27^{th} TJIMO

Alexandria, Virginia

Round: **Individual**

Problem 1. In the year 2010, Jennifer decided to start collecting stamps. Her collection increases by 7 stamps each year. In 2010, she had 7 stamps, in 2011, she had 14 stamps, and so on. How many stamps will she have in 2020?

Problem 2. Josh divides some number by 7. He adds the quotient, the dividend, and the remainder and gets an answer of 14. What is the value of his original number?

Problem 3. Saroja goes to the grocery store to buy mangoes and watermelons, where the mangoes are equally priced and the watermelons are equally priced. If she buys two mangoes and three watermelons, she will pay \$3.39. If she buys five mangoes and four watermelons, she will pay \$5.50. How much would she need to pay for one watermelon and one mango?

Problem 4. Arobin, Brobin, Crobin, and Drobin are standing in a line, from left to right. Arobin is to the left of Crobin, but not directly next to him. Crobin and Drobin have exactly one person between them. Who is farthest to the right?

Problem 5. A recipe for oreo truffles, which makes 400 truffles, calls for 10 bars of chocolate, 12 packs of oreos, and 16 sticks of cream cheese. If Josh wants to make 200 truffles, what is the sum of the number of bars of chocolate and the number of packs of oreos he will need?

Problem 6. Michael makes a pigeon shaped robot for his Robot Project. He tests his robot by making it run in a straight line. There are markings along this line with 6-inch spaces between each pair of markings. If his robot travels 80 spaces, calculate the speed of his robot in ft/sec.

Problem 7. A set of the first n odd integers has a sum of 64. How many integers are in the set?

Problem 8. A circle is inscribed into a square with side length 4. What is the area of the circle?

Problem 9. Elsa and Anna are building a snowman together. If Elsa can build a snowman in 7 hours and Anna can build a snowman in 5 hours, how long (in minutes) would it take them to build one snowman if they worked together?

Problem 10. Catherine, Franklin, and Lillian are trying to guess Jeffrey's test grade (out of a maximum of 100 points). He tells each of them one hint about his test grade.

Catherine: His test grade is above 90.

Franklin: His test grade is an odd integer.

Lillian: His test grade has 6 factors.

What was Jeffrey's test grade?

Problem 11. Pradeep's Pizza Parlor offers 10 different toppings for pizzas. If Aaditaya wants to get a pizza with 2 different toppings, how many different combinations of pizza toppings can Aaditaya get? Assume that order of toppings doesn't matter (Topping A and Topping B is the same as Topping B and Topping A).

Problem 12. A triangle with all integer side lengths has two sides of length 3 and 4. How many possible lengths are there for the third side of the triangle?

Problem 13. Alice, Bob, Charles, Diana, and Ethan participated in a race. Charles finished directly after Ethan, but did not finish last. Diana finished second, and Alice did not finish next to Diana. Assuming there were no ties, who finished first?

Problem 14. Lynne has 2 metronomes, one is set at 80 beats per minute, and the other is set at 120 beats per minute. Lynne switches on her 2 metronomes at the exact same time. In one minute, how many times will they share a common beat?

Problem 15. Two guests at dinner each received a slice of pie. A piece of pie was cut out for the first

guest, and the second guest received 3/8 of the remaining pie. If the second guests portion of pie was 1/3 of the entire pie, then what fraction of the entire pie did the first guest receive?

Problem 16. How many two-digit positive integers have an odd number of factors?

Problem 17. Emily has a playlist of Liszt's Transcendental Etudes, which there are 12 of, on shuffle. If she only likes 3 of them, what is the probability that those will be the first three she hears?

Problem 18. Franklyn the frog is doing combinatorics problems given to him by Akshaj the combinatorialist. Akshaj gives Franklyn 9 problems in the same form: "If Jame has n shirts and n pants, how many unique outfits of 1 shirt and 1 pair of pants can Jame dress up in?" Problem 1 has with $n = 1$, Problem 2 has $n = 2$, and so on up to Problem 9 has $n = 9$. What is the sum of the answers to these 9 problems?

Problem 19. In the land of Darnia, Darnians use the currency of Darns, and they have two kinds of bills: bills worth 2 Darns and bills worth 3 Darns. If Alec has plenty of both 2-Darn bills and 3-Darn bills, how many combinations of bills could Alec use to be worth 37 Darns? (One example would be 17 2-Darn bills and 1 3-Darn bill)

Problem 20. Given $x + \frac{1}{x} = 3$, find $x^3 + \frac{1}{x^3}$.

Problem 21. Hcir the dragon likes to collect gold pieces for his cave. Every day, he collects either 3 or 7 gold pieces to add to his collection. What is the largest possible number of gold pieces that Hcir could not own?

Problem 22. Sam1 lives on a Cartesian coordinate plane at $(1, 1)$. He wants to walk to his friend Sam5's house at $(5, 5)$, but he can only move either one unit right or one unit up at a time. However, Sam1 cannot move to the point at (2,3) because there is construction there. How many distinct paths can Sam1 take to get to Sam5's house?

Problem 23. Peter counts the number of times the hour hand and the minute hand of the clock cross each other on an analog clock. He starts watching the clock at exactly 11:30 AM and keeps watching until 11:30 AM the next day. How many times will the hands will have crossed eachother?

Problem 24. What are the last two digits of 47^{83}?

Problem 25. Given that the roots of the equation $x^2 - 15x + 36 = 0$ are p and q, and $p < q$, find the sum of the roots of the equation $x^2 - \frac{p}{3}x - q = 0$.

Problem 26. Geoffrey the Geometer is surveying his new plot of land, because he wants to make space for his pet giraffe. If his plot of land is a regular hexagon with side length of 40 meters, and his giraffe needs $400\sqrt{3}$ meters^2 of land to roam, how many square meters of land does he have left to build his house on?

Problem 27. Let p, q, and r be real numbers between 0 and 10, inclusive. What is the probability that $p + q + r$ is between 5 and 10, inclusive? Express your answer as a common fraction.

Problem 28. A unit circle is inscribed in a regular hexagon. What is the area between the hexagon and the circle?

Problem 29. A walrus and a whale are playing catch with a fish. They both have a $\frac{1}{3}$ chance of catching the fish and a $\frac{2}{3}$ chance of dropping the fish. The game ends whenever one of them drops the fish. What is the probability that the whale will win (meaning the walrus drops the fish first), if the whale is the first one to throw the fish?

Problem 30. Each minute, a broken robot has a $\frac{2}{3}$ chance of moving forward 1 meter and a $\frac{1}{3}$ chance of moving backward 1 meter. What is the probability that it reaches its destination 3 meters ahead of it before it falls off the cliff 2 meters behind it?

Time limit: 60 minutes.