Structural Finite Element Model Updating Using Ambient Vibration Test Results

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Abstract: This paper presents a practical and user-friendly finite element (FE) model updating technique for real structures using ambient vibration test results. The first case study of a simulated simply supported beam demonstrates a comparative study of the influence of different possible residuals in objective function. Frequency residual only, mode shape related function only, modal flexibility residual only, and their combinations are studied independently. In view of tuning as well as damage localization, full objective function that considers all three residuals is the best for FE updating. This objective function is implemented in a second case study of a real concrete-filled steel tubular arch bridge. The bridge was tested by ambient vibration measurements. Followed by the three-dimensional FE modeling of the bridge, an eigenvalue sensitivity study is carried out to see the most sensitive parameters to the concerned modes. FE model mass matrix obtained from Guyan reduction technique is used to the mass normalization of the mode shapes extracted from ambient modal test to calculate the modal flexibility. The updated FE model of the bridge is able to produce a sufficient improvement on modal parameters of the concerned modes which is in close agreement with the experimental results still preserving the physical meaning of updated parameters.

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Introduction

In modern analysis of structural dynamics, much effort is devoted to the derivation of accurate models. These accurate models are used in many applications of civil engineering structures like damage detection, health monitoring, structural control, structural evaluation, and assessment. In the development of finite element (FE) models of structures, it is usual to make simplifying assumptions. The FE model of a structure is constructed on the basis of highly idealized engineering blueprints and designs that may or may not truly represent all the physical aspects of an actual structure. When field dynamic tests are performed to validate the analytical model, inevitably their results, commonly natural frequencies and mode shapes, do not coincide with the expected results from the analytical model. These discrepancies originate from the uncertainties in simplifying assumptions of structural geometry, materials, as well as inaccurate boundary conditions. The problem of how to modify the analytical model from the dynamic measurements is known as the model updating in structural dynamics. Basically, the purpose of model updating is to modify the mass, stiffness, and damping parameters of the numerical model in order to obtain better agreement between numerical and test results.

A number of model updating methods in structural dynamics have been proposed (Mottershead and Friswell 1993; Friswell and Mottershead 1995; Maia and Silva 1997; Link 1999). Noniterative methods that directly update the elements of stiffness and mass matrices are one-step procedures (Baruch and Bar-Itzhack 1978; Berman and Nagy 1983). In the direct method, the resulting updated matrices reproduce the measured structural modal properties exactly but do not generally maintain structural connectivity and the corrections suggested are not always physically meaningful. The iterative parameter updating method involves using the sensitivity of the parameters to update the model (Friswell and Mottershead 1995; Link 1999). This sensitivity-based parameter updating approach has an advantage of identifying parameters that can directly affect the dynamic characteristics of the structure.

The selection of the objective function has a profound impact on the problem. It not only affects the interpretation of the best correlation, but also influences the behavior of the utilized optimization algorithm. The objective function is normally built up by the residuals between the measurement results and the numerical predictions. There are commonly three expressions possibly used for this purpose, which are frequency residual, mode shape related function, and modal flexibility residual. In addition to the eigenvalue residual and consideration of the modal assurance criterion (MAC) related function, the residual vector containing the deviation from the orthogonality of the experimental mode shapes to the analytical ones is also used in literature (Teughels et al. 2001). Most of the sensitivity-based approaches reported for FE updating of real case studies have considered only the eigenvalue

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(frequency) residual (Brownjohn and Xia 2000; Zhang et al. 2000, 2001).

