1.

a. $H_0: \bar{x} > 100$ (Avergge IQ is greater than Sarah's IQ)

b. $H_1: \bar{x} \leq 100$ (Not H_0)

c. test Statistic: $\frac{\bar{x} - \mu_0}{se} = \frac{104-100}{22/\sqrt{100}} = \frac{4}{22/10} = \frac{4}{2.2}$
$= 1.82$

d. I prefer One-sided test because in the null hypothesis, we want to test if average IQ is greater/less than Sarah's IQ, not equal/not equal.

e. I choose $\alpha = 0.05$, because $\alpha = 0.05$ is usually used. $t_{0.05}$-score $= 1.66$ (threshold). So the rejection region is $C: \{t: t > 1.66\}$.

f. Since $1.82 > 1.66$, so we can reject the null hypothesis.

g. $t_{0.025}$-score $= 1.984$. We have $1.82 < 1.984$ so we cannot reject the null hypothesis

h. CI for one-sided: $104 \pm \cancel{1.66 \cdot 22/\sqrt{100}}\ 1.66 \cdot \frac{22}{\sqrt{100}}$
$CI_{0.95} = (104 - 3.652, 104 + 3.652) = [100.348, 107.652]$

CI for two-sided: $104 \pm 1.984 \cdot \frac{22}{\sqrt{100}}$.

$CI_{0.975} = (104 - 4.3648,\ 104 + 4.3648)$
$= (99.6352,\ 108.3648)$

i. p-value $= 0.036$ (one-sided)

p-value $= 0.072$ (two-sided).

2.

a. $H_0: u_{men} = u_{women}$
$H_1: u_{men} \neq u_{women}$.

test-statistic: $\frac{u_{men} - u_{women}}{se_{diff}}$

$u_{men} = 1124$
$u_{women} = 1245$ $\Rightarrow u_{men} - u_{women} = 1124 - 1245 = -121$
$S_{men} = 200$
$S_{women} = 200$, $\Rightarrow se_{diff} = \sqrt{se^2_{men} + se^2_{women}}$

$= \sqrt{\left(\frac{200}{\sqrt{50}}\right)^2 + \left(\frac{200}{\sqrt{50}}\right)^2} = \sqrt{1600} = 40$.

test-statistic $= \left|\frac{-121}{40}\right| = |-3.025| = 3.025$

Since. $n_1 = n_2$ and $S_1 = S_2$, we have $df = 2n - 2 = 98$.
$t_{0.975, 98} = 1.98$, where $3.025 > 1.98$, so we

have to reject the null hypothesis. So the scores are statistically different.

b. Yes, we reject the null hypothesis, because our test statistic is 3.025, while $t_{0.975, 98} = 1.98$.
$3.025 > 1.98$, and thus we can conclude that we reject the null hypothesis and the means of two groups' are statistically different.

3.
a. test statistic: $\frac{78-75}{\sqrt{\frac{10^2}{50}+\frac{5^2}{50}}} = \frac{3}{\sqrt{2+0.5}} = 1.90$

$se_{diff} = \sqrt{\frac{10^2}{50}+\frac{5^2}{50}} = 1.58113.$

$df = \frac{se_{diff}^4}{se_1^4/(n_a-1) + se_2^4/(n_b-1)} = \frac{1.58113^4}{(\frac{10}{\sqrt{50}})^4/(50-1) + (\frac{5}{\sqrt{50}})^4/(50-1)} = 72.05$

$t_{0.975, 72} = 1.99$ and $1.90 < 1.99$, so we do not have enough evidence show that drinking helps the performance of exam.

