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Lecture 4 – Linear transformations

1.8 ~~11, 13, 14, 15, 17, 19, 21, 24, 34, 35, 36~~

1.9 ~~2, 4, 6, 8, 11, 12, 19, 20, 23(a,c), 24(a-c)~~

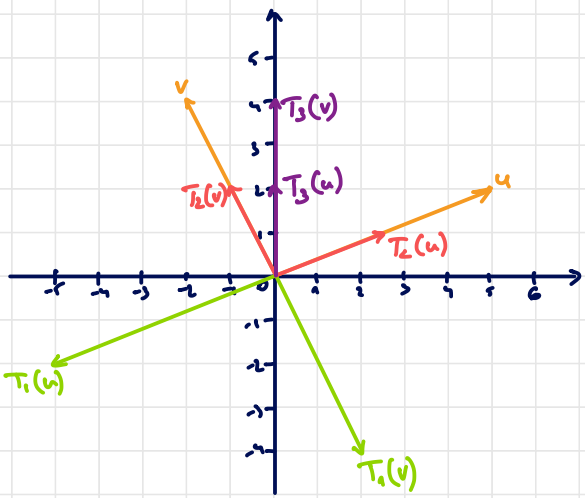
1.8./11. $A=\begin{bmatrix}1&-4&7&-5\\0&1&-4&3\\2&-6&6&-4\end{bmatrix}$ $B=\begin{bmatrix}-1\\1\\0\end{bmatrix}$ $[A\,|\,B]\to\begin{bmatrix}1&0&-9&7&3\\0&1&-4&3&1\\0&0&0&0&0\end{bmatrix}$ $Ax=B$ ✓

$u=\begin{bmatrix}5\\2\end{bmatrix}$ $v=\begin{bmatrix}-2\\4\end{bmatrix}$

13. $T_1(x)=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}x$ $T_1(u)=\begin{bmatrix}-5\\-2\end{bmatrix}$ $T_1(v)=\begin{bmatrix}2\\-4\end{bmatrix}$

14. $T_2(x)=\begin{bmatrix}.5&0\\0&.5\end{bmatrix}x$ $T_2(u)=\begin{bmatrix}2.5\\1\end{bmatrix}$ $T_2(v)=\begin{bmatrix}-1\\2\end{bmatrix}$

15. $T_3(x)=\begin{bmatrix}0&0\\0&1\end{bmatrix}x$ $T_3(u)=\begin{bmatrix}0\\2\end{bmatrix}$ $T_3(v)=\begin{bmatrix}0\\4\end{bmatrix}$



17. $T:\mathbb{R}^2\to\mathbb{R}^2$ linear

$u=\begin{bmatrix}5\\2\end{bmatrix}$ $T(u)=\begin{bmatrix}2\\1\end{bmatrix}$ $v=\begin{bmatrix}1\\3\end{bmatrix}$ $T(v)=\begin{bmatrix}-1\\3\end{bmatrix}$

$T(3u)=3T(u)=\begin{bmatrix}6\\3\end{bmatrix}$ $T(2v)=2T(v)=\begin{bmatrix}-2\\6\end{bmatrix}$ $T(3u+2v)=3T(u)+2T(v)=\begin{bmatrix}4\\9\end{bmatrix}$

19. $T:\mathbb{R}^2\to\mathbb{R}^2$ linear

$T(e_1)=\begin{bmatrix}2\\5\end{bmatrix}$ $T(e_2)=\begin{bmatrix}-1\\6\end{bmatrix}$ $T\left(\begin{bmatrix}5\\-3\end{bmatrix}\right)=\begin{bmatrix}13\\7\end{bmatrix}$ $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}2x_1-x_2\\5x_1+6x_2\end{bmatrix}$

21. a) F b) F c) F d) T e) T

1.9./2 $T:\mathbb{R}^3\to\mathbb{R}^2$ $T(e_2)=(4,-7)$ $T(x)=Ax$, $A=\begin{bmatrix}1&4&-5\\3&-7&4\end{bmatrix}$