The main objective of this paper is to present a practical and user-friendly sensitivity-based FE model updating technique in structural dynamics for real structures using ambient vibration test results. The objective function consisting of a combination of eigenvalue residual, mode shape related function, and modal flexibility residual is minimized using the least-squares algorithm which is the main contribution of this work. The first case study of a simulated beam demonstrates a comparative study of the influence of different possible residuals in the objective function for FE model updating and detection of damaged elements in structural dynamics problems. The simulated simply supported beam without damage and with several assumed damage elements are considered. Frequency residual only, mode shape related function only, modal flexibility residual only, and their combinations are studied independently, each having three cases depending upon the number of updating parameters. It is observed that the tuning on modal parameters alone can be achieved even if any one residual explained in this paper is used in the objective function. However, in the damage detection and localization part, the introduction of a modal flexibility residual in the objective function with other residuals considerably improves the detection capability of the updating procedure, which supports the fact that the modal flexibility term is more sensitive to the local changes of structures. In view of tuning as well as damage detection, the objective function considering frequency residual, MAC related function, and flexibility residual is the best for FE updating.

The objective function developed from the first case study is implemented in the second case study of a real concrete-filled steel tubular arch bridge. The bridge was tested by the ambient vibration technique. Followed by the three-dimensional FE modeling of the bridge, an eigenvalue sensitivity study is carried out to see the most sensitive parameters to the concerned modes. To use a modal flexibility residual in an objective function, the ambient vibration-based mode shapes need to be normalized to mass. The Guyan-reduced mass normalization technique (Doebling and Farrar 1996) is used for this purpose. It is demonstrated that the updated bridge FE model is able to produce a sufficient improvement on modal parameters of the concerned modes which are in close agreement with the experimental results and still preserve the physical meaning of updated parameters. Successful updating of the real bridge demonstrates that, even for a big model, the cost of calculation is not too high and this method can be practical for daily use by engineers.

Theoretical Background

Objective Functions and Constraints

An objective function Π reflects the deviation between the analytical prediction and the real behavior of a structure. The FE model updating can be posed as a minimization problem to find x^* design set such that

$$\begin{gathered}\Pi(x^*) \leqslant \Pi(x), \quad \forall\, x \\ x_{li} \leqslant x_i \leqslant x_{ui}, \quad i = 1,2,3,\ldots,N\end{gathered} \tag{1}$$

where the upper and lower bounds on the design variables should be set. The general objective function formulated in terms of the discrepancy between finite element and experimental eigenvalues and mode shapes are, respectively, shown below.

$$\Pi_1(x) = \sum_{i=1}^{m} \alpha_i \left(\frac{\lambda_{ai} - \lambda_{ei}}{\lambda_{ei}}\right)^2, \quad 0 \leqslant \alpha_i \leqslant 1 \tag{2}$$

$$\Pi_2(x) = \sum_{i=1}^{m} \beta_i f_i, \quad 0 \leqslant \beta_i \leqslant 1 \tag{3}$$

where α_i and β_i = weight factors to impose a relative difference between eigenvalue and mode shape deviations, respectively, because these entities may have been measured with different accuracy. λ_{ai} and λ_{ei} = finite element and experimental eigenvalue of the ith mode, respectively. f_i = mode shape related function. After trying several expressions, Moller and Friberg (1998) proposed the following expression:

$$f_i = f(\mathrm{MAC}_i) = \frac{(1 - \sqrt{\mathrm{MAC}_i})^2}{\mathrm{MAC}_i} \tag{4}$$

Modal assurance criterion (MAC) is defined by (Allemang and Brown 1982)

$$\mathrm{MAC}_i = \frac{(\phi_{ai}^T \phi_{ei})^2}{(\phi_{ai}^T \phi_{ai})(\phi_{ei}^T e_{ei})} \tag{5}$$

where ϕ_{ai} = analytical eigenmode that has been paired with the ith experimental mode ϕ_{ei}. It has been reported that the modal flexibility is more sensitive to local damage than the mode shapes and natural frequencies (Toksoy and Aktan 1994; Zhao and DeWolf 2002). The modal flexibility is the accumulation of the contribution from all available mode shapes and corresponding natural frequencies. The modal flexibility matrix $[F]_{n\times n}$ is defined as (Hoyos and Aktan 1987)

