However, since I found an unexpected problem in the process, I have to mention that part first. Maxwell's equations and the equations of the electromagnetic field are well-known equations that are relativistically consistent. However, there was an unresolved problem in the case of such an electromagnetic field, especially in the case of the force applied to a certain charge by the field. And, prior to the advent of Feynman's formula in the 1950s, such problems could not be addressed precisely. Nevertheless, it was a little imprudent to conclude that the equations of the electromagnetic field were relativistically consistent by looking only at Maxwell's equations. As a result, it was not wrong, but it was enough to make students mistake a problem that was not specifically solved as if it had been solved. I'm not sure who was actually the first to address such a problem, but the first generally available description is found in a section of Purcell's book "Electricity and Magnetism," published in the 1950s. It is also indirectly covered in Griffiths' "Introduction to Electrodynamics" book published in the 1990s.

However, they only dealt with the problem of charges running in parallel and did not mention the general cases. I cannot fathom the reason why this unfinished problem was not taken seriously, but fortunately, I was able to solve this problem immediately, I suppose it was due to my approach being fortuitously right. Therefore, I completed a missing brick of the theory of relativity and electromagnetism.

4.1 The force between parallel moving charges

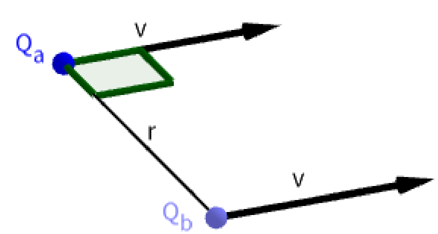

Figure 38: Two charges moving in parallel

First, I will look at the force between two charges running side by side with the same velocity v, separated by r perpendicular to the direction of travel, and the resulting motion. I will assume that the charge Qa is fixed, the motion is unaffected by the electromagnetic field, and that Qb has a mass m_0.

Earlier, Purcell's formula $\vec{E}_p = \frac{q}{4\pi\varepsilon_0 r_p^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\hat{r}_p$, which expresses the electric field due to charges moving at constant velocity, was introduced. Since this case is $\theta = \frac{\pi}{2} \to \sin\theta = 1$, when the electric field of Qa at rest is E_0, it can be seen that it is

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\frac{1-\beta^2}{(1-\beta^2)^{3/2}}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\frac{1}{\sqrt{1-\beta^2}}\hat{r} = \frac{\gamma q\hat{r}}{4\pi\varepsilon_0 r^2} = \gamma\vec{E}_0$$

At this point, when the rest mass of Qb is m_0 and the inertial mass is $m = \gamma m_0$, the acceleration will be looked to be $\vec{a} = \gamma\frac{\vec{E}_0}{m} = \frac{\vec{E}_0}{m_0}$. Let's consider the case of observing the movement of Qb in an inertial frame moving with Qa and Qb again. The electric field is E_0, and the rest mass of Qb is m_0. Therefore, the acceleration felt by Qb is $\frac{\vec{E}_0}{m_0} = \vec{a}$, which is calculated the same as the acceleration measured in the stationary inertial system above. However, this contradicts relativity. When viewed from a stationary inertial system, time seems to flow slowly at a rate of 1/γ in an inertial system moving at v speed, which means that in the case of acceleration, a change in speed by 1/γ occurs after γ times of the time has passed. Therefore, the acceleration should be observed as small as γ squared times. Nevertheless, the fact that it counts as the same amount means that for some reason it was calculated as larger as the γ-squared. This relationship can also be confirmed through the $\vec{a}' = \gamma^2((\gamma-1)(\vec{a}\cdot\hat{v})\hat{v} + \vec{a})$ equation used to derive the Larmor formula before. Exactly, it is the case in the opposite direction of the equation. If obtaining the opposite direction equation, from the previously obtained relativistic sum equation of velocities

$$\vec{u} = \frac{1}{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}\left(\frac{\vec{u}'}{\gamma} + \vec{v} + \frac{\gamma-1}{\gamma}(\vec{u}'\cdot\hat{v})\hat{v}\right)$$

and

$$\frac{dt'}{dt} = \frac{d}{dt}\left(\gamma t - \gamma\beta\frac{x}{c}\right) = \gamma\frac{dt}{dt} - \frac{1}{c^2}\gamma v\frac{dx}{dt} = \gamma - \gamma\frac{vv}{c^2} = \frac{1}{\gamma}$$

, it is

$$\begin{aligned}
\vec{u} &= \tfrac{1}{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}\left(\tfrac{\vec{u}'}{\gamma} + \vec{v} + \tfrac{\gamma-1}{\gamma}(\vec{u}'\cdot\hat{v})\hat{v}\right)\\
\vec{v}+d\vec{v} &= \tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\tfrac{d\vec{v}'}{\gamma} + \vec{v} + \tfrac{\gamma-1}{\gamma}(d\vec{v}'\cdot\hat{v})\hat{v}\right)\\
d\vec{v} &= \tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\tfrac{d\vec{v}'}{\gamma} + \vec{v} + \tfrac{\gamma-1}{\gamma}(d\vec{v}'\cdot\hat{v})\hat{v} - \vec{v} - \tfrac{\vec{v}\cdot d\vec{v}'}{c^2}\vec{v}\right)\\
&= \tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\tfrac{d\vec{v}'}{\gamma} + \left(\tfrac{\gamma-1}{\gamma} - \tfrac{v^2}{c^2}\right)(d\vec{v}'\cdot\hat{v})\hat{v}\right)\\
&= \tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\tfrac{d\vec{v}'}{\gamma} + \left(\tfrac{\gamma-1}{\gamma} + \tfrac{1}{\gamma^2} - 1\right)(d\vec{v}'\cdot\hat{v})\hat{v}\right)\\
&= \tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\tfrac{d\vec{v}'}{\gamma} + \tfrac{\gamma^2-\gamma+1-\gamma^2}{\gamma^2}(d\vec{v}'\cdot\hat{v})\hat{v}\right)\\
&= \tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\tfrac{d\vec{v}'}{\gamma} + \tfrac{1-\gamma}{\gamma^2}(d\vec{v}'\cdot\hat{v})\hat{v}\right)\\
&= \tfrac{1}{\gamma}\tfrac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(d\vec{v}' + \tfrac{1-\gamma}{\gamma}(d\vec{v}'\cdot\hat{v})\hat{v}\right)
\end{aligned}$$

, so when dv’ is infinitesimal, it becomes

$$\begin{aligned}
\frac{d\vec{v}}{dt} &= \frac{1}{\gamma}\frac{1}{1+\frac{\vec{v}\cdot d\vec{v}'}{c^2}}\left(\frac{d\vec{v}'}{dt'}\frac{dt'}{dt}+\frac{1-\gamma}{\gamma}\left(\frac{d\vec{v}'}{dt'}\frac{dt'}{dt}\cdot\hat{v}\right)\hat{v}\right)\\
\vec{a} &= \frac{1}{\gamma}\left(\frac{\vec{a}'}{\gamma}+\frac{1-\gamma}{\gamma}\left(\frac{\vec{a}'}{\gamma}\cdot\hat{v}\right)\hat{v}\right)\\
&= \frac{1}{\gamma^2}\left(\vec{a}'+\frac{1-\gamma}{\gamma}(\vec{a}'\cdot\hat{v})\hat{v}\right)
\end{aligned}$$

. Through this specific acceleration conversion equation, the relationship between a and a’ that is observed in different inertial frames can be confirmed. However, there is a caveat when using these relativistic acceleration conversion formulas. These formulas are conversion equations between an unmarked stationary inertial system that observes the object moving as a velocity v when it is moving with a velocity v and a primed inertial frame in which the object appears to be stationary. In other cases, physically correct results are not produced. In fact, different values are obtained depending on the transformation path, which is physically contradictory. Therefore, it can be seen that the entire formula cannot be used or it must be used within the above limitations, and when used within the limitations, a physically consistent result is obtained. Of course, it can be applied to deal with more general cases, and the method will be introduced later, but when dealing with relativity problems, we should always be careful which observer’s position we are in. If a theory of relativity cannot specify a specific observer’s position, it is likely a theory that started from an incorrect interpretation. The theory of relativity cannot be dealt with in mechanical mathematics, and we must always pay attention to the meaning of physical quantities and their changes according to the change of perspective and use mathematics according to the physically correct interpretation. If we treat relativity only as a mechanical mathematical logic, we will immediately arrive at things like the twin paradox.

Even using the above equation, there is a way to convert the acceleration from an inertial system in which an object is observed to be moving, even though it is not a more general stationary inertial system. I will introduce that method later, but before that, I will introduce an acceleration transformation formula that can directly handle a more general situation. It can be obtained by extending the earlier equation $\frac{dt'}{dt}=\frac{1}{\gamma}$ to more general cases.

From the Lorentz transformations of time $t'=\gamma t-\gamma\frac{\vec{v}\cdot\vec{r}}{c^2}$ and the inverse of it, $t=\gamma t'+\gamma\frac{\vec{v}\cdot\vec{r}'}{c^2}$, we can derive the equations

$$\frac{dt}{dt'}=\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\rightarrow\frac{dt'}{dt}=\frac{1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)}$$

and

$$\frac{dt'}{dt}=\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)\rightarrow\frac{dt}{dt'}=\frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)}$$

. These equations are the same as the formula used so far when $\vec{u}'=0$ and $\vec{u}=\vec{v}$ are satisfied. Using these, we can derive the acceleration transformation formula,

$$
\begin{aligned}
\vec{u} &= \frac{1}{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}\left(\frac{\vec{u}'}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}(\vec{u}'\cdot\hat{v})\hat{v}\right)\\
\vec{a} &= \frac{d\vec{u}}{dt}=\frac{d\vec{u}}{dt'}\frac{dt'}{dt}=\frac{1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)}\frac{d}{dt'}\left(\frac{1}{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}\left(\frac{\vec{u}'}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}'\cdot\vec{v}}{v^2}\vec{v}\right)\right)\\
&= \frac{1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)}\left(\left(\frac{\vec{u}'}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}'\cdot\vec{v}}{v^2}\vec{v}\right)\frac{d}{dt'}\left(\frac{1}{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}\right)+\frac{1}{\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)}\frac{d}{dt'}\left(\frac{\vec{u}'}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}'\cdot\vec{v}}{v^2}\vec{v}\right)\right)\\
&= \frac{1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)}\left(\left(\frac{\vec{u}'}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}'\cdot\vec{v}}{v^2}\vec{v}\right)\frac{-1}{c^2}\frac{0+\vec{v}\cdot\vec{a}'}{\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^2}+\frac{1}{\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)}\left(\frac{\vec{a}'}{\gamma}+\frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\vec{v}\right)\right)\\
&= \frac{-1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\left(\frac{\vec{u}'}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}'\cdot\vec{v}}{v^2}\vec{v}\right)\frac{\vec{v}\cdot\vec{a}'}{c^2}-\left(\frac{\vec{a}'}{\gamma}+\frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\vec{v}\right)\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\right)\\
&= \frac{-1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\frac{\vec{v}\cdot\vec{a}'}{c^2}\frac{\vec{u}'}{\gamma}-\frac{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}{\gamma}\vec{a}'+\left(\frac{\vec{v}\cdot\vec{a}'}{c^2}+\frac{\gamma-1}{\gamma}\frac{\vec{u}'\cdot\vec{v}}{v^2}\frac{\vec{v}\cdot\vec{a}'}{c^2}-\frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}-\frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\vec{v}\right)\\
&= \frac{-1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\frac{\vec{v}\cdot\vec{a}'}{c^2}\frac{\vec{u}'}{\gamma}-\frac{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}{\gamma}\vec{a}'+\left(\frac{\vec{v}\cdot\vec{a}'}{c^2}-\frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\right)\vec{v}\right)\\
&= \frac{-1}{\gamma\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\frac{\vec{v}\cdot\vec{a}'}{c^2}\frac{\vec{u}'}{\gamma}-\frac{1+\frac{\vec{v}\cdot\vec{u}'}{c^2}}{\gamma}\vec{a}'+\left(\frac{v^2}{c^2}-\frac{\gamma-1}{\gamma}\right)\frac{\vec{a}'\cdot\vec{v}}{v^2}\vec{v}\right)\\
&= \frac{1}{\gamma^2\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\vec{a}'-\frac{\vec{v}\cdot\vec{a}'}{c^2}\vec{u}'-\frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\vec{v}\right)
\end{aligned}
$$

