第三次作业 参考答案

10.16 - 10.22

习题 1.3 A 类

1

解：由于 $5 < \sqrt[n]{1+2^n+5^n} < \sqrt[n]{3\times 5^n} = 5\sqrt[n]{3}$. 且 $\lim_{n\to\infty} 5 = \lim_{n\to\infty} 5\sqrt[n]{3} = 5$.
由两边夹定理实 $\lim_{n\to\infty} \sqrt[n]{1+2^n+5^n} = 5$ 。

2

（2）解：由于 $\frac{n}{\sqrt{n^2+n}} < \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}} < \frac{n}{\sqrt{n^2+1}}$.
且 $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}} = 1, \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = 1$.
即 $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$.
由两边夹定理知 $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^22}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right) = 1$ 。

3

(4) 解: $\lim_{n\to\infty} n^2\left(1-\cos\frac{1}{n}\right) = \lim_{n\to\infty} n^2\cdot 2\sin^2\frac{1}{2n} = \lim_{n\to\infty} \frac{1}{2}\cdot\left(\frac{\sin\frac{1}{2n}}{\frac{1}{2n}}\right)^2 = \frac{1}{2}$.

4

（4）解：$\lim_{x\to 0}(1-2x)^{\frac{1}{x}} = \lim_{x\to 0}(1-2x)^{-\frac{1}{2x}\cdot(-2)} = \left[\lim_{x\to 0}(1-2x)^{-\frac{1}{2x}}\right]^{-2} = e^{-2}$.
（9）解：

$$\lim_{n\to\infty}(1+\sin\frac{2}{n})^n = \lim_{n\to\infty}(1+\sin\frac{2}{n})^{\frac{1}{\sin\frac{2}{n}}\cdot n\sin\frac{2}{n}} = \left[\lim_{n\to\infty}(1+\sin\frac{2}{n})^{\frac{1}{\sin\frac{2}{n}}}\right]^{\lim_{n\to\infty}\frac{\sin\frac{2}{n}}{\frac{2}{n}}\cdot 2} = e^2$$

习题 1.4 A 类

3

(4) 解：$\lim_{n\to\infty} \frac{n^2+n-1}{n+3} = \lim_{n\to\infty} \frac{n+1-\frac{1}{n}}{1+\frac{3}{n}} = \infty$.

4

(1) 解：由于 $\tan 3x \sim 3x, \sin 6x \sim 6x (x\to 0)$.

$$故 \lim_{x\to 0}\frac{\tan 3x}{\sin 6x} = \lim_{x\to 0}\frac{3x}{6x} = \frac{1}{2}.$$

(3) 解：由于 $\sqrt[3]{1+x}-1 \sim \frac{1}{3}x, \arctan x \sim x(x \to 0)$.

$$\text{故} \lim_{x\to 0} \frac{\sqrt[3]{1+x}-1}{\arctan x} = \lim_{x\to 0} \frac{\frac{1}{3}x}{x} = \frac{1}{3}.$$

(5) 解：由于 $\sin(x^n) \sim x^n, \sin x \sim x(x \to 0)$.

$$\text{故} \lim_{x\to 0} \frac{\sin(x^n)}{(\sin x)^m} = \lim_{x\to 0} \frac{x^n}{x^m} = \begin{cases} 0, & n > m \\ 1, & n = m \\ \infty, & n < m. \end{cases}.$$

(6) 解：

由于 $\arcsin \frac{1}{n} \sim \frac{1}{n}(n \to \infty)$.

$$\text{故} \lim_{n\to\infty} n \arcsin \frac{1}{n} = \lim_{n\to\infty} n \cdot \frac{1}{n} = 1.$$