$$[F]_{n\times n} = [\Phi]_{n\times m} \left[\frac{1}{\omega^2}\right] [\Phi]_{n\times m}^T \tag{6}$$

in which $[\Phi]$ = mode shape matrix and ω = circular frequency. Similarly, n and m = number of the measurement degrees of freedom (DOFs) and number of mode shapes considered, respectively. If the deflection vector u_i under uniformly distributed unit load, called the uniform load surface (ULS), is defined, the objective function Π_3 considering the flexibility residual can be presented as

$$u_i = \sum_{k=1}^{m} \frac{(\Phi_{ik}) \sum_{j=1}^{n} (\Phi_{kj})}{\omega_k^2} \tag{7}$$

$$\Pi_3(x) = \frac{m}{n} \cdot \sum_{j=1}^{n} \left[\frac{u_{aj} - u_{ej}}{u_{ej}}\right]^2 \tag{8}$$

where u_{aj} and u_{ej} = analytical and experimental uniform load surface, respectively. It is necessary to have mass-normalized mode shapes to use the measured flexibility matrix in the FE updating which is the major drawback of the procedure. For a force vibration test, the mass normalization can be implemented from the driving point inertance measurement. However, for a modal test that uses an ambient excitation source, the mass-normalized mode shapes are not straightforward. To realize this, the Guyan-reduced mass normalization technique is used in the paper (Doebling and Farrar 1996).

$$\Phi_{ij} = \frac{\varphi_{ij}}{\sqrt{\{\varphi_i\}^T [M] \{\varphi_i\}}} \tag{9a}$$

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$$\Phi_{ij} = \frac{\varphi_{ij}}{\sqrt{\sum_{k=1}^{n} m_k \varphi_{kj}^2}} \tag{9b}$$

This method uses a FE model mass matrix, reduced to measured DOFs, to normalize the mode shapes obtained from ambient vibration measurements. The reduction is performed according to Guyan (1965), which assumes that the inertial forces at the eliminated degree of freedom are negligible. This assumption typically makes this method valid for only the lower frequency modes. The expression shown in Eq. (9*a*) is an especially convenient normalization for a general system and for a system having a diagonal mass matrix, it may be written as shown in Eq. (9*b*).

Eqs. (2), (3), and (8) are the objective functions considering frequency residual only, mode shape related function only, and modal flexibility residual only. Hence full objective function is their combination with the constraints to be imposed on objective functions.

$$\Pi(x) = \Pi_1(x) + \Pi_2(x) + \Pi_3(x) \tag{10}$$

$$0 \leqslant |\lambda_{ai} - \lambda_{ei}| \leqslant \mathrm{UL} \tag{11}$$

$$L_1 \leqslant \mathrm{MAC} \leqslant 1 \tag{12}$$

where UL=upper limit whose value can be set as absolute error of the ith eigenvalue and L_1 represents the lower limit to constrain the MAC value.

Sensitivity Analysis

Sensitivity analysis is carried out to see the most sensitive parameters for FE updating. Sensitivity analysis computes the sensitivity coefficient S_j which is defined as

$$\delta z = S_j \delta x \tag{13a}$$

$$S_j = \frac{\delta z}{\delta x} = \frac{z_m - z_j}{x - x_j} \tag{13b}$$

In Eq. (13*b*), the numerator shows the change in the measured output and the denominator shows the perturbation in the parameter. The sensitivity matrix can be computed for all related properties (material, geometrical, boundary, etc.) by using direct derivation or perturbation techniques. In this paper, the eigenvalue sensitivity matrix is approximated using the forward difference of the function with respect to each parameter considered.

$$\frac{\partial \lambda_r}{\partial x_i} = \frac{\lambda(x + \Delta x_i e) - \lambda(x)}{\Delta x_i} \tag{14}$$

$$\Delta x_i = \frac{\Delta D}{100}(\overline{x_i} - \underline{x}_i) \tag{15}$$

where ΔD=forward difference step size (in percent), taken to be 0.2 in this study and $\overline{x_i}$, $\underline{x}_i$=upper and lower limit for the design variable x.

