0. If $a \cdot b \cdot c \cdot d \cdot e = 1$, and a, b, c, d, and e are all positive real numbers, what is the minimum value of $a + b + c + d + e$?

   Answer. 5.

   Solution. Since a,b,c,d, and e all must be positive, the only way to minimize the sum of a,b,c,d, and e is for all the values to be 1. Therefore, the answer is $a + b + c + d + e = 5$

1. There is a soaked watermelon weighing 10 kilograms and is found to be 99% water. It was left out to dry for a set time and it was found to be 95% water. What is the new weight of the watermelon in kilograms?

   Answer. 2.

   Solution. From the introduction of the problem, we see that 1% of the weight is not water, so $0.01 * 10 = 0.1$kg is not water. Since this part of the watermelon cannot evaporate, it is the 5% that is not water in the new weight of the watermelon. Therefore the final weight is $0.1 * 20 = 2$ kg.

2. If you multiply all primes less than 100, what would the last digit be?

   Answer. 0.

   Solution. If we look at the first few prime numbers, we immediately see that the first prime number is 2, and the third prime number is 5. Since the produce of all the prime numbers less than 100 now must be a multiple of 10, the last digit must be 0.

3. There are 4 mangoes, 3 apples and 2 oranges in a bag, fruits of the same variety being identical. In how many ways can a selection of fruits can be made if at least one of each kind is selected ?

   Answer. 24

   Solution. There are 4 options for the number of mangoes in the selection (1,2,3, or 4). There are 3 options for the number of apples in the selection (1,2, or 3). There are 2 options for the number of oranges in the selection (1 or 2). The total number of possible selections is $4 * 3 * 2 = 24$

4. Find the smallest positive integer such that it is a multiple of 9 and it has no odd digits.

   Answer. 288.

   Solution. The sum of the digits of any multiple of 9 must also be a multiple of 9. If the desired number has no odd digits, that means the sum of the digits must be even. The smalled even multiple of 9 is 18. The smallest permutation of even digits that sum to 18 is 288.

5. Thomas Jefferson rolls 2 standard 6-sided dice and writes down the sum of the top faces. How many different values could the sum have?

   Answer. 11.

   Solution. The smallest sum of the two faces is $1 + 1 = 2$. We can create the sums 2 to 7 by setting one of the dice to have rolled as 1 and changing the value rolled on the second dice. Then, setting one of the dice to have rolled to 6 and rotating the value rolled on the second dice, we can make the numbers 8 to 12. Therefore, there are 11 possible sums.

6. A square is inscribed in the circle. If the area of the square is 36 what is the area of the circle? Express your answer in terms of π.

   Answer. 18π.

   Solution. From the area of the square, we know that the side length of the square is 6. Using the Pythagorean Theorem, we can find the diagonal of the square to be $6\sqrt{2}$. That means the radius of the circle is $3\sqrt{(2)}$ and the area is $\pi * r^2 = 18\pi$

7. At any given time, there are two angles formed by the hands of the clock. What is the larger angle, in degrees, formed by the two hands at 12:48?

   Answer. 264.

   Solution. The hour hand of a normal 12-hour analogue clock turns 360 in 12 hours (720 minutes) or 0.5 per minute. The minute hand rotates through 360 in 60 minutes or 6 per minute. The degrees traveled by the hour hand at 12:48 is $0.5(0 + 48) = 24$. The degrees traveled by the minute hand is $6(48) = 288$. Therefore, the angle between the two hands is $288 - 24 = 264$.

8. Given $2^x + 2^{13} + 2^{10}$ is a perfect square. Determine the value of x.

   Answer. 14.

   Solution. We can start out by factoring out 2^10 from the expression.

$$\begin{aligned} 2^x + 2^{13} + 2^{10} &= 2^{10}(2^{x-10} + 2^3 + 1) \\ &= 2^{10}(2^{x-10} + 9) \\ x - 10 &= 4 \\ x &= 14 \end{aligned}$$