27^{th} TJIMO

Alexandria, Virginia

Round: **Guts**

Set 1

Problem 1. What is 50% of 10% of 1000?

Problem 2. I had 50 cookies yesterday but now I only have 30. If only two people — my brother, Frank, and my sister, Gabriella — ate my cookies, and I know Frank ate 8 of them, how many did my sister eat?

Problem 3. If the area of a square is 64, what is its perimeter?

Problem 4. Simplify $\frac{\sqrt{15^2+8^2}}{\sqrt{13^2-5^2}}$

27^{th} TJIMO

Alexandria, Virginia

Round: **Guts**

Set 1

Problem 1. What is 50% of 10% of 1000?

Problem 2. I had 50 cookies yesterday but now I only have 30. If only two people — my brother, Frank, and my sister, Gabriella — ate my cookies, and I know Frank ate 8 of them, how many did my sister eat?

Problem 3. If the area of a square is 64, what is its perimeter?

Problem 4. Simplify $\frac{\sqrt{15^2+8^2}}{\sqrt{13^2-5^2}}$

Set 2

Problem 5. How many primes are there that are less than 60?
Problem 6. Find the area of a circle with radius 6. Express your answer in terms of π.
Problem 7. Let $x \star y = x^2 - y^2$, and $x \uplus y = \sqrt{x^2 + y^2}$. Find $8 \star (5 \uplus 12)$.
Problem 8. In how many ways can I order the four letters in "WORD"? For example, one way is "ORWD".

27^{th} TJIMO
Alexandria, Virginia

Round: **Guts**

Set 2

Problem 5. How many primes are there that are less than 60?
Problem 6. Find the area of a circle with radius 6. Express your answer in terms of π.
Problem 7. Let $x \star y = x^2 - y^2$, and $x \uplus y = \sqrt{x^2 + y^2}$. Find $8 \star (5 \uplus 12)$.
Problem 8. In how many ways can I order the four letters in "WORD"? For example, one way is "ORWD".

Set 3

Problem 9. Compute $733^2 - 267^2$

Problem 10. In a bizarre world far, far away, an apple is worth three bananas, an orange is worth 8 coconuts, and a coconut is worth 7 bananas. If an apple costs \$1.50, how many dollars does an orange cost?

Problem 11. What is the value of $\sqrt{x^3 + 3y}$ if $x = 7$ and $y = 6$?

Problem 12. Find the smallest four digit number that is divisible by 2, 3, and 11.

27^{th} TJIMO
Alexandria, Virginia

Round: **Guts**

Set 3

Problem 9. Compute $733^2 - 267^2$

Problem 10. In a bizarre world far, far away, an apple is worth three bananas, an orange is worth 8 coconuts, and a coconut is worth 7 bananas. If an apple costs \$1.50, how many dollars does an orange cost?

Problem 11. What is the value of $\sqrt{x^3 + 3y}$ if $x = 7$ and $y = 6$?

Problem 12. Find the smallest four digit number that is divisible by 2, 3, and 11.

Set 4

Problem 13. How many non-congruent isosceles triangles with positive area and integer side lengths have perimeter 14?
Problem 14. What are the sum of the prime factors of $3^8 - 1$?
Problem 15. How many integer solutions of x exist for $|2x - 21| \leq 7$?
Problem 16. A stock began at \$1000. If throughout the year it increased in value by 10%, then increased by 30%, and finally decreased by 40%, what was its final value in dollars?

27^{th} TJIMO
Alexandria, Virginia

Round: **Guts**

Set 4

Problem 13. How many non-congruent isosceles triangles with positive area and integer side lengths have perimeter 14?
Problem 14. What are the sum of the prime factors of $3^8 - 1$?
Problem 15. How many integer solutions of x exist for $|2x - 21| \leq 7$?
Problem 16. A stock began at \$1000. If throughout the year it increased in value by 10%, then increased by 30%, and finally decreased by 40%, what was its final value in dollars?

Set 5

Problem 17. Three VMT officers can write a lecture in two seconds. How many seconds would it take 5 VMT officers to write 10 lectures?
Problem 18. Fill in the blank in the following sequence: $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \_\_\_\_$.
Problem 19. Samel is a crazy kid who is only happy if he has his favorite number of camels. Four of these are purple, and the others are white. Samel knows that when he has his favorite number of camels, three times the number of white camels equals the sum of the number of white camels and the total number of camels. What is Samel's favorite number of camels?
Problem 20. Compute the number of factors of 900.

27^{th} TJIMO
Alexandria, Virginia

Round: **Guts**

Set 5

Problem 17. Three VMT officers can write a lecture in two seconds. How many seconds would it take 5 VMT officers to write 10 lectures?
Problem 18. Fill in the blank in the following sequence: $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \_\_\_\_$.
Problem 19. Samel is a crazy kid who is only happy if he has his favorite number of camels. Four of these are purple, and the others are white. Samel knows that when he has his favorite number of camels, three times the number of white camels equals the sum of the number of white camels and the total number of camels. What is Samel's favorite number of camels?
Problem 20. Compute the number of factors of 900.