Optimization Techniques

Finite element model updating is carried out to solve a minimization problem whose aim is the minimization of the objective function Π under the constraints. Design variables are subjected to constraints with upper and lower limits, that is

$$x = [x_1\ x_2\ x_3 \cdots x_N] \tag{16a}$$

$$\underline{x}_i \leqslant x_i \leqslant \overline{x_i} \quad (i = 1,2,3,\ldots,N) \tag{16b}$$

Then the constrained optimization problem can be formed as follows:

$$\text{Minimize} \quad \Pi = \Pi(x) \tag{16c}$$

Subjected to

$$g_i(x) \leqslant \overline{g_i} \quad (i = 1,2,3,\ldots,m_1) \tag{16d}$$

$$\underline{h}_j \leqslant h_j(x) \quad (j = 1,2,3,\ldots,m_2) \tag{16e}$$

$$\underline{w}_k \leqslant w_k(x) \leqslant \overline{w_k} \quad (k = 1,2,3,\ldots,m_3) \tag{16f}$$

where g_i, h_j, and w_k represent the state variables containing the design, with the under bar and over bar representing lower and upper bounds, respectively, and $m_1+m_2+m_3$=number of state variables constrained with various upper and lower limits.

There are several techniques available to solve the constrained optimization problem. Two methods, subproblem approximation method and first order optimization method (Powell 1964; Denn 1969; Morris 1982), are utilized in this paper. In these optimization algorithms, the penalty function concept is used. Penalty function methods generally use a truncated Taylor series expansion of the modal data in terms of unknown parameters. Three main steps of the subproblem optimization method are described as follows:

1. The dependent variables are first replaced with approximations as shown below using notation ˆ by means of least-squares fitting.

$$\hat{\Pi}(x) = \Pi(x) + \text{error} \tag{17a}$$

$$\hat{g}(x) = g(x) + \text{error}$$

$$\hat{h}(x) = h(x) + \text{error} \tag{17b}$$

$$\hat{w}(x) = w(x) + \text{error}$$

$$\hat{\Pi} = a_0 + \sum_i^n a_i x_i + \sum_i^n \sum_j^n b_{ij} x_i x_j \tag{18}$$

The most complex form that the approximations can take on is a fully quadratic representation with cross terms as shown in Eq. (18), where a_i and b_{ij}=coefficients whose value is determined by the weighted least-squares technique. With the function approximation available, the constrained minimization problem can be recast as follows:

$$\text{Minimize} \quad \hat{\Pi} = \Pi(x) \tag{19a}$$

subject to

$$\underline{x}_i \leqslant x_i \leqslant \overline{x_i} \quad (i = 1,2,3,\ldots,n)$$

$$\hat{g}_i(x) \leqslant \overline{g_i} + A_i \quad (i = 1,2,3,\ldots,m_1)$$

$$\underline{h}_j - B_j \leqslant \hat{h}_j(x) \quad (j = 1,2,3,\ldots,m_2) \tag{19b}$$

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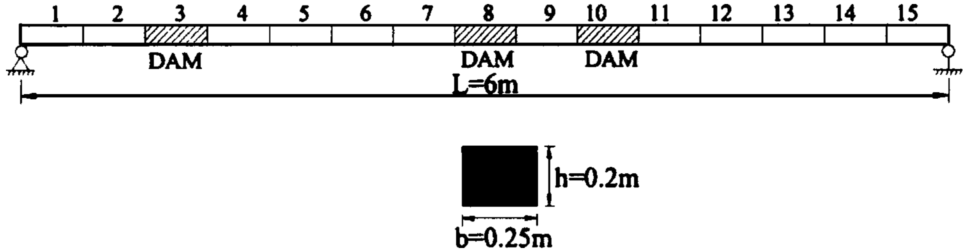

Fig. 1. Simulated simply supported beam

$$\underline{w}_k - C_k \leqslant \hat{w}_k(x) \leqslant \overline{w_k} + C_k \quad (k = 1,2,3, \ldots, m_3)$$

where A_i, B_j, C_k represent the state variables related parameters after function approximation available during optimization.

