

第四次作业 参考答案

10.23 - 10.30

习题 1.5 A 类

1

(2) 解:

$$f(0-0) = \lim_{x \rightarrow 0-} (a + e^x) = a + 1, f(0+0) = \lim_{x \rightarrow 0+} x \sin \frac{1}{x} = 0.$$

由 $f(x)$ 连续知 $f(0-0) = f(0+0)$ 即 $a + 1 = 0$. 得 $a = -1$.

2

(4) 解:

由 $\sin x = 0$ 知间断点为 $x_n = n\pi, n \in \mathbb{Z}$.

$$\text{而 } f(n\pi-0) = \lim_{x \rightarrow n\pi-} \frac{x}{\sin x} = \begin{cases} 1, & n = 0 \\ +\infty, & n \text{ 为正奇数或负偶数} \\ -\infty, & n \text{ 为正偶数或负奇数} \end{cases}.$$

$$f(n\pi+0) = \lim_{x \rightarrow n\pi+} \frac{x}{\sin x} = \begin{cases} 1, & n = 0 \\ +\infty, & n \text{ 为正偶数或负奇数} \\ -\infty, & n \text{ 为正奇数或负偶数} \end{cases}.$$

故 0 为 $f(x)$ 的可去间断点, 可补充定义 $f(0) = 1$.

当 $n \neq 0$ 时, $n\pi$ 为 $f(x)$ 的第二类间断点, 且为无穷间断点。

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解:

由 $f(x)$ 在 (a, b) 内连续知 $f(x)$ 在 $[x_1, x_n]$ 上连续, 其在 $[x_1, x_n]$ 上存在最大最小值.

设 $M = \max_{x_1 \leq x \leq x_n} f(x), m = \min_{x_1 \leq x \leq x_n} f(x)$.

则 $m \leq \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n} \leq M$. 即 $\frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n} \in [m, M]$.

故由介值定理知 $\exists \xi \in [x_1, x_n] \subseteq (a, b)$, 使

$$f(\xi) = \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n}.$$

习题 2.1 A 类

1

(4) 解:

$$f'_-(1) = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{1-x}{x-1} = -1.$$

$$f'_+(1) = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{(1-x)(2-x)}{x-1} = -1. \text{ 故 } f'(1) = -1.$$

$$f'_-(2) = \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^-} \frac{(1-x)(2-x)}{x-2} = 1.$$

$$f'_+(2) = \lim_{x \rightarrow 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^+} \frac{-(2-x)}{x-2} = 1. \text{ 故 } f'(2) = 1.$$

$$f'(3) = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \rightarrow 3} \frac{x-3}{x-3} = 1.$$

3

(4) 解:

$$\text{由于 } f(0-0) = \lim_{x \rightarrow 0^-} \ln(1+x) = 0,$$

$$f(0+0) = \lim_{x \rightarrow 0^+} (\sqrt{1+x} - \sqrt{1-x}) = \lim_{x \rightarrow 0^+} \frac{2x}{\sqrt{1+x} + \sqrt{1-x}} = 0,$$

$$\text{且 } f(0) = 0.$$

故 $f(0-0) = f(0) = f(0+0)$. 于是 $f(x)$ 在 $x=0$ 连续.

$$f'_-(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^-} \frac{\ln(1+x)}{x} = 1.$$

$$f'_+(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^+} \frac{2}{\sqrt{1+x} + \sqrt{1-x}} = 1.$$

故 $f'_+(0) = f'_-(0) = 1$, 于是 $f(x)$ 在 $x=0$ 可导且 $f'(0) = 1$.

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解:

$$f'_+(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0^+} - \left(\frac{f(-x) - f(0)}{-x} \right) = - \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x} = -f'_-(0).$$

$$\text{且 } f'_+(0) = f'_-(0) = f'(0)$$

由于 $f'(0)$ 存在,

$$\text{所以 } f'_+(0) = f'_-(0) = f'(0) = 0.$$

习题 2.2 A 类

3

(4) 解:

$$y' = 2 \sin(\cos 3x) \cdot \cos(\cos 3x) \cdot (-\sin 3x) \cdot 3 = -3 \sin 3x \sin(2 \cos 3x).$$

(10) 解:

$$y' = \frac{1}{\tan x} \cdot \sec^2 x = \frac{1}{\sin x \cos x}.$$

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(2) 解:

两边对 x 求导得 $y' \sin x + y \cos x + \sin(x - y)(1 - y') = 0$.

整理即 $y' = \frac{y \cos x + \sin(x - y)}{\sin(x - y) - \sin x}$.

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解:

两边对 x 求导得 $\cos(xy)(y + xy') - \frac{y}{x+1} \cdot \frac{y - (x+1)y'}{y^2} = 0. (*)$

由 $\sin(xy) - \ln \frac{x+1}{y} = 1$ 知, 当 $x = 0$ 时, $y = e$ 。

代入 $(*)$ 式得 $e - \frac{e - \left. \frac{dy}{dx} \right|_{x=0}}{e} = 0$.

即 $\left. \frac{dy}{dx} \right|_{x=0} = e - e^2$.