Set 6

Problem 21. What is the smallest positive integer that has a remainder of 2 when divided by 7 and 1 when divided by 9?
Problem 22. On a 5×7 grid, Charles can only move upwards one unit or rightwards one unit at a time and can only move along gridlines. In how many ways can he move from the lower left corner to the top right corner?
Problem 23. What are the last two digits of 7^{2015}?
Problem 24. If $3^{2x-4} = 4$, where x is a positive number, what is the value of 3^x?

27th TJIMO
Alexandria, Virginia

Round: **Guts**

Set 6

Problem 21. What is the smallest positive integer that has a remainder of 2 when divided by 7 and 1 when divided by 9?
Problem 22. On a 5×7 grid, Charles can only move upwards one unit or rightwards one unit at a time and can only move along gridlines. In how many ways can he move from the lower left corner to the top right corner?
Problem 23. What are the last two digits of 7^{2015}?
Problem 24. If $3^{2x-4} = 4$, where x is a positive number, what is the value of 3^x?

Set 7

Problem 25. Bo Schmoe is raising farm animals, but only has mos and joes. A *mo* has 2 heads and a *joe* has 3 heads. If Bo has 29 animals and these animals have 71 heads total, what is the absolute value of the difference between the number of mos and the number of joes on Bo Schmoe's farm?

Problem 26. Victoria and Katherine are sharing a circular pizza. However, they only have a very bad knife that can only making 3 straight cuts before breaking. If they cannot rearrange the pizza pieces until they have made all the cuts, what is the largest number of pieces Victoria and Katherine can cut the pizza into?

Problem 27. What is the 5^{th} smallest positive number with 5 factors?

Problem 28. In how many ways can the letters in "EFFERSON" be arranged?

27^{th} TJIMO
Alexandria, Virginia

Round: **Guts**

Set 7

Problem 25. Bo Schmoe is raising farm animals, but only has mos and joes. A *mo* has 2 heads and a *joe* has 3 heads. If Bo has 29 animals and these animals have 71 heads total, what is the absolute value of the difference between the number of mos and the number of joes on Bo Schmoe's farm?

Problem 26. Victoria and Katherine are sharing a circular pizza. However, they only have a very bad knife that can only making 3 straight cuts before breaking. If they cannot rearrange the pizza pieces until they have made all the cuts, what is the largest number of pieces Victoria and Katherine can cut the pizza into?

Problem 27. What is the 5^{th} smallest positive number with 5 factors?

Problem 28. In how many ways can the letters in "EFFERSON" be arranged?

Set 8

Problem 29. Jendy is shipping herself down a river. Paddling furiously in her cardboard box, she travels downstream to her chosen man in 3 hours. However, when she realizes that she chose the wrong man, it takes her 5 hours to paddle upstream to her starting point. Her paddling speed relative to the water and the current speed are constant. What is the ratio of Jendy's paddling speed in still water to the speed of the current?

Problem 30. Everyone in VMT is either an Akshaj, who always tells the truth, or a Franklyn, who always lies. A group of confused freshmen meets VMT members Will, Michael, Ruiran, and Alex. The VMT members say the following:

- Will says, "Only Franklyns get sweaty."
- Michael says, "Will is so rich that he's a Franklyn."
- Ruiran says, "Will does not get sweaty."
- Alex says, "Both Will and I are Franklyns."

How many of the 4 VMT members are Franklyns?

Problem 31. How many convex hexagons are there drawn below? A polygon is said to be convex if all of its interior angles have measure less than 180°.

27th TJIMO
Alexandria, Virginia

Round: **Guts**

Set 8

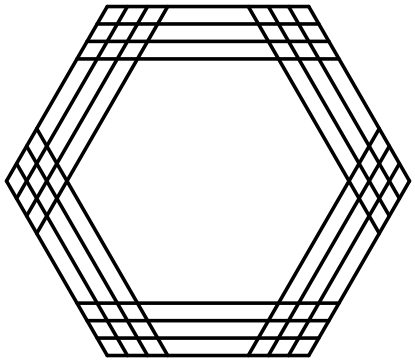
Problem 29. Jendy is shipping herself down a river. Paddling furiously in her cardboard box, she travels downstream to her chosen man in 3 hours. However, when she realizes that she chose the wrong man, it takes her 5 hours to paddle upstream to her starting point. Her paddling speed relative to the water and the current speed are constant. What is the ratio of Jendy's paddling speed in still water to the speed of the current?

Problem 30. Everyone in VMT is either an Akshaj, who always tells the truth, or a Franklyn, who always lies. A group of confused freshmen meets VMT members Will, Michael, Ruiran, and Alex. The VMT members say the following:

- Will says, "Only Franklyns get sweaty."
- Michael says, "Will is so rich that he's a Franklyn."
- Ruiran says, "Will does not get sweaty."
- Alex says, "Both Will and I are Franklyns."