2. The constrained minimization problem in Eq. (19*a*) is converted to the unconstrained problem using penalty functions leading to the following subproblem statement:
Minimize

$$F(x,P_k) = \hat{\Pi} + \Pi_0 P_k \left[\sum_{i=1}^{n} X(x_i) + \sum_{i=1}^{m_1} G(\hat{g}_i) + \sum_{j=1}^{m_2} H(\hat{h}_j) + \sum_{k=1}^{m_3} W(\hat{w}_k) \right] \quad (20)$$

where $F(x,p_k)$ represents the unconstrained objective function that varies with the design variables and parameter P_k. X=penalty function used to enforce design variable constraints, G,H,W=penalty functions for the state variable constraints. A sequential unconstrained minimization technique (Fiacco and McCormick 1968) is used to solve Eq. (20) at each design iteration.

3. Minimization is then performed at every iteration on the approximated penalized function until convergence is achieved. Convergence is assumed when either the present design set $x^{(j)}$, or the previous design set $x^{(j-1)}$, or the best design set $x^{(b)}$ is feasible and one of the following conditions is satisfied.

$$|\Pi^{(j)} - \Pi^{(j-1)}| \leqslant \tau \quad (21a)$$

$$|\Pi^{(j)} - \Pi^{(b)}| \leqslant \tau \quad (21b)$$

$$|x_i^{(j)} - x_i^{(j-1)}| \leqslant \rho_i \quad (i = 1,2,3, \ldots, n) \quad (21c)$$

$$|x_i^{(j)} - x_i^{(b)}| \leqslant \rho_i \quad (i = 1,2,3, \ldots, n) \quad (21d)$$

Eqs. (21*a*) and (21*b*) correspond to a difference in objective function values, and Eqs. (21*c*) and (21*d*) to design variable difference. If the satisfaction of Eqs. (21*a*)–(21*d*) is not realized, then termination occurs. With regard to the first order optimization method, three major steps involved are shown below.

1. The constrained problem statement expressed in Eq. (16) is transformed into an unconstrained one using penalty functions. An unconstrained form of Eq. (16) is formulated as follows.

$$Q(x,q) = \frac{\Pi}{\Pi_0} + \sum_{i=1}^{n} P_x(x_i) + q\left[\sum_{i=1}^{m1} P_g(g_i) + \sum_{j=1}^{m2} P_h(h_j) + \sum_{k=1}^{m3} P_w(w_k) \right] \quad (22)$$

where $Q(x,q)$=dimensionless unconstrained objective function; P_x, P_g, P_h, P_w=penalties applied to the constrained design and state variables, and Π_0 refers to the objective function value that is selected from the current group of design sets.

2. Derivatives are formed for the objective function and the state variable penalty functions leading to the search direction in design space. For each optimization iteration (j) a search direction vector $d^{(i)}$ is devised. The next iteration $(j+1)$ is obtained from Eq. (23). In this equation, measured from $x^{(j)}$, the line search parameter S_j corresponds to the minimum value of Q in the direction $d^{(j)}$.

$$x^{(j+1)} = x^{(j)} + S_j d^{(j)} \quad (23)$$

3. Various steepest descent and conjugated direction searches are performed during each iteration until the convergence is reached. Convergence is assumed when comparing the current iterations design set (j) to the previous $(j+1)$ set and the best (b) set as shown in Eq. (24), in which τ=objective function tolerance.

$$|\Pi^{(j)} - \Pi^{(j-1)} \leqslant \tau| \text{ and } |\Pi^{(j)} - \Pi^{(b)} \leqslant \tau| \quad (24)$$

