Mercury's Perihelion Advancement Due to Special Relativistic Effects

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Abstract

I discovered that the cause of Mercury's perihelion advance could be explained by the sum of Maxwellian gravity and several special relativity factors. The simulation results, taking into account all these factors, were demonstrated to align precisely with the outcomes predicted by the traditional Gerber-Einstein formula.

1 Introduction

The perihelion advance of Mercury's orbit in the direction of its rotation, often cited as one of the most prominent proofs of general relativity, was a fairly famous riddle of classical celestial mechanics. Since it was first posed by French astrophysicist Le Verrier in 1859, it remained unsolved for decades as a mystery that has spawned numerous hypotheses.

The first person to discover the correct formula was German physicist Paul Gerber. In 1902, he published the results $\Psi = \frac{6\,\pi\,\mu}{a\,(1-e^2)\,c^2}$, $\mu = \frac{4\,\pi^2\,a^3}{\tau^2}$, and $\therefore \Psi = \frac{24\,\pi^3\,a^2}{\tau^2\,c^2\,(1-e^2)}$, which accurately predicted the perihelion shift of Mercury's orbit for the first time, based on his study of retarded fields, which were one of the main concerns of the physics community at the time, even before the advent of special relativity. However, this result, for some reason, did not gain much fame and remained relatively unknown, and in 1915, Einstein's general theory of relativity, published with equation $\epsilon = \frac{24\,\pi^3\,a^2}{T^2\,c^2\,(1-e^2)}$, was mistakenly attributed as the first theory to describe the perihelion shift of Mercury's orbit.

The two expressions are so similar that around 1920, Einstein had to leave an article arguing that he did not plagiarize, but discovered them independently. "Mr. Gerber's work is therefore completely useless, an unsuccessful and erroneous theoretical attempt. I maintain that the theory of general relativity has provided the first real explanation of the perihelion motion of Mercury. I did not mention the work by Gerber initially, because I did not know about it when I wrote my work on the perihelion motion of Mercury; even if I had been aware of it, I would not have had any reason to mention it." A very defensive Einstein can be seen here.

2 Simulation

The way I found the result was through trial and error and chance, so there will be no need to introduce the detailed process. Instead, I will only explain the results. I discovered the results through simulation and intend to leave the mathematical interpretation to mathematicians. The simulation itself was a very basic simulation, keeping the sun's position fixed and only simulating the planet's orbit. Based on the $\vec{r}_{\text{next}} = \vec{r}_{\text{now}} + \vec{v}\,t + \frac{1}{2}\,\vec{a}\,t^2$ formula, the following modifications were made only to the acceleration term.

$\vec{a}_0 = -\frac{G\,M}{r^2}\,\hat{r}$, $\vec{a}_1 = \vec{a}_0\left(1 + \frac{G\,M}{r\,c^2}\right)^3$, $\vec{a}_2 = \vec{a}_1 + \frac{1}{2}\,\frac{\vec{v}}{c} \times \vec{a}_1 \times \frac{\vec{v}}{c} - \frac{\vec{v}}{c^2}\,(\vec{v} \cdot \vec{a}_1)$, and $\vec{a} = \frac{\vec{a}_2}{\gamma}$

The factors applied are as follows. The initial consideration involves the phenomenon where the mass of the sun seems to increase as one gets closer. When an object of a certain mass in free space enters a gravitational field, it acquires kinetic energy from the gravitational field, causing its relativistic inertial mass to increase accordingly. However, there should be no change in mass when observed from outside the gravitational field. Subsequently, when the object collides with the gravitational source and emits light, the same amount of energy gained from gravity is emitted as light. Consequently, the mass of the object appears reduced by an amount equivalent to that energy when observed from outside the gravitational field. However, upon entering the gravitational field and measuring the object up close, it must be assessed as its original mass. In essence, within a gravitational field, there exists an effect that makes the mass of the source of gravity appear larger than when observed from a distance. Various formulas can be considered for how much it will be, but I selected the simplest form,

$$m_r = m_\infty \left(1 + \frac{G\,m_\infty}{r\,c^2}\right)$$

which is approximately certain under the weak gravity around the sun. This factor is responsible for 1/6 of the total orbital rotation.

