EMTH210 Tutorial 1: Revision

For the week starting Monday 17 February.

1. Differentiate the following functions with respect to t.

   (a) $(t-4)e^{3t}$

   (b) $(t^2+t)\cos(5t)$

   (c) $\sin(t^2)\ln(t^3)$

   (d) $\cos(3t)\sin(2t)$

   (e) $\Big(\sin\big(\ln(t)\big)\Big)^2$

2. Solve $\exp(x^2) = 8\exp(-x)$ for x.

3. Write the following equations in the form $(s+a)^2 \pm b = 0$ with appropriate choices for the constants a and b, and the $\pm$ sign. Solve for s.

   (a) $s^2 + 4s + 20 = 0$

   (b) $\dfrac{2s+1}{s-3} = \dfrac{s+1}{s-1}$

   (c) $\ln(s+5) = \ln(5) - \ln(s+1)$

4. (a) Find and sketch the level curve of $f(x,y) = 3x^2 + 2y^2$ that goes through $(2,3)$.
   (b) Find and sketch the level curve of $f(x,y) = y^2 - 2x$ that goes through $(6,4)$.

5. Solve $s^2Y + 5sY + 4Y = 2s + 5$ for Y and then use partial fraction decomposition to write Y as the sum of two terms.

6. In this question $i^2 = -1$.

   (a) Simplify $(1+2i)(1-2i)$.

   (b) Write $\dfrac{i}{1-2i}$ in the form $a+bi$.

   (c) Write $\exp\left(\dfrac{i\pi}{6}\right)$ in the form $a+bi$.

7. Use implicit differentiation to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the following functions.

   (a) $x^3 + 5x^2y + 2y^2 \;=\; 0$ (b) $x^2y^3 - xy \;=\; 6$ (c) $xe^{xy} - y^2 \;=\; 4x$

8. Solve the following systems of linear equations.

   (a)
$$\begin{pmatrix} 2 & 1 & 2 \\ 4 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 3 \end{pmatrix}$$

   (b)
$$\begin{pmatrix} -1 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

9. Find the partial fractions expansions of the following expressions.

(a) $\dfrac{1}{s^2(s+1)}$ (b) $\dfrac{3s^2-1}{s(s^2-1)}$ (c) $\dfrac{s^2+6s}{(s+4)(s^2+4)}$

10. Use the double angle formula

$$\cos(2x) = 2\cos^2 x - 1$$

to find the indefinite integral of $\cos^2(x)$.

11. Find the following indefinite integrals.

(a) $\displaystyle\int t\sin(2t)\,\mathrm{d}t$

(b) $\displaystyle\int \ln(t)\,\mathrm{d}t$

(c) $\displaystyle\int \frac{4t}{t^2-1}\,\mathrm{d}t$

(d) $\displaystyle\int 6t\sqrt{4-t^2}\,\mathrm{d}t$

(e) $\displaystyle\int 12te^{3t}\,\mathrm{d}t$

(f) $\displaystyle\int \frac{\cos(t)}{\sin(t)}\,\mathrm{d}t$

(g) $\displaystyle\int \frac{(2+t)^2}{1+t^2}\,\mathrm{d}t$

12. Use the two trigonometric identities

- $2\sin(A)\sin(B) = \cos(A-B) - \cos(A+B)$
- $2\sin(A)\cos(B) = \sin(A+B) + \sin(A-B)$

to evaluate

(a) $\displaystyle\int_{-\pi}^{\pi} \sin(x)\sin(3x)\,\mathrm{d}x$ (b) $\displaystyle\int_{-\pi}^{\pi} \cos(x)\sin(2x)\,\mathrm{d}x$

13. Use $e^{ix} = \cos(x) + i\sin(x)$ to show that

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}.$$