, and the inverse transformation for acceleration,

$$
\begin{aligned}
\vec{u}' &= \frac{1}{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}\left(\frac{\vec{u}}{\gamma}-\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\vec{v}\right)\\
\vec{a}' &= \frac{d\vec{u}'}{dt'}=\frac{d\vec{u}'}{dt}\frac{dt}{dt'}=\frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)}\frac{d}{dt}\left(\frac{1}{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}\left(\frac{\vec{u}}{\gamma}-\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\vec{v}\right)\right)\\
&= \frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)}\left(\frac{\vec{u}}{\gamma}-\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\vec{v}\right)\frac{d}{dt}\left(\frac{1}{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}\right)+\frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^2}\frac{d}{dt}\left(\frac{\vec{u}}{\gamma}-\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\vec{v}\right)\\
&= \frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)}\left(\frac{\vec{u}}{\gamma}-\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\vec{v}\right)\frac{-1}{c^2}\frac{0-\vec{v}\cdot\vec{a}}{\left(1-\frac{\vec{u}\cdot\vec{v}}{c^2}\right)^2}+\frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^2}\left(\frac{\vec{a}}{\gamma}+\frac{\gamma-1}{\gamma}\frac{\vec{a}\cdot\vec{v}}{v^2}\vec{v}\right)\\
&= \frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^3}\left(\left(\frac{\vec{u}}{\gamma}-\vec{v}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\vec{v}\right)\frac{\vec{v}\cdot\vec{a}}{c^2}+\left(\frac{\vec{a}}{\gamma}+\frac{\gamma-1}{\gamma}\frac{\vec{a}\cdot\vec{v}}{v^2}\vec{v}\right)\left(1-\frac{\vec{u}\cdot\vec{v}}{c^2}\right)\right)\\
&= \frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^3}\left(\frac{\vec{v}\cdot\vec{a}}{c^2}\frac{\vec{u}}{\gamma}+\frac{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}{\gamma}\vec{a}+\left(-\frac{\vec{v}\cdot\vec{a}}{c^2}+\frac{\gamma-1}{\gamma}\frac{\vec{u}\cdot\vec{v}}{v^2}\frac{\vec{v}\cdot\vec{a}}{c^2}+\frac{\gamma-1}{\gamma}\frac{\vec{a}\cdot\vec{v}}{v^2}-\frac{\gamma-1}{\gamma}\frac{\vec{a}\cdot\vec{v}}{v^2}\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\vec{v}\right)\\
&= \frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^3}\left(\frac{\vec{v}\cdot\vec{a}}{c^2}\frac{\vec{u}}{\gamma}+\frac{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}{\gamma}\vec{a}+\left(-\frac{\vec{v}\cdot\vec{a}}{c^2}+\frac{\gamma-1}{\gamma}\frac{\vec{a}\cdot\vec{v}}{v^2}\right)\vec{v}\right)\\
&= \frac{1}{\gamma\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^3}\left(\frac{\vec{v}\cdot\vec{a}}{c^2}\frac{\vec{u}}{\gamma}+\frac{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}{\gamma}\vec{a}+\left(-\frac{v^2}{c^2}+\frac{\gamma-1}{\gamma}\right)\frac{\vec{a}\cdot\vec{v}}{v^2}\vec{v}\right)\\
&= \frac{1}{\gamma^2\left(1-\frac{\vec{v}\cdot\vec{u}}{c^2}\right)^3}\left(\left(1-\frac{\vec{u}\cdot\vec{v}}{c^2}\right)\vec{a}+\frac{\vec{v}\cdot\vec{a}}{c^2}\vec{u}-\frac{\gamma-1}{\gamma}\frac{\vec{a}\cdot\vec{v}}{v^2}\vec{v}\right)
\end{aligned}
$$

. Again, when $\vec{u}'=0$ and $\vec{u}=\vec{v}$ are satisfied, they match the previously used formulas.

Now, if analyzing the motion in more detail based on these formulas, only an electric field exists in ′ inertial frame running side by side with Qs at v speed, but in a stationary inertial frame, there is also a magnetic field due to the moving charge Qa, and this magnetic field also affects the movement of Qb. Thus, the motion of Qb is the sum of the force due to the electric field and the force due to the magnetic field at its position, and the principle of relativity is maintained by showing that the result of the sum of the electric and magnetic fields, rather than the result of the electric field alone, is consistent with the result of the electric field alone in the ′ inertial system. In other words, the phenomenon of the magnetic field itself is a part of the essential factors to establish the principle of relativity. Of course, the factor due to this magnetic field is in the direction of decreasing in the opposite direction to the effect of the electric field. I will add the effect of this magnetic field and calculate it again.

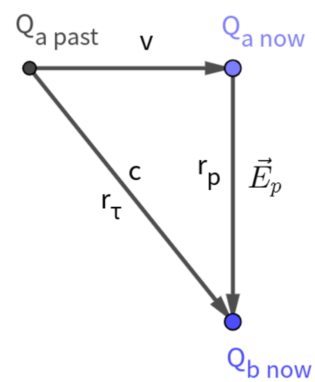

Figure 39: Relationship between r and E in a moving charge

The force exerted on a charge by electric and magnetic fields is traditionally expressed by the Lorentz force formula $\vec{F} = q(\vec{E}+\vec{v}\times\vec{B})$. The deflection term added earlier will be discussed later. At this time, the magnetic field B is $\vec{B} = \frac{\hat{r}}{c} \times \vec{E}$ and $|\hat{r}_\tau \times \vec{E}_p| = \frac{v}{c}E_p$ when a point charge is the source, so it can be confirmed that it is

$$\vec{F} = q(\vec{E}_p + \vec{v}\times\vec{B}) = q\left(\vec{E}_p + \vec{v}\times\left(\frac{\hat{r}_\tau}{c}\times\vec{E}_p\right)\right) = q\vec{E}_p\left(1-\frac{v^2}{c^2}\right)$$

. Therefore, when adding the effect of the magnetic field, it is confirmed that the principle of relativity is satisfied by weakening the effect of the electric field by the required $1-\frac{v^2}{c^2} = \frac{1}{\gamma^2}$ times.

So far is the part covered in exactly the same way in Purcell's book. By the way, Purcell only dealt directly up to this point and only indirectly mentioned the part I will deal with from now on. That part of the problem is the case of two charges moving in a line series.

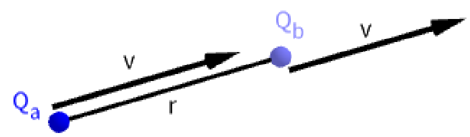

Figure 40: Two charges moving in a series line

In the case of two charges running in a line, one additional factor different from the case of running side by side is added, which is the relativistic length contraction effect. And, in the case of θ=0 in Purcell's formula $\frac{q}{4\pi\varepsilon_0 r_p^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\hat{r}_p$, the denominator is 1 and the numerator is $1/\gamma^2$, so the strength of the electric field becomes γ^2 times weaker than when felt in an inertial frame moving together, but since the distance is reduced to $1/\gamma$, the total electric field strength cancels out and becomes equal. At this time, since the inertial mass increases by γ times, the acceleration will decrease by that much. However, comparing the accelerations in the stationary inertial frame and in the moving inertial frame, it was pointed out earlier that the acceleration is at a rate of γ times slower in γ times more time than the stationary inertial frame, so that a is γ^2 times smaller than a′. And, if the acceleration is in the direction of travel, the slowing effect is added by the length contraction effect that will make it γ times smaller again. The relational expression is $\vec{a}' = \gamma^2((\gamma-1)(\vec{a}\cdot\hat{v})\hat{v}+\vec{a})$ and $\vec{a} = \frac{1}{\gamma^2}\left(\vec{a}' + \frac{1-\gamma}{\gamma}(\vec{a}'\cdot\hat{v})\hat{v}\right)$ as obtained earlier, which in this case is $\vec{a} = \frac{1}{\gamma^3}\vec{a}'$.

In this case, as in the horizontal case, it can be seen that the effect of reducing the acceleration by γ^2 times is required in addition to the effect of increasing the relativistic inertial mass by γ times.

So, I came up with the following idea. It's actually a simple solution. I just considered the addition of a new force that would play the same role as the magnetic force in the horizontal direction as a factor making the force due to the electric field as small as γ^2 in all directions. That's rather neat mathematically.

Earlier, the force due to the magnetic field caused by the moving charge was expressed as $\vec{v}\times\vec{B} = \vec{v}\times\left(\frac{\hat{r}_\tau}{c}\times\vec{E}_p\right)$. Expressing this with the familiar BAC-CAB rule, it can be decomposed into two terms with $\vec{v}\times\left(\frac{\hat{r}_\tau}{c}\times\vec{E}_p\right) = \frac{\hat{r}_\tau}{c}(\vec{v}\cdot\vec{E}_p) - \vec{E}_p\left(\vec{v}\cdot\frac{\hat{r}_\tau}{c}\right)$. Among them, if analyzing the $-\vec{E}_p\left(\vec{v}\cdot\frac{\hat{r}_\tau}{c}\right)$ term using the 'Relationship between r and E for a moving charge' figure, we can see that it is $-\vec{E}_p\left(\vec{v}\cdot\frac{\hat{r}_\tau}{c}\right)=-\vec{E}_p\left(\vec{v}\cdot\frac{v}{c^2}\hat{v}\right)=-\vec{E}_p\frac{v^2}{c^2}$ and therefore

$$\vec{E}_p\left(1-\frac{v^2}{c^2}\right)=\vec{E}_p-\vec{E}_p\left(\vec{v}\cdot\frac{\hat{r}_\tau}{c}\right)=\vec{E}_p+\vec{v}\times\left(\frac{\hat{r}_\tau}{c}\times\vec{E}_p\right)-\frac{\hat{r}_\tau}{c}(\vec{v}\cdot\vec{E}_p)$$

.

Here, it is $\frac{\hat{r}_\tau}{c}(\vec{v}\cdot\vec{E}_p)=0$ in the case where two charges run side by side, which is a familiar situation. We see that when the two charges run in a line, the corresponding term remains as $\vec{v}\cdot\vec{E}_p=vE_p$. And, even when running in a line, the size of the necessary correction term is also $\frac{1}{\gamma^2}$ times of multiplication, which is an adding $-\vec{E}_p\frac{v^2}{c^2}$ as in the case of parallel, which is an addition of $-\frac{\hat{r}_\tau}{c}(\vec{v}\cdot\vec{E}_p)$ in the case of $\frac{\hat{r}_\tau}{c}=\frac{v}{c}$.

This led me to speculate that the original expression for the total force due to relativistic effects was not $\vec{v}\times\left(\frac{\hat{r}_\tau}{c}\times\vec{E}_p\right)$, but rather $-\vec{E}_p\left(\vec{v}\cdot\frac{\vec{v}}{c^2}\right)=\vec{v}\times\left(\frac{\hat{r}_\tau}{c}\times\vec{E}_p\right)-\frac{\vec{v}}{c^2}(\vec{v}\cdot\vec{E}_p)$, as a way to express both case of the parallel and the straight-line movements simultaneously.

In order to actually verify this conjecture, I will have to deal with the case where there is a difference in velocity between the two charges.

4.2 The force between two charges with different velocities

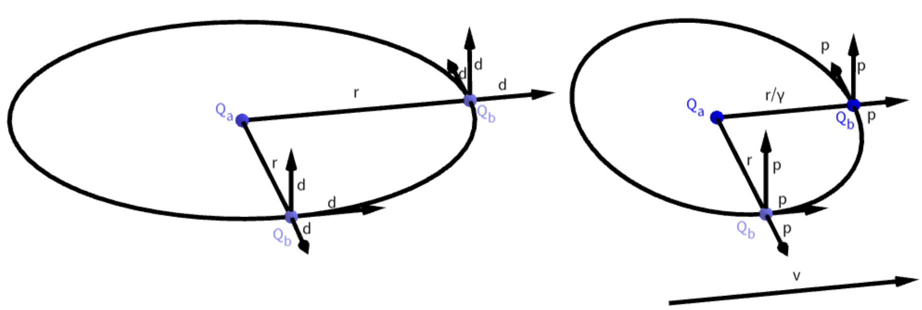

Figure 41: Two charges with different velocities

The diagram on the left shows Qb orbiting in a circular orbit around the charge Qa at a speed d. Since this problem was initially devised to deal with the orbital case, a circular orbit was drawn, but in fact, the purpose is to deal with Qb moving at a speed d in an arbitrary direction in an arbitrary position around Qa.

Three inertial frames appear in this analysis. One is the point of view of a stationary observer and will usually be denoted by a physical quantity labeled 0 or p. The second is from the perspective of the v inertial system, which is the velocity at which the Qa charge is moving, and there will be no mark or v mark. Finally, it is the point of view of the Qb charge moving with a velocity difference of d relative to the Qa and v inertial systems, and since this speed will be marked as u based on the stationary observer, its physical quantities will be generally marked u or d. For example, the case of the unlabeled Lorentz factor γ is $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which is needed to deal with the v inertial system. However, sometimes a name different from the previous rule is given, in which case it will be specified separately.

For convenience, assume that the distance between Qa and Qb is r=1. This structure will look like the picture on the left from the point of view of the v inertial system itself, but it will look like the picture on the right from the standpoint of a stationary observer, an observer passing by at a speed of -v to the left with respect to the v inertial system. Qa moves with the speed of v, and Qb is seen as moving with the relative speed of Qa and p. In fact, d can be called p′, but since I plan to use it often, I gave it a separate name d. And, the distance r between Qa and Qb on the left side is contracted by a factor of 1/γ in the forward direction component in the right figure.