The second factor involves the phenomenon of time dilation within a gravitational field, which also becomes a contributing factor to perihelion rotation. Using the formula for acceleration transformation between inertial frames, $\vec{a}' = \gamma^2\,((\gamma - 1)\,(\vec{a}\cdot\hat{v})\hat{v} + \vec{a})$, it can be affirmed that on the side where time passes slowly, the acceleration appears significantly larger on the side where time passes quickly, in proportion to the square of the rate of time passage. This is due to the observation, from the perspective of slower time passage, of faster changes in velocity over shorter periods of time. If this phenomenon also applies to time dilation caused by gravity, then as one gets closer to a gravitational source, there will be a perception that gravity becomes stronger than the inverse-square law would predict. Consequently, this contributes to the cause of orbital rotation. When observed from outside the gravitational field, as an object approaches a gravitational source, there's a slowing of time. If the effect of gravity follows the inverse-square law just as it does outside, then from the object's point of view, gravity would appear to strengthen more than the inverse-square law predicts as it approaches the gravitational source. The formula can be applied similarly to the formula used for the effect of increasing the gravity field due to the increase in mass.

$$m_r = m_\infty \left(1 + \frac{G\,m_\infty}{r\,c^2}\right)^2$$

This factor is responsible for 1/3 of the total orbital rotation. Therefore, $\vec{a}_1 = \vec{a}_0\left(1 + \frac{G\,M}{r\,c^2}\right)^3$ accounts for half of the total orbital rotation.

The third factor is the term due to the Wigner rotation expressed as $\frac{1}{c^2}\frac{\gamma}{\gamma+1}\vec{v}\times\vec{a}\times\vec{v}$ and was applied as $+\frac{1}{2}\frac{\vec{v}}{c}\times\vec{a}_1\times\frac{\vec{v}}{c}$. This term is responsible for 1/3 of the total orbital rotation.

The fourth factor is the well-known induced acceleration term expressed as $-\frac{\vec{v}}{c^2}\,(\vec{v}\cdot\vec{a}_1)$'. This term arises due to the following special relativistic effects.

$$\begin{aligned}
\frac{d\vec{p}}{dt} &= \gamma^3\,m\,\frac{\vec{a}\cdot\vec{v}}{c^2}\,\vec{v} + \gamma\,m\,\vec{a} \\
&= q\,(\vec{E} + \vec{v}\times\vec{B}) \\
\vec{a} &= \frac{q}{\gamma m}\left(\vec{E} + \vec{v}\times\vec{B} - \frac{1}{c^2}\,\vec{v}\,(\vec{v}\cdot\vec{E})\right)
\end{aligned}$$

This term is responsible for 1/3 of the total orbital rotation. The sum of factors so far exceeds the required rotation amount by 1/6. Therefore, the following reverse factors are required.

The final factor is the abandonment of the equivalence principle. The force acting on the acceleration of an object should be viewed as determined by a constant rest mass, such as a constant charge in electromagnetic force, rather than a relativistically corrected inertial mass. This is the effect of dividing the final acceleration by γ, that is $\vec{a} = \frac{\vec{a}_2}{\gamma}$. This term is responsible for -1/6 of the total orbital rotation.

Interestingly, all factors accounted for an integer fraction of the actual rotation amount. This appears to be the cause of many misinterpretations so far.

The outcome of the simulation, which combines these effects, is 5.018808203222051e-7 radians per rotation based on 66,296 seconds of simulation on my computer using actual Mercury data. The result from the Gerber-Einstein formula is 5.018881066308666e-7 radians, confirming a very close match between the two results.

3 Conclusion

This result shows that Mercury's perihelion movement is completely explained by Maxwellian gravity, which applies the principles of special relativity and electromagnetism to gravity, without the need for a completely different theory. And, by predicting that gravitational mass and inertial mass do not coincide, it directly denies the assumption of the equivalence principle of general relativity. Therefore, it can be confirmed as truth without big difficulty. As long as special relativity is a fundamental truth that is more strictly defined than general relativity, there is no other possibility.

Bibliography

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