28th TJIMO

Alexandria, Virginia

Round: **Guts**

Problem 1. What is 50% of 10% of 1000?

Problem 2. I had 50 cookies yesterday but now I only have 30. If only two people—my brother, Frank, and my sister, Gabriella—ate my cookies, and I know Frank ate 8 of them, how many did my sister eat?

Problem 3. If the area of a square is 64, what is its perimeter?

Problem 4. Simplify $\frac{\sqrt{15^2+8^2}}{\sqrt{13^2-5^2}}$

Problem 5. How many primes are there that are less than 60?

Problem 6. Find the area of a circle with radius 6. Express your answer in terms of π.

Problem 7. Let $x \star y = x^2 - y^2$, and $x \uplus y = \sqrt{x^2 + y^2}$. Find $8 \star (5 \uplus 12)$.

Problem 8. In how many different orders can I write the 4 letters in"WORD"?

Problem 9. Compute $733^2 - 267^2$

Problem 10. In a bizarre world far, far away, an apple is worth three bananas, an orange is worth 8 coconuts, and a coconut is worth 7 bananas. If an apple costs \$1.50, how much does an orange cost?

Problem 11. What is the value of $\sqrt{x^3 + 3y}$ if $x = 7$ and $y = 6$?

Problem 12. Find the smallest four digit number that is divisible by 2, 3, and 11.

Problem 13. What is the least number of line segments I need to draw in a circle to separate it into 11 distinct regions?

Problem 14. What are the sum of the prime factors of $3^8 - 1$?

Problem 15. How many integer solutions of x exist for $|2x - 21| \leq 7$?

Problem 16. A stock began at \$1000. If throughout the year it increased in value by 10%, then increased by 30

Problem 17. Three VMT officers can write a lecture in two seconds. How many second would it take 5 VMT officers to write 10 lectures?

Problem 18. Find the next term in the following series: $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, ???$.

Problem 19. Samel is a crazy kid who is only happy if he has his favorite number of camels. Four of these are purple and the others are white. Samel knows when he has his favorite number of camels, three times the number of white camels equals the sum of the number of white camels and the total number of camels. What is Samel's favorite number of camels?

Problem 20. Compute the number of factors of 900.

Problem 21. What is the smallest positive integer that has a remainder of 2 when divided by 7 and 1 when divided by 9?

Problem 22. On a 5×7 grid, Charles can only move upwards or to the right and can only move along gridlines. In how many ways can he move from the lower left corner to the top right corner?

Problem 23. What are the last two digits of 7^{1685}?

Problem 24. If $3^{2x-4} = 4$, what is the value of 3^x?

Problem 25. Shya LeBuff is raising farm animals, but only has mos and joes. A *mo* has 2 heads and a *joe* has 3 heads. A punch is worth two points and a kick is worth 3. If Shya has 29 animals and these animals have 71 heads total, what is the absolute value of the difference between the number of mos and the number of joes on Shya LeBuff's farm?

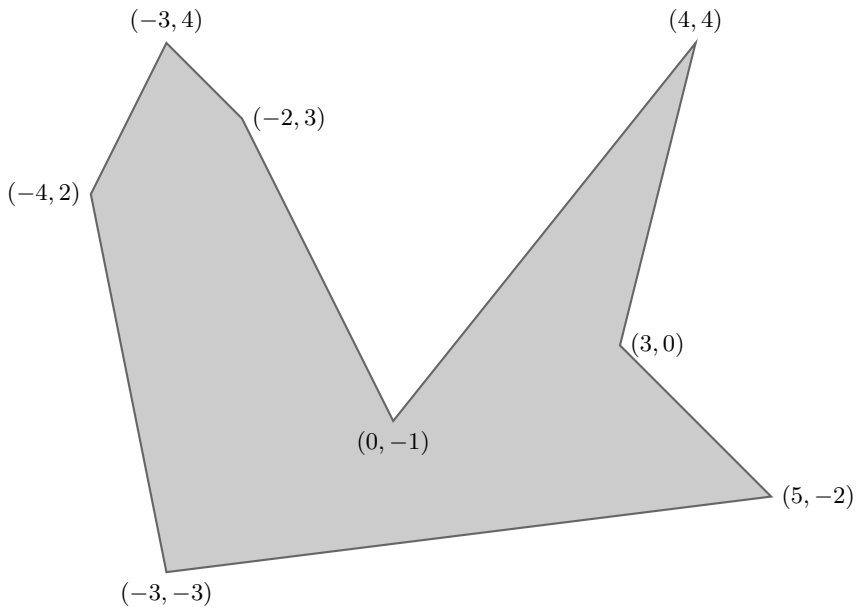
Problem 26. Victoria and Katherine are sharing a circular pizza. However, they only have a very bad knife that can only making 3 straight cuts before breaking. If they cannot rearrange the pizza pieces until they have made all the cuts, what is the largest number of pieces Victoria and Katherine can cut the pizza into?

Time limit: 30 minutes.

Problem 27. What is the 5^{th} smallest number with 5 factors?
Problem 28. In how many ways can the letters in "EFFERSON" be arranged?
Problem 29. Jendy is shipping herself down a river. Paddling furiously in her cardboard box, she travels downstream to her chosen man in 3 hours. However, when she realizes that she chose the wrong man, it takes her 5 hours to paddle upstream to her starting point. What is the ratio of Jendy's paddling speed in still water to the speed of the current?
Problem 30. Everyone in VMT is either an Akshaj, who always tells the truth, or a Franklyn, who always lies. A group of confused freshmen meets VMT members Will, Michael, Ruiran, and Alex. The VMT members say the following:

- Will says, "Only Franklyns get cancer."
- Michael says, "Will is so rich that he's a Franklyn."
- Ruiran says, "Will does not have cancer."
- Alex says, "Both Will and I are Franklyns."

How many of the 4 VMT members are Franklyns?
Problem 31. How many convex hexagons are there drawn below? A polygon is said to be convex if all of its interior angles have measure less than $180°$.
Problem 32. Find the area of the shaded region.



Problem 33. If $\frac{1}{x} + 3 = \sqrt{10}$, what is x to the nearest integer?
Problem 34. The time is now 4:00 on an analog clock (a normal clock with hands). In how many minutes will the minute and hour hands of this clock be exactly together?
Problem 35. Evaluate $1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \cdots + 12 \cdot 2^{11}$
Problem 36. What is the remainder when 3^{178} is divided by 30?

Time limit: 30 minutes.