The velocity of Qa is v and the velocity of Qb is $\vec{u} = \vec{v} + \vec{p}$. And, according to the relativistic velocity sum formula obtained earlier, it is also $\vec{u} = \frac{1}{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}\left(\frac{\vec{d}}{\gamma} + \vec{v} + \frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\hat{v}\right)$. Therefore, applying similar calculations as previously done for dv, we have

$$\begin{aligned}\vec{v}+\vec{p} &= \frac{1}{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}\left(\frac{\vec{d}}{\gamma} + \vec{v} + \frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\hat{v}\right)\\ \vec{p} &= \frac{1}{\gamma}\frac{1}{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}\left(\vec{d} + \frac{1-\gamma}{\gamma}(\vec{d}\cdot\hat{v})\hat{v}\right)\end{aligned}$$

Note that p is not used and is just for interest, but d is useful, and using the previous relativistic velocity difference formula, d is

$$\vec{d} = \frac{1}{1-\frac{\vec{u}\cdot\vec{v}}{c^2}}\left(\frac{\vec{u}}{\gamma} - \vec{v} + \frac{\gamma-1}{\gamma}(\vec{u}\cdot\hat{v})\hat{v}\right)$$

Continuing, prepare some basic formulas in advance. These are $\gamma = \gamma_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, $\gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$, and $\gamma_d = \frac{1}{\sqrt{1-\frac{d^2}{c^2}}}$. At this point, since it is

$$\begin{aligned}
\vec{u}\cdot\vec{u} &= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{\vec{d}}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\hat{v}\right)\cdot\left(\frac{\vec{d}}{\gamma}+\vec{v}+\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\hat{v}\right)\\
&= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{\vec{d}}{\gamma}+\left(v+\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\right)\hat{v}\right)\cdot\left(\frac{\vec{d}}{\gamma}+\left(v+\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\right)\hat{v}\right)\\
&= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{d^2}{\gamma^2}+\left(v+\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\right)^2+2\frac{1}{\gamma}\left(v+\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\right)(\vec{d}\cdot\hat{v})\right)\\
&= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{d^2}{\gamma^2}+\left(v^2+\frac{(\gamma-1)^2}{\gamma^2}(\vec{d}\cdot\hat{v})^2+2v\frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\right)+\left(2\frac{v}{\gamma}(\vec{d}\cdot\hat{v})+2\frac{\gamma-1}{\gamma^2}(\vec{d}\cdot\hat{v})^2\right)\right)\\
&= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{d^2}{\gamma^2}+v^2+\frac{2v\gamma-2v+2v}{\gamma}(\vec{d}\cdot\hat{v})+\frac{\gamma^2-2\gamma+1+2\gamma-2}{\gamma^2}(\vec{d}\cdot\hat{v})^2\right)\\
&= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{d^2}{\gamma^2}+v^2+2v(\vec{d}\cdot\hat{v})+\frac{\gamma^2-1}{\gamma^2}(\vec{d}\cdot\hat{v})^2\right)\\
&= \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}\left(\frac{d^2}{\gamma^2}+v^2+2v(\vec{d}\cdot\hat{v})+(\vec{d}\cdot\hat{v})^2-\frac{1}{\gamma^2}(\vec{d}\cdot\hat{v})^2\right)\\
&= \frac{(v+\vec{d}\cdot\hat{v})^2+\frac{d^2-(\vec{d}\cdot\hat{v})^2}{\gamma^2}}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}
\end{aligned}$$

it can be seen that it is

$$\begin{aligned}
\gamma_u &= \frac{1}{\sqrt{1-\frac{\vec{u}\cdot\vec{u}}{c^2}}}\\
&= \frac{1}{\sqrt{1-\frac{1}{c^2}\frac{(v+\vec{d}\cdot\hat{v})^2+\frac{d^2-(\vec{d}\cdot\hat{v})^2}{\gamma^2}}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}}}\\
&= \frac{1}{\sqrt{\frac{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2-\left(\frac{v+\vec{d}\cdot\hat{v}}{c}\right)^2-\frac{d^2-(\vec{d}\cdot\hat{v})^2}{c^2\gamma^2}}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2}}}\\
&= \frac{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}{\sqrt{1+2\frac{\vec{v}\cdot\vec{d}}{c^2}+\left(\frac{\vec{v}\cdot\vec{d}}{c^2}\right)^2-\frac{v^2}{c^2}-2\frac{\vec{v}\cdot\vec{d}}{c^2}-\frac{(\vec{d}\cdot\hat{v})^2}{c^2}-\left(1-\frac{v^2}{c^2}\right)\frac{d^2-(\vec{d}\cdot\hat{v})^2}{c^2}}}\\
&= \frac{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}{\sqrt{1+\frac{v^2(\vec{d}\cdot\hat{v})^2}{c^4}-\frac{v^2}{c^2}-\frac{(\vec{d}\cdot\hat{v})^2}{c^2}-\frac{d^2}{c^2}+\frac{(\vec{d}\cdot\hat{v})^2}{c^2}+\frac{v^2d^2}{c^4}-\frac{v^2(\vec{d}\cdot\hat{v})^2}{c^4}}}\\
&= \frac{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}-\frac{d^2}{c^2}+\frac{v^2d^2}{c^4}}}\\
&= \frac{1+\frac{\vec{v}\cdot\vec{d}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{d^2}{c^2}}}\\
&= \gamma\gamma_d\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)
\end{aligned}$$

Based on these, the following analyses become possible.

These motions can be defined from three different perspectives as mentioned earlier. One is the perspective of the stationary inertial system, where we observe Qa to be moving with velocity v and Qb to be moving with velocity u. The other is the perspective of the v inertial system, where we observe Qa to be at rest and Qb to be moving with velocity d. The third is the perspective of the u inertial system, where we observe Qb to be at rest and Qa to be moving with velocity -d (-d is just a temporary name, not exact, and will be discussed later). In classical mechanics, the relations between the three inertial systems and their transformations are simple additions, but in relativity, the transformations are Lorentz transformations and the relations must be expressed using relativistic velocity sums. For the electromagnetic force to be relativistically consistent, it must be described without contradiction in all these inertial systems. The absence of contradiction can be defined as the motion by electromagnetic force obtained in one inertial system must be the same as the motion by electromagnetic force in that inertial system when described in another inertial system.

Earlier, when describing the motion of an object in a stationary inertial system, I identified the possibility of a new force/acceleration in addition to the electric and magnetic fields. As a way of characterizing these forces, I devised the following process.

Considering the observation in the u inertial frame specified earlier, since Qb is stationary in the u inertial frame, the acceleration due to the magnetic field or the new acceleration term previously estimated that requires the motion term of Qb does not occur, and the force received by Qb is the force only by the electric field generated by Qa moving at -d speed. And, since the mass of Qb in the u inertial frame is the rest mass, the acceleration of Qb can also be known if only the electric field is known. If this is called a′, it can be converted to the acceleration in the stationary inertial system by the previously prepared acceleration conversion formula $\vec{a} = \frac{1}{\gamma^2}\left(\vec{a}' + \frac{1-\gamma}{\gamma}(\vec{a}' \cdot \hat{v})\hat{v}\right)$. By multiplying this by the relativistic inertial mass γ_u of Qb in a stationary inertial frame, the total acceleration applied to Qb by fields originating from Qa in the stationary inertial frame can be obtained. Other accelerations could be obtained by subtracting the pure acceleration due to the Coulomb force of the expression of the stationary inertial system of the electric field generated by Qa from this total acceleration. Let's check to see if this is the correct idea.

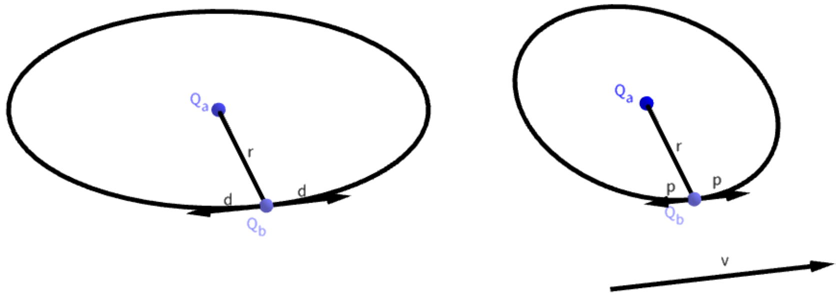

Figure 42: Side-by-side parallel movement

I will first analyze the case where Qa and Qb are positioned horizontally to the direction of travel v, Qb has a relative velocity of d with respect to Qa, and the direction of d is the same as the direction of v.

In this case, since the directions of v and d are identical, $\vec{v} \cdot \vec{d} = vd$ and $\vec{u} = \frac{v+d}{1+\frac{vd}{c^2}}\hat{v}$ respectively. And since the acceleration is perpendicular to the direction of motion, $\vec{a}' \cdot \hat{v} = 0$. However, as mentioned earlier, the acceleration conversion formula is completely reliable only between the inertial frame in which the object to be analyzed is stationary and the inertial frame in which the object is observed, so the starting point of the calculation should be the acceleration in the u inertial frame, which is the inertial frame of the object.

The electric field at Qb in the u inertial frame is, according to Purcell's formula $\vec{E} = \frac{q}{4\pi\varepsilon_0 r_p^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\hat{r}_p$, $\theta = \frac{\pi}{2} \rightarrow \sin\theta = 1$, and r is unchanged as $r_u = r_v$. And, the relative velocity between the v inertial frame and the u inertial frame is d. As will be explained later, Wigner rotation is applied between the two inertial frames, so the direction of a physical quantity such as the direction of d may look different in both inertial frames, but the size or the relative relationship between each physical quantity does not change. But, in this case, since the directions of d, v, and u are all the same, Wigner rotation does not occur. Based on this, if the electric field due to Qa felt in the u inertial frame is calculated, if the magnitude of the electric field due to Qa measured in the v inertial frame is $\vec{E}_v = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} = 1\hat{r}$, it can be seen that the magnitude at the Qb position in the u inertial frame is $\vec{E}_d = \frac{q}{4\pi\varepsilon_0 r^2}\frac{1}{\sqrt{1-\beta_d^2}}\hat{r} = \gamma_d\vec{E}_v = \gamma_d\hat{r}$.

At this time, if Qb's charge and rest mass are set to 1, the acceleration that Qb receives becomes $\vec{a}' = \gamma_d\hat{r}$ in the u inertial frame.

If the acceleration of Qb observed in the stationary inertial frame is calculated by applying the previously obtained acceleration conversion formula $\vec{a} = \frac{1}{\gamma^2}\left(\vec{a}' + \frac{1-\gamma}{\gamma}(\vec{a}' \cdot \hat{u})\hat{u}\right) \rightarrow \vec{a} = \frac{\vec{a}'}{\gamma_u^2}$, the magnitude is $a = \frac{\gamma_d}{\gamma_u^2}$. On the other hand, looking at the motion of Qb by the electric field

E_p generated by Qa in the stationary inertial frame, the charge of Qb is 1, the inertial mass of Qb is γ_u, and the electric field is $\vec{E}_p = \gamma_v \vec{E}_v = \gamma \hat{r}$ because the velocity of Qa is v. Therefore, the acceleration acting on Qb becomes $\frac{\gamma}{\gamma_u}$. What to find here is the component obtained by subtracting the acceleration by the Coulomb force from the total acceleration, which is the term corresponding to $q\frac{v^2}{c^2}\vec{E}_p$ in $\vec{F} = q\vec{E}_p\left(1 - \frac{v^2}{c^2}\right)$, and the acceleration due to this term is $\frac{\gamma_d}{\gamma_u^2} - \frac{\gamma}{\gamma_u}$ obtained by subtracting the acceleration $\frac{\gamma}{\gamma_u}$ due to the Coulomb force from the total acceleration $\frac{\gamma_d}{\gamma_u^2}$ experienced by Qb. The component of the acceleration is obtained by multiplying this by the inertial mass in the stationary inertial frame, and then dividing by the magnitude of the electric field in the stationary inertial frame, that is, $\left(\frac{\gamma_d}{\gamma_u^2} - \frac{\gamma}{\gamma_u}\right)\frac{\gamma_u}{\gamma} = \frac{\gamma_d}{\gamma\gamma_u} - 1$ is the component of the acceleration due to the induced field to obtain. By calculating this,

$$\begin{aligned}
\frac{\gamma_d}{\gamma\gamma_u} - 1 &= \frac{\gamma_d}{\gamma\gamma\gamma_d\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)} - 1 \\
&= \frac{1}{\gamma^2\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)} - 1 \\
&= \frac{1-\gamma^2\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)}{\gamma^2\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)} \\
&= \frac{1-\frac{v^2}{c^2}-1-\frac{\vec{v}\cdot\vec{d}}{c^2}}{1+\frac{\vec{v}\cdot\vec{d}}{c^2}} \\
&= \frac{-\frac{v^2}{c^2}-\frac{vd}{c^2}}{1+\frac{vd}{c^2}} \\
&= \frac{-v(v+d)}{c^2\left(1+\frac{vd}{c^2}\right)}
\end{aligned}$$

is obtained.

For now, the interpretation of this will be postponed for a while, the following case will be analyzed first, and then the two results will be interpreted together.

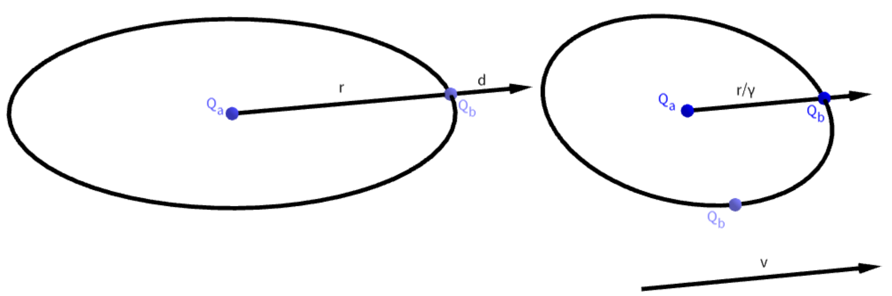

Figure 43: Motion in a straight line

This is the case where Qa and Qb exist on a straight line in the direction of travel and move with relative speed d in the same direction as v. In the v inertial system which is the moving velocity of Qa, the distance between the two is r=1. If each physical quantity at this time is measured in the stationary coordinate system, they are as follows.

