第十次作业 参考答案

12.4 - 12.10

习题 5.3 A 类

1

(2) 解:

$$\begin{aligned}\int_0^\pi \left(1-\sin^3 x\right) dx &= x|_0^\pi - \int_0^\pi \left(\cos^2 x - 1\right) d(\cos x) \\ &= \pi - \left(\frac{1}{3}\cos^3 x - \cos x\right)\Big|_0^\pi \\ &= \pi - \frac{4}{3}\end{aligned}$$

2

(5) 解:

$$\begin{aligned}\int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx &= \int_0^{\frac{\pi}{2}} \frac{x\sin x}{1+\cos^2 x}dx + \int_{\frac{\pi}{2}}^\pi \frac{x\sin x}{1+\cos^2 x}dx \\ &= \int_0^{\frac{\pi}{2}} \frac{x\sin x}{1+\cos^2 x}dx + \int_{\frac{\pi}{2}}^0 \frac{(\pi-x)\sin x}{1+\cos^2 x}d(\pi-x) \\ &= \int_0^{\frac{\pi}{2}} \frac{x\sin x}{1+\cos^2 x}dx + \int_0^{\frac{\pi}{2}} \frac{(\pi-x)\sin x}{1+\cos^2 x}dx \\ &= \int_0^{\frac{\pi}{2}} -\frac{\pi}{1+\cos^2 x}d(\cos x) \\ &= -\pi\arctan(\cos x)|_0^{\frac{\pi}{2}} \\ &= \frac{\pi^2}{4}\end{aligned}$$

(8) 解:

$$\begin{aligned}\int_1^2 \frac{\sqrt{x-1}}{x}dx &\xlongequal{令\sqrt{x-1}=t} \int_0^1 \frac{t}{t^2+1}\cdot 2tdt \\ &= \int_0^1 \left(2-\frac{2}{t^2+1}\right)dt \\ &= (2t-2\arctan t)|_0^1 \\ &= 2-\frac{\pi}{2}\end{aligned}$$

(13) 解:

$$\begin{aligned}\int_1^3 f(x-2)dx &\xlongequal{\text{令}x-2=t} \int_{-1}^1 f(t)dt \\ &= \int_{-1}^0 \left(1+t^2\right)dt + \int_0^1 e^{-t}dt \\ &= \left(t+\frac{1}{3}t^3\right)\Big|_0^1 + \left(-e^{-t}\right)\Big|_0^1 \\ &= \frac{7}{3} - e^{-1}.\end{aligned}$$

3

(3) 解:

$$\begin{aligned}\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{8-2x^2}dx &= 2\int_0^{\sqrt{2}} \sqrt{8-2x^2}dx \\ &\xlongequal{\text{令}x=2sint,t\in\left[0,\frac{\pi}{4}\right]} 2\int_0^{\frac{\pi}{4}} 2\sqrt{2}\cos t \cdot 2\cos t dt \\ &= \sqrt{2}(\pi+2)\end{aligned}$$

(6) 解:

$$\begin{aligned}\int_{-1}^1 \frac{x+(\arctan x)^2}{1+x^2}dx &= \int_{-1}^1 \frac{x}{1+x^2}dx + \int_{-1}^1 \frac{(\arctan x)^2}{1+x^2}dx \\ &= 2\int_0^1 (\arctan x)^2 d(\arctan x) \\ &= 2\cdot\frac{1}{3}(\arctan x)^3\Big|_0^1 \\ &= \frac{\pi^3}{96}.\end{aligned}$$

4

解:

由 $f(t)$ 是奇函数知 $f(-t)=-f(t)$ 恒成立。

$$\begin{aligned}\text{所以}\int_0^{-x} f(t)dt &\xlongequal{\text{令}-t=k} \int_0^x f(-k)d(-k) \\ &= \int_0^x -f(-k)dk = \int_0^x f(k)dk = \int_0^x f(t)dt.\end{aligned}$$

故 $\int_0^x f(t)dt$ 为偶函数。

6

(2) 解:

$$
\begin{aligned}
\int_0^1(\arcsin x)^2dx &\xlongequal{令arcsinx=t} \int_0^{\frac{\pi}{2}} t^2 d(\sin t) \\
&= t^2\sin t\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2t\sin t d(\sin t) \\
&= \frac{\pi^2}{4} + \int_0^{\frac{\pi}{2}} 2tdt(\cos t) \\
&= \frac{\pi^2}{4} + 2t\cos t|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2\cos t dt \\
&= \frac{\pi^2}{4} - 2\sin t|_0^{\frac{\pi}{2}} \\
&= \frac{\pi^2}{4} - 2
\end{aligned}
$$

（8）解:

$$
\begin{aligned}
\int_0^1 e^{\sqrt{x}}dx &\xlongequal{令\sqrt{x}=t} \int_0^1 2te^t dt = \int_0^1 2td\left(e^t\right) \\
&= 2te^t\Big|_0^1 - \int_0^1 2e^t dt \\
&= 2e - 2e^t\Big|_0^1 \\
&= 2.
\end{aligned}
$$

7

解:

$$
\begin{aligned}
\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(x)dx &= [xf(x)]|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} xf'(x)dx \\
&= \frac{3\pi}{2}f(\frac{3\pi}{2}) - \frac{\pi}{2}f(\frac{\pi}{2}) - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x\cdot\frac{\cos x}{x}dx \\
&= \frac{3\pi}{2}b - \frac{\pi}{2}a - \sin x|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \\
&= \frac{3\pi}{2}b - \frac{\pi}{2}a + 2
\end{aligned}
$$