$T(e_1)=(1,3)$ $T(e_3)=(-5,4)$

4. $T: \mathbb{R}^2 \to \mathbb{R}^2$ $\frac{\pi}{4}$ rotation

$T(e_1) = (\cos\frac{\pi}{4}, \sin\frac{\pi}{4}) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$T(e_2) = (-\sin\frac{\pi}{4}, \cos\frac{\pi}{4}) = (\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

6. $T: \mathbb{R}^2 \to \mathbb{R}^2$

$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \to \begin{bmatrix} e_1 \\ e_1 - 2e_2 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}$

8. $T: \mathbb{R}^2 \to \mathbb{R}^2$ 1) reflect on x_1 2) reflect on $x_2 = x_1$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{1)} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \xrightarrow{2)} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

11. $T: \mathbb{R}^2 \to \mathbb{R}^2$ 1) reflect on x_1 2) reflect on x_2

3) rotate around origin

4) rotate $180°/\pi$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

1) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \xrightarrow{2)} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

3) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

4) $\begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix} \to \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

12. $T: \mathbb{R}^2 \to \mathbb{R}^2$ from ⑧ $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

1) rotate $90°/\frac{\pi}{2}$ $\xrightarrow{1)} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

19. $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3,\ x_2 - 6x_3)$

$A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

20. $T: \mathbb{R}^4 \to \mathbb{R}$ $T(x_1, x_2, x_3, x_4) = 2x_1 + 3x_3 - 4x_4$

$A = \begin{bmatrix} 2 & 3 & -4 & 0 \end{bmatrix}$

23. a) F b) T c) F

24. a) F b) F c) T

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Lecture 5 – Matrix operations

2.1 ~~8~~, ~~10~~, ~~11~~, ~~13~~, ~~16~~, ~~20~~, ~~22~~, ~~23~~, ~~24~~

2.1/8. BC – 3×4 matrix $\Rightarrow$ B – 3 rows

10. $A=\begin{bmatrix}2 & -3\\ -4 & 6\end{bmatrix}$ $B=\begin{bmatrix}8 & 4\\ 5 & 5\end{bmatrix}$ $C=\begin{bmatrix}5 & -2\\ 3 & 1\end{bmatrix}$ $AB=\begin{bmatrix}1 & -7\\ -2 & 14\end{bmatrix}$ $AC=\begin{bmatrix}1 & -7\\ -2 & 14\end{bmatrix}$ $AB=AC$, $B\neq C$

11. $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 4 & 5\end{bmatrix}$ $D=\begin{bmatrix}2 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 5\end{bmatrix}$ $AD=\begin{bmatrix}2 & 3 & 5\\ 2 & 6 & 15\\ 2 & 12 & 25\end{bmatrix}$ $DA=\begin{bmatrix}2 & 2 & 2\\ 3 & 6 & 9\\ 5 & 20 & 25\end{bmatrix}$ $AB=BA$, $B\neq I_3, O_3$ $\Rightarrow B=A^{-1}$

$\Rightarrow B=\begin{bmatrix}1 & \frac{1}{2} & -\frac{1}{2}\\ 1 & -2 & 1\\ -1 & \frac{3}{2} & -\frac{1}{2}\end{bmatrix}$ $AB=BA=I_3$

17. $r_1,\dots,r_p$ – vectors in $\mathbb{R}^n$ $[Qr_1 \dots Qr_p]=Q[r_1\dots r_p]$ ($\to n\times p$)
Q – $m\times n$ matrix

16. a) F b) F c) F d) F e) T

20. $\exists$ column of $B=0 \Rightarrow \exists$ column of $AB=0$

22. $B=[b_1 \dots b_n]$ $\{b_1 \dots b_n\}$ – linearly dependent
$\Rightarrow AB=[Ab_1 \dots Ab_n]$, $\{Ab_1 \dots Ab_n\}$ – linearly dependent

23. $CA=I_n \Rightarrow C=A^{-1}$, A – invertible
$Ax=0 \Rightarrow$ only has trivial solution

24. $AD=I_n \Rightarrow D=A^{-1}$, A – invertible
$Ax=b \Rightarrow \exists x$ for $\forall b\in\mathbb{R}^n$

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Lecture 6 – Invertibility of matrices