First, in this case, since the directions of v, d, and u are the same, they are also $\vec{v} \cdot \vec{d} = vd$ and $\vec{u} = \frac{v+d}{1+\frac{vd}{c^2}} \hat{v}$. However, the acceleration is the same as the direction of motion, so $\vec{a}' \cdot \hat{u} = a'$. Therefore, it is the case of $\vec{a} = \frac{1}{\gamma^2} \left(\vec{a}' + \frac{1-\gamma}{\gamma} (\vec{a}' \cdot \hat{u}) \hat{u} \right) \to \vec{a} = \frac{\vec{a}'}{\gamma_u^3}$. From the perspective of the u inertial system, the distance between Qa and Qb is measured as reduced by $1/\gamma_d$, but since θ=0, therefore, the electric field felt by Qb in the u inertial system is $\vec{E}_d = \frac{q}{4\pi\varepsilon_0 \left(\frac{r}{\gamma_d} \right)^2} \frac{1-\beta_d^2}{1} \hat{r} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} = \vec{E}_v = 1\hat{r}$, and the acceleration is $a' = 1$.

Accordingly, the total acceleration seen in the stationary inertial system is $a = \frac{a'}{\gamma_u^3} = \frac{1}{\gamma_u^3}$. On the other hand, the magnitude of the electric field due to Qa for the Qb position seen in the stationary inertial frame is $\vec{E}_0 = \frac{q}{4\pi\varepsilon_0 \left(\frac{r}{\gamma} \right)^2} \frac{1-\beta^2}{1} \hat{r} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} = \vec{E}_v = 1\hat{r}$ without change, as in the u inertial frame, by γ according to v and the corresponding distance reduction. Therefore, $\frac{1}{\gamma_u}$ is the acceleration due to the electric field alone because only the inertial mass needs to be considered, and the value obtained by subtracting the acceleration due to the electric field only from the total acceleration obtained above and multiplying this value by the inertial mass is the force term corresponding to the acceleration due to the induced field, which is

$$\begin{aligned} \left(\frac{1}{\gamma_u^3} - \frac{1}{\gamma_u} \right) \gamma_u &= \frac{1}{\gamma_u^2} - 1 \\ &= 1 - \frac{u^2}{c^2} - 1 \\ &= \frac{-1}{c^2} \frac{(v+d)^2}{\left(1+\frac{vd}{c^2}\right)^2} \end{aligned}$$

By comparing these results with the result of

$$-\vec{E}_p \left(\vec{v} \cdot \frac{\vec{v}}{c^2} \right) = \vec{v} \times \left(\frac{\hat{r}_\tau}{c} \times \vec{E}_p \right) - \frac{\vec{v}}{c^2} (\vec{v} \cdot \vec{E}_p)$$

in the case of the force between charges moving in parallel, which was examined earlier section, it can be inferred that they correspond to

$$\frac{-v(v+d)}{c^2 \left(1 + \frac{vd}{c^2}\right)} = \frac{-vu}{c^2} \to \vec{u} \times \left(\frac{\vec{v}}{c^2} \times \vec{E}_p \right) = \vec{u} \times \left(\frac{\hat{r}_\tau}{c} \times \vec{E}_p \right) = \vec{u} \times \vec{B}$$

and

$$\frac{-1}{c^2} \frac{(v+d)^2}{\left(1+\frac{vd}{c^2}\right)^2} = -\frac{u^2}{c^2} \to \frac{-\vec{u}}{c^2} (\vec{u} \cdot \vec{E}_p)$$

respectively. These are more straightforward expressions than I expected.

Furthermore, this term $\frac{-\vec{u}}{c^2}(\vec{u} \cdot \vec{E}_p)$ is actually an expression that has already been discovered in a different way. This term appears in the exercises of Griffiths' electromagnetism book. Of course, it can be seen that the starting point, the definition of force $\vec{F} = \frac{d\vec{p}}{dt}$, appeared quite early before Griffiths. However, I do not know who first derived this simple result,

$$\begin{aligned}
\frac{d\vec{p}}{dt} &= \frac{d}{dt}(\gamma m\vec{v}) = \frac{d}{dt}\frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \\
&= m\vec{v}\left(-\frac{1}{2}\right)\frac{-2\vec{a}\cdot\vec{v}}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}} + \frac{m\vec{a}}{\sqrt{1-\frac{v^2}{c^2}}} \\
&= \gamma^3 m\frac{\vec{a}\cdot\vec{v}}{c^2}\vec{v} + \gamma m\vec{a}
\end{aligned}$$

, and for now, it should be attributed to Griffiths himself.

The meaning of this equation is not trivial. It's because it shows that

$$\vec{F} = \frac{d\vec{p}}{dt} \neq m\vec{a}$$

is the conclusion of relativity. This will be considered again later. And, applying the definition of Lorentz force to this equation results in

$$\begin{aligned}
\vec{F} &= q(\vec{E} + \vec{v}\times\vec{B}) = \gamma^3 m\frac{\vec{a}\cdot\vec{v}}{c^2}\vec{v} + \gamma m\vec{a} \\
\vec{a} &= \frac{q}{\gamma m}(\vec{E} + \vec{v}\times\vec{B}) - \gamma^2\frac{\vec{a}\cdot\vec{v}}{c^2}\vec{v}\ldots(1) \\
\vec{a}\cdot\vec{v} &= \frac{q}{\gamma m}\vec{E}\cdot\vec{v} - \gamma^2\frac{v^2}{c^2}\vec{a}\cdot\vec{v} \\
\vec{a}\cdot\vec{v} &= \frac{q(\vec{E}\cdot\vec{v})}{\gamma m\left(1+\gamma^2\frac{v^2}{c^2}\right)} = \frac{q(\vec{E}\cdot\vec{v})}{\gamma^3 m}
\end{aligned}$$

, and applying this final result to (1) results in

$$\begin{aligned}
\vec{a} &= \frac{q}{\gamma m}(\vec{E} + \vec{v}\times\vec{B}) - \gamma^2\frac{q(\vec{E}\cdot\vec{v})}{\gamma^3 m}\frac{\vec{v}}{c^2} \\
&= \frac{q}{\gamma m}\left(\vec{E} + \vec{v}\times\vec{B} - \frac{\vec{E}\cdot\vec{v}}{c^2}\vec{v}\right)
\end{aligned}$$

, where $\frac{-\vec{u}}{c^2}(\vec{u}\cdot\vec{E})$ reappears. According to the definition of Newtonian mechanics, $\vec{F} = m\vec{a}$, it can be seen as if a new force-like term has emerged. Therefore, looking at the results so far, it can be understood that this acceleration term is the element that completes the relativistic consistency of electromagnetism.

This result is an analysis of the case that is easiest to analyze, and it is necessary to check whether this relationship actually holds in any magnitude and direction of $\vec{d}$. Even in the results, if it is confirmed that the acceleration due to this $\frac{-\vec{u}}{c^2}(\vec{u}\cdot\vec{E})$ term corrects the relativistic transformation of the acceleration for any direction and magnitude in all inertial systems, its correctness may be considered to be proven.

However, the analysis of arbitrary velocities is a very complicated computation, which is too difficult for symbolic computation, so I will deal with it as numerical computation.

4.3 Numerical computation for arbitrary velocity differences

The first step is to enter the basic functions. Since friCAS does not have vector multiplication and scalar multiplication operators, define and input them as functions. Readers who have followed the calculations in this book in the meantime should restart friCAS and input. The numbering of calculation inputs may not be consistent if re-entry is required, so we ignore it. A Cartesian coordinate system was used, and the origin of the position vector [0,0,0] is the position of Qa.

(1) -> cX (a, b) == vector [a. 2 b. 3 - a. 3 b. 2, a. 3 b. 1 - a. 1 b. 3, a. 1 b. 2 - a. 2 b. 1]

Type: Void

(2) -> cX ([1, 0, 0], [0, 1, 0]), cX ([0, 1, 0], [0, 0, 1]), cX ([0, 0, 1], [1, 0, 0])
Compiling function cX with type (List(NonNegativeInteger), List(
NonNegativeInteger)) -> Vector(Integer)

$[[0, 0, 1], [1, 0, 0], [0, 1, 0]]$
Type: Tuple(Vector(Integer))

I defined cX as a function responsible for the $\times$ operator in vector product $\vec{a} \times \vec{b}$ and simply tested it.

(3) -> dX (a, b) == a. 1 b. 1 + a. 2 b. 2 + a. 3 b. 3

Type: Void

(4) -> sq (v) == dX (v, v)

Type: Void

(5) -> dX ([1, 2, 3], [2, 3, 4])

Compiling function dX with type (List(PositiveInteger), List(
PositiveInteger)) -> PositiveInteger

20

Type: PositiveInteger

dX was defined as a function responsible for the $\cdot$ operator in scalar product $\vec{a} \cdot \vec{b}$, and the sq function was defined and tested simply to express $v^2 = \vec{v} \cdot \vec{v}$.

(6) -> $Gm(v, c) == \frac{1}{\sqrt{1 - \frac{dX(v,v)}{c^2}}}$

Type: Void

(7) -> Gm ([x, y, z], c)

$$\frac{1}{\sqrt{\frac{-z^2 - y^2 - x^2 + c^2}{c^2}}}$$

Type: Expression(Integer)

A function to calculate $\gamma_{v,c} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ was entered and tested.

(9) -> $E(p, v, c) == \frac{1 - \frac{sq(v)}{c^2}}{\left(sq(p) - \frac{sq(cX(v,p))}{c^2}\right)^{3/2}} p$

Type: Void

(11) -> E (vector [1, 1, 1], vector [0.3, 0.2, - 0.1], 1),

E (vector [1.0, 1.0, - 1.0], vector [0, 0, 0], 1)

$[[0.1896149635\_9998254822, 0.1896149635\_9998254822, 0.1896149635\_9998254822],$
$[0.1924500897\_2987525484, 0.1924500897\_2987525484, -0.1924500897\_2987525484]]$

Type: Tuple(Vector(Expression(Float)))

This function calculates the electric field according to Purcell's formula. p is the position vector, v is the velocity vector, and c is the scalar value of the speed of light. $\frac{\hat{r}_p}{r_p^2} = \frac{\vec{r}_p}{r_p^3} = \frac{x\hat{x}+y\hat{y}+z\hat{z}}{(x^2+y^2+z^2)^{3/2}} = \frac{\vec{p}}{(\vec{p}\cdot\vec{p})^{3/2}}$ and $\vec{p} \times \vec{v} = pv\sin\theta \rightarrow \sin^2\theta = \frac{(\vec{p}\times\vec{v})^2}{p^2v^2}$ were applied to Purcell's formula $\vec{E} = \frac{q}{4\pi\varepsilon_0 r_p^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\hat{r}_p$, constants were excluded, and

$$\begin{aligned}
\vec{E} &= \frac{q}{4\pi\varepsilon_0 r_p^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\hat{r}_p \\
&= \frac{q}{4\pi\varepsilon_0}\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\frac{(\vec{p}\times\vec{v})^2}{p^2v^2}\right)^{3/2}}\frac{\vec{p}}{(\vec{p}\cdot\vec{p})^{3/2}} \\
&= \frac{q}{4\pi\varepsilon_0}\frac{1-\frac{v^2}{c^2}}{\left(p^2-\frac{(\vec{p}\times\vec{v})^2}{c^2}\right)^{3/2}}\vec{p} \\
&\rightarrow \frac{1-\frac{v^2}{c^2}}{\left(p^2-\frac{(\vec{p}\times\vec{v})^2}{c^2}\right)^{3/2}}\vec{p}
\end{aligned}$$

was organized and entered as a Cartesian coordinate expression.

$(12) -> U(v, d, c) ==$

if (v = [0, 0, 0]) then d

else $\left(\frac{1}{1+\frac{dX(d,v)}{c^2}}\left(\frac{1}{Gm(v,c)}d + v + \left(1-\frac{1}{Gm(v,c)}\right)\frac{dX(d,v)}{sq(v)}v\right)\right)$

Type: Void

$(13) -> U(vector[0.1, 0.2, 0.3], vector[0.1, 0.1, 0.1], 1), U(vector[0.0, 0.0, 0.0], vector[0.1, 0.1, 0.1], 1)$

$Cannot compile map : dX$

We will attempt to interpret the code.

$$[[0.1847634420\_2423559974, 0.2820399171\_0983248484, 0.3793163921\_9542936994], [0.1, 0.1, 0.1]]$$

Type: Tuple(Vector(Float))

The relativistic velocity sum formula $\vec{u} = \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)}\left(\frac{\vec{d}}{\gamma} + \vec{v} + \frac{\gamma-1}{\gamma}(\vec{d}\cdot\hat{v})\hat{v}\right) = \frac{1}{\left(1+\frac{\vec{v}\cdot\vec{d}}{c^2}\right)}\left(\frac{\vec{d}}{\gamma} + \vec{v} + \left(1-\frac{1}{\gamma}\right)\frac{\vec{d}\cdot\vec{v}}{v^2}\vec{v}\right)$ was entered and tested. The case of v=0, which is divided by 0 and causes an error, is handled separately with an if statement.

