

all other principles. Making and attaching a reason as such a cumbersome name like bizarre space here is an unnecessary act to satisfy human psychological vanity, and it is not at all an attitude to face and appreciate nature as it is. There is no reason here, and no understanding is needed. This is just the truth of this universe we have to accept. Life doesn't disappear just because there's no reason for life, or because we don't understand it. Life flows regardless of reason or understanding. So is the existence of nature. Own understanding is only up to each person, and the essence is beyond the phenomenon of understanding or transmission of understanding through language. However, which can only be different is whether we are seeing exactly what is actually happening or not. I introduce as an example the concept of the relative view of a circle and an ellipse as a very good tool for analyzing events from that perspective.

Using this, I will first find the traditional Lorentz transformation.

1.2 Lorentz Transformation

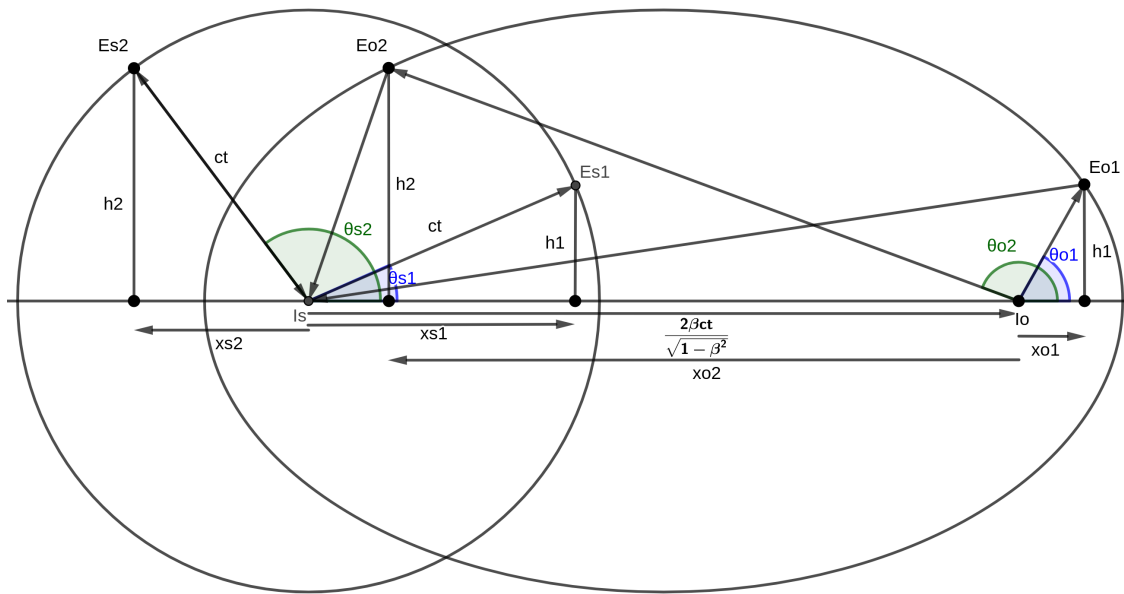


Figure 3: The Lorentz transformation

The Lorentz transformation can be expressed as shown in the figure above. Specifically, I described the I_s inertial system with I_s as the center all the time, and the inertial system I_o with relative velocity $v = \beta c$ whose center coincided at I_s at time 0 and whose center shifted to I_o separated by $\frac{2\beta ct}{\sqrt{1-\beta^2}}$ at time $\frac{2t}{\sqrt{1-\beta^2}}$. Let us think of these events, when mirrors are arranged in a circle around I_s , light shines on the surrounding mirrors from the central focus I_s of the circle at time 0, and the lights are reflected back to the central focus at the same time t . Let

us call these events Es1 and Es2, and think of the events of lights converging to the center again at 2t. If all these events are observed in the Io inertial frame, the rays departing from one focal point of the ellipse at time 0, Io, are respectively reflected back through different times events Eo1 and Eo2, which exist on the ellipse, and we shall observe the event converging again at the other focus Is of the ellipse at time $\frac{2t}{\sqrt{1-\beta^2}}$. In other words, as shown in the Lorentz transformation figure above, it is the Lorentz transformation geometrically described to change events observed in the Is inertial frame into events observed in the Io inertial frame.

Using this to derive the traditional Lorentz transformation, since the equation of the ellipse in the general form is $R = a \frac{1-e^2}{1-e \cos(\theta)} = \frac{A}{2} \frac{1-\frac{f^2}{A^2}}{1-\frac{f}{A} \cos(\theta)}$ and since $A = \frac{2ct}{\sqrt{1-\beta^2}}$, $f = \frac{2\beta ct}{\sqrt{1-\beta^2}}$, so the equation of the ellipse in which the events Eo are located in the Is inertial frame can be calculated using

$$R_{\theta o} = \frac{ct}{\sqrt{1-\beta^2}} \frac{1-\beta^2}{1+\beta \cos \theta o}$$

in polar coordinates. First, enter this basic expression into firCAS.

$$(5) - > R_{\theta o} := \frac{ct}{\sqrt{1-\beta^2}} \frac{1-\beta^2}{1+\beta \cos(\theta o)}$$

Type: Expression(Integer)

To find the relationship between θo and θs by using the fact that the Y-axis positions h of the events Eo and Es do not change in both inertial frames, we can enter a command to solve equation $R_{\theta o} \sin(\theta o) = ct \sin(\theta s)$ for θo in CAS as follows.

$$(6) - > \text{solve}(R_{\theta o} \sin(\theta o) = ct \sin(\theta s), \theta o)$$

$$[\theta o = -2 \operatorname{atan}\left(\frac{\sqrt{-\beta^2+1}}{(\beta-1) \tan(\frac{\theta s}{2})}\right), \theta o = -2 \operatorname{atan}\left(\frac{\tan(\frac{\theta s}{2}) \sqrt{-\beta^2+1}}{\beta-1}\right)]$$

Type: List(Equation(Expression(Integer)))

Since the cosine form is easier to handle than the tangent form in the result, the equation is modified and re-entered.

$$(7) - > \text{solve}\left(R_{\theta o} \sqrt{1-\cos(\theta o)^2} = ct \sqrt{1-\cos(\theta s)^2}, \theta o\right)$$

$$[\theta o = \operatorname{acos}\left(\frac{-\cos(\theta s)+\beta}{\beta \cos(\theta s)-1}\right), \theta o = \operatorname{acos}\left(\frac{-\cos(\theta s)-\beta}{\beta \cos(\theta s)+1}\right)]$$

Type: List(Equation(Expression(Integer)))

Two solutions are computed, but the left one will be selected because it fits the definition of the Lorentz transformation figure above. The solution on the right is a figure in which the

left and right are reversed.

$$\cos(\theta_o) = \frac{\cos(\theta_s) - \beta}{1 - \beta \cos(\theta_s)}$$

The transformation of the space-time coordinates ct and X_s of E_s in the inertial frame I_s into the space-time coordinates $R\theta_o$ and X_o of the events E_o in the inertial frame I_o is the Lorentz transformation in the time and X axes, respectively. Since $R\theta_o$ is the distance that light travels in an ellipse, it is ct' of E_o 's in a moving inertial frame. To substitute the above expression here, use the substitution or substitution command 'eval' or 'subst' to change the $\cos(\theta_o)$ part of the $R\theta_o$ expression to $\frac{\cos(\theta_s) - \beta}{1 - \beta \cos(\theta_s)}$ using θ_s .

$$(8) - >eval \left(R\theta_o, \cos(\theta_o) = \frac{\cos(\theta_s) - \beta}{1 - \beta \cos(\theta_s)} \right)$$

$$\frac{-ct\beta \cos(\theta_s) + ct}{\sqrt{-\beta^2 + 1}}$$

Type: Expression(Integer)

According to the Lorentz transformation figure, $X_s = ct \cos(\theta_s) \rightarrow \cos(\theta_s) = \frac{X_s}{ct}$, and substitute it into the above result. The '%' symbol in the input means that the previous result is the corresponding input.

$$(9) - >eval \left(\%, \cos(\theta_s) = \frac{X_s}{ct} \right)$$

$$\frac{-X_s\beta + ct}{\sqrt{-\beta^2 + 1}}$$

Type: Expression(Integer)

This is the Lorentz transformation regarding time. However, CAS programs still have limitations when it comes to formatting such expressions neatly. Therefore, using $\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$, and expressing $ct' = \gamma ct - \gamma \beta x_s$ is a more familiar expression.

I will now find the Lorentz transformation in the x -direction. Since it is $X_o = R\theta_o \cos(\theta_o)$ in the Lorentz transformation figure, applying the same substitution as in Calculation 8 here to change the expression θ_o to the expression θ_s ,

$$(10) - >eval \left(R\theta_o \cos(\theta_o), \cos(\theta_o) = \frac{\cos(\theta_s) - \beta}{1 - \beta \cos(\theta_s)} \right)$$

$$\frac{ct \cos(\theta_s) - ct\beta}{\sqrt{-\beta^2 + 1}}$$

Type: Expression(Integer)

If substitute $\cos(\theta_s) = \frac{X_s}{ct}$ into the above result as in Calculation 9, changing the polar coordinate expression to a xy -coordinate expression,

$$(11) - >X_o = eval \left(\%, \cos(\theta_s) = \frac{X_s}{ct} \right)$$

$$X_o = \frac{-ct\beta + X_s}{\sqrt{-\beta^2 + 1}}$$

Type: Equation(Expression(Integer))

This is $X_o = \gamma X_s - \gamma \beta ct$, which confirms that it is a Lorentzian transformation about the X axis.

Therefore, it was confirmed that the Lorentz transformation can be obtained from the geometrical expression of the relativity principle.

Now, through these calculations using ellipses, we can depict the expanding universe as observed from the center of each primordial inertial system.

1.3 The universe viewed from other primordial inertial systems

Geometrically, a circle is a set of points that are at a constant distance from the center. However, this is the case where both the center and the points at a certain distance are stationary. In our real universe, the expanding universe, we have to find a way to define how moving points compose equal distances around some moving primordial inertial system points. In this regard, the principle of relativity, which looks different in circles and ellipses depending on the point of view, suggests the following method of defining distance.

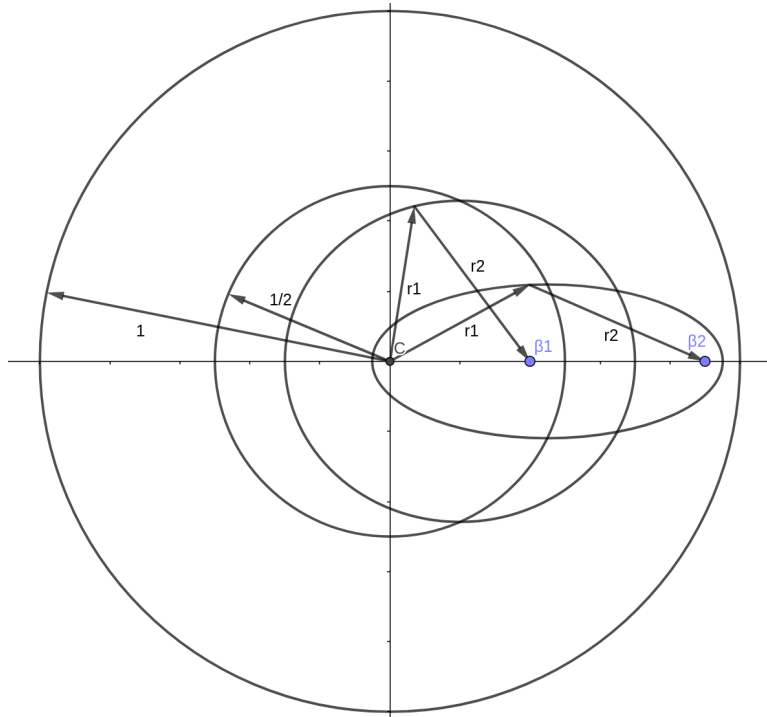


Figure 4: The observed universe

As a new distance measurement method suitable for the moving dynamic universe, the 'distance measurement method by radar radio wave round-trip time' is possible. If the radar

waves that simultaneously departed from one point to two places return at the same time, the distances between the radar device and the two places can be said the same. This relationship can be used as it is in a moving inertial frame as well as in a stationary inertial frame. If the events of measuring radio waves traveling two equal distances in this one inertial frame are viewed in another inertial frame having a relative velocity, they'll be seen as reflections at different distances from the original focal point, and then, they'll be seen as events that have returned to the focal point that has reached a different location, since it also moved while the radio waves travel. However, even in this case, the phenomenon that the two radio waves return 'simultaneously' to the starting focal point that has moved so far does not change. This phenomenon can be used to define the same distance in different inertial systems. A set of events that are neither simultaneous nor equidistant in this inertial system but simultaneous and equidistant in that inertial system can be defined in this way, and it can be seen that they form an ellipse.

Just since the outermost part of the universe is moving away at the speed of light from every observer looking from the center of any primordial inertial system, the light from there can be thought of exactly the same as the case of radar reflection. Describing the outermost shell of the universe as seen from different inertial systems in this way, is the same as the previous 'The observed Universe' figure.

