

第八次作业 参考答案

11.20 - 11.26

习题 4.2 A 类

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(9) 解:

$$\begin{aligned}\int \cos 3x \sin x dx &= \int -\cos 3x d(\cos x) \\&= \int (3 \cos x - 4 \cos^3 x) d(\cos x) \\&= \frac{3}{2} \cos^2 x - \cos^4 x + C \\&= \left(-\frac{1}{8} \cos 4x + \frac{1}{4} \cos 2x + C \right)\end{aligned}$$

(14) 解:

$$\begin{aligned}\int \frac{\arctan \sqrt{x}}{\sqrt{x}(1+x)} dx &= \int \frac{2 \arctan \sqrt{x}}{1+x} d(\sqrt{x}) \\&= \int 2 \arctan \sqrt{x} d(\arctan \sqrt{x}) \\&= \arctan^2 \sqrt{x} + C\end{aligned}$$

(24) 解:

$$\begin{aligned}\int x \ln x dx &= \int \ln x \left(\frac{x^2}{2} \right) = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x dx \\&= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C\end{aligned}$$

(29) 解:

$$\begin{aligned}\int \arccos x dx &= x \arccos x - \int -\frac{x}{\sqrt{1-x^2}} dx \\&= x \arccos x - \int \frac{1}{2\sqrt{1-x^2}} d(1-x^2) \\&= x \arccos x - \sqrt{1-x^2} + C\end{aligned}$$

(34) 解:

$$\begin{aligned}\int e^{2x} (1 + \tan x)^2 dx &= \int e^{2x} (\sec^2 x + 2 \tan x) dx = \int e^{2x} d(\tan x) + \int 2e^{2x} \tan x dx \\&= e^{2x} \tan x - \int 2e^{2x} \tan x dx + \int 2e^{2x} \tan x dx \\&= e^{2x} \tan x + C\end{aligned}$$

(39) 解:

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(44) 解:

$$\begin{aligned}\int \frac{1}{1+x^4} dx &= \frac{1}{2} \int \frac{x^2+1}{1+x^4} dx - \frac{1}{2} \int \frac{x^2-1}{1+x^4} dx \\&= \frac{1}{2} \int \frac{d(x-\frac{1}{x})}{(x-\frac{1}{x})^2+2} - \frac{1}{2} \int \frac{d(x+\frac{1}{x})}{(x+\frac{1}{x})^2-2} \\&= \frac{1}{2\sqrt{2}} \int \frac{d(\frac{x-\frac{1}{x}}{\sqrt{2}})}{1+(\frac{x-\frac{1}{x}}{\sqrt{2}})^2} - \frac{1}{2\sqrt{2}} \int \frac{d(\frac{x+\frac{1}{x}}{\sqrt{2}})}{(\frac{x+\frac{1}{x}}{\sqrt{2}})^2-1} \\&= \frac{1}{2\sqrt{2}} \arctan(\frac{x-\frac{1}{x}}{\sqrt{2}}) - \frac{1}{4\sqrt{2}} \ln \left| \frac{1-\frac{x+\frac{1}{x}}{\sqrt{2}}}{1+\frac{x+\frac{1}{x}}{\sqrt{2}}} \right| + C \\&= \frac{1}{2\sqrt{2}} \arctan(\frac{x^2-1}{\sqrt{2}x}) + \frac{1}{4\sqrt{2}} \ln \left| \frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1} \right| + C\end{aligned}$$

(49) 解:

$$\begin{aligned}\int \frac{\cos x}{\sin x(1+\sin x)} dx &= \int \frac{d(\sin x)}{\sin x(1+\sin x)} \\&= \int \frac{d(\sin x)}{\sin x} - \int \frac{d(1+\sin x)}{1+\sin x} \\&= \ln|\sin x| - \ln|1+\sin x| + C \\&= \ln \left| \frac{\sin x}{1+\sin x} \right| + C\end{aligned}$$

(54) 解:

$$\begin{aligned}\int \frac{\arcsine^x}{e^x} dx &= \int \frac{\arcsine^x}{e^{2x}} d(e^x) \quad \text{令 } e^x = t \\&= \int \frac{\arcsint}{t^2} dt = \int \arcsint d(-\frac{1}{t}) \\&= -\frac{\arcsint}{t} + \int \frac{1}{t\sqrt{1-t^2}} dt \\&= -\frac{\arcsint}{t} - \int \frac{1}{t^2} d(\sqrt{1-t^2})\end{aligned}$$

令 $\sqrt{1-t^2} = k$, 则

$$\begin{aligned}\int \frac{1}{t^2} d(\sqrt{1-t^2}) &= \int \frac{1}{1-k^2} dk = \int \frac{1}{(1+k)(1-k)} dk \\&= \int \frac{1}{2(1+k)} d(1+k) - \int \frac{1}{2(1-k)} d(1-k) \\&= \frac{1}{2} \ln \left| \frac{1+k}{1-k} \right| + C_1\end{aligned}$$

故 原式 $= -\frac{\arcsine^x}{e^x} - \frac{1}{2} \ln \left| \frac{1+\sqrt{1-e^{2x}}}{1-\sqrt{1-e^{2x}}} \right| + C$

(59) 解:

令 $\sqrt{\frac{1-x}{1+x}} = t$, 则 $x = \frac{1-t^2}{1+t^2}$, $dx = -\frac{4t}{(1+t^2)^2} dt$

则 $\int \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{x} dx = \int t \cdot \frac{1+t^2}{1-t^2} \cdot (-\frac{4t}{(1+t^2)^2}) dt = \int \frac{4t^2}{(t^2+1)(t^2-1)} dt$

设 $\frac{4t^2}{(t^2+1)(t^2-1)} dt = \frac{A}{t-1} + \frac{B}{t+1} + \frac{Ct+D}{t^2+1}$

则 $A = \lim_{t \rightarrow 1} \frac{4t^2}{(t^2+1)(t^2-1)} \cdot (t-1) = 1$, $B = \lim_{t \rightarrow -1} \frac{4t^2}{(t^2+1)(t^2-1)} \cdot (t+1) = -1$, 比较系数可得 $C=0, D=2$.

$$\text{故 原式} = \int \frac{1}{t-1} d(t-1) - \int \frac{1}{t+1} d(t+1) + \int \frac{2}{t^2+1} d(t) = \ln \left| \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}} \right| + 2 \arctan \sqrt{\frac{1-x}{1+x}} + C$$