(14) -> rr (d, r, c) ==

if (d = [0, 0, 0]) then r

else $\left(r - \left(1-\frac{1}{Gm(d,c)}\right)\frac{dX(d,r)}{dX(d,d)}d\right)$

Type: Void

(15) -> rr (vector [0.8, 0, 0], vector [0.8, 4, 4], 1.0), rr (- vector [0.8, 0, 0], - vector [0.8, 4, 4], 1)
Cannot compile map: Gm
We will attempt to interpret the code.

$$[[0.48, 4.0, 4.0], [-0.48, -4.0, -4.0]]$$

Type: Tuple(Vector(Float))

Function $\vec{r} - \frac{\gamma-1}{\gamma}(\vec{r} \cdot \hat{d})\hat{d}$ to handle relativistic length contraction of coordinates. It is a formula that selectively applies relativistic length contraction to the components of the coordinate r in the direction of relative velocity d only. As mentioned earlier, the basic coordinates will be given in the v inertial system with Qa as the origin, so we need to take relativistic length contraction into account when converting them to values in the rest and u coordinate systems. This function will take care of that. The actual usage will be explained later along with usage examples.

(16) -> A (a, u, c) ==
if (u = [0, 0, 0]) then a
else $\left(\frac{1}{Gm(u,c)^2}\left(a + \left(\frac{1}{Gm(u,c)} - 1\right)\frac{dX(a,u)}{sq(u)}u\right)\right)$

Type: Void

(17) -> rA (a, v, c) ==
if (v = [0, 0, 0]) then a
else $\left(Gm(v,c)^2\left(a + (Gm(v,c) - 1)\frac{dX(a,v)}{sq(v)}v\right)\right)$

$$Type : Void$$

(18) -> dA (a, u, d, c) ==
if (u = [0, 0, 0]) then a
else $\left(\frac{1}{Gm(u,c)^2\left(1+\frac{dX(u,d)}{c^2}\right)^3}\left(\left(1 + \frac{dX(u,d)}{c^2}\right)a + \left(\frac{1}{Gm(u,c)} - 1\right)\frac{dX(a,u)}{sq(u)}u - \frac{dX(a,u)}{c^2}d\right)\right)$

$$Type : Void$$

These are the relativistic acceleration conversion formulas $\vec{a} = \frac{1}{\gamma^2}\left(\vec{a}' + \frac{1-\gamma}{\gamma}(\vec{a}' \cdot \hat{v})\hat{v}\right)$, $\vec{a}' = \gamma^2((\gamma-1)(\vec{a} \cdot \hat{v})\hat{v} + \vec{a})$ and $\vec{a} = \frac{1}{\gamma^2\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\left(1 + \frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\vec{a}' - \frac{\vec{v}\cdot\vec{a}'}{c^2}\vec{u}' - \frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\vec{v}\right)$. Use it by entering $u \leftarrow \vec{v}$ and $d \leftarrow \vec{u}'$. When converting inversely, simply enter $u \leftarrow -\vec{v}$ and $d \leftarrow \vec{u} = \vec{v} \oplus \vec{u}'$ in the same formula, so one formula can be used in both cases.

Also, since we have to deal with Wigner rotation, we need to prepare for that as well.

(19) -> $\Lambda matrix := \begin{pmatrix} \gamma & -\beta_x\gamma & -\beta_y\gamma & -\beta_z\gamma \\ -\beta_x\gamma & 1+\frac{(\gamma-1)\beta_x^2}{\beta^2} & \frac{(\gamma-1)\beta_x\beta_y}{\beta^2} & \frac{(\gamma-1)\beta_x\beta_z}{\beta^2} \\ -\beta_y\gamma & \frac{(\gamma-1)\beta_x\beta_y}{\beta^2} & 1+\frac{(\gamma-1)\beta_y^2}{\beta^2} & \frac{(\gamma-1)\beta_y\beta_z}{\beta^2} \\ -\beta_z\gamma & \frac{(\gamma-1)\beta_x\beta_z}{\beta^2} & \frac{(\gamma-1)\beta_y\beta_z}{\beta^2} & 1+\frac{(\gamma-1)\beta_z^2}{\beta^2} \end{pmatrix}$

$$\begin{pmatrix} \gamma & -\beta_x\gamma & -\beta_y\gamma & -\beta_z\gamma \\ -\beta_x\gamma & \frac{\beta_x^2\gamma+\beta^2-\beta_x^2}{\beta^2} & \frac{\beta_x\beta_y\gamma-\beta_x\beta_y}{\beta^2} & \frac{\beta_x\beta_z\gamma-\beta_x\beta_z}{\beta^2} \\ -\beta_y\gamma & \frac{\beta_x\beta_y\gamma-\beta_x\beta_y}{\beta^2} & \frac{\beta_y^2\gamma+\beta^2-\beta_y^2}{\beta^2} & \frac{\beta_y\beta_z\gamma-\beta_y\beta_z}{\beta^2} \\ -\beta_z\gamma & \frac{\beta_x\beta_z\gamma-\beta_x\beta_z}{\beta^2} & \frac{\beta_y\beta_z\gamma-\beta_y\beta_z}{\beta^2} & \frac{\beta_z^2\gamma+\beta^2-\beta_z^2}{\beta^2} \end{pmatrix}$$

Type: Matrix(Fraction(Polynomial(Integer)))

(20) -> $\Lambda Meval\left(\Lambda matrix, \left[\gamma = \frac{1}{\sqrt{1-\beta_x^2-\beta_y^2-\beta_z^2}}, \beta = \sqrt{\beta_x^2+\beta_y^2+\beta_z^2}\right]\right)$

$$\begin{pmatrix} \frac{1}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & -\frac{\beta_x}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & -\frac{\beta_y}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & -\frac{\beta_z}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} \\ -\frac{\beta_x}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{(\beta_z^2+\beta_y^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_x^2}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{-\beta_x\beta_y\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_x\beta_y}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{-\beta_x\beta_z\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_x\beta_z}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} \\ -\frac{\beta_y}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{-\beta_x\beta_y\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_x\beta_y}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{(\beta_z^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_y^2}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{-\beta_y\beta_z\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_y\beta_z}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} \\ -\frac{\beta_z}{\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{-\beta_x\beta_z\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_x\beta_z}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{-\beta_y\beta_z\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_y\beta_z}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} & \frac{(\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}+\beta_z^2}{(\beta_z^2+\beta_y^2+\beta_x^2)\sqrt{-\beta_z^2-\beta_y^2-\beta_x^2+1}} \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Integer))

A Lorentz transformation matrix ΛM was prepared by substituting an expression for a more specific calculation into the transformation matrix obtained while introducing the 3-dimensional Lorentz transformation.

From now on, I will explain by making specific calculations based on these.

(21) -> digits (20) ;
c := 1 ;
r := vector [0.6, 1.0, 0] ;
d := vector [0, 0, 0.8] ;
v := vector [0.8, 0, 0] ;
u := U (v, d, c)

$[0.8, 0.0, 0.48]$

Type: Vector(Float)

First, the number of significant digits for calculation was appropriately determined to be 20. We don't always have to type this in, but I'll continue to do it as a custom as it will come in handy later. Assuming 1 as the speed of light, enter it. Then, enter the coordinates r of Qb based on Qa in the v inertial system, its relative velocity d, and the velocity v of Qa as arbitrary values, and based on these, find the velocity u of Qb.

We can see that the appropriate values are calculated and printed out. And, I will do the following calculation.

(22) -> U (u, - d, c)

$[0.9320091673\_0328495034, 0.0, 0.0916730328\_4950343774\_4]$

Type: Vector(Float)

The value obtained by adding d to v through the relativistic sum of velocities is u, but when -d is added to u, we can see that the original v does not come out. This property is called non-linearity in mathematics, but it seems a bit broad here. And it's not always the case.

```
(23) -> digits (20) ;
c := 1 ;
r := vector [0.6, 1.0, 0] ;
d := vector [0.4, 0, - 0.4] ;
v := vector [0.3, 0, - 0.3] ;
u := U (v, d, c)
```

$[0.5645161290\_3225806452, 0.0, -0.5645161290\_3225806452]$

Type: Vector(Float)

(24) -> U (u, - d, c)

$[0.3, 0.0, -0.3]$

Type: Vector(Float)

When v, d, and u are in the same direction, we can see that the original v is obtained by adding -d to u. This is because the relativistic sum of velocities of two velocities that are not in the same direction corresponds to two Lorentz transformations, and a phenomenon represented by one Lorentz transformation and one Wigner rotation transformation is involved.

I will check this out. Earlier, to obtain the Wigner rotation, a Lorentz transformation matrix for each velocity was obtained and multiplied, and then an inverse matrix for the sum of the velocities was obtained and the product of the former two matrices was multiplied to leave a rotation matrix. I will use it again. I will continue to explain the order of matrix multiplication and the order of application of rotation matrices during the calculation, but in fact, it's worth mentioning that in the actual process of discovery, I tested all possible combinations of orders and identified the correct working combination, which was the original method I found.

First, enter values that are not in the same direction again.

(25) -> digits (20) ;
c := 1 ;
r := vector [0.6, 1.0, 0] ;
d := vector [0, 0, 0.8] ;
v := vector [0.8, 0, 0] ;
u := U (v, d, c)

$$[0.8, 0.0, 0.48]$$

Type: Vector(Float)

Then, find the Lorentz transform matrix Λv for velocity v, the Lorentz transform matrix Λd for velocity d, and the inverse of the Lorentz transform matrix iΛu for the relativistic velocity sum u.

(26) -> $\Lambda v := eval\left(\Lambda M, \left[\beta_x = \frac{v.1}{c}, \beta_y = \frac{v.2}{c}, \beta_z = \frac{v.3}{c}\right]\right)$

$$\begin{pmatrix} 1.6666666666\_666666667 & -1.3333333333\_333333333 & 0.0 & 0.0 \\ -1.3333333333\_333333333 & 1.6666666666\_666666667 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

(27) -> $\Lambda d := eval\left(\Lambda M, \left[\beta_x = \frac{d.1}{c}, \beta_y = \frac{d.2}{c}, \beta_z = \frac{d.3}{c}\right]\right)$

$$\begin{pmatrix} 1.6666666666\_666666667 & 0.0 & 0.0 & -1.3333333333\_333333333 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -1.3333333333\_333333333 & 0.0 & 0.0 & 1.6666666666\_666666667 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

(28) -> i $\Lambda u := eval\left(\Lambda M, \left[\beta_x = -1\frac{u.1}{c}, \beta_y = -1\frac{u.2}{c}, \beta_z = -1\frac{u.3}{c}\right]\right)$

$$\begin{pmatrix} 2.7777777777\_777777778 & 2.2222222222\_222222222 & 0.0 & 1.3333333333\_333333333 \\ 2.2222222222\_222222222 & 2.3071895424\_836601307 & 0.0 & 0.7843137254\_9019607844 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 1.3333333333\_333333333 & 0.7843137254\_9019607844 & 0.0 & 1.4705882352\_941176471 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

In order to rotate only within the xz plane and make it easy to see, v and d are set to the values of x and z only. There are four possible orders of multiplication using these values.

(29) -> $\Lambda R := i\Lambda u \Lambda v \Lambda d$

$$\begin{pmatrix} 1.0 & -0.1E-19 & 0.0 & 0.3E-19 \\ -0.7E-20 & 0.8823529411\_764705882 & 0.0 & 0.4705882352\_941176471 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.1E-19 & -0.4705882352\_941176471 & 0.0 & 0.8823529411\_764705882 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

(30) -> r$\Lambda R := \Lambda d \Lambda v i \Lambda u$

$$\begin{pmatrix} 1.0 & -0.7E-20 & 0.0 & 0.2E-19 \\ -0.1E-19 & 0.8823529411\_764705882 & 0.0 & -0.4705882352\_941176471 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.3E-19 & 0.4705882352\_941176471 & 0.0 & 0.8823529411\_764705882 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

(31) -> i$\Lambda u \Lambda d \Lambda v$

$$\begin{pmatrix} 1.7901234567\_90123457 & -0.0987654320\_987654321 & 0.0 & -1.4814814814\_81481481 \\ 1.3536673928\_83079158 & 0.3013798111\_837327524 & 0.0 & -1.6557734204\_793028323 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -0.6100217864\_9237472765 & 0.9586056644\_8801742918 & 0.0 & 0.6732026143\_790849673 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

(32) -> $\Lambda v \Lambda d i \Lambda u$

$$\begin{pmatrix} 1.7901234567\_90123457 & 1.3536673928\_830791576 & 0.0 & -0.6100217864\_923747276 \\ -0.0987654320\_987654321 & 0.3013798111\_837327524 & 0.0 & 0.9586056644\_880174292 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -1.4814814814\_81481481 & -1.6557734204\_793028323 & 0.0 & 0.6732026143\_790849673 \end{pmatrix}$$

Type: SquareMatrix(4,Expression(Float))

The results show that only the combinations named ΛR and rΛR contain rotations in the xz plane, which means that when two Lorentz transformations are combined into one Lorentz transformation for the relativistic sum of two velocities and one related rotation transformation, the order of application is Lorentz transformation -> rotation transformation. In fact, since the u inertial system is an inertial system obtained by adding d velocity to the v inertial system, the order of the velocity sum is v->d, and if this is expressed as vector and matrix multiplication, it can be seen that the matrix multiplication order of the second ΛdΛviΛu is the rotational transformation by removing a Lorentz transformation once from the order of v->d. However, the reason why the reverse rotation iΛuΛvΛd was chosen as ΛR, was because that was practically needed more often in my calculations. This will be presented by the following calculations.