Let us consider a universe with an age of 1, a speed of light of 1, and a size of 1 that expands around C, which will be called the observation center, one of the primordial inertial system points in the universe. The outermost shell of the universe is expanding at the speed of light, and the light observing this shell from the center at time 1 will be the light that departed from the outer shell, which is circular at time $\frac{1}{2}$ and size $\frac{1}{2}$, toward the center. At this time, the cosmic shell which is observed in another primordial inertial system with relative speed β with a primordial inertial system C is as described in the figure of the observed universe. Since the light of the cosmic shell that began to expand at the speed of light with the center as the focus at time 0 was observed at another focus β , therefore, the shell of the universe from which lights simultaneously reached B forms an ellipse. In three-dimensional space, it will be an ellipsoid in the shape of a rugby ball. In this case, the sum of the distances from each focus of the ellipse is $r_1 + r_2 = 1$. If $\beta = 0$, the distance between the two focal points is 0, that is a circle, $r_1 = r_2 = \frac{1}{2}$. The distance r , closer than the outermost, observed in the primordial inertial system β , at time 1, from focus β , describing the space inside this cosmic shell, can be defined using the following distance definition figure

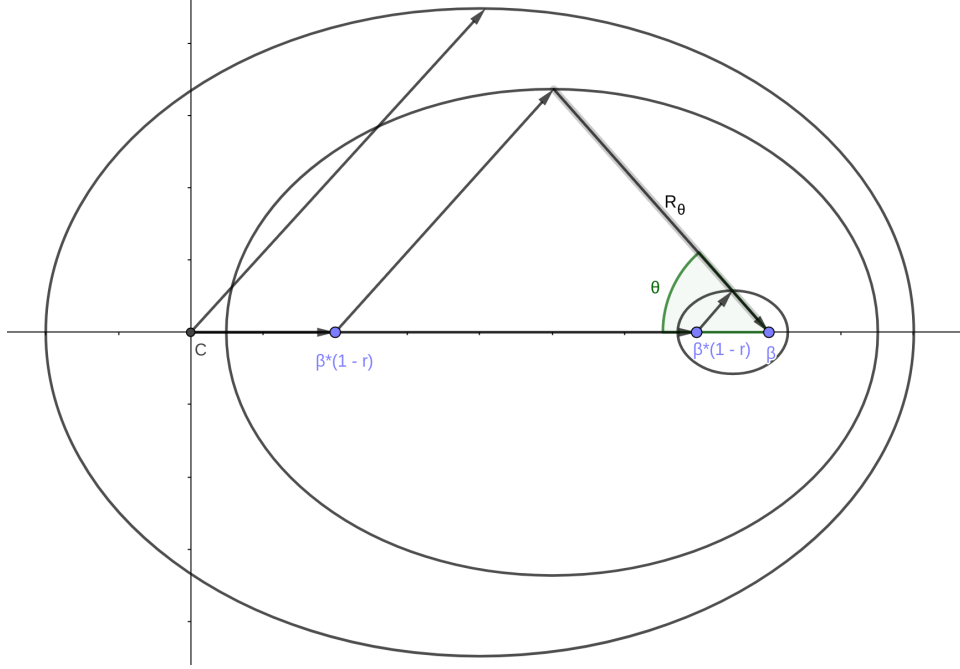


Figure 5: Definition of distance

In a universe of age 1, if we try to describe a value r smaller than 1 which is inside the outer shell of the universe, based on the β inertial system, it can be described as a set of points that reflect the lights to converge when the lights departed from the β inertial system point located at $\beta(1 - r)$ at the time $(1-r)$, is reflected along the way, and return to converge β at the same time at time 1. These points form an ellipse, and the equation of the ellipse describing it is

$$R_\theta = \frac{r}{2} \frac{1 - \beta^2}{1 - \beta \cos(\theta)}$$

This will be referred to as the observation ellipse.

In this equation, r represents the apparent distance observed in a primordial inertial system and is a value corresponding to the 'angular diameter distance' in general relativity-based cosmology. However, here, it is expressed as a relative ratio to the total size of the universe rather than an absolute distance.

Using this, I will analyze the universe observed in the inertial system β . First, I will analyze the age of each point in the universe located on the observation ellipse observed in the inertial system β . The self-perceived age of the universe at all positions on the same observational ellipse should be the same. I will check whether this expectation holds true.

1.4 The age structure of the universe

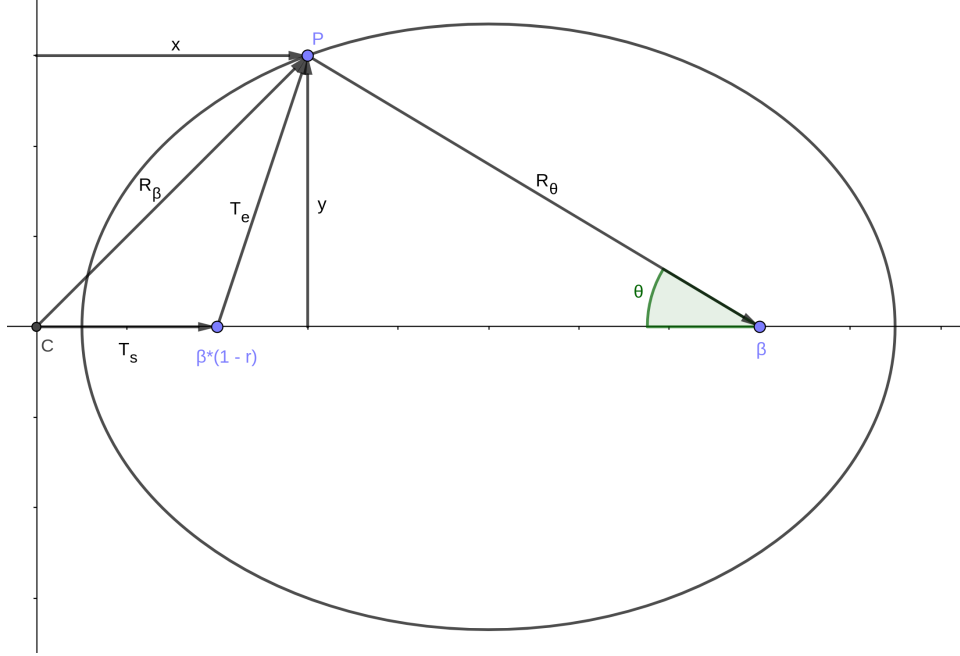


Figure 6: Age Calculation

When a light departs from point P located at (R_θ, θ) to β with respect to the inertial frame β , let us call this event P. At this time, the event P exists on a circle drawing a radius $\frac{r}{2\gamma}$ with the view of the β inertial frame. In addition, based on the observation center C inertial system, it exists on an ellipse with major axis diameter r and points $\beta(1-r)$ and β as focal points. In this case, the age of the universe around oneself felt at the event P is independent of the inertial frame observing it, and is a unique age felt by itself.

To express this based on center C, we need to know the relative speed β_p between the primordial inertial system corresponding to the event point P and the observation center C of the universe. When the speed of light is set to 1, its relative speed is the distance R_β between event P and observation center C divided by the size of the universe at that point in time. In this case, since the speed of light is 1, the size of the universe is the time of the event P, $T_s + T_e$. It is

$$\beta_p = \frac{R_\beta}{T_s + T_e}$$

. At this time, T_s is the time when the β inertial system reaches the distance of $\beta(1-r)$ at the speed of β , so it is $T_s = \frac{\beta(1-r)}{\beta}$. And, since P is a point on an ellipse with major axis diameter r , T_e satisfies $r = T_e + R_\theta$ in terms of distance. In this case, the speed of light is assumed to be 1, so the time is $T_e = \frac{r-R_\theta}{c} = r - R_\theta$.

At this time, point P is moving away from point C at the speed of β_p , so according to special relativity, time passes more slowly relatively than point C. Thus the age of the universe at point P is

$$\sqrt{1 - \beta_p^2}(T_s + T_e)$$

In order to satisfy the uniformity and isotropy of the universe according to the cosmological principle, all P events with the same r and arbitrary θ constituting the celestial sphere observed at an arbitrary β must all share the same proper age that felt by themselves.

x I will check if this holds. First, the definitions of all values in the age calculation figure are arranged through the observed universe figure and the age calculation figure as described previously and assigned to the firCAS variables. In the previous explanation, the transformation of the P point position to xy coordinates was not discussed, but it will be used later, so it will be input together.

$$(12) - > \begin{aligned} R\theta &:= \frac{r}{2} \frac{1-\beta^2}{1-\beta \cos(\theta)}, \\ T_s &:= \frac{\beta(1-r)}{\beta}, \\ T_e &:= r - R\theta, \\ x &:= \beta - R\theta \cos(\theta), \\ y &:= R\theta \sqrt{1 - \cos(\theta)^2} \\ & \left[\frac{r\beta^2-r}{2\beta \cos(\theta)-2}, -r+1, \frac{2r\beta \cos(\theta)-r\beta^2-r}{2\beta \cos(\theta)-2}, \right. \\ & \left. \frac{((-r+2)\beta^2+r) \cos(\theta)-2\beta}{2\beta \cos(\theta)-2}, \frac{(r\beta^2-r)\sqrt{-\cos(\theta)^2+1}}{2\beta \cos(\theta)-2} \right] \end{aligned}$$

Type: Tuple(Any)

Now, I will proceed to calculate the age equation for the universe.

To make it easier for automatic computation, I will use the form $\sqrt{(1 - \beta_p^2)(T_s + T_e)^2}$ instead of $\sqrt{1 - \beta_p^2}(T_s + T_e)$. And, since $R_\beta^2 = x^2 + y^2$, it is $\beta_p^2 = \left(\frac{R_\beta}{T_s+T_e}\right)^2 = \frac{x^2+y^2}{(T_s+T_e)^2}$. Therefore, the final form of the expression for the age of the universe at point P to be used is

$$\sqrt{\left(1 - \frac{x^2 + y^2}{(T_s + T_e)^2}\right) (T_s + T_e)^2}$$

. The values necessary for the expression have been assigned to variables in previous Calculation 12, so the calculation is performed automatically just by entering the corresponding expression, and the calculation result of

$$(13) - > \sqrt{\left(1 - \frac{x^2+y^2}{(Ts+Te)^2}\right) (Ts+Te)^2} \\ \sqrt{(r-1)\beta^2 - r + 1}$$

Type: Expression(Integer)

$\sqrt{1-\beta^2}\sqrt{1-r}$ is obtained. Since this is a value dependent only on r and β , which is unrelated to θ where the θ information that existed at the input time disappeared, it satisfies the isotropy that the universe should be seen equally in all θ directions. In addition, this is the age of the universe around it felt in all arbitrary primordial inertial systems β where time is slowly flowing at the speed of $\sqrt{1-\beta^2}$. Therefore, all observers of the primordial inertial system β see the age of the universe in area r distance away from themselves as $\sqrt{1-r}$, which is based on the standard of feeling that their own age is 1. And, this satisfies the uniformity of the universe that the universe viewed from all primordial inertial systems must be the same.

In the limited case of $\beta = 0$, the relationship between the distance r and the running away speed β_p can be obtained directly by calculating a different approach to the above. When looking at the movement of β_p , which departed from the observation center C at time 0, and the movement of light that departed at the time $(1-r)$, caught up with it, is reflected, and returned to the origin at time 1. Since the round-trip time given for the light to catch up and return was r , the time taken for β_p to run before being caught up with the light was $(1-r) + \frac{r}{2}$, and the time taken for the light to chase was $\frac{r}{2}$, the equation can be set up as follows.

$$(14) - > \text{solve} \left(\frac{r}{2} = \beta_p \left(1 - r + \frac{r}{2}\right), \beta_p \right) \\ [\beta_p = -\frac{r}{r-2}]$$

Type: List(Equation(Fraction(Polynomial(Integer))))

As the speed at some distance r is obtained, substituting this result into the well-known relativistic redshift formula for frequency, $\sqrt{\frac{1-\beta_p}{1+\beta_p}}$. By doing so, we obtain the rate at which time passes at the point r as observed from C . Continuing to observe this until the age of the universe at point C reaches 1 provides us with the proper age experienced by point r as observed from C . This gives the following,

$$(15) - > \text{eval} \left(\sqrt{\frac{1-\beta_p}{1+\beta_p}}, \% \right) \\ \sqrt{-r+1}$$

Type: Expression(Integer)

From this, it can be confirmed that the age obtained by different methods is the same for observer C whose β is 0. In other words, it was confirmed that the two methods are not contradictory and that the result obtained by Calculation 13 is justified.

It is not enough for the universe to be uniform and isotropic with respect to characteristics of the proper age of the observed universe area, it also must be observed uniformly and isotropically in the density of the universe that is gradually diminishing because of the expansion. Now that we know the proper age of the universe observed in every inertial system, we can calculate the density distribution of the universe based on this.

1.5 Matter density distribution of the universe

First, let's make an initial assumption. Classically, around the center of an expanding universe, the particle density should decrease with time. This can be inferred because the radius of the universe increases with time, and the volume of the universe, which is proportional to the cube of the radius, should also increase. Therefore, the density would decrease inversely with the cube of time. According to the assumption of the universe's uniformity, this should apply uniformly in all primordial inertial systems. However, since the age of the universe is different in each primordial inertial system, the density at each point in the primordial inertial systems should be $\frac{1}{\sqrt{1-\beta^2}^3}$ when the density at the observation center C is 1. In addition, since all primordial inertial systems are moving away from the observation center C with a velocity β , the matter density at the center points of the other primordial inertial systems, as observed from C, should increase by a factor of $\frac{1}{\sqrt{1-\beta^2}}$ due to the length contraction in special relativity. Therefore, the final density should be $\frac{1}{(1-\beta^2)^2}$. This consideration of length contraction can also be inferred from the perspective that if the distance between two particles is L when measured by an observer at rest, it should be $\frac{L}{\sqrt{1-\beta^2}}$ when measured by the two particles themselves.

