27_{10}}\ \underbrace{3\ 5}_{3\times6+5=23_{10}}$

$12_{10} = c_{36}$ $\quad$ $27_{10} = r_{36}$ $\quad$ $23_{10} = n_{36}$

5. (8 mins) You are given the following recipe for Super Student Pizza:

- $0101\ 0110\ 0011_2$ grams of flour
- 1970(Excess-1894) grams of salt
- FD_{16} grams of cheese
- 605_8 grams of tomato sauce
- GA_{23} grams of Mozzarella
- 1562(Excess-1439) grams of oil
- $0010\ 1010\ 1010_2$ grams of ham

What is the combined weight of the ingredients in grams (radix-10)?

5. 3280_{10} grams

11 10 9 8 7 6 5 4 3 2 1 0 ; 1 2 32 64 256 1024

$010101100011_2 = 1\times2^0 + 1\times2^1 + 1\times2^5 + 1\times2^6 + 1\times2^8 + 1\times2^{10} = 1379_{10}$

$1970_{10} - 1894_{10} = 76_{10}$

$FD_{16} = 13\times16^0 + 15\times16^1 = 253_{10}$

$605_8 = 5\times8^0 + 0\times8^1 + 6\times8^2 = 389_{10}$

$GA_{23} = 10\times23^0 + 16\times23^1 = 378_{10}$

$1562_{10} - 1439_{10} = 123_{10}$

9 8 7 6 5 4 3 2 1 0

$0010\ 10101010_2 = 2^9 + 2^7 + 2^5 + 2^3 + 2^1 = 682_{10}$

512 128 32 8 2

6. (10 mins) Consider the following three numbers in radix-10: 34, -23, and 212. Write down their 8-bit binary forms for the following. If it is not possible, state why.

(a) Sign & Magnitude

34 = 32+2 34 → 00100010

23 = 16+4+2+1 23 → 10010111

212 → impossible 8-bit S&M has range $[-(2^7-1), 2^7-1] = [-127, 127]$

(b) 1's complement

34 → same as S&M → 00100010

$-23 \to \neg(23_{10}) = \neg(10111_2) \to 11101000$

212 → impossible, same range as S&M $[-127, 127]$

(c) 2's complement

34 → same as S&M → 00100010

-23 → 1C + 1 → 11101001

212 → impossible, range $[-2^7, 2^7-1] = [-128, 127]$

(d) Excess-63

$34_{10} \to 34 + 63 = 97 \to 01100001$

$-23_{10} \to -23 + 63 = 40 \to 00101000$

$212_{10} \to 212 + 63 = 275 \to$ imposible range $[-63, 2^8-64] = [-63, 192]$

7. (6 mins) Levi wants to prepare for the Computer Organisation Midterm and he wants to determine the time he needed to finish the practice exam. His clock was showing the value 0001 0010 1001 0101 (BCD) when he started the exam and the value 0001 1111 0001 0111 (Excess-4096) when he finished it. How many seconds (in 2's complement) did Levi spend doing the practice exam if the values shown by his clock represent seconds?

2^{12} 7. 0000101000001000_2

start (BCD) end (Excess-4096)

0001 0010 1001 0101 — 0001 1111 0001 0111

1 2 9 5 — 0000 1111 0001 0111_2

1295_{10} = 0000 0101 0000 1111_2

$$
\begin{array}{r}
0000\ 1111\ 0001\ 0111_2 \\
+\ 1111\ 1010\ 1111\ 0001_2 \\
\hline
1\ 0000\ 1010\ 0000\ 1000_2
\end{array}
$$

2 Floating Points

1. (10 mins) Convert the following numbers from IEEE-754 to radix-10:

   (a) 1 10001000 00000000000000000000000

   (a) $-1 \times 2^9_{10}$

   s = 1 -> negative

   $e = 1000\,1000_2 = 136 - 127 = 9_{10}$ } $-1 \times 2^9_{10}$

   m = 1.00..

   (b) 0 10000100 00110010000000000000000

   (b) 38.25_{10}

   s = 0 -> positive

   $ex = 1000\,1000_2 = 132 - 127 = 5_{10}$

   m = 1.0011001

   $\Rightarrow 1.0011001_2 \times 2^5 = 100110.01_2 =$

   $= 2^5 + 2^2 + 2^1 + 2^{-2} = 32 + 4 + 2 + \frac{1}{4} = 38.25_{10}$

   (c) 1 10000100 01111110000000000000000

   (c) -47.75_{10}

   s = 1 -> negative

   $e = 10000100_2 = 132 - 127 = 5$

   m = 1.0111111

   $-1.0111111_2 \times 2^5 = -101111.11 =$

   $= -(2^5 + 2^3 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2}) =$

   $= -32 - 8 - 4 - 2 - 1 - 0.5 - 0.25 = -47.75$

2. (8 mins) Convert the following numbers from radix-10 to IEEE-754:

   (a) 12.50

positive => s=0

$12.50 = 8+4+\frac{1}{2} = 2^3+2^2+2^{-1} = 1100.1_2 = 1.1001_2 \times 2^3$

=> $e = 3+127 = 130_{10} = 10000010_2$

m = 1001000...