First, extracting only the rotational transformations for space excluding the time term from the Lorentz transformation, it is

(33) -> rM := $\begin{pmatrix} \Lambda R(2,2) & \Lambda R(2,3) & \Lambda R(2,4) \\ \Lambda R(3,2) & \Lambda R(3,3) & \Lambda R(3,4) \\ \Lambda R(4,2) & \Lambda R(4,3) & \Lambda R(4,4) \end{pmatrix}$

$$\begin{pmatrix} 0.8823529411\_764705882 & 0.0 & 0.4705882352\_941176471 \\ 0.0 & 1.0 & 0.0 \\ -0.4705882352\_941176471 & 0.0 & 0.8823529411\_764705882 \end{pmatrix}$$

Type: Matrix(Expression(Float))

(34) -> rrM := $\begin{pmatrix} r\Lambda R(2,2) & r\Lambda R(2,3) & r\Lambda R(2,4) \\ r\Lambda R(3,2) & r\Lambda R(3,3) & r\Lambda R(3,4) \\ r\Lambda R(4,2) & r\Lambda R(4,3) & r\Lambda R(4,4) \end{pmatrix}$

$$\begin{pmatrix} 0.8823529411\_764705882 & 0.0 & -0.4705882352\_941176471 \\ 0.0 & 1.0 & 0.0 \\ 0.4705882352\_941176471 & 0.0 & 0.8823529411\_764705882 \end{pmatrix}$$

Type: Matrix(Expression(Float))

Among these rotational transformations, If rM(-d) obtained by applying rM to -d is obtained to obtain the relativistic speed sum with u, it is,

(35) -> U (u, rM - d, 1)

$[0.8, 0.0, 0.1882295438\_3428896424E-19]$

Type: Vector(Expression(Float))

then the original v=[0.8,0,0] is obtained again with little floating point error. This means that an inertial system applied with d Lorentz transformation to the v inertial system with v Lorentz transformation was a transformation applied with u Lorentz transformation and certain rotational transformation. And, it can be interpreted that in order to return to the v inertial system from the result, it is necessary to apply the inverse transformation of the final rotation transformation to the last result. Of course, I found the combination that gave the right answers by trying all possible ways with the possibility of this and that interpretation in mind rather than this interpretation, so the importance of these interpretations is left to each person's judgment. By the way, the interpretation of the application of the calculation order is only important in the end which gives the correct answer, but there is a more important interpretation. It's a matter of what case rotation is applied. There is no rotational relation between the physical quantities in the stationary inertial frame and the v inertial frame. And, this should be the same between the physical quantities of the stationary inertial frame and the u inertial frame. If so, the rotation must exist only in the physical quantity change between the v inertial frame and the u inertial frame. This should be kept in mind. With these calculations, we now have a rough idea of how to apply the Wigner rotation.

Now it's time to proceed with the detailed electric field calculations. Since Qb is in a stationary state in the u inertial system, the applied force is only the force due to the electric field caused by the moving charge Qa, and no force from the magnetic field or other induced forces exists. Thus, the acceleration in the u inertial system will be converted to the acceleration in the stationary inertial system. At this point, if the newly added force is called the induced electric force, in the stationary inertial frame, the sum of the acceleration due to this force and the acceleration due to the electric and magnetic fields is obtained. If this result is always the same as the acceleration converted from the u inertial system, the existence of the new induced electric force is proved, and at the same time, the relativistic acceleration conversion problem that has not been solved so far is solved.

For this purpose, first, the electric field in u inertial frame must be obtained, and to do so, the following calculation is required.

(36) -> id := rM - d

$$[-0.3764705882\_3529411765, 0.0, -0.7058823529\_4117647058]$$

Type: Vector(Expression(Float))

And, the relative velocity rM(-d) of Qa observed from Qb obtained earlier is assigned to the id variable.

(37) -> dr := rr (id, rM r, c)
$[0.5294117647\_0588235293, 1.0, -0.2823529411\_7647058824]$

Type: Vector(Expression(Float))

This function applies the previously prepared relativistic length contraction to coordinates. The relative velocity observed by Qb is id as obtained above, and the relative position is entered by applying the Wigner rotation to r in the inertial frame based on Qa. This result, dr, is the position of Qb based on Qa determined by Qb.

(38) -> ed := E (dr, id, c)

Compiling function cX with type (Vector(Expression(Float)), Vector(
Expression(Float))) -> Vector(Expression(Float))

$[0.5563319154\_7094129771, 1.0508491736\_673335624, -0.2967103549\_1783535879]$

Type: Vector(Expression(Float))

Now that we know the position and relative velocity, we can input it into Purcell's formula to calculate the electric field felt by Qb.

Now it's time to calculate the electric and magnetic fields in a stationary inertial frame.

(39) -> p := rr (v, r, c)
$[0.36, 1.0, 0.0]$

Type: Vector(Float)

P is the position of Qb relative to the origin of Qa observed in a stationary inertial frame. Apply relativistic length contraction in the direction of motion to r of the v inertial frame criterion.

(40) -> ep := E (p, v, c)

Compiling function cX with type (Vector(Float), Vector(Float)) ->
Vector(Float)

$[0.3783057025\_2024008245, 1.0508491736\_673335624, 0.0]$
Type: Vector(Float)

The electric field by Purcell's formula at the Qb position observed in the stationary inertial frame was obtained and stored in ep.

(41) -> ev := E (r, [0, 0, 0], c)

Compiling function cX with type (List(NonNegativeInteger), Vector(
Float)) -> Vector(Float)

$[0.3783057025\_2024008245, 0.6305095042\_0040013742, 0.0]$
Type: Vector(Float)

The electric field for the Qb's position r in the v inertial frame, which is not used right away but is needed later, is stored in ev.

In the v inertial frame, since Qa is stationary, there is no magnetic field by Qa, and in the u inertial frame, since Qb is stationary, it is not affected by any magnetic field. However, in the stationary inertial frame, there is a magnetic field and it is affected, so it must be calculated. The magnetic field calculation uses the $\vec{B} = \frac{\hat{r}}{c} \times \vec{E}$ obtained when working with Feynman's formula earlier. Since the electric field is known, $\hat{r}$ must be calculated, $\hat{r}_\tau$ will be used here. To do this, it is necessary to know the distance vector $\vec{r}_\tau$ of the electric field reaching Qb at the speed of light in a stationary inertial frame. For the calculation, I will start by first finding the time required for the electric field to reach Qb from the origin.

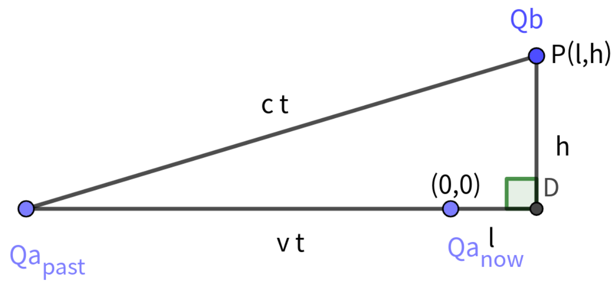

Figure 44: The origin calculation

If h is the height of the vertical line drawn from the position of Qb to the axis of motion of Qa, and l is the distance between the point where the line meets the axis of motion and the coordinate origin, that is, the current position of Qb, then if the distance through which the electric field is transmitted is ct, then we can know that $c^2t^2 = (vt + l)^2 + h^2$. By solving this quadratic equation for t, it is

$$\begin{aligned}
c^2t^2 &= (vt + l)^2 + h^2 \\
&= v^2t^2 + 2lvt + l^2 + h^2 \\
(c^2 - v^2)t^2 - 2lvt - (l^2 + h^2) &= 0 \\
t &= \frac{2lv \pm \sqrt{4l^2v^2 + 4(c^2 - v^2)(l^2 + h^2)}}{2(c^2 - v^2)} \\
&= \frac{\gamma^2}{2c^2}\left(2lv \pm 2\sqrt{l^2v^2 + c^2\frac{l^2 + h^2}{\gamma^2}}\right) \\
&= \frac{\gamma^2}{c^2}\left(lv + \sqrt{l^2v^2 + \frac{c^2p^2}{\gamma^2}}\right)
\end{aligned}$$

Since t is meaningful only when it is positive, only the + root is used. The + root is always positive. Entering this formula is

(42) -> $l := \frac{dX(p,v)}{sq(v)}v$
$[0.36, 0.0, 0.0]$

Type: Vector(Float)

l is the position $\vec{l} = (\vec{p} \cdot \hat{v})\hat{v}$ at which the perpendicular at the p position meets the v axis of motion. Since already know the location of p, there is no need to find h separately. If t is calculated using this value of l, it is

(43) -> $t := \frac{Gm(v,c)^2}{c^2}\left(dX(l,v) + \sqrt{dX(l,v)^2 + c^2\frac{sq(p)}{Gm(v,c)^2}}\right)$
$2.7436506316\_15100157$

Type: Float

Next, $\vec{r}_\tau$ can be simply obtained as $\vec{v}t + \vec{p}$. store this in o.

(44) -> o := p + tv
$[2.5549205052\_920801256, 1.0, 0.0]$
Type: Vector(Float)

And, since it is $\hat{r}_\tau = \frac{\vec{r}_\tau}{\sqrt{\vec{r}_\tau \cdot \vec{r}_\tau}}$, I will compute it and store it in ro.

(45) -> $ro := \frac{1}{\sqrt{sq(o)}} o$
$[0.9312120413\_0428202934, 0.3644778925\_1189452594, 0.0]$
Type: Vector(Float)

Now, the electric and magnetic fields and the induced electric force components acting on the stationary inertial frame criterion Qb can be completely calculated.

(46) -> $\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c} cX(u, cX(ro, ep)) - \frac{1}{c^2} udX(ep, u)\right)$
$[0.0490284190\_4662311468\_5, 0.1361900529\_0728642968, -0.0522969803\_1639798899\_8]$
Type: Vector(Float)

This is the calculation of $\frac{1}{\gamma_u}\left(\vec{E}_p + \vec{u} \times \left(\frac{\hat{r}_\tau}{c} \times \vec{E}_p\right) - \frac{\vec{u}}{c^2}(\vec{u} \cdot \vec{E}_p)\right)$, assuming that the charge of Qb and all related constants are 1, this is the acceleration of Qb measured in a stationary inertial frame. $\vec{E}_p$ is the term due to the electric field, $\vec{u} \times \left(\frac{\hat{r}_\tau}{c} \times \vec{E}_p\right)$ is the term due to the magnetic field, and $-\frac{\vec{u}}{c^2}(\vec{u} \cdot \vec{E})$ is the induced term.

Meanwhile,

(47) -> A (ed, u, c)   Cannot compile map: Gm
We will attempt to interpret the code.

$[0.0490284190\_4662311468\_3, 0.1361900529\_0728642968, -0.0522969803\_1639798899\_6]$
Type: Vector(Expression(Float))

In this calculation, ed is the electric field applied to Qb in u inertial frame. Since the rest mass of Qb is 1, this is the acceleration received by Qb in the u inertial frame. This acceleration is converted into acceleration in the stationary inertial system through the A function, which implements the $\vec{a} = \frac{1}{\gamma^2}\left(\vec{a}' + \frac{1-\gamma}{\gamma}(\vec{a}' \cdot \hat{v})\hat{v}\right)$ conversion as a function.

(48) -> $A(ed, u, c) - \frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep, u)\right)$
$[-0.1E-20, -0.8E-21, 0.1E-20]$
Type: Vector(Expression(Float))

Comparing the two accelerations by subtraction,

$$\frac{1}{\gamma_u^2}\left(\vec{E}_d + \frac{1-\gamma_u}{\gamma_u}(\vec{E}_d \cdot \hat{u})\hat{u}\right) - \frac{1}{\gamma_u}\left(\vec{E}_p + \vec{u} \times \left(\frac{\hat{r}_\tau}{c} \times \vec{E}_p\right) - \frac{\vec{u}}{c^2}(\vec{u} \cdot \vec{E}_p)\right) = 0$$

, it confirms that they are exactly equal.