This prediction can be verified by calculating it more rigorously in the following way.

To explain this method, first, it needs a concept that I call a normalized universe. As an observer located at observation center C, if we look at the density of matter around us, we will see that the density decreases with time in the case of an expanding universe. Here, density may be defined as the number of particles existing within a certain distance from the center. However, the concept of a certain distance from the center may be an absolute distance, but may also be a relative distance. In the case of a constant-expanding universe, the radius r of the universe will increase uniformly with time. In this case, if we set a distance of $1/n$ of r and count the number of particles closer than that, the number of particles within that distance

will always be constant without changing with time. Here, I will call the universe where r is always assumed to be 1 a normalized universe. At this time, according to the hypothesis of uniformity among the cosmological principles, for any primordial inertial system, the number of particles present in the same microscopic radius dr must be equal in the normalized universe of that inertial system reference. If this is observed in some other primordial inertial system, the nearby primordial inertial system moving away from the observer slowly will show similar matter density to the observer, while the rapidly moving away primordial inertial system far from the observer will show an appearance from long ago and also the time delay in special relativity will make it appear to have higher matter density. This is the calculation to find the particle density distribution according to the phenomenon.

In the previous explanation of the age structure of the universe, I have shown that the points forming the celestial sphere at a distance r , based on a certain primordial inertial system β , create an ellipsoid when observed from the center C . These points on the celestial sphere of the β primordial inertial system represent a set of events where light is sent to β points at different times, not simultaneously, when observed from the observation center C , resulting in the formation of an ellipsoid. For now, I will refer to this as the "ellipsoid of events." At this point, the overall size of the universe at the time of each event is also different. When all these events are projected into the normalized universe, the major axis of the ellipsoid is distorted into a more flattened disk shape. In this case, the compression ratio is not uniform across different parts of the ellipsoid, but if we consider a fine dr as the radius of the celestial sphere, the compression ratio of the resulting disk can be considered relatively constant. In the normalized universe, there is always the same number of particles within these microscopic spheres. Therefore, the particle distribution observed at center C will be directly proportional to the rate of shrinkage of these microscopic spheres. I will now proceed to project these microscopic spheres from the β inertial system into the normalized universe based on the observation center C .

First, I will transform the microsphere in the β inertial system into an ellipsoid of events at the C observation center. According to the age calculation figure, when θ is 0 and π , it is the direction of motion in the β inertial system in the direction of the x -axis, and when $\cos \theta = \beta$, the y -axis height of the ellipse becomes the maximum which is the height of the minor axis radius of the ellipse. These will be referred to as an X -axis direction radius and a Y -axis direction radius, respectively. When obtaining their projection into the normalized universe, regarding the problem of deformation, as mentioned above, since it is a deformation in a microscopic domain, the rate of distortion can be considered constant for all parts. So it is sufficient to divide the position where the event occurred by the time the event occurred for the projection of this ellipsoid onto the normalized universe. To extract the sizes of the compressed disk shape after projection, in the x -axis direction, subtract the smaller value from the larger nor-

malized value on the x-axis. The y-axis and z-axis directions are the same and symmetrical, so it is enough to double the radius of the disk. The x and y values of the ellipse required for this calculation have already been entered in Calculation 12, so only the corresponding θ value should be substituted here. This is expressed as a ratio with the original microsphere dr and is as shown in the calculation below. Since we can use 'eval' instead of 'limit',

$$(16) \rightarrow \frac{\text{eval}\left(D\left(\text{eval}\left(\frac{x}{T_s+T_e}, \theta = 0\right) - \text{eval}\left(\frac{x}{T_s+T_e}, \theta = \pi\right), r\right), r = 0\right)}{\text{eval}\left(D\left(2\text{eval}\left(\frac{y}{T_s+T_e}, \theta = \text{acos}(\beta)\right), r\right), r = 0\right)}$$

Type: Tuple(Expression(Integer))

The result is a sphere squeezed by $\sqrt{1 - \beta^2}$ in both the y and z directions and by $1 - \beta^2$ in the x-axis direction, and since there is the same number of particles as in the observation center C in its volume, the density ratio with the observation center C is the same as before, it can be confirmed that

$$\frac{1}{(1 - \beta^2)^2}$$

. Now that we have established a plausible density distribution for the universe, it's time to examine whether this density distribution is observed to be the same not only at observation center C but also at all the centers of primordial inertial systems β , and whether it satisfies the homogeneity and isotropy of the universe.

1.6 Confirmation of uniformity and isotropy of the universe

In order to confirm the uniformity and isotropy of the universe, what we actually need to look at is whether the particle density formula of the universe observed on the celestial sphere at a distance r from the center β of each primordial inertial system is a constant form in all primordial inertial systems and whether it is an isotropic form that does not contain directional components. To put this concretely, it is to verify whether the number of particles is a function of only distance r_0 , regardless of the direction angles θ_0 and ϕ_0 , in a small area with a fine thickness of dr_0 and fine widths of $d\theta_0$ and $d\phi_0$, at a distance r_0 from the center point β of an inertial system on the celestial sphere of β .

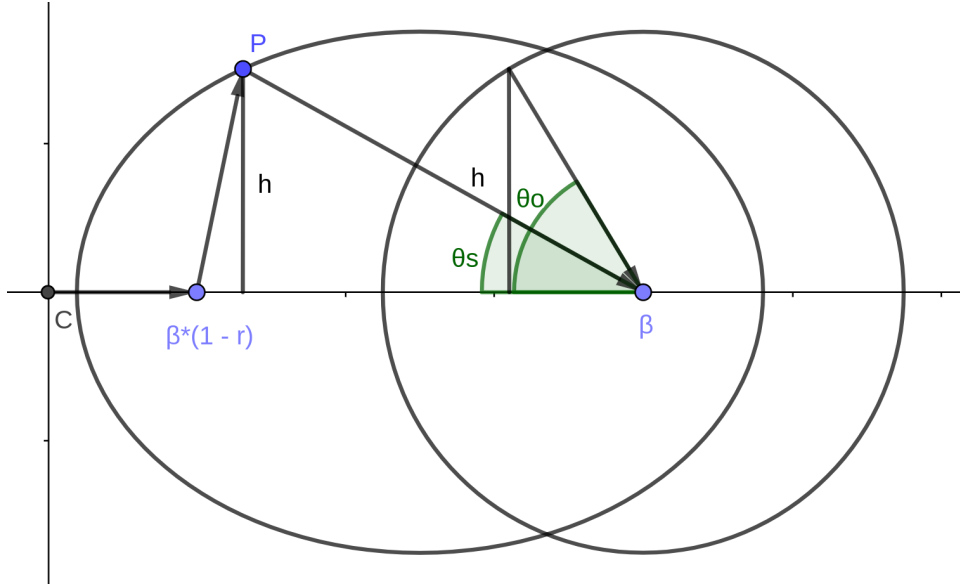


Figure 7: Relative celestial sphere

First, it needs to define the observation ellipse. Since the used angle, θ_s is the angle based on the center C inertial system, a process of converting it to the angle θ_o in the β inertial system is required. The observation ellipse, which was defined as $R_\theta = \frac{r}{2} \frac{1-\beta^2}{1-\beta \cos(\theta)}$ at the observation center, is observed as a circle with radius $\frac{r}{2} \sqrt{1-\beta^2}$ in the β inertial system. And as similar work done in the Lorentz transformation derivation previously, it will use the point that the y-axis height h of the event is the same regardless of whether it is an ellipse or a circle. Thus, θ_s on the observation ellipse (in actual calculations, it is expressed as θ) is converted into an expression of θ_o in the β inertial frame. This is similar to the calculation of correcting the velocity aberration of the light.

$$(17) \rightarrow \text{solve} \left(R_\theta \sqrt{1 - \cos(\theta)^2} = \frac{r}{2} \sqrt{1 - \beta^2} \sqrt{1 - \cos(\theta_o)^2}, \theta \right)$$

$$[\theta = \text{acos}(\frac{\cos(\theta_o) + \beta}{\beta \cos(\theta_o) + 1}), \theta = \text{acos}(\frac{\cos(\theta_o) - \beta}{\beta \cos(\theta_o) - 1})]$$

Type: List(Equation(Expression(Integer)))

The equation on the left is consistent with the defining figure of the relative celestial sphere, but when $\theta=0$, r is directed in the -X direction, so the value of the determinant to be calculated later has a correct magnitude but a negative sign, so instead of multiplying by -1 at the end. With the equation on the right where the value is the same and the sign is reversed, I will convert $T_s + T_e$, x , y , etc. of Calculation 12 in the age structure of the universe into the form using θ_o , and input.

$$\begin{aligned}
(18) \rightarrow & \text{eq} := \theta = \text{acos} \left(\frac{\beta - \cos(\theta_o)}{1 - \beta \cos(\theta_o)} \right), \\
& T_o := T_s + \text{eval}(T_e, \text{eq}), \\
& x_o := \text{eval}(x, \text{eq}), \\
& \text{eval}(y, \text{eq}) = \frac{r}{2} \sqrt{1 - \beta^2} \sqrt{1 - \cos(\theta_o)^2}, \\
& y_o := \frac{r}{2} \sqrt{1 - \beta^2} \sqrt{1 - \cos(\theta_o)^2} \\
& [\theta = \text{acos} \left(\frac{\cos(\theta_o) - \beta}{\beta \cos(\theta_o) - 1} \right), \frac{r\beta \cos(\theta_o) - r + 2}{2}, \frac{r \cos(\theta_o) + (-r + 2)\beta}{2}, \\
& \frac{(-r\beta \cos(\theta_o) + r) \sqrt{\frac{(\beta^2 - 1) \cos(\theta_o)^2 - \beta^2 + 1}{\beta^2 \cos(\theta_o)^2 - 2\beta \cos(\theta_o) + 1}}}{2} = \frac{r \sqrt{-\beta^2 + 1} \sqrt{-\cos(\theta_o)^2 + 1}}{2}, \\
& \left. \frac{r \sqrt{-\beta^2 + 1} \sqrt{-\cos(\theta_o)^2 + 1}}{2} \right]
\end{aligned}$$

Type: Tuple(Any)

Since I am going to find the density, the volume element must be defined. In polar coordinates, θ_o is usually used as latitude, but here, latitude is given to ϕ_o , which will be used only once as the value of 0 near the equator. ϕ_o is the same as ϕ . Since it is near the equator, $\cos(\phi_o) = 1$, which makes the calculation easier. In order to find the micro volume element based on the C inertial system, which is the form of $dr \times r \times \cos(\phi) \times d\theta \times r \times d\phi \rightarrow r^2 dr d\theta d\phi_o$ in the β inertial system, I will enlarge the vicinity of point P on the relative celestial sphere figure. Point P is expressed as $\vec{R}(r, \theta)$ from β to P.

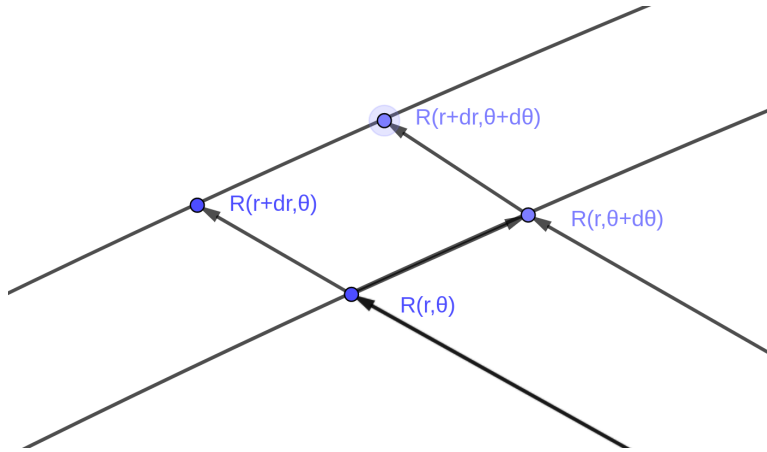


Figure 8: Enlarged P-point

Since it is a small area of an ellipse rather than a circle, simple definitions such as dr , $r d\theta_o$, and $r d\phi_o$ that are orthogonal to each other cannot be seen, but instead, definitions such as $\vec{R}(r + dr, \theta) - \vec{R}(r, \theta) = \frac{d\vec{R}(r, \theta)}{dr} dr$ and $\vec{R}(r, \theta + d\theta) - \vec{R}(r, \theta) = \frac{d\vec{R}(r, \theta)}{d\theta} d\theta$ from the parallelogram can be used.

Since $Rd\phi$ is in the z direction perpendicular to the xy plane, the definition of $rd\phi$ in a circle can be used as is. To calculate the density of the universe seen by the β observer, the needed area corresponding to $dr \times rd\theta$ from the micro volume $dr \times rd\theta \times rd\phi$ is $\frac{d\vec{R}(r,\theta)}{dr}dr \times \frac{d\vec{R}(r,\theta)}{d\theta}d\theta$. What we want to know is not the absolute density, but the ratio of the density of the point P to the density of the center β of the inertial system. So what is needed is not the actual micro volume, but the ratio of micro-volumes based on r and θ . So, I will compute $\frac{d\vec{R}(r,\theta)}{dr} \times \frac{d\vec{R}(r,\theta)}{d\theta}$.