2.2 ~~1~~, ~~8~~, ~~11~~, ~~13~~, ~~15~~, ~~16~~, ~~17~~, ~~20~~, ~~33~~, ~~35~~

2.3 ~~8~~, ~~7~~, ~~11~~, ~~12(a-c)~~, ~~18~~, 20, ~~21~~, ~~22~~, ~~23~~, ~~28~~, ~~34~~

2.2./7. $A=\begin{bmatrix}1&2\\5&12\end{bmatrix}$ $b_1=\begin{bmatrix}-1\\3\end{bmatrix}$ $b_2=\begin{bmatrix}1\\-5\end{bmatrix}$ $b_3=\begin{bmatrix}2\\6\end{bmatrix}$ $b_4=\begin{bmatrix}3\\5\end{bmatrix}$

$\det(A)=2$ $Ax=b_1$ $Ax=b_2$ $Ax=b_3$ $Ax=b_4$

$A^{-1}=\begin{bmatrix}6&-1\\-\frac{5}{2}&\frac{1}{2}\end{bmatrix}$ $x=A^{-1}b_1=\begin{bmatrix}-9\\4\end{bmatrix}$ $x=A^{-1}b_2=\begin{bmatrix}11\\-5\end{bmatrix}$ $x=A^{-1}b_3=\begin{bmatrix}6\\-2\end{bmatrix}$ $x=A^{-1}b_4=\begin{bmatrix}13\\-5\end{bmatrix}$

9. a) T b) F c) T d) T e) T

11. $AX=B$ $X=A^{-1}B$

13. $AB=AC \Rightarrow A^{-1}AB=A^{-1}AC \Rightarrow IB=IC \Rightarrow B=C$

15. $D(ABC)^{-1}=C^{-1}B^{-1}A^{-1} \Rightarrow D(ABC)=I^3=I$ $(ABC)D=I^3=I$

16. $C=AB$ $C^{-1}=B^{-1}A^{-1} \Rightarrow IA^{-1}$

17. $AB=BC \Rightarrow A=BCB^{-1}$

20. $A-AX=B^{-1}X \Rightarrow IB^{-1}$

$AX^{-1}-A=B^{-1}$ $AX^{-1}=B^{-1}+A$ $X^{-1}=B^{-1}A^{-1}+I$ $X=AB+I$

33. inverse → $\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}$

$\begin{matrix}0&0&1&4\\0&1&0&-6\\&&0&3\\1&0&4&19\end{matrix}$

35. $A=\begin{bmatrix}-2&-7&-9\\2&5&6\\1&3&4\end{bmatrix}$ $Ax=e_3 \rightarrow \begin{bmatrix}-2&-7&-9&0\\2&5&6&0\\1&3&4&1\end{bmatrix} \rightarrow \begin{bmatrix}1&0&0&3\\0&1&0&-6\\0&0&1&4\end{bmatrix} \Rightarrow A^{-1}=\begin{bmatrix}&&3\\a_1^{-1}&a_2^{-1}&-6\\&&4\end{bmatrix}$

2.2 / 6. $A = \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ $\det(A) = 1\begin{bmatrix} 3 & 4 \\ 6 & 0 \end{bmatrix} - 3\begin{bmatrix} -5 & -4 \\ 3 & 4 \end{bmatrix} = -24 - 3(-20-(-12)) = -24 - 3(-8) = 0$ $\Rightarrow$ not invertible

7. $A = \begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 2 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$ $\det(A) = -\begin{bmatrix} 5 & 2 & -3 \\ -6 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix} + 3\begin{bmatrix} 3 & 2 & -3 \\ -2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 2 \\ -2 & -6 & 3 \\ 0 & -1 & 2 \end{bmatrix}$

$= -(5(3-4) + 6(2+6) - (16+9)) + 3(3(3-4) + 2(2+6)) - (3(-12+3) + 2(10+8)) = 12 \Rightarrow$ inv.

11. a) T b) F c) F d) F e) T

12. a) T b) T c) T

16. No

21. No, pivots $< n$

23. linearly dependent

28. $(AB)^{-1} = B^{-1}A^{-1} \Rightarrow \exists\, B^{-1}$

34. $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$

$A = \begin{bmatrix} 6 & -8 \\ -5 & 7 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} \frac{7}{2} & 4 \\ \frac{5}{2} & 3 \end{bmatrix}$

$\det(A) = 2$