第五次作业 参考答案

10.30 - 11.5

习题 2.3 A 类

1

（6）解:

$$
\begin{aligned}
y' &= -\frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{2x}{\sqrt{1-x^4}} \\
y'' &= -\frac{2\sqrt{1-x^4} - 2x \cdot \frac{-4x^3}{2\sqrt{1-x^4}}}{1-x^4} = -\frac{2x^4+2}{(1-x^4)^{\frac{3}{2}}}
\end{aligned}
$$

2

（3）解:

两边对 x 求导得 $y' = e^y + xe^y \cdot y'$ 得 $y' = \frac{e^y}{1-xe^y}$.

$$
\begin{aligned}
\text{故} y'' &= \frac{e^y y' \left(1 - xe^y\right) - e^y \left(-e^y - xe^y \cdot y'\right)}{\left(1-xe^y\right)^2} \\
&= \frac{e^{2y}\left(2 - xe^y\right)}{\left(1-xe^y\right)^3} = \frac{e^{2y}(3-y)}{(2-y)^3}
\end{aligned}
$$

4

（3）解:

$$
\begin{aligned}
&\frac{dx}{d\varphi} = a(1-\cos\varphi), \frac{d^2x}{d\varphi^2} = a\sin\varphi. \\
&\frac{dy}{d\varphi} = a\sin\varphi, \frac{d^2y}{d\varphi^2} = a\cos\varphi. \\
&\text{故} \frac{d^2y}{dx^2} = \frac{\frac{d^2y}{d\varphi^2} \cdot \frac{dx}{d\varphi} - \frac{d^2x}{d\varphi^2} \cdot \frac{dy}{d\varphi}}{\left(\frac{dx}{d\varphi}\right)^3} = \frac{a^2(\cos\varphi - 1)}{a^3(1-\cos\varphi)^3} = -\frac{1}{a(1-\cos\varphi)^2}.
\end{aligned}
$$

5

（3）解:

$y' = \ln x + 1, y'' = \frac{1}{x}$.

当 $n \geqslant 2$ 时，$y^{(n)} = (-1)^n \cdot \frac{(n-2)!}{x^{n-1}}$.

$$
\text{故} y^{(n)} = \begin{cases} lnx + 1, & n = 1. \\ (-1)^n \cdot \frac{(n-2)!}{x^{n-1}}, & n \geqslant 2. \end{cases}
$$

6

解:

对 $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$ 两边求导得

$y'=e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}+e^{-\sqrt{x}}\cdot\left(-\frac{1}{2\sqrt{x}}\right)=\frac{1}{2\sqrt{x}}\left(e^{\sqrt{x}}-e^{-\sqrt{x}}\right)$

所以 $2\sqrt{x}y'=e^{\sqrt{x}}-e^{-\sqrt{x}}$. 两边求导得

$\frac{1}{\sqrt{x}}y'+2\sqrt{x}y''=\frac{1}{2\sqrt{x}}e^{\sqrt{x}}+\frac{1}{2\sqrt{x}}e^{-\sqrt{x}}=\frac{1}{2\sqrt{x}}\left(e^{\sqrt{x}}+e^{-\sqrt{x}}\right)=\frac{1}{2\sqrt{x}}y.$

即 $xy''+\frac{1}{2}y'-\frac{1}{4}y=0$ 。

得证。

9

解:

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{1}{\cos t}\cdot\frac{dy}{dt}.$

$\frac{d^2y}{dx^2}=\frac{d(\frac{dy}{dx})}{dx}=\frac{d(\frac{1}{cost}\cdot\frac{dy}{dt})}{dx}=\frac{\frac{d(\frac{1}{cost}\cdot\frac{dy}{dt})}{dt}}{\frac{dx}{dt}}=\frac{\frac{sint}{\cos^2 t}\cdot\frac{dy}{dt}+\frac{1}{\cos t}\cdot\frac{d^2y}{dt^2}}{cost}=\frac{sint}{cos^3t}\cdot\frac{dy}{dt}+\frac{1}{cos^2t}\cdot\frac{d^2y}{dt^2}$

代入原方程得

$$\begin{aligned}0&=\left(1-\sin^2 t\right)\cdot\left(\frac{\sin t}{cos^3t}\cdot\frac{dy}{dt}+\frac{1}{\cos^2 t}\cdot\frac{d^2y}{dt^2}\right)-\sin t\cdot\frac{1}{\cos t}\cdot\frac{dy}{dt}+a^2y\\&=\tan t\cdot\frac{dy}{dt}+\frac{d^2y}{dt^2}-\tan t\cdot\frac{dy}{dt}+a^2y\\&=\frac{d^2y}{dt^2}+a^2y\end{aligned}$$

得证。

习题 2.4 A 类

1

(4) 解:

$dy=d\left(\arctan e^{\sqrt{x}}\right)=\frac{1}{1+e^{2\sqrt{x}}}d\left(e^{\sqrt{x}}\right)=\frac{1}{1+e^{2\sqrt{x}}}e^{\sqrt{x}}d(\sqrt{x})=\frac{1}{1+e^{2\sqrt{x}}}\cdot e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}dx=\frac{e^{\sqrt{x}}}{2\left(1+e^{2\sqrt{x}}\right)\sqrt{x}}dx.$

2

(2) 解:

$$\begin{aligned}dy&=d(\sin(uvw))=\cos(uvw)d(uvw)\\&=\cos(uvw)[vwdu+ud(vw)]\\&=\cos(uvw)(vwdu+uwdv+uvdw)\end{aligned}$$

5

解:

$y'|_{x=x_0}=2x|_{x=x_0}=2$

(1) 当 $\Delta x=0.1$ 时, $\Delta y=y\left(x_0+\Delta x\right)-y(x_0)=1.1^2-1^2=0.21.$ $dy=y'|_{x=x_0}dx=0.2.$

故 $\Delta y - dy = 0.21 - 0.2 = 0.01$ 。

(2) 当 $\Delta x = 0.01$ 时, $\Delta y = y\left(x_0 + \Delta x\right) - y\left(x_0\right) = 1.01^2 - 1^2 = 0.0201.$

$dy = \left.y'\right|_{x=x_0} dx = 0.02.$

故 $\Delta y - dy = 0.0201 - 0.02 = 0.0001.$