In this case, since each point P is the same distance from point β and is arbitrary θ , the observed events of particles around point P are not simultaneous with respect to the observation center C. Hence, the size of the universe at each P event based on C is different. Therefore, the area that is a parallelogram in this microdomain must be projected onto the normalized universe, and since this is the case of an infinitesimal domain, the characteristics of a parallelogram do not change, only the size changes.

The expression corrected for that is $\frac{d\vec{R}(r,\theta)/T}{dr} \times \frac{d\vec{R}(r,\theta)/T}{d\theta}$, and since the z-axis is the celestial sphere based on the β inertial system, it becomes $rd\phi$, and since it only needs to be considered around $\phi \rightarrow 0$, it is actually only the value of the z-axis proportional to r. However, since the time of the point P event differs according to θ as mentioned above, the z-axis term must also be divided by the size (time) of the universe at that time point in the same way. Since the z term is always perpendicular to the previous parallelogram, it can be expressed as a scalar product without need of vector product, and the proportional expression of the final micro volume including this is

$$\frac{d\vec{R}(r,\theta)/T}{dr} \times \frac{d\vec{R}(r,\theta)/T}{d\theta} \cdot \frac{d\vec{R}(r,\theta,\phi)/T}{d\phi}$$

. This can be expressed conveniently in a determinant.

$$\begin{vmatrix} \frac{d\vec{R}(r,\theta)/T}{dr} \cdot \hat{X} & \frac{d\vec{R}(r,\theta)/T}{dr} \cdot \hat{Y} & 0 \\ \frac{d\vec{R}(r,\theta)/T}{d\theta} \cdot \hat{X} & \frac{d\vec{R}(r,\theta)/T}{d\theta} \cdot \hat{Y} & 0 \\ 0 & 0 & \frac{d\vec{R}(r,\theta,\phi)/T}{d\phi} \cdot \hat{Z} \end{vmatrix}$$

This can be expressed by the C-referenced Cartesian coordinate expression $X(r,\theta)\hat{X} + Y(r,\theta)\hat{Y}$ prepared in Calculation 12, rather than the β -referenced polar coordinate expression R vector,

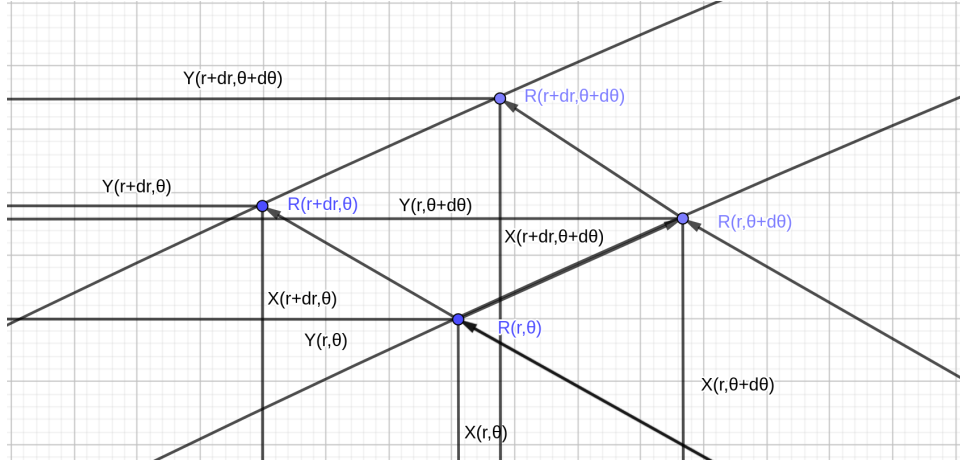


Figure 9: Orthogonal coordinate representation

And, as for the Z-axis, ϕ -direction, since it is a value only of the z-axis, it simply can be obtained directly from the definition and is $\frac{d\vec{R}(r, \theta, \phi)/T}{d\phi} \cdot \hat{Z} = \frac{dZ(r, \theta, \phi)/T}{d\phi} = \frac{r/2}{T}$. The reason why $1/2$ is entered is that it is the celestial sphere in the β inertial system, and this has been explained through the figure of 'The observed universe'. With this and the Cartesian coordinate position expression $X(r, \theta)\hat{X} + Y(r, \theta)\hat{Y}$, the ratio of the micro volume is expressed as follows.

$$\begin{vmatrix} \frac{dX(r, \theta)/T}{dr} & \frac{dY(r, \theta)/T}{dr} & 0 \\ \frac{dX(r, \theta)/T}{d\theta} & \frac{dY(r, \theta)/T}{d\theta} & 0 \\ 0 & 0 & \frac{r/2}{T} \end{vmatrix}$$

What is being attempted to confirm is the uniformity and isotropy of the universe as seen from the β inertial system, so X_0 and Y_0 , which are the functions of θ_0 used in the β inertial system, must be used. And these have been input in Calculation 18.

At this point, the size r of the universe seen from point β is the age of the universe at point β , $\sqrt{1 - \beta^2}$, which is smaller than compared to point C. Multiply this by $r/2$ is $\frac{r}{2}\sqrt{1 - \beta^2}$, and divide this by T_0 to project onto the normalized universe. Of course, when looking at C from β , the age of the universe on the C side will be $\sqrt{1 - \beta^2}$, but this is relativistic, and here I am basically describing the universe viewed from the viewpoint of the observation center C.

I will now find the determinant of unit volume by applying all of these. However, since the machine's ability to organize formulas is still a bit lacking, the 'tsimp' function that helps to organize formulas is manually defined and substituted, artificially intervening in the process of simplifying formulas a little, making it easier to confirm. Without this, the formula would be overly long and complex.

Function definition in friCAS uses $==$, and $:=$ is a variable value assignment, and $=$ is sim-

ply an equation notation. Symbolic arithmetic can also be used in function definitions. In the case of friCAS linked with TeXmacs, the determinant can be input using the determinant symbol directly as shown below, or using a matrix in the 'determinant' function. The 'map' function is a higher-order function 'map' familiar to functional programming languages. It was used to apply the 'tsimp' function to all the constituent values of a matrix. The applied result is also too long to display directly, so it is easy to see the result of the input matrix calculation by calculating the difference with the manually arranged result to show that it is a 0 or 0 matrix. It was a shortcut to show the output neatly. If you are a reader who is following the input yourself, it is also recommended to check the output before it is simplified.

$$(19) - >tsimp(x) == eval \left(x, \sqrt{1 - \cos(\theta o)^2} = \sin(\theta o) \right)$$

Type: Void

$$(20) - >map \left(tsimp, \begin{bmatrix} D\left(\frac{x_o}{T_o}, r\right) & D\left(\frac{y_o}{T_o}, r\right) & 0 \\ D\left(\frac{x_o}{T_o}, \theta o\right) & D\left(\frac{y_o}{T_o}, \theta o\right) & 0 \\ 0 & 0 & \frac{r}{2} \frac{\sqrt{1-\beta^2}}{T_o} \end{bmatrix} \right) -$$

$$\begin{pmatrix} \frac{(-2\beta^2+2)\cos(\theta o)}{(r\beta\cos(\theta o)-r+2)^2} & \frac{2\sin(\theta o)\sqrt{-\beta^2+1}}{(r\beta\cos(\theta o)-r+2)^2} & 0 \\ \frac{((-r^2+2r)\beta^2+r^2-2r)\sin(\theta o)}{(r\beta\cos(\theta o)-r+2)^2} & \frac{((-r^2+2r)\cos(\theta o)+r^2\beta)\sqrt{-\beta^2+1}}{(r\beta\cos(\theta o)-r+2)^2} & 0 \\ 0 & 0 & \frac{r\sqrt{-\beta^2+1}}{r\beta\cos(\theta o)-r+2} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Type: Matrix(Expression(Integer))

Therefore, it can be seen that the value of the determinant, which is the normalized micro volume ratio, is

$$(21) - >detro := \begin{vmatrix} D\left(\frac{x_o}{T_o}, r\right) & D\left(\frac{y_o}{T_o}, r\right) & 0 \\ D\left(\frac{x_o}{T_o}, \theta o\right) & D\left(\frac{y_o}{T_o}, \theta o\right) & 0 \\ 0 & 0 & \frac{r}{2} \frac{\sqrt{1-\beta^2}}{T_o} \end{vmatrix};$$

$$\frac{2r^2(1-\beta^2)^2}{(r\beta\cos(\theta o)-r+2)^4} - tsimp(detro)$$

0

Type: Expression(Integer)

$$\frac{2r^2(1-\beta^2)^2}{(r\beta\cos(\theta o)-r+2)^4}$$

The particle density ratio of the universe in polar coordinates of $r \theta o \phi$ viewed from point β is the product of the normalized density distribution $\frac{1}{(1-\beta^2)^2}$ and the normalized micro volume ratio at the observed point. To calculate this, the micro volume ratio will be multiplied

by $\frac{1}{(1-\beta^2)^2} = \frac{1}{\left(1-\frac{x_o^2+y_o^2}{T_o^2}\right)^2}$ as done in Calculation 13.

$$(22) \rightarrow \text{simp} \left(\frac{1}{\left(1-\frac{x_o^2+y_o^2}{T_o^2}\right)^2} \begin{vmatrix} D\left(\frac{x_o}{T_o}, r\right) & D\left(\frac{y_o}{T_o}, r\right) & 0 \\ D\left(\frac{x_o}{T_o}, \theta_o\right) & D\left(\frac{y_o}{T_o}, \theta_o\right) & 0 \\ 0 & 0 & \frac{r}{2} \frac{\sqrt{1-\beta^2}}{T_o} \end{vmatrix} \right)$$

$$\frac{r^2}{8r^2-16r+8}$$

Type: Expression(Integer)

Summarizing this result a little further, it becomes $\frac{1}{8} \left(\frac{r}{1-r} \right)^2$. Since this is the same expression for all directions irrelevant to θ_o , it was confirmed that the isotropy of the universe was satisfied.

As in the case of the age equation of the universe, this equation also must be possible to obtain in a direct way when at observation center C. Briefly, when $\beta p = \frac{r}{2-r}$ is the result of Calculation 14 and the density function of the form βp is $\frac{1}{(1-\beta p^2)^2}$, the polar coordinate definition of the volume element with radius βp in the normalized universe is $\beta p^2 d\beta p d\theta d\phi$, and when $f(r)$ is the density function in $r \theta \phi$ polar coordinates, it is $\frac{1}{(1-\beta p^2)^2} \beta p^2 d\beta p d\theta d\phi = f(r) dr d\theta d\phi$, thus we can see that it is $f(r) = \frac{1}{(1-\beta p^2)^2} \beta p^2 \frac{d\beta p}{dr}$. Deriving this, it is

$$(23) \rightarrow \text{eval} \left(\frac{1}{(1-\beta p^2)^2} \beta p^2 D\left(\frac{r}{2-r}, r\right), \beta p = \frac{r}{2-r} \right)$$

$$\frac{r^2}{8r^2-16r+8}$$

Type: Fraction(Polynomial(Integer))

, and as a result, the density distribution of the universe viewed from the central inertial system C and the density distribution of the universe viewed from an arbitrary β inertial system is the same

$$\frac{1}{8} \left(\frac{r}{1-r} \right)^2$$

in the polar coordinate system, and it is confirmed that the universe is uniform and isotropic.

It can be seen that, here, the three are noteworthy as ways of describing distance in the universe. They are r , the distance corresponding to the redshift $z+1$, and βp . The relationship between each distance can be obtained using the results of calculations 14 and 15. Since $z+1$ is more convenient than z , I will refer to this as z_n and will use it.