How many of the 4 VMT members are Franklyns?

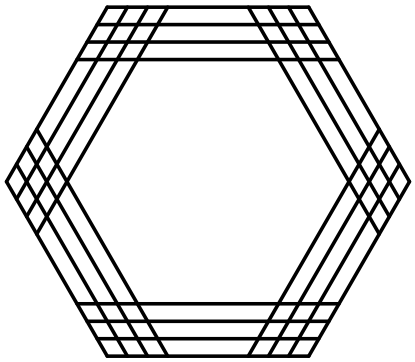
Problem 31. How many convex hexagons are there drawn below? A polygon is said to be convex if all of its interior angles have measure less than 180°.



Problem 32. If $\frac{1}{x} + 3 = \sqrt{10}$, what is x to the nearest integer?

27th TJIMO
Alexandria, Virginia

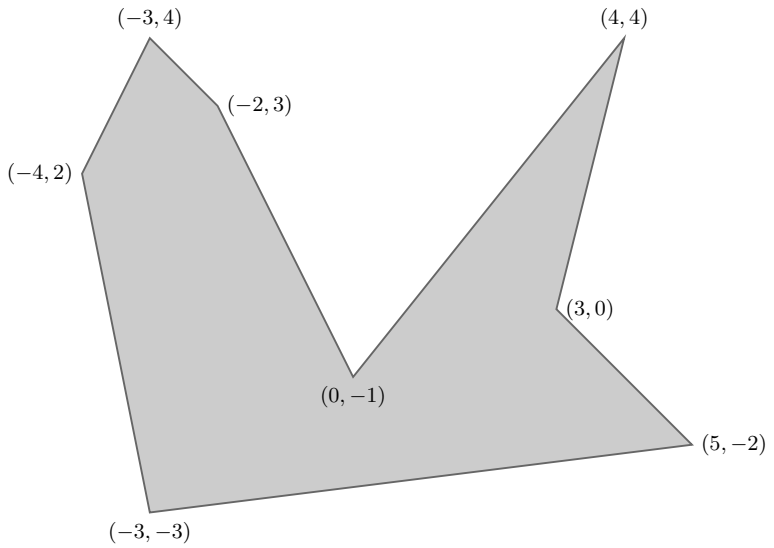
Round: **Guts**



Problem 32. If $\frac{1}{x} + 3 = \sqrt{10}$, what is x to the nearest integer?

Set 9

Problem 33. Find the area of the shaded region.



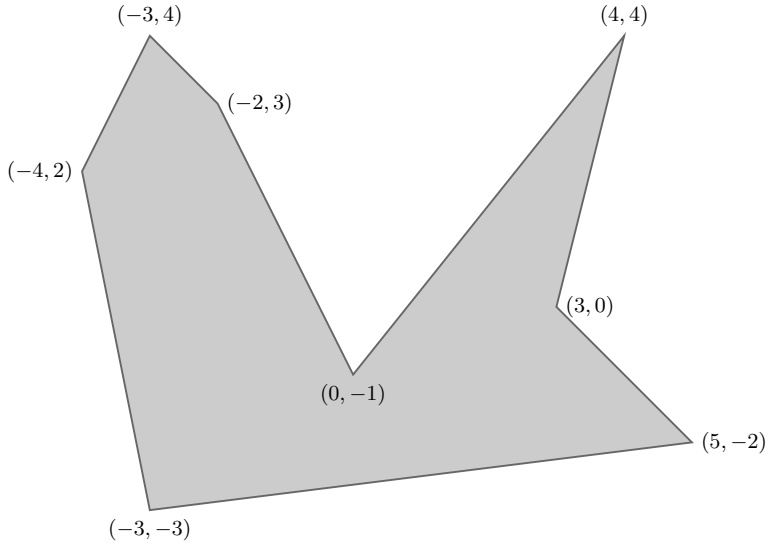
Problem 34. The time is now 4:00 on an analog clock (a normal clock with hands). In how many minutes

27th TJIMO
Alexandria, Virginia

Round: **Guts**

Set 9

Problem 33. Find the area of the shaded region.



Problem 34. The time is now 4:00 on an analog clock (a normal clock with hands). In how many minutes

will the minute and hour hands of this clock be exactly together for the first time? Give your answer to the nearest minute.

Problem 35. Evaluate $1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \cdots + 12 \cdot 2^{11}$

Problem 36. What is the remainder when 3^{178} is divided by 30?

27th TJIMO
Alexandria, Virginia

Round: **Guts**

will the minute and hour hands of this clock be exactly together for the first time? Give your answer to the nearest minute.

Problem 35. Evaluate $1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \cdots + 12 \cdot 2^{11}$

Problem 36. What is the remainder when 3^{178} is divided by 30?