Case Studies

Case 1—Simulated Simply Supported Beam

A simulated simply supported beam is aimed at demonstrating a comparative study of the influence of different possible residuals on the objective function for the FE model updating and their sensitivity to the detection of damaged elements. The simulated simply supported beam with a length of 6 m is discretized as shown in Fig. 1. The density and modulus of elasticity of the beam are 2,500 kg/m^3 and 3.2×10^4 MPa, respectively. Similarly, the area and moment of inertia of the cross section are 0.05 m^2 and 1.66×10^{-4} m^4, respectively. Modal analysis is first carried out to get the FE frequency and mode shapes. To get the simulated experimental modal parameters, three damage locations are assumed in the beam as shown in Fig. 1 where the elastic modulus and moment of inertia of beam elements 3, 8, and 10 are reduced by 20, 50, and 30%, respectively. The modal analysis is

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Table 1. Frequencies and Modal Assurance Criterion (MAC) of Simulated Beam before Updating

Mode	Frequencies of undamaged beam (Hz)	Frequencies of damaged beam (Hz)	Differences in frequencies (%)	MAC values
1	8.990	7.257	23.8	0.995
2	35.915	33.683	6.62	0.994
3	80.632	68.706	17.35	0.962
4	142.93	131.1	9.02	0.977
5	149.14	141.2	5.62	0.998
6	222.53	193.56	14.96	0.903
7	319.16	305.21	4.57	0.977
8	432.53	382.46	13.09	0.924
9	449.05	420.96	6.67	0.993
10	562.42	522.3	7.68	0.943

again carried out on this damaged beam to get the assumed experimental modal parameters. The initial values of frequencies and corresponding errors and MAC of the first ten modes selected in this study are shown in Table 1. The maximum error that appeared in frequency is 23.8% and the minimum MAC value is 0.903.

Updating of the FE model of the undamaged beam is to correlate the modal parameters with the damaged beam and to identify the damage severity and location. The moment of inertia and elastic modulus of individual elements are chosen as updating parameters (UPs). The numbers of UPs are selected with respect to the numbers of modes considered according to Eq. (13*a*) where three cases are considered:

1. UPs = 6 case where *E* and *I* of damaged elements 3, 8, and 10 are selected as UPs and numbers of UPs are less than the mode numbers considered. It is the case that the structural damaged locations are exactly known.
2. UPs = 10 case where *E* and *I* of elements 5 and 13 in addition to damaged elements 3, 8, and 10 are selected as UPs and numbers of UPs are equal to the mode numbers considered. It is the case that the structural damaged locations are partly known.
3. UPs = 30 case where *E* and *I* of all 15 elements are selected as UPs and numbers of UPs are larger than the mode numbers considered. It is the case that the structural damaged locations are not known.

Four different cases of objective function consisting of frequency residual only, MAC related function only, flexibility residual only, and combination of frequency, MAC, and flexibility residuals are studied independently, each having three cases depending upon the number of updating parameters. The first order method is used to do the optimization and results are also checked by using the subproblem method so that the algorithm does not trap in local minima. The tuning results of frequency and MAC with the objective function considering the combination of frequency residual, MAC related function, and flexibility residuals for the conceived three UPs cases are shown in Fig. 2. It is shown that there is significant improvement on the tuning of frequencies in the three cases laying all points of frequencies in the diagonal line [Fig. 2(a)]. It is also demonstrated that there is enough improvement on MAC values with more than 0.96 for each mode [Fig. 2(b)].