First, when $z_n = \sqrt{\frac{1+\beta p}{1-\beta p}}$, the values already defined from calculations 14 and 15 are $\beta p = \frac{r}{2-r}$ and $z_n = \frac{1}{\sqrt{1-r}}$. Using these to compute the remaining missing relationships,

$$(24) \rightarrow \begin{aligned} & \text{solve} \left(\beta p = \frac{r}{2-r}, r \right), \\ & \text{solve} \left(zn = \frac{1}{\sqrt{1-r}}, r \right), \\ & \text{solve} \left(zn = \sqrt{\frac{1+\beta p}{1-\beta p}}, \beta p \right) \\ & [[r = \frac{2\beta p}{\beta p+1}], [r = \frac{zn^2-1}{zn^2}], [\beta p = \frac{zn^2-1}{zn^2+1}]] \end{aligned}$$

Type: Tuple(List(Equation(Expression(Integer))))

Summarizing all the relationships between the three distance definitions,

$$\begin{aligned} \beta_p &= \frac{r}{2-r} \\ \beta_p &= \frac{zn^2-1}{zn^2+1} \\ r &= \frac{2\beta_p}{\beta_p+1} \\ r &= \frac{zn^2-1}{zn^2} \\ zn &= \frac{1}{\sqrt{1-r}} \\ zn &= \sqrt{\frac{1+\beta_p}{1-\beta_p}} \end{aligned}$$

The density function in the polar coordinate of calculations 22 and 23 can be obtained with even the changed distance notation to zn or βp instead of r. From

$$\frac{1}{(1-\beta p^2)^2} \beta p^2 d\beta p d\theta d\phi = f(zn) dzn d\theta d\phi \rightarrow f(zn) = \frac{1}{(1-\beta p^2)^2} \beta p^2 \frac{d\beta p}{dzn}$$

and from

$$\frac{1}{(1-\beta p^2)^2} \beta p^2 d\beta p d\theta d\phi = f(\beta p) d\beta p d\theta d\phi \rightarrow f(\beta p) = \frac{1}{(1-\beta p^2)^2} \beta p^2$$

$$(25) \rightarrow \text{eval} \left(\frac{1}{(1-\beta p^2)^2} \beta p^2, \beta p = \frac{zn^2-1}{zn^2+1} \right) D \left(\frac{zn^2-1}{zn^2+1}, zn \right),$$

$$\frac{1}{(1-\beta p^2)^2} \beta p^2$$

$$\left[\frac{zn^4-2zn^2+1}{4zn^3}, \frac{\beta p^2}{\beta p^4-2\beta p^2+1} \right]$$

Type: Tuple(Fraction(Polynomial(Integer)))

We can also use the result of Calculation 22 as the density function.

$$(26) \rightarrow \text{eval} \left(\frac{1}{8} \left(\frac{r}{1-r} \right)^2, r = \frac{zn^2-1}{zn^2} \right) D \left(\frac{zn^2-1}{zn^2}, zn \right),$$

$$\text{eval} \left(\frac{1}{8} \left(\frac{r}{1-r} \right)^2, r = \frac{2\beta p}{\beta p+1} \right) D \left(\frac{2\beta p}{\beta p+1}, \beta p \right)$$

$$\left[\frac{zn^4-2zn^2+1}{4zn^3}, \frac{\beta p^2}{\beta p^4-2\beta p^2+1} \right]$$

Type: Tuple(Fraction(Polynomial(Integer)))

For fun, I will also introduce a calculation based on the zn distance for the previous calculation 22.

$$\begin{aligned}
(27) \rightarrow & \quad eq := r = \frac{zn^2-1}{zn^2}, \\
& \quad Tzn := eval(To, eq), \\
& \quad Xzn := eval(xo, eq), \\
& \quad Yzn := eval(yo, eq) \\
& \quad \left[r = \frac{zn^2-1}{zn^2}, \frac{(zn^2-1)\beta \cos(\theta o) + zn^2+1}{2zn^2}, \right. \\
& \quad \left. \frac{(zn^2-1) \cos(\theta o) + (zn^2+1)\beta}{2zn^2}, \frac{(zn^2-1)\sqrt{-\beta^2+1}\sqrt{-\cos(\theta o)^2+1}}{2zn^2} \right] \\
& \quad \text{Type: Tuple(Any)}
\end{aligned}$$

$$\begin{aligned}
(28) \rightarrow & \quad map \left(tsimp, \begin{pmatrix} D\left(\frac{Xzn}{Tzn}, zn\right) & D\left(\frac{Yzn}{Tzn}, zn\right) & 0 \\ D\left(\frac{Xzn}{Tzn}, \theta o\right) & D\left(\frac{Yzn}{Tzn}, \theta o\right) & 0 \\ 0 & 0 & eval\left(\frac{r}{2} \frac{\sqrt{1-\beta^2}}{To}, eq\right) \end{pmatrix} \right) - \\
& \quad \begin{pmatrix} \frac{(-4zn\beta^2+4zn)\cos(\theta o)}{((zn^2-1)\beta \cos(\theta o) + zn^2+1)^2} & \frac{4zn \sin(\theta o) \sqrt{-\beta^2+1}}{((zn^2-1)\beta \cos(\theta o) + zn^2+1)^2} & 0 \\ \frac{((zn^4-1)\beta^2 - zn^4+1) \sin(\theta o)}{((zn^2-1)\beta \cos(\theta o) + zn^2+1)^2} & \frac{((zn^4-1) \cos(\theta o) + (zn^4-2zn^2+1)\beta) \sqrt{-\beta^2+1}}{((zn^2-1)\beta \cos(\theta o) + zn^2+1)^2} & 0 \\ 0 & 0 & \frac{(zn^2-1)\sqrt{-\beta^2+1}}{(zn^2-1)\beta \cos(\theta o) + zn^2+1} \end{pmatrix} \\
& \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
& \quad \text{Type: Matrix(Expression(Integer))}
\end{aligned}$$

$$\begin{aligned}
(29) \rightarrow & \quad \frac{4zn \frac{(1-\beta^2)^2}{(zn^2-1)^2}}{\left(\beta \cos(\theta o) + \frac{zn^2+1}{zn^2-1}\right)^4} - tsimp \left(\begin{vmatrix} D\left(\frac{Xzn}{Tzn}, zn\right) & D\left(\frac{Yzn}{Tzn}, zn\right) & 0 \\ D\left(\frac{Xzn}{Tzn}, \theta o\right) & D\left(\frac{Yzn}{Tzn}, \theta o\right) & 0 \\ 0 & 0 & eval\left(\frac{r}{2} \frac{\sqrt{1-\beta^2}}{To}, eq\right) \end{vmatrix} \right) \\
& \quad 0 \\
& \quad \text{Type: Expression(Integer)}
\end{aligned}$$

$$\begin{aligned}
(30) \rightarrow & \quad \frac{4zn \frac{(1-\beta^2)^2}{(zn^2-1)^2}}{\left(\beta \cos(\theta o) + \frac{zn^2+1}{zn^2-1}\right)^4} \frac{1}{\left(1 - \frac{Xzn^2+Yzn^2}{Tzn^2}\right)^2} \\
& \quad \frac{zn^4-2zn^2+1}{4zn^3} \\
& \quad \text{Type: Expression(Integer)}
\end{aligned}$$

Summarizing the polar coordinate universe density function with three distance expressions, they are as follows,

First, for a small volume area defined by $drd\theta d\phi$, it is

$$\frac{1}{8} \left(\frac{r}{1-r} \right)^2$$

, for a small volume area defined by $dznd\theta d\phi$, it is

$$\frac{1}{4zn} \left(\frac{zn^2 - 1}{zn} \right)^2$$

, and for a small volume area defined by $d\beta_p d\theta d\phi$, it is

$$\left(\frac{\beta p}{1 - \beta p^2} \right)^2$$

. Now that the uniformity and isotropy in the structure of cosmic particle density have been confirmed, the cosmic background radiation will be analyzed.

1.7 Doppler beaming, Doppler shift, and the blackbody radiation

As a theoretical basis for analyzing the cosmic background radiation, the Doppler beaming will be first calculated. Doppler beaming or Relativistic beaming is the opposite concept of velocity aberration of light, and refers to a phenomenon in which light emitted from an object moving at a speed close to the speed of light is concentrated forward.

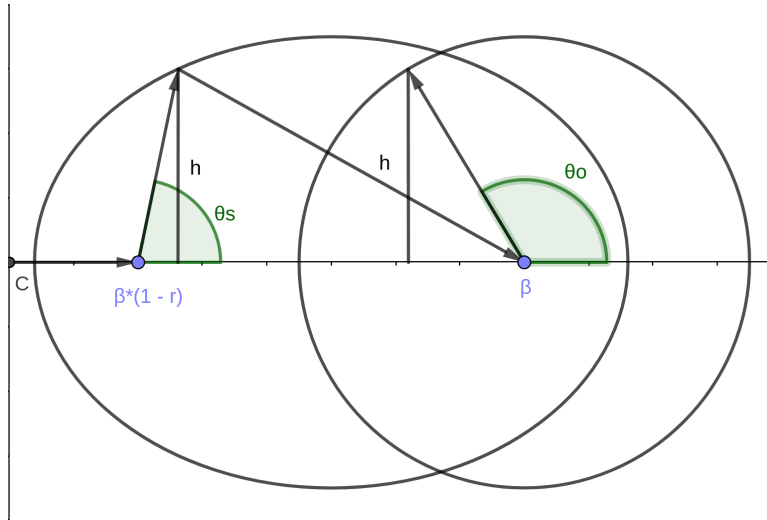


Figure 10: Doppler beaming : The opposite of velocity aberration of light

The figure of the relative celestial sphere can be used to depict Doppler beaming which is similar to the expression of velocity aberration of light. The light radiated at an angle of θ_o in the β inertial frame will appear to be radiated at an angle of θ_s more forward in the C inertial frame of reference.

In this case, in the case of the sphere in the β inertial system, the number of photons flowing out through the area forming the ring with the fine angle $d\theta_o$ at θ_o is as much as $\frac{2\pi r \sin(\theta_o) r d\theta_o}{4\pi r^2} = \frac{\sin(\theta_o) d\theta_o}{2}$ out of the total photons. Since this is observed to flow out through the micro-area of $\frac{\sin(\theta_s) d\theta_s}{2}$ in the C inertial frame, the beaming intensity ratio is

$$\frac{\sin(\theta_o) d\theta_o}{\sin(\theta_s) d\theta_s}$$

. Calculating this,

First, since the equation of the ellipse is $\frac{r}{2} \frac{1-\beta^2}{1-\beta \cos(\theta_s)}$ based on the $\beta(1-r)$ point and θ_s ,

(31)– > solve $\left(\frac{r}{2} \frac{1-\beta^2}{1-\beta \cos(\theta_s)} \sin(\theta_s) = \frac{r}{2} \sqrt{1-\beta^2} \sin(\theta_o), \theta_o \right)$
 $[\theta_o = \text{asin}(\frac{(\beta^2-1) \sin(\theta_s)}{(\beta \cos(\theta_s)-1) \sqrt{-\beta^2+1}})]$
 Type : List (Equation (Expression (Integer)))

Therefore, it can be seen that

$$\frac{\sin(\theta_o)}{\sin(\theta_s)} = \frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta_s)}$$

. Now, finding $\frac{d\theta_o}{d\theta_s}$, it is

(32)– > $\frac{d\theta_o}{d\theta_s} = \text{eval} \left(D \left(\text{asin}(\frac{(\beta^2-1) \sin(\theta_s)}{(\beta \cos(\theta_s)-1) \sqrt{-\beta^2+1}}), \theta_s \right), \sin(\theta_s) = \sqrt{1-\cos(\theta_s)^2} \right)$
 $\frac{d\theta_o}{d\theta_s} = \frac{(\cos(\theta_s)-\beta) \sqrt{-\beta^2+1}}{(\beta^2 \cos(\theta_s)^2 - 2\beta \cos(\theta_s) + 1) \sqrt{\frac{\cos(\theta_s)^2 - 2\beta \cos(\theta_s) + \beta^2}{\beta^2 \cos(\theta_s)^2 - 2\beta \cos(\theta_s) + 1}}}$
 Type : Equation (Expression (Integer))

Summarizing the above result,

$$\frac{d\theta_o}{d\theta_s} = \frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta_s)} = \frac{\sin(\theta_o)}{\sin(\theta_s)}$$

Therefore, the beaming intensity ratio is

$$\frac{\sin(\theta_o) d\theta_o}{\sin(\theta_s) d\theta_s} = \frac{1-\beta^2}{(1-\beta \cos(\theta_s))^2}$$

. The θ_s to be used for the Doppler shift is defined by the following figure.

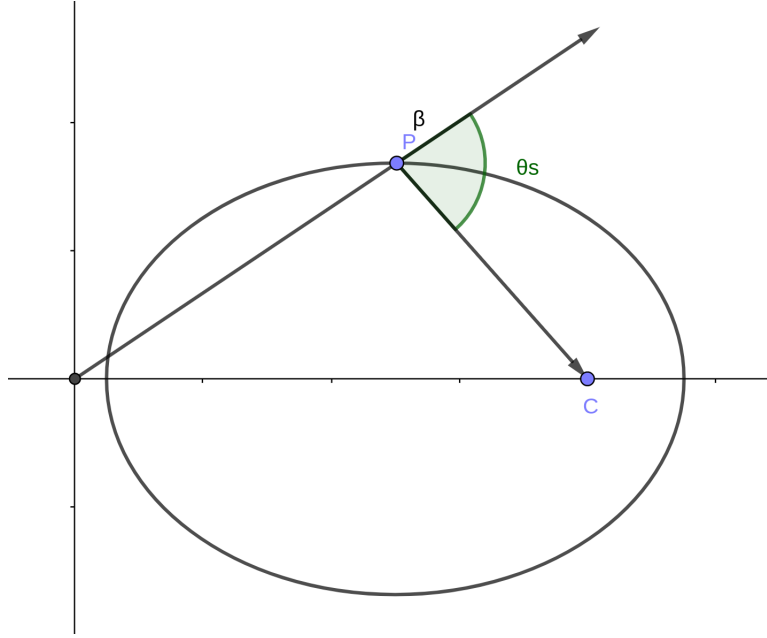


Figure 11: The angle of radiation

Assume that a stationary observer at point C sees the light emitted by an object moving at a speed of β at point P at an angle of θ_s . This is in order to match the meaning of θ_s with the ratio of beaming intensities.