=> 12.50 -> 0 10000010 1001000...$_2$

   (b) -79.0625

negative => s=1

$79.0625 = 64+8+4+2+1+\frac{1}{16} = 2^6+2^3+2^2+2^1+2^0+2^{-4} = 1001111.0001 =$

$= 1.0011110001_2 \times 2^6$

=> $e = 6+127 = 133_{10} = 10000101_2$

=> -79.0625_{10} = 1 10000101 0011110001000...$_2$

3. (6 mins) John was surprised that at TU Delft, students use the CO-20C standard more than the IEEE-754. CO-20C works the same way as IEEE-754, with the following specifications:

   - there are 18 bits in total
   - the exponent is 8 bits representing the exponent in excess-127
   - 1 sign bit
   - 9 bits for mantissa

   As a curious student, he also wants to use the CO-20 standard, but he is not good at computation and he asks for your help. What is the decimal value represented by 110000100101110110?

1 10000100 101110110

negative 132-127=5 1.101110110

$-1.101110110_2 \times 2^5 = -110111.0110_2 = -(2^5+2^4+2^2+2^1+2^0+2^{-2}+2^{-3}) = -(32+16+4+2+1+0.25+0.125)$ =>

$110000100101110110_2 = -55.375_{10}$

4. (6 mins) A digital barometer stores its values in fixed-point format, using 7 bits for the unsigned integer part and 3 bits for the fractional part. Because the barometer stores the value in a limited number of bits, there can be an error in the measurement. Furthermore, the barometer is used in special labs where the air pressure lies in the range [0, 127], and it rounds the real value to the nearest representable value. What is the maximum error in the measurements of the air pressure (in radix-10)?

$x_2 \in [0_2; 1111111.111_2] \Rightarrow x_{10} \in [0; 127.875]$

maximal precision error $= 1.0001_2 = 2^{-4} = 0.0625$

5. (8 mins) The star of our show, Mando, keeps track of how much time Grogu, the child he adopted, spends on average on eating a frog. Since this value is very small, Mando saves it on one of his fancy devices, that uses a custom **floating point** representation, which begins with a sign bit, followed by a 6-bit exponent (in Excess-64), and a 9-bit mantissa (which has a hidden bit, just like IEEE-754).

   Before losing Grogu, Mando knew that he ate 6 frogs in total and it took him 0 111000 100110000 minutes on average to eat a frog. Now he asks you to calculate the total time it took Grogu to eat all of the frogs, while he goes on to rescue the child.

   Show your work and make sure to do the calculations in floating point notation!

$x = + m \times 2^e \ (s=0)$

$e = 111000_2 - 64_{10} = 2^5 + 2^4 + 2^3 - 64 = -8$

$x = 1.100111 \times 2^{-8} = 0.00000001100111$

$+ \ 2x = 0.0000001100110$

$3x = 0.000001001101\,01$

$\Rightarrow 6x = 2 \cdot 3x = 0.000010011001 = 1.0011001 \times 2^{-5}$

$m = 001100100 \qquad e = -5 + 64 = 59 = 32 + 16 + 8 + 2 + 1 =$

$= 2^5 + 2^4 + 2^3 + 2^1 + 2^0 = 111011$

$6x = 0 \ 111011 \ 001100100$

3 Addition

1. (5 mins) Use **binary addition** to perform the following operations. All numbers are expressed in 2's complement and all results must also be in 2's complement. This means that you cannot convert the numbers to any different representation.

   (a) `1100 0001 + 0001 0010` = 1101 0011

   ```
   + 1100 0001
     0001 0010
   -----------
     1101 0011
   ```

   (b) `1101 0110 + 0101 1110` = 0011 0100

   ```
   + 1101 0110
     0101 1110
   -----------
   1 0011 0100
   ```

2. (10 mins) Use **floating point addition** to perform the following operations. All numbers are expressed in IEEE-754 and all results must also be in IEEE-754. This means that you cannot convert the numbers to any different representation.

   (a) `0 10000011 00000100000000000000000 (= 16.25) + 0 10000101 00010010000000000000000 (= 68.5)`

   1) exponent: $10000011_2 = 131_{10} - 127 = 4_{10}$

   2) exponent: $10000101_2 = 133_{10} - 127 = 6_{10}$

   $1.000001 \times 2^4 = 0.01000001 \times 2^6$

   1.00010010×2^6

   1.01010011×2^6

   0 10000101 010100110...

   (b) `1 10000011 10111000000000000000000 (= -27.5) + 1 10000110 01110101100000000000000 (= -186.75)`

   1) exponent: $10000011_2 = 131_{10} - 127 = 4_{10}$ $\quad 1.10111 \times 2^4 =$

   $= 0.00110111 \times 2^7$

   2) exponent: $10000110_2 = 134_{10} - 127 = 7_{10}$

   $+\ 0.001101110 \times 2^7$

   1.011101011×2^7

   1.101011001×2^7

   1 10000110 101011001000...