4. See codes.

IntroComp_Weixuan_HW5

Weixuan Chen

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4. Using data of your choosing (or using simulated data), use R to conduct the following tests, and explain the results you get:

a. A standard one-sample hypothesis test.

```
datasets::cars
```

```
##    speed dist
## 1      4    2
## 2      4   10
## 3      7    4
## 4      7   22
## 5      8   16
## 6      9   10
## 7     10   18
## 8     10   26
## 9     10   34
## 10    11   17
## 11    11   28
## 12    12   14
## 13    12   20
## 14    12   24
## 15    12   28
## 16    13   26
## 17    13   34
## 18    13   34
## 19    13   46
## 20    14   26
## 21    14   36
## 22    14   60
## 23    14   80
## 24    15   20
## 25    15   26
## 26    15   54
## 27    16   32
## 28    16   40
## 29    17   32
## 30    17   40
## 31    17   50
## 32    18   42
## 33    18   56
```

```
## 34    18   76
## 35    18   84
## 36    19   36
## 37    19   46
## 38    19   68
## 39    20   32
## 40    20   48
## 41    20   52
## 42    20   56
## 43    20   64
## 44    22   66
## 45    23   54
## 46    24   70
## 47    24   92
## 48    24   93
## 49    24  120
## 50    25   85
```

```
t.test(cars$dist,alternative="two.sided",mu=60)
```

```
##
##  One Sample t-test
##
## data:  cars$dist
## t = -4.6703, df = 49, p-value = 2.372e-05
## alternative hypothesis: true mean is not equal to 60
## 95 percent confidence interval:
##  35.65642 50.30358
## sample estimates:
## mean of x
##     42.98
```

Here, in the cars dataset, we have, Null Hypothesis $H_0 : \mu = 60$ Since, p value < alpha (0.05), we reject the null hypothesis.

b. A difference-in-means test with independent samples.

```
s1 <- sample(15:45, 10)
s2 <- sample(20:50, 10)
t.test(s1,s2,mu=0,conf=0.95,alternative="two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  s1 and s2
## t = 0.071982, df = 15.943, p-value = 0.9435
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -8.537758  9.137758
## sample estimates:
## mean of x mean of y
##      31.9      31.6
```

Here, the Null hypothesis, $H_0 : \mu = 0$ However, since the p-value = 0.0539, we cannot reject the null hypothesis at 0.05 level.

c. A difference-in-means test with dependent samples (ie., a paired t-test)

Here, we'll use an example data set, which contains the weight of 10 mice before and after the treatment.

```
# Weight of the mice before treatment
before <-c(200.1, 190.9, 192.7, 213, 241.4, 196.9, 172.2, 185.5, 205.2, 193.7)
# Weight of the mice after treatment
after <-c(392.9, 393.2, 345.1, 393, 434, 427.9, 422, 383.9, 392.3, 352.2)
# Compute t-test
t.test(before, after, paired = TRUE)
```

```
##
##  Paired t-test
##
## data:  before and after
## t = -20.883, df = 9, p-value = 6.2e-09
## alternative hypothesis: true mean difference is not equal to 0
## 95 percent confidence interval:
##  -215.5581 -173.4219
## sample estimates:
## mean difference
##         -194.49
```

As we can see that the p value is significantly less than alpha, hence we reject Null hypothesis.

d. Manually verify the results in (a) using the mean and sd as calculated by R (ie, you don't have to manually calculate the mean or sd by hand!).

```
x_bar <- mean(cars$dist)
x_bar
```

```
## [1] 42.98
```

```
std_dev <- sd(cars$dist)
std_dev
```

```
## [1] 25.76938
```

```
n <- length(cars$dist)
n
```

```
## [1] 50
```

```
se <- std_dev/(sqrt(n))
se
```

```
## [1] 3.64434
```

```r
mu <- 60
z <- (x_bar - mu)/se
z
```

```
## [1] -4.670255
```

```r
low_int <- qt(0.025,n-1)
low_int
```

```
## [1] -2.009575
```

```r
high_int <-qt(0.975,n-1)
high_int
```

```
## [1] 2.009575
```

```r
low_CI <- ((low_int)*se)+x_bar
low_CI
```

```
## [1] 35.65642
```

```r
high_CI <- ((high_int)*se)+x_bar
high_CI
```

```
## [1] 50.30358
```

This verifies the results from part a and Null hypothesis can be rejected since mu lies in the rejected area.