Since the magnitude of the relative velocity based on the center of observation of P and C is $-\beta \cos(\theta_s)$, the classical Doppler shift equation for the frequency is $\frac{1}{1 - \beta \cos(\theta_s)}$. The relativistic Doppler shift is a value multiplied this by the speed of time flow at P, so the relativistic Doppler shift is

$$\frac{1}{z + 1} = \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos(\theta_s)}$$

. At this time, When looking at the change in the intensity of the emitted light due to the Doppler shift, the energy of the emitted photons is proportional to $\frac{1}{z+1}$, and the emission rate of photons is also proportional to $\frac{1}{z+1}$, so the energy emission rate due to the Doppler shift is proportional to the square of them. Therefore,

$$\frac{1}{(z + 1)^2} = \frac{1 - \beta^2}{(1 - \beta \cos(\theta_s))^2}$$

this exactly matches the effect due to Doppler beaming calculated earlier. Therefore, the energy emission rate, taking into account both Doppler beaming effect and Doppler shift is determined solely by the observed redshift and is equal to

$$\frac{1}{(z+1)^4} = \frac{(1-\beta^2)^2}{(1-\beta \cos(\theta_s))^4}$$

. Here, considering the law of inverse square according to distance, it suggests that the luminosity of a certain star or galaxy will appear to follow the luminosity according to equation

$$\frac{1}{r^2} \frac{1}{(z+1)^4}$$

proportional to $\frac{1}{r^2}$ which is the inverse square of its distance to that star or galaxy, and the $\frac{1}{(z+1)^4}$ above.

This result shows that, if considering the emissivity per unit area when blackbody radiation from some substance at position P is Doppler-shifted, it will be found that the frequency directly proportional to temperature T is shifted according to $\frac{1}{zn} = \frac{1}{z+1}$. Therefore, the observed emissivity per unit area will follow the fourth power of the observed temperature, so it can be seen that it always satisfies the emission rate according to the Stefan-Boltzmann formula $\frac{2\pi^5 k^4}{15c^2 h^3} T^4$, which represents the energy emission rate according to the fourth power of the temperature in ideal blackbody radiation.

If the above expression is expressed as r or z only, they are

$$\left(\frac{1-r}{r} \right)^2$$

, and

$$\frac{1}{z^2(z+2)^2}$$

. These expressions represent relative distance and luminous intensity, so they should be used with care.

It has now been confirmed that cosmic background radiation emitted from the omnidirectional universe will be observed as perfect blackbody radiation with partial temperature distribution. Based on this, I will deal with the cosmic microwave background radiation.

1.8 Cosmic microwave background radiation

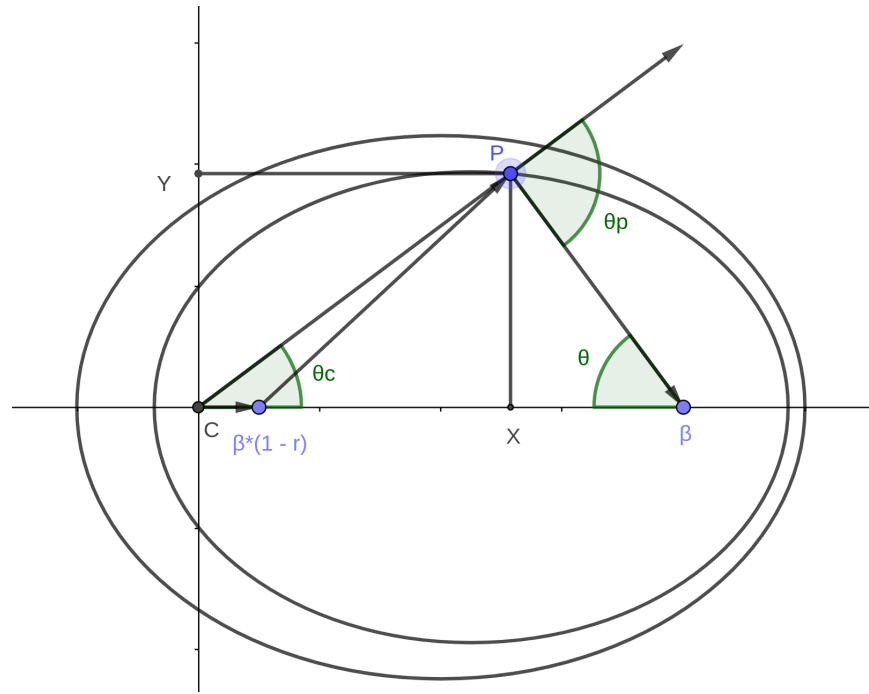


Figure 12: Cosmic microwave background radiation

The motion relative to the cosmic background radiation is described through the cosmic microwave background radiation diagram. The point β inertial system is a primordial inertial system and is moving away from the center C at a speed of β . In this case, if there is an observer located at β and stationary with respect to the observation center C , the observer can be seen as moving toward the center C at a speed of β when viewed from the primordial inertial system β at that position. Here, the cosmic microwave background radiation is light generated at point P on the observation ellipse $\frac{r}{2} \frac{1-\beta^2}{1-\beta \cos(\theta)}$ and traveling from P to β at an angle of $\theta_c + \theta = \theta_p$. Pre-enter the basic values as in Calculation 12, calculate the new additional values

$$\begin{aligned}
 (33) - & \quad R\theta := \frac{r}{2} \frac{1-\beta^2}{1-\beta \cos(\theta)}, \\
 & \quad Ts := 1 - r, \\
 & \quad Te := r - R\theta, \\
 & \quad x := \beta - R\theta \cos(\theta), \\
 & \quad y = R\theta \sin(\theta), \\
 & \quad y := R\theta \sqrt{1 - \cos(\theta)^2} \\
 & \quad \left[\frac{r\beta^2 - r}{2\beta \cos(\theta) - 2}, -r + 1, \frac{2r\beta \cos(\theta) - r\beta^2 - r}{2\beta \cos(\theta) - 2}, \frac{((-r+2)\beta^2 + r) \cos(\theta) - 2\beta}{2\beta \cos(\theta) - 2}, \right. \\
 & \quad \left. \frac{(r\beta^2 - r) \sqrt{-\cos(\theta)^2 + 1}}{2\beta \cos(\theta) - 2} = \frac{(r\beta^2 - r) \sin(\theta)}{2\beta \cos(\theta) - 2}, \frac{(r\beta^2 - r) \sqrt{-\cos(\theta)^2 + 1}}{2\beta \cos(\theta) - 2} \right] \\
 & \quad \text{Type : Tuple (Any)}
 \end{aligned}$$

, and get

$$\frac{1}{z+1} = \frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta_p)} = fr$$

, then

$$(34) \rightarrow \theta_c := \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right),$$

$$\theta_p := \theta_c + \theta,$$

$$\beta_p := \frac{\sqrt{x^2+y^2}}{Ts+Te}$$

$$\left[\arccos\left(\frac{((-r+2)\beta^2+r)\cos(\theta)-2\beta}{(2\beta\cos(\theta)-2)\sqrt{\frac{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2}{4\beta^2\cos(\theta)^2-8\beta\cos(\theta)+4}}}\right), \right.$$

$$\arccos\left(\frac{((-r+2)\beta^2+r)\cos(\theta)-2\beta}{(2\beta\cos(\theta)-2)\sqrt{\frac{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2}{4\beta^2\cos(\theta)^2-8\beta\cos(\theta)+4}}}\right) + \theta,$$

$$\left. \frac{(2\beta\cos(\theta)-2)\sqrt{\frac{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2}{4\beta^2\cos(\theta)^2-8\beta\cos(\theta)+4}}}{2\beta\cos(\theta)-r\beta^2+r-2} \right]$$

Type : Tuple (Expression (Integer))

$$(35) \rightarrow fr := \frac{\sqrt{1-\beta_p^2}}{1-\beta_p \cos(\theta_p)} ;$$

$$ar := (2\beta\cos(\theta)-2)\sqrt{\frac{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2}{4\beta^2\cos(\theta)^2-8\beta\cos(\theta)+4}};$$

$$fr - \frac{(-2\beta\cos(\theta)+r\beta^2-r+2)\sqrt{\frac{4(1-\beta\cos(\theta))^2(1-\beta^2)(1-r)}{(-2\beta\cos(\theta)+r\beta^2-r+2)^2}}}{ar \cos(\arccos(\frac{((-r+2)\beta^2+r)\cos(\theta)-2\beta}{ar})+\theta)-2\beta\cos(\theta)+r\beta^2-r+2} = 0$$

$$0 = 0$$

Type : Equation (Expression (Integer))

The original output display was blocked because it was too long, and the expression was manually abbreviated to check whether it is the same as the original expression. This expression will be dealt with as a numerical calculation for now.

Here, unlike the case in the cosmic microwave background radiation figure where a stationary observer observes blackbody radiation from a moving object at point P, I will calculate the Doppler shift of the light that a moving object receives based on the incoming blackbody radiation from all directions. Using the notation from Calculation 34, It can be expressed as follows,

$$\frac{1}{z+1} = \frac{1+\beta \cos(\theta_i)}{\sqrt{1-\beta^2}}$$

The numerator is a term according to the classical Doppler shift, and the denominator is a term according to the relativistic time delay of a moving observer. And since θ_i is the angle of incidence at the point of view of the stationary observer C, it must be changed to θ applied

with the velocity aberration of light when observed by a moving object.

For the relationship between θ_i and θ , refer to the Doppler beaming figure, but only the sign of the cosine term of the equation of the ellipse needs to be accurately determined, so if we assume an appropriately imaginary diagram to match the notation with the Calculation 34, it is

$$(36) - > \text{solve} \left(\frac{1-\beta^2}{1+\beta \cos(\theta_i)} \sqrt{1-\cos(\theta_i)^2} = \sqrt{1-\beta^2} \sqrt{1-\cos(\theta)^2}, \theta_i \right)$$

$$[\theta_i = \text{acos}(\frac{-\cos(\theta)+\beta}{\beta \cos(\theta)-1}), \theta_i = \text{acos}(\frac{-\cos(\theta)-\beta}{\beta \cos(\theta)+1})]$$

Type : List (Equation (Expression (Integer)))

In this case, since the equation on the left must be used, the Doppler shift observed from the point of view of a moving object considering the velocity aberration of light is

$$(37) - > \text{eval} \left(\frac{1+\beta \cos(\theta_i)}{\sqrt{1-\beta^2}}, \theta_i = \text{acos} \left(\frac{\cos(\theta)-\beta}{1-\beta \cos(\theta)} \right) \right)$$

$$\frac{\beta^2-1}{(\beta \cos(\theta)-1)\sqrt{-\beta^2+1}}$$

Type : Expression (Integer)

in summary,

$$\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}$$

The first equation in Calculation 35 represents the Doppler shift influenced by β_p , combining the Doppler shift due to the expansion of the universe and the Doppler shift resulting from the relative motion with respect to β . In contrast, the second equation in Calculation 37 considers only the Doppler shift observed by a moving object, expressed in terms of β . To offset the Doppler shift effect due to the expansion of the universe in the first equation, $\frac{1}{\sqrt{1-r}}$ is multiplied to retain only the terms related to relative motion, and then it will be compared with the second equation.

The significant digits of numerical calculations of friCAS can be set using the 'digits' function. The 'eval' function can perform multiple assignments at once, and if multiple relationships are assigned, they are entered as a list. The 'cons' function is a function that creates a list by adding $r = 0.1$ items to the first head of the given input list of pn , which is familiar to Lisp language users.

$$(38) - > \text{digits}(256),$$

$$pn := [\beta = 0.7, \theta = 0.3\pi]$$

$$[20, [\beta = 0.7, \theta =$$

0.9424777960 7693797153 8793014983 8508652591 5081981253 1746292483 3776923449
2188586269 9588410447 6026351203 9464442595 3984691994 1281533828 6517466951 7607822438

5443352350 8523081058 1556331667 8933868846 8647911458 9328643292 6997800338 3854269447
0136034949 5813605727 436946]]

Type : Tuple (Any)

$$(39) - > \begin{aligned} & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.1, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.2, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.5, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.75, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.9, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right) \end{aligned}$$

[- 0.1 E - 254, 0.0, 0.7 E - 256, - 0.4 E - 255, - 0.1 E - 255]

Type : Tuple (Expression (Float))

$$(40) - > pn[\beta = 0.5, \theta = 0.9\pi]$$

$[\beta = 0.5, \theta = 2.8274333882 \ 3081391461 \ 6379044951 \ 5525957774 \ 5245943759 \ 5238877450$
1330770347 6565758809 8765231342 8079053611 8393327786 1954075982 3844601485 9552400855
2823467315 6330057052 5569243174 4668995003 6801606540 5943734376 7985929878 0993401015
1562808341 0408104848 7440817182 31084]

Type : List (Equation (Polynomial (Float)))

$$(41) - > \begin{aligned} & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.1, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.2, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.5, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.75, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right), \\ & eval\left(\frac{fr}{\sqrt{1-r}}, cons(r = 0.9, pn)\right) - eval\left(\frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}, pn\right) \end{aligned}$$

[-0.1 E - 255, 0.3 E - 256, 0.0, 0.0, - 0.3 E - 256]

Type : Tuple (Expression (Float))

The first expression may appear complex in form, but it has been confirmed that the values of the two expressions are identical within a significant digit range of 255. This consistency holds even when a larger number of significant digits is considered. In other words, concerning the cosmic microwave background radiation, the observations align with what an observer moving relative to a stationary background would perceive, irrespective of the distance represented by r or $z+1$ where the background radiation originates.

In summary, all the results indicate that cosmic microwave background radiation can be considered as perfect blackbody radiation occurring at the edge of the universe. This conclusion holds regardless of the distance at which it occurs, the relative motion of the observer to the background radiation, or the motion of the radiation source.