Let's compare it at other arbitrary positions and velocities. We can run the entire process at once,

(49) -> digits (20) ;
c := 1 ;
r := vector [0.6, 1.0, - 0.9] ;
d := vector [0.3, - 0.4, 0.5] ;
v := vector [0.2, 0.3, 0.2] ;
u := U (v, d, c) ;
$\Lambda v := eval\left(\Lambda M, \left[\beta_x = \frac{v.1}{c}, \beta_y = \frac{v.2}{c}, \beta_z = \frac{v.3}{c}\right]\right)$;
$\Lambda d := eval\left(\Lambda M, \left[\beta_x = \frac{d.1}{c}, \beta_y = \frac{d.2}{c}, \beta_z = \frac{d.3}{c}\right]\right)$;
i $\Lambda u := eval\left(\Lambda M, \left[\beta_x = -1\frac{u.1}{c}, \beta_y = -1\frac{u.2}{c}, \beta_z = -1\frac{u.3}{c}\right]\right)$;
$\Lambda R := i\Lambda u \Lambda v \Lambda d;$
r $\Lambda R := \Lambda d \Lambda v i \Lambda u;$
rM := $\begin{pmatrix} \Lambda R(2,2) & \Lambda R(2,3) & \Lambda R(2,4) \\ \Lambda R(3,2) & \Lambda R(3,3) & \Lambda R(3,4) \\ \Lambda R(4,2) & \Lambda R(4,3) & \Lambda R(4,4) \end{pmatrix}$;
rrM := $\begin{pmatrix} r\Lambda R(2,2) & r\Lambda R(2,3) & r\Lambda R(2,4) \\ r\Lambda R(3,2) & r\Lambda R(3,3) & r\Lambda R(3,4) \\ r\Lambda R(4,2) & r\Lambda R(4,3) & r\Lambda R(4,4) \end{pmatrix}$;
id := - rM d ;
dr := rr (id, rM r, c) ;
ed := E (dr, id, c) ;
p := rr (v, r, c) ;

ep := E (p, v, c) ;
ev := E (r, [0, 0, 0], c) ;
l := $\frac{dX(p,v)}{sq(v)}v$;
t := $\frac{Gm(v,c)^2}{c^2}\left(dX(l,v)+\sqrt{dX(l,v)^2+c^2\frac{sq(p)}{Gm(v,c)^2}}\right)$;
o := p + tv ;
ro := $\frac{1}{\sqrt{sq(o)}}o$;

$A(ed,u,c)-\frac{1}{Gm(u,c)}\left(ep+\frac{1}{c}cX(u,cX(ro,ep))-\frac{1}{c^2}udX(ep,u)\right)$
$[-0.4E-21, 0.8E-21, 0.8E-21]$

Type: Vector(Expression(Float))

It confirms that the two forces are completely equal even in other positions and directions.

(50) -> digits (200) ;
c := 1 ;
r := vector [0.6, 1.0, 1.7] ;
d := vector [0.02, 0.4, 0.7] ;
v := vector [0.8, 0.2, 0.1] ;
u := U (v, d, c) ;
$\Lambda v := eval\left(\Lambda M, \left[\beta_x=\frac{v.1}{c}, \beta_y=\frac{v.2}{c}, \beta_z=\frac{v.3}{c}\right]\right)$;
$\Lambda d := eval\left(\Lambda M, \left[\beta_x=\frac{d.1}{c}, \beta_y=\frac{d.2}{c}, \beta_z=\frac{d.3}{c}\right]\right)$;
i $\Lambda u := eval\left(\Lambda M, \left[\beta_x=-1\frac{u.1}{c}, \beta_y=-1\frac{u.2}{c}, \beta_z=-1\frac{u.3}{c}\right]\right)$;
$\Lambda R := i\Lambda u\Lambda v\Lambda d;$
r $\Lambda R := \Lambda d\Lambda vi\Lambda u;$
rM := $\begin{pmatrix} \Lambda R(2,2) & \Lambda R(2,3) & \Lambda R(2,4) \\ \Lambda R(3,2) & \Lambda R(3,3) & \Lambda R(3,4) \\ \Lambda R(4,2) & \Lambda R(4,3) & \Lambda R(4,4) \end{pmatrix}$;
rrM := $\begin{pmatrix} r\Lambda R(2,2) & r\Lambda R(2,3) & r\Lambda R(2,4) \\ r\Lambda R(3,2) & r\Lambda R(3,3) & r\Lambda R(3,4) \\ r\Lambda R(4,2) & r\Lambda R(4,3) & r\Lambda R(4,4) \end{pmatrix}$;
id := - rM d ;
dr := rr (id, rM r, c) ;
ed := E (dr, id, c) ;
p := rr (v, r, c) ;
ep := E (p, v, c) ;
ev := E (r, [0, 0, 0], c) ;
l := $\frac{dX(p,v)}{sq(v)}v$;

t := $\frac{Gm(v,c)^2}{c^2}\left(dX(l,v)+\sqrt{dX(l,v)^2+c^2\frac{sq(p)}{Gm(v,c)^2}}\right)$;
o := p + tv ;
ro := $\frac{1}{\sqrt{sq(o)}}o$;

$A(ed,u,c)-\frac{1}{Gm(u,c)}\left(ep+\frac{1}{c}cX(u,cX(ro,ep))-\frac{1}{c^2}udX(ep,u)\right)$
$[0.3E-201,-0.3E-201,-0.1E-202]$
Type: Vector(Expression(Float))

Even if increase the calculation precision to 200 digits with other values, we can confirm that it matches exactly up to that limit.

(51) -> digits (20) ;
c := 1 ;
r := vector [- 0.6, 1.0, - 0.7] ;
d := vector [- 0.02, 0.4, - 0.5] ;
v := vector [0.7, - 0.3, 0.1] ;
u := U (v, d, c) ;
$\Lambda v := eval\left(\Lambda M,\left[\beta_x=\frac{v.1}{c},\beta_y=\frac{v.2}{c},\beta_z=\frac{v.3}{c}\right]\right)$;
$\Lambda d := eval\left(\Lambda M,\left[\beta_x=\frac{d.1}{c},\beta_y=\frac{d.2}{c},\beta_z=\frac{d.3}{c}\right]\right)$;
i $\Lambda u := eval\left(\Lambda M,\left[\beta_x=-1\frac{u.1}{c},\beta_y=-1\frac{u.2}{c},\beta_z=-1\frac{u.3}{c}\right]\right)$;
$\Lambda R := i\Lambda u\Lambda v\Lambda d;$
r $\Lambda R := \Lambda d\Lambda vi\Lambda u;$
rM := $\begin{pmatrix} \Lambda R(2,2) & \Lambda R(2,3) & \Lambda R(2,4) \\ \Lambda R(3,2) & \Lambda R(3,3) & \Lambda R(3,4) \\ \Lambda R(4,2) & \Lambda R(4,3) & \Lambda R(4,4) \end{pmatrix}$;
rrM := $\begin{pmatrix} r\Lambda R(2,2) & r\Lambda R(2,3) & r\Lambda R(2,4) \\ r\Lambda R(3,2) & r\Lambda R(3,3) & r\Lambda R(3,4) \\ r\Lambda R(4,2) & r\Lambda R(4,3) & r\Lambda R(4,4) \end{pmatrix}$;
id := rM - d ;
dr := rr (id, rM r, c) ;
ed := E (dr, id, c) ;
p := rr (v, r, c) ;
ep := E (p, v, c) ;
ev := E (r, [0, 0, 0], c) ;
l := $\frac{dX(p,v)}{sq(v)}v$;
t := $\frac{Gm(v,c)^2}{c^2}\left(dX(l,v)+\sqrt{dX(l,v)^2+c^2\frac{sq(p)}{Gm(v,c)^2}}\right)$;
o := p + tv ;

ro := $\frac{1}{\sqrt{sq(o)}} o$;

$A(ed,u,c) - \frac{1}{Gm(u,c)} \left(ep + \frac{1}{c} cX(u, cX(ro, ep)) - \frac{1}{c^2} udX(ep,u)\right)$
$[0.8E-21, -0.8E-21, 0.8E-21]$
Type: Vector(Expression(Float))

Tried another value once more.

And, the acceleration in the u inertial system was converted to the stationary inertial system and compared, but the opposite direction is also possible.

(53) -> A (ed, u, c) - $\left(ep + \frac{1}{c} cX(u, cX(ro, ep)) - \frac{1}{c^2} udX(ep,u)\right) \frac{1}{Gm(u,c)}$,
ed - rA $\left(\left(ep + \frac{1}{c} cX(u, cX(ro, ep)) - \frac{1}{c^2} udX(ep,u)\right) \frac{1}{Gm(u,c)}, u, c\right)$
$Cannotcompilemap : Gm$
We will attempt to interpret the code.

$[[0.8E-21, 0.2E-20, 0.0], [0.8E-21, 0.2E-20, 0.2E-20]]$
Type: Tuple(Vector(Expression(Float)))

The conversion between the u inertial system and the stationary inertial system was compared in both directions, and both directions were 0 to confirm that there was no abnormality.

(54) -> rM rA $\left(\frac{1}{Gm(d,c)} \left(ev - \frac{1}{c^2} ddX(ev,d)\right), d, c\right) - ed,$
$rrMA(ed, id, c) - \frac{1}{Gm(d,c)} \left(ev - \frac{1}{c^2} ddX(ev,d)\right)$

$[[-0.2E-20, -0.7E-20, -0.5E-20], [-0.3E-20, 0.8E-20, 0.0]]$
Type: Tuple(Vector(Expression(Float)))

The conversion relationship between v inertial system and u inertial system was compared. In v inertial system, Qa is stationary and there is no magnetic field, so only the induced acceleration term was entered. When converting between two inertial frames, Wigner rotation must be taken into account.\$\

(55) -> rM rA $\left(\frac{1}{Gm(d,c)} \left(ev + \frac{1}{c} cX(v, cX(r, ev)) - \frac{1}{c^2} ddX(ev,d)\right), d, c\right) - ed,$
$\frac{1}{c} cX(v, cX(r, ev))$
$[[-0.2E-20, -0.7E-20, -0.5E-20], [0.0, 0.0, 0.0]]$
Type: Tuple(Vector(Expression(Float)))

Since the force term $\frac{1}{c}cX(v, cX(r, ev))$ due to the magnetic field is 0, it can be omitted.

Below is the bilateral transformation between the v inertial frame and the resting inertial frame, which is the most complex because it goes through the u inertial frame.

(56) -> A $\left(rMrA\left(\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev,d)\right), d, c\right), u, c\right)$
- $\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right)\frac{1}{Gm(u,c)},$

rrM A $\left(rA\left(\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right)\frac{1}{Gm(u,c)}, u, c\right), id, c\right)$
- $\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev,d)\right)$

$[[-0.8E-21, -0.3E-20, -0.3E-20], [-0.4E-20, 0.8E-20, -0.2E-20]]$

Type: Tuple(Vector(Expression(Float)))

Using the dA function, which implements the more general acceleration conversion formula $\vec{a} = \frac{1}{\gamma^2\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)^3}\left(\left(1+\frac{\vec{v}\cdot\vec{u}'}{c^2}\right)\vec{a}' - \frac{\vec{v}\cdot\vec{a}'}{c^2}\vec{u}' - \frac{\gamma-1}{\gamma}\frac{\vec{a}'\cdot\vec{v}}{v^2}\vec{v}\right)$ presented previously, conversion can be done directly without going through the u inertial frame. The inverse conversion of the dA conversion function requires only changing the sign of v and entering u instead of u′ in d, so there is no need to define it separately.

(57) -> dA $\left(\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev,d)\right), v, d, c\right) - \frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right)$

Cannot compile map: Gm
We will attempt to interpret the code.

$[0.8E-20, -0.2E-20, -0.8E-20]$

Type: Vector(Float)

Of course, the same is true when comparing the value converted from the acceleration in the u inertial frame to the stationary inertial frame and the value directly converted to the stationary inertial frame from the acceleration observed in the v inertial frame.

(58) -> A (ed, u, c) - dA $\left(\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev,d)\right), v, d, c\right)$
$[0.0, 0.3E-20, 0.8E-20]$
Type: Vector(Expression(Float))

The relationship between the acceleration conversion formula A, its inverse conversion formula rA, and the more general acceleration conversion formula dA is as follows.

(59) -> A (ed, u, c) - dA (ed, u, vector [0, 0, 0], c),
rA $\left(\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right), u, c\right)$
- dA $\left(\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right), -u, u, c\right)$
$[[0.0, 0.0, 0.0], [-0.2E-20, -0.3E-20, -0.3E-20]]$
Type: Tuple(Vector(Expression(Float)))

The direct conversion formula dA can be employed to directly transfer and compare the acceleration in the inertial frame v to the stationary inertial frame. Conversely, it can also be used to transfer the acceleration in the stationary inertial frame to the inertial frame v and compare it.

(60) -> dA $\left(\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev,d)\right), v, d, c\right)$
$-\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right)$,

dA $\left(\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep,u)\right), -v, u, c\right)$
$-\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev,d)\right)$
$[[0.8E-21, -0.2E-20, -0.8E-21], [-0.8E-21, 0.3E-20, 0.0]]$
Type: Tuple(Vector(Float))

The acceleration received by the charge Qb due to the electromagnetic field observed in each of the three inertial systems is different because the electromagnetic fields and the inertial masses of Qb in the three inertial systems are all different. However, when that acceleration is relativistically converted to another inertial system, that is, when the acceleration is observed in another inertial system, it is confirmed that it is always consistent with the acceleration received by Qb due to the electromagnetic field in its own inertial system. These results show that the expression of the electromagnetic field is relativistically consistent.

Let the acceleration of Qb observed in a stationary inertial frame be

$$\vec{a}_r = \frac{1}{\gamma_u}\left(\vec{E}_p + \vec{u}\times\vec{B}_p - \frac{\vec{u}}{c^2}(\vec{u}\cdot\vec{E}_p)\right)$$

, the acceleration of Qb observed in a v inertial frame be

$$\vec{a}_v = \frac{1}{\gamma_d}\left(\vec{E}_v - \frac{\vec{d}}{c^2}(\vec{d}\cdot\vec{E}_v)\right)$$

, and the acceleration of Qb observed in a u inertial frame be

$$\vec{a}_u = \vec{E}_d$$

. And, given the acceleration conversion formula

$$A(\vec{a},\vec{u},c) = \frac{1}{\gamma_u^2}\left(\vec{a} + \frac{1-\gamma_u}{\gamma_u}(\vec{a}\cdot\vec{u})\vec{u}\right)$$

, the inverse conversion formula

$$rA(\vec{a},\vec{u},c) = \gamma_u^2(\vec{a} + (\gamma_u - 1)(\vec{a}\cdot\vec{u})\vec{u})$$

, and the direct acceleration conversion formula

$$dA(\vec{a},\vec{u},\vec{d},c) = \frac{1}{\gamma_u^2\left(1+\frac{\vec{u}\cdot\vec{d}}{c^2}\right)^3}\left(\left(1+\frac{\vec{u}\cdot\vec{d}}{c^2}\right)\vec{a} - \frac{\vec{a}\cdot\vec{u}}{c^2}\vec{d} - \frac{\gamma_u - 1}{\gamma_u}\frac{\vec{a}\cdot\vec{u}}{u^2}\vec{u}\right)$$

, the relationship between each acceleration can be summarized in the acceleration conversion formulas as follows. Additionally, rM and rrM represent the Wigner rotation matrix and its inverse rotation matrix, respectively.

