

EMTH210 Tutorial 9: Laplace Transforms

For the week starting Monday 11 May.

The homework questions this week are **6(a)** and **9(a)**.

1. Use the definition of the Laplace Transform to find the transforms of the following functions.

$$(a) f(t) = \begin{cases} -1 & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases} \quad (b) f(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$$

2. Find the Laplace transforms of the following functions.

$$(a) f(t) = e^{t+8} \quad (b) f(t) = 1 + 4t - 2e^t \quad (c) f(t) = 12t^5$$

3. Find the inverse Laplace transforms of the following functions.

$$(a) F(s) = \frac{1}{s^3} \quad (c) F(s) = \frac{1}{2s+1} \quad (e) F(s) = \frac{7s+3}{s^2+9}$$
$$(b) F(s) = \frac{(s+1)^2}{s^3} \quad (d) F(s) = \frac{7}{s^2+36} \quad (f) F(s) = \frac{1}{s^2-16}$$

4. Find the Laplace transforms of the following functions. For all functions, assume $t \geq 0$.

$$(a) f(t) = t^2 e^{3t} \quad (b) f(t) = e^{3t} \sin(2t) \quad (c) f(t) = e^{3t} \cos(2t)$$

5. Find the inverse Laplace transforms of the following functions.

$$(a) F(s) = \frac{1}{(s-3)^2} \quad (c) F(s) = \frac{s}{(s-2)^2-4} \quad (e) F(s) = \frac{4}{s^2(s+1)}$$
$$(b) F(s) = \frac{1}{(s-2)^2-4} \quad (d) F(s) = \frac{2s+5}{s^2+6s+10} \quad (f) F(s) = \frac{s^2+1}{s(s+1)(s-1)}$$

6. Solve the following differential equations by Laplace Transforms.

$$(a) \text{ (Homework) } \frac{dy}{dt} + y = 1, \quad y(0) = 0.$$

$$(b) \frac{dy}{dt} + 3y = e^{-3t}, \quad y(0) = 6.$$

$$(c) \frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 0, \quad y(0) = 1 \quad \text{and} \quad y'(0) = 0.$$

7. Solve the following differential equations by Laplace Transforms.

$$(a) \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = t, \quad y(0) = 0 \quad \text{and} \quad y'(0) = 1.$$

$$(b) \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 1 + t, \quad y(0) = 0 \quad \text{and} \quad y'(0) = 4.$$

8. Find the Laplace transforms of the following functions using the second shift theorem. In each case sketch the function.

(a) $f(t) = (t - 1) H(t - 1)$

(b) $f(t) = H(t - 3) - H(t - 5)$

(c) $f(t) = e^t H(t - 2)$

(d) $f(t) = \sin(t) H(t - 2\pi)$

(e) $f(t) = t H(t - 3)$

(f) $f(t) = (1 + t)(H(t - 2) - H(t - 5))$

(g) $f(t) = \begin{cases} -1 & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$

(h) $f(t) = \begin{cases} t & 0 \leq t < 1 \\ 1 & t \geq 1 \end{cases}$

(i) $f(t) = \begin{cases} e^t + t & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$

9. Solve the following differential equations.

(a) **(Homework)** $\frac{dy}{dt} + y = 4t H(t - 2), \quad y(0) = 0.$

(b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = H(t - 2) - H(t - 3), \quad y(0) = 0 \quad \text{and} \quad y'(0) = 0.$