Numerical calculations have confirmed the equivalence of the two expressions, but it's also possible to demonstrate their equivalence through manual simplification of expressions. While this process may be a bit laborious, I will perform it using friCAS to show the step-by-step simplification.

$$(42) \rightarrow f_{n1} := \frac{((-r+2)\beta^2 + r) \cos(\theta) - 2\beta}{(2\beta \cos(\theta) - 2) \sqrt{\frac{((-4r+4)\beta^4 + 4r\beta^2) \cos(\theta)^2 + ((4r-8)\beta^3 - 4r\beta) \cos(\theta) + r^2\beta^4 + (-2r^2+4)\beta^2 + r^2}{4\beta^2 \cos(\theta)^2 - 8\beta \cos(\theta) + 4}}},$$

$$f_{r2} := \text{eval}\left(\frac{fr}{\sqrt{1-r}}, \cos(\text{acos}(f_{n1}) + \theta)\right) = \left(\cos(\theta) f_{n1} - \sin(\theta) \sqrt{1 - (f_{n1})^2}\right);$$

Type : Tuple (Expression (Integer))

The complex expression here is fr.

$$\begin{aligned} & (-2*\beta*\cos(\theta)+r*\beta^2-r+2)*\text{sqrt}((((4*r-4)*\beta^4+(-4*r+4)*\beta^2)*\cos(\theta)^2+((-8*r+8)*\beta^3+(8*r-8)*\beta)* \\ & \cos(\theta)+(4*r-4)*\beta^2-4*r+4)/(4*\beta^2*\cos(\theta)^2+(-4*r*\beta^3+(4*r-8)*\beta)*\cos(\theta)+r^2*\beta^4+(-2*r^2+4*r)* \\ & \beta^2+r^2-4*r+4)))/((2*\beta*\cos(\theta)-2)*\text{sqrt}(((((-4*r+4)*\beta^4+4*r*\beta^2)*\cos(\theta)^2+((4*r-8)*\beta^3-4*r*\beta)* \\ & \cos(\theta)+r^2*\beta^4+(-2*r^2+4)*\beta^2+r^2)/(4*\beta^2*\cos(\theta)^2-8*\beta*\cos(\theta)+4))*\cos(\text{acos}(((((-r+2)*\beta^2+r)* \\ & \cos(\theta)-2*\beta)/((2*\beta*\cos(\theta)-2)*\text{sqrt}(((((-4*r+4)*\beta^4+4*r*\beta^2)*\cos(\theta)^2+((4*r-8)*\beta^3-4*r*\beta)*\cos(\theta) \\ & + \\ & r^2*\beta^4+(-2*r^2+4)*\beta^2+r^2)/(4*\beta^2*\cos(\theta)^2-8*\beta*\cos(\theta)+4))))+\theta)-2*\beta*\cos(\theta)+r*\beta^2-r+2) \end{aligned}$$

The expression is too complex to display directly, so I've provided it as a string. If you're following along with the calculations, you can view the friCAS output directly. To simplify this expression, I applied substitution rules to eliminate the arccos term using trigonometric identities,

$$\cos(\text{acos}\left(\frac{((-r+2)\beta^2 + r) \cos(\theta) - 2\beta}{(2\beta \cos(\theta) - 2) \sqrt{\frac{((-4r+4)\beta^4 + 4r\beta^2) \cos(\theta)^2 + ((4r-8)\beta^3 - 4r\beta) \cos(\theta) + r^2\beta^4 + (-2r^2+4)\beta^2 + r^2}{4\beta^2 \cos(\theta)^2 - 8\beta \cos(\theta) + 4}}}\right) + \theta)$$

I used trigonometric rules such as,

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

and

$$\sin y = \sqrt{1 - \cos^2 y}$$

to create substitution rules to eliminate the arccos term, and I saved the result in fr2.

$$(43) - > fn2 := \sqrt{\frac{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2}{4\beta^2\cos(\theta)^2-8\beta\cos(\theta)+4}},$$

$$fn3 := \frac{\sqrt{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2}}{(2-2\beta\cos(\theta))},$$

$$fr3 := eval(fr2, fn2 = fn3);$$

Type : Tuple (Expression (Integer))

Using the previous results as new inputs, I simplified the expression in fn2 to create fn3, and I saved this simplified expression in fr3. Since the denominator is always positive, this simplification is valid. The result is also too long to display directly.

$$(44) - > fn4 := \sqrt{\frac{(r^2\beta^4-2r^2\beta^2+r^2)\cos(\theta)^2-r^2\beta^4+2r^2\beta^2-r^2}{((4r-4)\beta^4-4r\beta^2)\cos(\theta)^2+((-4r+8)\beta^3+4r\beta)\cos(\theta)-r^2\beta^4+(2r^2-4)\beta^2-r^2}},$$

$$fn5 := \sqrt{((-4r+4)\beta^4+4r\beta^2)\cos(\theta)^2+((4r-8)\beta^3-4r\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4)\beta^2+r^2},$$

$$fn6 := \sqrt{-((r^2\beta^4-2r^2\beta^2+r^2)\cos(\theta)^2-r^2\beta^4+2r^2\beta^2-r^2)},$$

$$fr4 := eval(fr3, [fn4 = fn6, fn5 = 1]);$$

Type : Tuple (Expression (Integer))

When looking at fr3, it is noticeable that there are parts where fn4 and fn5 are multiplied in the denominator. Since these parts were not automatically simplified, I manually set fn5 to 1 and kept only the numerator of fn4 to eliminate that part.

$$(45) - > fn7 := \sqrt{(-r^2\beta^4+2r^2\beta^2-r^2)\cos(\theta)^2+r^2\beta^4-2r^2\beta^2+r^2},$$

$$fn8 := r(1-\beta^2)\sqrt{1-\cos(\theta)^2},$$

$$fn9 := \sqrt{1-\cos(\theta)^2},$$

$$fr5 := eval(fr4, [fn7 = fn8, \sin(\theta) = fn9])$$

$$\frac{[\sqrt{(-r^2\beta^4+2r^2\beta^2-r^2)\cos(\theta)^2+r^2\beta^4-2r^2\beta^2+r^2},$$

$$(-r\beta^2+r)\sqrt{-\cos(\theta)^2+1},$$

$$\sqrt{-\cos(\theta)^2+1},$$

$$(-2\beta\cos(\theta)+r\beta^2-r+2)\sqrt{\frac{((4r-4)\beta^4+(-4r+4)\beta^2)\cos(\theta)^2+((-8r+8)\beta^3+(8r-8)\beta)\cos(\theta)+(4r-4)\beta^2-4r+4}{4\beta^2\cos(\theta)^2+(-4r\beta^3+(4r-8)\beta)\cos(\theta)+r^2\beta^4+(-2r^2+4r)\beta^2+r^2-4r+4}}}]$$

Type : Tuple (Expression (Integer))

Since it's guaranteed to always be positive, fn7 can be simplified to fn8, and the remaining sine functions have been expressed in terms of cosine functions. From this point on, the results are simplified enough to be displayed, so they are displayed.

$$\begin{aligned}
(46) \rightarrow \quad & fn10 := \sqrt{\frac{((4r-4)\beta^4 + (-4r+4)\beta^2) \cos(\theta)^2 + ((-8r+8)\beta^3 + (8r-8)\beta) \cos(\theta) + (4r-4)\beta^2 - 4r + 4}{4\beta^2 \cos(\theta)^2 + (-4r\beta^3 + (4r-8)\beta) \cos(\theta) + r^2\beta^4 + (-2r^2+4r)\beta^2 + r^2 - 4r + 4}}, \\
& fn11 := \frac{2(1-\beta \cos(\theta))\sqrt{1-\beta^2}\sqrt{1-r}}{(-2\beta \cos(\theta) + r\beta^2 - r + 2)}, \\
& fr6 := eval(fr5, fn10 = fn11) \\
& \left[\sqrt{\frac{((4r-4)\beta^4 + (-4r+4)\beta^2) \cos(\theta)^2 + ((-8r+8)\beta^3 + (8r-8)\beta) \cos(\theta) + (4r-4)\beta^2 - 4r + 4}{4\beta^2 \cos(\theta)^2 + (-4r\beta^3 + (4r-8)\beta) \cos(\theta) + r^2\beta^4 + (-2r^2+4r)\beta^2 + r^2 - 4r + 4}}, \frac{(2\beta \cos(\theta) - 2)\sqrt{-\beta^2+1}\sqrt{-r+1}}{2\beta \cos(\theta) - r\beta^2 + r - 2}, \right. \\
& \left. - \frac{\sqrt{-\beta^2+1}}{\beta \cos(\theta) - 1} \right] \\
& \text{Type : Tuple (Expression (Integer))}
\end{aligned}$$

The value of fn10 is also guaranteed to always be positive, so it can be simplified to fn11. Applying this, we can confirm that $\frac{fr}{\sqrt{1-r}} = \frac{\sqrt{1-\beta^2}}{1-\beta \cos(\theta)}$. This may be solvable through a simpler process in other CAS systems, and friCAS may also improve in the future. Here, I introduced the process of simplifying equations through a combination of manual and CAS-based methods when automatic simplification is not straightforward. CAS can be a significant aid in performing complex and tedious calculations accurately, even when equations don't simplify automatically.

1.9 Total number of galaxies in the universe

It is possible to estimate the total number of galaxies in the universe to some extent. Since the number of galaxies in a region of the universe shall be proportional to the number of particles in that region, integrating the particle density function and using the known number of galaxies within a certain distance and the distance to the farthest known primordial galaxy, it can be estimated in some extent.

Since the particle density function in Calculation 22 is expressed in polar coordinates, it can be directly integrated without additionally calculating and integrating the area according to the distance. Integration in friCAS uses the 'integrate' function. The expression to be integrated, the variable to be integrated, and, if necessary, the integration interval is displayed using '.', the two ',', and it might be entered as three in the formula input mode of TeXmacs, but there is no problem. The notation "noPole" indicates that the integral interval does not include any poles of the function. Sometimes, integration fails with a "potential pole" message. If we are confident that there are no poles within the integration interval, entering "noPole" can resolve the issue. In this case, $\frac{1}{8} \left(\frac{r}{1-r} \right)^2$ is used. This has a pole at $r=1$, However, since there is no need to integrate up to that point, it can still be used.

$$(47) \rightarrow digits(50)$$

256

Type : PositiveInteger

$$(48) - > \text{integrate}\left(\frac{1}{8} \left(\frac{r}{1-r}\right)^2, r = 0..r, \text{"noPole"}\right)$$

$$\frac{(r-1) \log(r^2-2r+1)+r^2-2r}{8r-8}$$

Type : Union (f 1 : OrderedCompletion (Expression (Integer)), ...)

$$\frac{(r-1) \log(r^2-2r+1)+r^2-2r}{8r-8} = \frac{1}{4} \log(1-r) + \frac{r}{8} \frac{2-r}{1-r}$$

The result of this integral will be prepared as a function named TG.

$$(49) - > TG(r) == \frac{(r-1) \log(r^2-2r+1)+r^2-2r}{8r-8}$$

Type : Void

Since polar coordinates were used from the outset, there is no need to consider the increasing surface area with distance when calculating the integral. Just enter the following information, the number of known galaxies that exist within 12 million light-years is 152, the age of the known universe is 13.8 billion years, and as of prior to 2021, the redshift of the farthest known galaxy is $z=11$.

Since TG is a function of r , I will use $r = \frac{zn^2-1}{zn^2}$ of Calculation 24 and the definition in Figure 43 The observed universe. Although it is not exact, by dividing the total number of particles up to the distance where galaxies begin to appear by the total number of particles up to the distance where the number of galaxies is known, it is possible to estimate the total number of galaxies of the universe. Since the TG function is the integration of the relative density, which is 1 based on the observer, not the actual total number of particles. This is a calculation to find the number of galaxies up to a far distance by calculating the ratio of the total number of particles between two distances and multiplying the ratio by the number of known galaxies within a short distance.

When r equals 1, it corresponds to the outermost. This distance corresponds to half the age of the universe, which is actually 13.8/2 billion light-years. Therefore, substitute $2\frac{0.12}{138}$ for r in the denominator of TG, and substitute $\frac{12^2-1}{12^2}$, which is the value of r when $zn=12$, for r in the numerator of TG. The result is,

$$(50) - > 152 \frac{TG\left(\frac{12^2-1}{12^2}\right)}{TG\left(2\frac{0.12}{138}\right)}$$

1159_0774970220.4039828181_3924816361_3966209381_533438

Type : Expression (Float)

It is estimated that there are about 12 trillion galaxies in the entire universe. In reality, the

z at which galaxies begin to actively form will be a little smaller and the age of the universe will be a bit older. If substitute the more plausible values,

$$(51) - > 152 \frac{TG\left(\frac{7.5^2-1}{7.5^2}\right)}{TG\left(2^{\frac{0.12}{145}}\right)}$$

483_2326279935.1814775736_4809891602_9694219114_6098787

Type : Expression (Float)

It's approximately 5 trillion galaxies, which is about 2-3 times larger than the latest estimate known from observations in 2016 (arXiv:1607.03909). This rough estimate seems to align reasonably well, but for a more accurate value, one would need to refine the assumptions made in this calculation, such as when galaxy formation started and ended in terms of the age of the universe. Additionally, a more precise calculation would require consideration of variables like our knowledge of galaxy mergers and whether our location in the universe is in a region with a relatively rich or sparse distribution of galaxies. Applying weights based on different galaxy formation models to the original density function and performing integrations would also be necessary. The result of this calculation is a very rough estimate.

$$(52) - > 152 \frac{TG\left(\frac{6^2-1}{6^2}\right)}{TG\left(2^{\frac{0.12}{138}}\right)}$$

249_0607008872.4504104383_5344625355_4488173356_316798

Type : Expression (Float)

The lower limit estimated in this calculation appears to be similar to the 2016 observational/estimated value. This suggests that more galaxies are expected to be discovered.

