ASSIGNMENT 2

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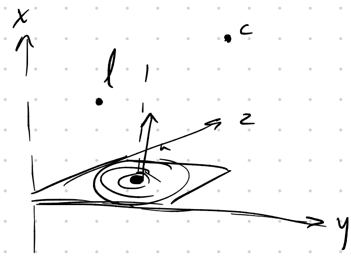
Exercise 2.

plane $y = 0$ $\Rightarrow$ the normal $= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

camera position $= \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix}$

direction towards the light $= \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$



a) $r = 2\underbrace{n}_{\text{normal}} \cdot \langle n, l \rangle - \underbrace{l}_{\text{light}}$

$$= 2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right\rangle - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$= 2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot 2 - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

$$r = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

$\Rightarrow$ we can construct a lin. comb of $p = c - (r)f$, introd. f and $y = 0$

$$\begin{cases} 4 - f(-1) = x \\ 6 - f(2) = 0 \\ 7 - f(-2) = z \end{cases} \Rightarrow \begin{cases} 4 + f = x \\ 6 - 2f = 0 \\ 7 + 2f = z \end{cases} \Rightarrow \begin{cases} x = 7 \\ f = 3 \\ z = 13 \end{cases} \Rightarrow p = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 13 \end{pmatrix}$$

intersection point

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b) $P_s = \frac{1}{2}$ $\quad k = 2$

$P_d = \frac{1}{2}$ $\quad I = 1$

$P_a = 0$

$$I_s = P_s \cdot (\cos \alpha)^k \cdot I$$

$$= \frac{1}{2} \cdot 1^2 \cdot 1 = \frac{1}{2}$$

$$I_d = P_d \cdot \langle n, l \rangle \cdot I$$

$$= \frac{1}{2} \cdot \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right\rangle \cdot 1 = 1$$

Atten. in dir. light is 0
$\Rightarrow$ 1.

$$I_{tot} = (I_s + I_d) \cdot \text{atten.} = \left(\frac{1}{2} + 1\right) \cdot 1 = 1{,}5$$

Exercise 3

Prove:

$$\underbrace{\angle \text{ between } h \,\&\, n}_{\text{LHS: } \varphi} = (1/2)^{*} \underbrace{\angle \text{ between } v \text{ and } r}_{\text{RHS: } \theta}$$

LHS: $\longrightarrow$

$$\cos(\varphi) = \frac{\langle h, n \rangle}{\|h\|\|n\|}$$

$$h = \frac{(l+v)}{2}$$

$$\Rightarrow \langle h, n \rangle = \frac{l+v}{2} \cdot n$$

$$\langle h, n \rangle = \underbrace{\frac{1}{2} \cdot \langle l, n \rangle}_{\cos \text{ of } \angle\, l,n} + \underbrace{\frac{1}{2} \langle v, n \rangle}_{\cos \text{ of } \angle\, v,n}$$

since vectors coplanar, angle between l and n eq. $\angle$ between v and n $\Rightarrow \frac{1}{2} \langle l, n \rangle = \frac{1}{2} \langle v, n \rangle$

$$\therefore \text{LHS } (\varphi) = \cos(\varphi) = \left(\tfrac{1}{2}\right)\langle l, n \rangle + \left(\tfrac{1}{2}\right)\langle v, n \rangle$$

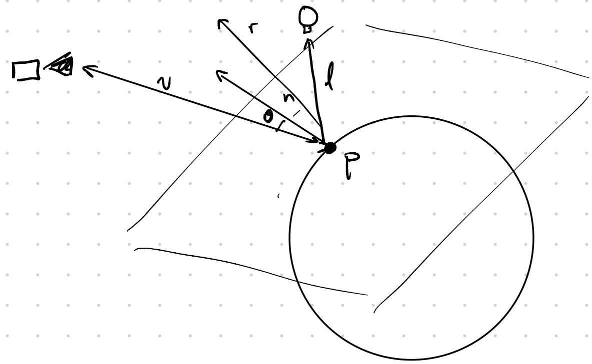
$$\text{RHS } (\theta) = \cos(\theta) = \frac{\langle v, r \rangle}{\|v\|\|r\|}$$

since, $\frac{1}{2}\langle l, n \rangle = \frac{1}{2}\langle v, n \rangle$

rewrite LHS $= \langle v, n \rangle$

now, $\langle v, n \rangle = \dfrac{\langle v, r \rangle}{\|v\|\|r\|} \quad \Big| \; / \|v\|\|r\|$

$\Rightarrow \cos(\varphi) = \cos(\theta) \Rightarrow \theta = \varphi$



coplaner either $x=0$, $y=0$, $z=0$ of v.

$$\Rightarrow \frac{(l+v)}{2} \text{ and } n$$

$$= \frac{1}{2} \angle \text{ between } v \,\&\, r$$