(61) -> ar := $\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep, u)\right)$,
av := $\frac{1}{Gm(d,c)}\left(ev - \frac{1}{c^2}ddX(ev, d)\right)$,
au := ed

[[0.1505706189_0053737395, 0.2778490282_0134569344, 0.1873977469_020411621],
[0.2363192506_84838996, 0.3909135356_617518724,0.2767389512_1685508766],
[0.2277506744_1102550144, 0.4056332444_851947955, 0.2772075752_01946528]]

Type: Tuple(Vector(Expression(Float)))

(62) -> A (au, u, c) - ar = dA (au, u, vector [0, 0, 0], c) - ar,
A (rM rA (av, d, c), u, c) - ar = dA (av, v, d, c) - ar,
rA (ar, u, c) - au = dA (ar, - u, u, c) - au,
rM rA (av, d, c) - au = rM dA (av, - d, d, c) - au,
dA (dA (av, v, d, c), - u, u, c) - au,
rrM A (au, - rM d, c) - av = rrM dA (au, - rM d, vector [0, 0, 0], c) - av,
dA (dA (au, u, vector [0, 0, 0], c), - v, u, c) - av,
rrM A (rA (ar, u, c), - rM d, c) - av = dA (ar, - v, u, c) - av

$[[0.8E-21, 0.2E-20, 0.0] = [0.8E-21, 0.2E-20, 0.0], [-0.8E-21, -0.3E-20, -0.3E-20] = [0.8E-21, -0.2E-20, -0.8E-21], [-0.8E-21, -0.2E-20, -0.2E-20] = [0.8E-21, 0.2E-20, 0.2E-20], [-0.2E-20, -0.7E-20, -0.5E-20] = [0.0, -0.3E-20, -0.3E-20], [0.4E-20, -0.2E-20, 0.0], [-0.3E-20, 0.8E-20, 0.0] = [-0.3E-20, 0.8E-20, 0.0], [0.8E-21, 0.5E-20, 0.0], [-0.4E-20, 0.8E-20, -0.2E-20] = [-0.8E-21, 0.3E-20, 0.0]]$

Type: Tuple(Any)

$$\begin{aligned}
\vec{a}_r &= A(\vec{a}_u, \vec{u}, c) = dA(\vec{a}_u, \vec{u}, 0, c) \\
\vec{a}_r &= A(rMrA(\vec{a}_v, \vec{d}, c), \vec{u}, c) = dA(\vec{a}_v, \vec{v}, \vec{d}, c) \\
\vec{a}_u &= rA(\vec{a}_r, \vec{u}, c) = dA(\vec{a}_r, -\vec{u}, \vec{u}, c) \\
\vec{a}_u &= rMrA(\vec{a}_v, \vec{d}, c) = rMdA(\vec{a}_v, -\vec{d}, \vec{d}, c) \\
&= dA(dA(\vec{a}_v, \vec{v}, \vec{d}, c), -\vec{u}, \vec{u}, c) \\
\vec{a}_v &= rrMA(\vec{a}_u, -rM\vec{d}, c) = rrMdA(\vec{a}_u, -rM\vec{d}, 0, c) \\
&= dA(dA(\vec{a}_u, \vec{u}, 0, c), -\vec{v}, \vec{u}, c) \\
\vec{a}_v &= rrMA(rA(\vec{a}_r, \vec{u}, c), -rM\vec{d}, c) = dA(\vec{a}_r, -\vec{v}, \vec{u}, c)
\end{aligned}$$

And, through these processes, in order to consistently describe the field described by the potential theory relativistically, it was proved that the induced acceleration according to the $-\frac{\vec{u}}{c^2}(\vec{u} \cdot \vec{E})$ term, and the force $\vec{u} \times \left(\frac{\hat{r}_\tau}{c} \times \vec{E}\right)$ due to the magnetic field, must exist.

By the way, I just realized that I overlooked something. When calculating the electromagnetic field, I only addressed the case where Qa undergoes uniform rectilinear motion and did not consider cases involving accelerated motion. It would be beneficial to address and supplement this aspect before moving forward.

The formula for the electric field to account for accelerated motion can no longer use the Purcell formula, instead, it should adopt the practical form of the Feynman formula,

$$\vec{E} = \frac{q_a}{4\pi\varepsilon_0 r^2 \left(1 + \frac{\dot{r}}{c}\right)^3} \left(\left(1 - \frac{v^2}{c^2} + \frac{\vec{a} \cdot \vec{r}}{c^2}\right) \left(\hat{r} - \frac{\vec{v}}{c}\right) - \left(1 + \frac{\dot{r}}{c}\right) \frac{r\vec{a}}{c^2} \right)$$

This should be Prepared as a function, excluding all constant terms, and then input it as needed.

(63) -> Ea (r, v, a, c) ==
$$\frac{1}{sq(r)\left(1-\frac{dX(v,r)}{c\sqrt{sq(r)}}\right)^3}\left(\left(1-\frac{sq(v)}{c^2}+\frac{dX(a,r)}{c^2}\right)\left(\frac{1}{\sqrt{sq(r)}}r-\frac{1}{c}v\right)-\left(1-\frac{dX(v,r)}{c\sqrt{sq(r)}}\right)\frac{\sqrt{sq(r)}}{c^2}a\right)$$
Type: Void

And then input the process of calculating the electric fields.

(64) -> digits (20) ;
c := 2 ;
r := vector [0.6, 1.0, 0.7] ;
d := vector [0.02, 0.1, 0] ;
v := vector [- 0.5, 0.2, - 0.1] ;
u := U (v, d, c) ;
$\Lambda v := eval\left(\Lambda M, \left[\beta_x = \frac{v.1}{c}, \beta_y = \frac{v.2}{c}, \beta_z = \frac{v.3}{c}\right]\right)$;
$\Lambda d := eval\left(\Lambda M, \left[\beta_x = \frac{d.1}{c}, \beta_y = \frac{d.2}{c}, \beta_z = \frac{d.3}{c}\right]\right)$;
$\Lambda u := eval\left(\Lambda M, \left[\beta_x = \frac{u.1}{c}, \beta_y = \frac{u.2}{c}, \beta_z = \frac{u.3}{c}\right]\right)$;
i $\Lambda u := eval\left(\Lambda M, \left[\beta_x = -1\frac{u.1}{c}, \beta_y = -1\frac{u.2}{c}, \beta_z = -1\frac{u.3}{c}\right]\right)$;
$\Lambda R := i\Lambda u\Lambda v\Lambda d;$
r $\Lambda R := \Lambda d\Lambda vi\Lambda u;$
rM := $\begin{pmatrix} \Lambda R(2,2) & \Lambda R(2,3) & \Lambda R(2,4) \\ \Lambda R(3,2) & \Lambda R(3,3) & \Lambda R(3,4) \\ \Lambda R(4,2) & \Lambda R(4,3) & \Lambda R(4,4) \end{pmatrix}$;
rrM := $\begin{pmatrix} r\Lambda R(2,2) & r\Lambda R(2,3) & r\Lambda R(2,4) \\ r\Lambda R(3,2) & r\Lambda R(3,3) & r\Lambda R(3,4) \\ r\Lambda R(4,2) & r\Lambda R(4,3) & r\Lambda R(4,4) \end{pmatrix}$;
id := rM - d ;
dr := rr (id, rM r, c) ;
p := rr (v, r, c) ;
l := $\frac{dX(p,v)}{sq(v)}v$;
t := $\frac{Gm(v,c)^2}{c^2}\left(dX(l,v)+\sqrt{dX(l,v)^2+c^2\frac{sq(p)}{Gm(v,c)^2}}\right)$;
o := p + tv ;
ro := $\frac{1}{\sqrt{sq(o)}}o$;
aqv := vector [0.2, 0.1, - 2.3],
aqd := A (rM aqv, id, c),

aqp := A (aqv, v, c),
ep := Ea (o, v, aqp, c) ;
ev := Ea (r, vector [0, 0, 0], aqv, c) ;
tmp := $\Lambda u transpose([cons(ct, o)])$;
ed := Ea ([tmp (2, 1), tmp (3, 1), tmp (4, 1)], id, aqd, c) ;

Cannot compile map: sq
We will attempt to interpret the code.

Type: Vector(Expression(Float))

The additional elements are, first, the Lorentz transformation matrix $\Lambda u\ :=\ eval\left(\Lambda M, \left[\beta_x = \frac{u.1}{c}, \beta_y = \frac{u.2}{c}, \beta_z = \frac{u.3}{c}\right]\right)$ for the transformation to the u inertial frame. Next,

aqv := vector [0.2, 0.1, - 2.3],
aqd := A (rM aqv, id, c),
aqp := A (aqv, v, c),

the acceleration aqv of Qb in the v inertial frame is the information of the electromagnetic field affecting the motion of Qa at the moment under consideration. aqd is the value of this acceleration when observed in the u inertial frame. Wigner rotation is applied. aqp is the acceleration of Qa as seen from the rest inertial frame.

ep := Ea (o, v, aqp, c);
ev := Ea (r, vector [0, 0, 0], aqv, c);

Using the Feynman formula, the electric fields in the stationary inertial frame and the v inertial frame are calculated. o is the path vector from Qa, where the electric field information originates, to Qb. In the v inertial frame, it is simply r.

tmp := $\Lambda u transpose([cons(ct, o)])$;
ed := $Ea([tmp(2, 1), tmp(3, 1), tmp(4, 1)], id, aqd, c)$;

By Lorentz-transforming o with Λ u, the spatial components can be extracted, allowing the determination of the distance vector in the u inertial frame. The velocity of Qa in the u inertial frame at the moment when the electric field information departs is id.

(65) -> ar := $\frac{1}{Gm(u,c)}\left(ep + \frac{1}{c}cX(u, cX(ro, ep)) - \frac{1}{c^2}udX(ep, u)\right)$,
av := $\frac{1}{Gm(d,c)}\left(ev + \frac{1}{c}cX\left(d, cX\left(\frac{1}{\sqrt{sq(r)}}r, ev\right)\right) - \frac{1}{c^2}ddX(ev, d)\right)$,
au := ed

$$[[0.1061811242\_9138400705, 0.2239412002\_4608670304, 0.5380086502\_1665292439],$$
$$[0.1194845433\_8388336285, 0.2397685784\_157310356, 0.5854368139\_6836798568],$$
$$[0.1183498170\_446227071, 0.2422921291\_332650608, 0.5866247876\_5702919082]]$$

Type: Tuple(Vector(Expression(Float)))

ar represents the acceleration experienced by Qb in the stationary inertial frame due to the electromagnetic field. av is the acceleration experienced by Qb in the v inertial frame due to the electromagnetic field. Unlike when Qa is moving at a constant velocity, in the case of Qa's accelerated motion, there is a magnetic field due to this acceleration that must be taken into account. au is the acceleration experienced by Qb due to the electric field in the u inertial frame. In the u inertial frame, Qb is at rest, so there is no need to consider magnetic fields or induced accelerations.

(66) -> A (au, u, c) - ar = dA (au, u, vector [0, 0, 0], c) - ar,
A (rM rA (av, d, c), u, c) - ar = dA (av, v, d, c) - ar,
rA (ar, u, c) - au = dA (ar, - u, u, c) - au,
rM rA (av, d, c) - au = rM dA (av, - d, d, c) - au,
dA (dA (av, v, d, c), - u, u, c) - au,
rrM A (au, - rM d, c) - av = rrM dA (au, - rM d, vector [0, 0, 0], c) - av,
dA (dA (au, u, vector [0, 0, 0], c), - v, u, c) - av,
rrM A (rA (ar, u, c), - rM d, c) - av = dA (ar, - v, u, c) - av

$$[[-0.3E-20, -0.8E-21, -0.3E-20] = [-0.3E-20, -0.8E-21, -0.3E-20], [0.2E-20, 0.5E-20, 0.7E-20] = [-0.4E-21, 0.8E-21, 0.0], [0.3E-20, 0.0, 0.3E-20] = [0.6E-20, 0.4E-20, 0.1E-19], [0.6E-20, 0.7E-20, 0.1E-19] = [0.6E-20, 0.7E-20, 0.7E-20], [0.6E-20, 0.6E-20, 0.1E-19], [-0.4E-20, -0.4E-20, -0.1E-19] = [-0.4E-20, -0.4E-20, -0.1E-19], [-0.4E-20, -0.3E-20, -0.1E-19], [-0.8E-21, -0.4E-20, -0.7E-20] = [-0.4E-21, -0.2E-20, -0.3E-20]]$$

Type: Tuple(Any)

I checked all possible transformations of acceleration between different inertial frames at once and confirmed that they all cancel out.