As of spring 2023, the farthest galaxy known is JADES-GS-z13-0 with a redshift of $z=13.2$. After the James Webb Space Telescope started operating, there were rumors about many high z -value galaxies being discovered (up to $z=16-20$). However, as of February 2023, the officially reported record is $z=13.2$ (arXiv:2212.04480). If the average position of galaxy formation exceeds $z = 10$, the total number of galaxies according to the above formula will exceed 10 trillion. When $z_n = 14.2$ is substituted into the above formula, it is calculated that approximately 16.5 trillion galaxies can be observed. However, I don't think there will be that many galaxies. The biggest reason is that galaxies are not evenly distributed in this universe, and matter and galaxies exist densely packed in a mesh-like structure in some areas. Therefore, there is a high probability that our own Milky Way galaxy is located in an area with a much higher density of galaxies than the average density of the entire universe. However, on the other hand, if the age of the universe, when galaxies begin to actively form, is a bit earlier, the number of galaxies increases dramatically. In the end, the estimate for the total number of galaxies in the universe is expected to be slightly over 5 trillion based on observed trends and estimates of galaxy formation, although there may be significant variations depending on the accuracy

of the data. I am concluding my estimate at this level. More accurate estimates with more sophisticated models should be left to the realm of astronomy.

1.10 Temperature changes in the universe over time

The temperature distribution over time in the cosmology of constant velocity expansion based on special relativity is obtained by simple logic. The space of the universe is not treated as a medium such as air, water, or ether. It is sufficient to consider only the thermal equilibrium conditions of the universe.

The age of the universe at a fixed distance background r observed in a certain inertial system is

$$t = \sqrt{1 - r} = time_{there}$$

, and the corresponding observed redshift is

$$zn = \frac{1}{\sqrt{1 - r}}$$

. That is, it has the form of $1/t$. In this case, the condition for the temperature of the surrounding universe of the observer to reach thermal equilibrium with the past universe that has the observed redshift is

$$Temp_{here} = Temp_{there} \frac{Time_{there}}{Time_{here}}$$

, or according to the expression of 'The Dirac-Milne cosmology' currently advocated by a group of French scientists, it is

$$t = \frac{1}{H_0} \frac{T_0}{T}$$

(T_0 : temperature of the current universe, t : age of the universe, H_0 : Hubble constant, T : temperature of the universe at that age),

if the temperature change over time of the universe follows this function, then observers will see that any region they observe, regardless of the direction and distance, is in thermal equilibrium with them. They will also observe that the temperature of every part of the universe, including the background, is in thermal equilibrium. Since this holds regardless of location and direction, any observer in any primordial inertial system can see that the universe is uniform and isotropic.

This applies even if the universe is opaque to radiation due to either being hotter than

3000°K the temperature at which hydrogen atoms become ionized and the universe becomes opaque to visible light, or due to the high density of other opaque substances. That is, even if the observable radius of the celestial sphere is a small distance, it is possible to ensure thermal equilibrium throughout the entire universe if only this condition holds within that range. In other words, as long as the temperature distribution of the universe follows a distribution of $1/t$, regardless of the size of the visible region, this satisfies the condition of thermal equilibrium.

This explanation alone is sufficient for describing the temperature transitions in the universe, but some may want a more microscopic explanation. To do so, locally, when the tiny regions of each primordial inertial system of the universe expand, the high-temperature particles leave the region more quickly, leaving only the low-temperature particles behind and thus lowering the temperature of the remaining part. On the other hand, particles that enter from outside enter with energy loss due to the speed difference between the inertial systems. Therefore, the temperature of all regions decreases due to the expansion. For the size of each tiny region, it can be defined as small as possible so that only one particle can exist on average in that region. Therefore, the speed at which the temperature decreases in this tiny region is simply the speed at which all temperatures are lost instantly since the particles that were in the region have all gone elsewhere. However, particles from the surroundings fly in at the same rate and define a new temperature in the region. The temperature is lowered by a Doppler redshift proportional to the relative speed between the inertial systems. Therefore, the decreasing rate follows the equilibrium condition of $1/t$ according to the redshift in real time.

No other strange assumptions are needed here. However, temperature calculations based on general relativity are different.

The formula for calculating the temperature by the expansion time of the Big Bang claimed by general relativity-based cosmology is as follows. (Source 2020.10.2 <http://hyperphysics.phy-astr.gsu.edu/hbase/Astro/expand.html#c3>)

$$t_{exp} = \frac{1}{H} = \sqrt{\frac{3C^2}{8\pi G\rho_0}}$$

When $T \gg 3000^\circ\text{K}$, most of the energy density is in the form of radiation energy and is said to follow the next equation.

$$\rho_{radiation}(T) \approx (0.4\text{MeV}/m^3) \left[\frac{T}{2.7\text{K}} \right]^4$$

The final equation based on this is as follows.

$$t_{exp} = \left[\frac{2.7K}{T} \right]^2 \sqrt{\frac{3C^2}{8\pi G(0.4MeV/m^3)}}$$

If $T \ll 3000^\circ K$, the contribution of radiation energy to the energy density is negligible.

$$\rho_{mass} = (0.5GeV/m^3) \left[\frac{T}{2.7K} \right]^3$$

Therefore, the expansion time calculation formula is as follows.

$$t_{exp} = \left[\frac{2.7K}{T} \right]^{3/2} \sqrt{\frac{3C^2}{8\pi G(0.5GeV/m^3)}}$$

(t_{exp} : time elapsed during expansion, T : temperature, k : Boltzmann constant, c : speed of light, G : gravitational constant)

This formula demonstrates that the temperature of an observable universe below $3000^\circ K$ is proportional to the power of $-3/2$ of time. However, it is incomprehensible what this actually means. They claim that a vacuum undergoes a phase transition, but there is no evidence to support this. Even at the temperature that humans have easily produced on Earth for thousands of years, which is only $3000^\circ K$, they discuss the phase transition of space without any experimental or physical evidence to support their claims, and merely make their argument based on their own needs.

This implies the conclusion that the observed redshifted temperature of the cold, uncondensed interstellar gas in the observable universe must be different from the average temperature of the universe. If so, even if the phase transition cannot be proven, it is a theory that should be asserted after at least observing that. It will be impossible to explain with dark energy. In order to tune the redshifted temperature of interstellar gas with the average temperature of the universe, another dark angel should have to appear again. I don't believe that such ad hoc explanations are even remotely true.

The fundamental difference between the two ideas is that general relativity-based cosmology treats the thermodynamics of the universe as a closed system and special relativity-based cosmology treats it as an open system even though it is a finite universe. Is our universe, in which infinite matter and energy are irreversibly expanding, a thermally open universe or a closed universe? If we take what is observed as the truth, it is either an open universe or a universe indistinguishable from the open universe. Cosmology based on general relativity is nothing more than the theory that denies what is observed and imagines that there must be some hidden closure.

1.11 The cosmological principle and constant velocity expansion

A basic assumption of general relativity-based cosmology is that gravity will affect the expansion of the universe. However, this is not the case in special relativity. If the universe is isotropic, there is no bias of forces in any particular direction, so there can be no acceleration or deceleration due to such a bias.

It's not just can't feel it, even though actually accelerating or decelerating. In addition to gravity, there is another force in the universe that is actually acting between extremely distant stars, and that is the force exchanged through the pressure of light between stars. Although it is even weaker than the weak gravity, it is a force transmitted at the speed of light, a force that must be acting between stars, and it is also acting between the Sun and Earth. The same is true for the momentum transferred by neutrinos and, although slower than the speed of light, high-energy particles are also transferring some momentum. These forces, unlike gravity, can be felt by the object receiving the force and can be used to test this question. Considering whether these forces can affect the expansion of the universe, it can be seen that they cannot because they are transmitted in the same amount in all directions according to the isotropic principle of the universe. There is no reason to believe that the situation is different for gravity, another force transmitted at the speed of light.

Furthermore, apart from the discussion of forces, in this cosmology, the outermost shell of the universe possesses infinite mass and is expanding at the speed of light. It's impossible to either slow down or accelerate this expansion. Consequently, we must acknowledge that the outermost shell is expanding at a constant speed. When examining this issue, it raises questions like, "Then, can the interior accelerate or decelerate?" or "When only the outermost part of the universe is expanding at a constant speed, can the density distribution inside the universe be rearranged over time due to gravity?" The answer is "no," unless we abandon the uniformity of the universe. This is fundamentally a matter of geometry, akin to Galileo's principle of relativity. Imagine a set of points that all start at the same moment from a single point and then explode and scatter in arbitrary directions at arbitrary speeds. Regardless of which point you choose, as long as all these points are expanding at constant velocities, the observation will appear as if other points are moving away from that chosen point. No angular velocity can be measured for these moving points. However, if the expansion velocity varies due to gravitational forces centered on a point, other points will detect this through the generation of angular velocities at certain observed points. This would violate the uniformity of the universe.

However, it is possible to question whether the overall change in energy within the universe shell's interior, as a result of the expansion, might eventually influence the rate of time flow, and consequently, the overall expansion rate. While this is a matter that should be con-

sidered alongside a more rigorous theory of gravity, qualitative speculation can be offered. The idea is that any changes in positional energy would not only affect the expansion rate but also the speed of light as time progresses. Therefore, the expansion rate, relative to the speed of light, remains constant. In other words, it would appear as a uniform expansion to an observer inside. This provides a potential excuse, although a more thorough investigation would be required in conjunction with a more precise gravitational theory.

The question of whether this change in potential energy affects the cosmic microwave background radiation or the spectroscopic properties of distant stars is difficult to answer. It is that there is no effect, or if there is, it is too small to be observed, which is the result of observations of the cosmic microwave background radiation to date. It has already been observed that if it does change, the change must be negligible such that by the time of the cosmic microwave background radiation, there will be very little observable difference from today.

This special relativity-based cosmology describes a universe with a finite size but at the same time infinite space and the idea that the rate of expansion of the universe can be affected by gravity is only valid if we assume that the universe is finite after all. The cosmologies based on general relativity cannot adequately describe the infinite aspects of the universe, and I think that only this cosmology based on special relativity can provide a complete depiction of the deeper aspects of the universe.

I conclude that gravity cannot affect the speed of expansion of the universe, and leave further discussion of the remaining issues, such as spectroscopic properties, to the next generation of theories.

1.12 The cosmological principle and relativity

From the point of view of special relativity, right now, here is the oldest part of the universe, with the fastest flow of time.

Other regions of the observable universe are always new regions younger than this one. If so, let's see what happens when we go there ourselves. For example, let's consider what it would look like if we flew at the speed of light to a distant place about $z+1=100$ and looked back at the earth.

Logically, the answer is straightforward. The earth seen from there would be $z+1=100$, so the flow of time would appear to be $1/100$. Therefore, the age of the universe felt in there should be 1.4 trillion years, which is 100 times greater than the current age of about 14 billion years felt by the earth here, and the earth still would be seen to be in the area of 14 billion

years old.

This is a conjecture based on the principle of relativity. Let's see if it holds true in actual calculations.

First, we need to find the value of β for the region running away at speed $z+1=100$. We can use the expression for $\beta_p = \frac{zn^2-1}{zn^2+1}$ from earlier. We can see that it is simply $\frac{9999}{10001}$.

This region is currently $\frac{9999}{10001}14 = \frac{139986}{10001}$ billion light years away from Earth. Therefore, the time it takes to catch up with this region at the speed of light is $vt+l = ct \rightarrow t = \frac{l}{c-v}$, so $\frac{\frac{139986}{10001}}{1 - \frac{9999}{10001}} = 69993$ billion years. The time that will pass in that area during that time is $69993\sqrt{1 - \left(\frac{9999}{10001}\right)^2}$. And, the current age of that area is $14\sqrt{1 - \left(\frac{9999}{10001}\right)^2}$.

Therefore, the age of the region when pursued at the speed of light is

$$(69993 + 14)\sqrt{1 - \left(\frac{9999}{10001}\right)^2} = 1400$$

billion years, which is exactly 100 times 14 billion years. Of course, at that point, the age of this place seen from there is 14 billion years.

Thus, we can see that the relativity between the primordial inertial systems of the universe is maintained.

Among the cosmological theories proposed so far, there is no other theory that mathematically explains the uniformity and isotropy of the universe. While theories based on general relativity indirectly address the curvature of the universe, the curvature has been negated in current observations. Therefore, as of now, the theory I am presenting is the only mathematical explanation for the uniformity of the universe. It is not difficult to speculate on which theory, among those that can provide such an explanation and those that cannot, might be closer to the truth.

1.13 Some issues and criticisms

1.13.1 The issue of accelerated expansion and dark energy

Cosmology based on special relativity is a theory of constant velocity expansion, but it has an answer to the question of the accelerating expansion of the universe or dark energy, which has become a major issue recently.