In the three cases of updating parameters with four different objective functions, the maximum errors in frequency tuning and minimum MAC values within the first 10 modes as well as maximum error in damage detection are shown in Table 2. It is clearly seen in each case of four objective functions that as the number of UPs in the FE model goes on increasing, tuning on the frequency and MAC values go on decreasing. Similarly, an accurate identification of damage location and severity becomes difficult if a large number of parameters and elements are selected. It is observed that the tuning on modal parameters alone can be achieved even if any one residual explained in this paper is used in the objective function. However, in the damage detection part, the introduction of a modal flexibility residual in the objective function with other residuals considerably improves the damage detection, which supports the fact that the modal flexibility term is sensitive to local damage. For example, when the number of updating parameter is 30, maximum error in damage detection in the last case is 18.3% which is considerably less than the remaining three cases as shown in Table 2. Therefore, in view of tuning as well as damage detection, the objective function considering frequency residual, MAC related function, and flexibility residual is the best for FE updating.

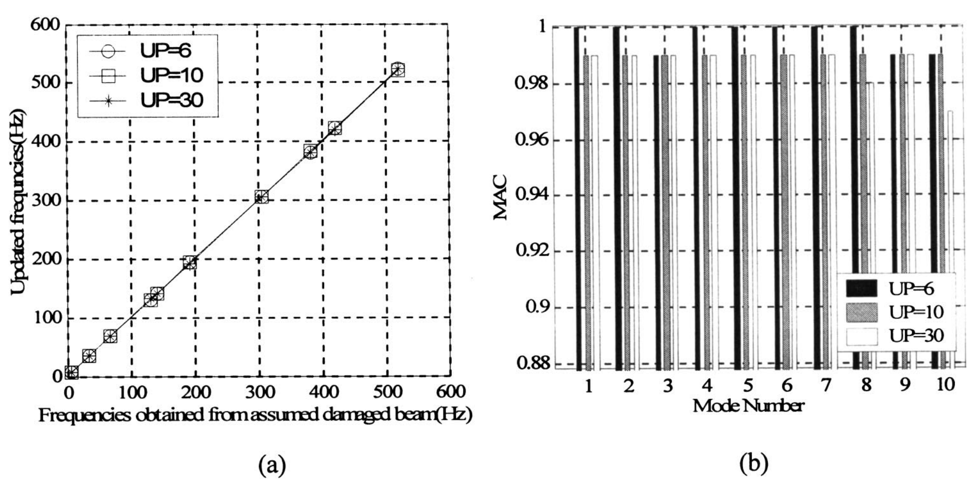

Fig. 2. Correlation of simulated beams with frequency residual, Modal Assurance Criterion (MAC) function, and flexibility residual in objective function

Table 2. Results of Simulated Beam after Updating with Different Residuals in Objective Function

Residuals in objective function	Updating parameters	Maximum error (%) in frequency tuning	Minimum values in MAC tuning	Maximum error (%) in damage detection
Frequency residualonly	6	0.10	0.99	3.50
	10	0.30	0.98	18.5
	30	3.5	0.97	26.7
MAC residual only	6	0.18	0.99	21.1
	10	0.20	0.99	22.5
	30	0.42	0.99	23.1
Flexibility residualonly	6	0.29	0.99	3.23
	10	0.40	0.99	14.9
	30	2.36	0.97	24.5
Frequency+MAC + flexibility	6	0.26	0.99	3.18
	10	0.27	0.99	12.3
	30	1.29	0.97	18.3

Case 2—Concrete-Filled Steel Tubular Arch Bridge

Bridge Description

The target Beichuan River Bridge, located in Xining city, the Capital of Ningxia Province, China, is a half-through arch bridge, with the span of 90 m as shown in Fig. 3. The cross-sectional dimension of two main concrete-filled steel tubular arch ribs is 650×10 mm. The remaining connecting tubes of superstructure are hollow steel tubes. There are 32 main suspenders of steel wire ropes that are vertically attached on the main arch rib and the floor system is suspended through it. The floor system consists of a 250 mm thick concrete slab supported by cross girders at a spacing of 5 m center-to-center. The typical rectangular cross section of the cross girder is 0.36×1.361 m. The main arch ribs are fixed at two abutments and connected by prestressed strands in the longitudinal direction.

Finite Element Modeling

A three-dimensional linear elastic finite element model of the bridge was constructed using *ANSYS*(*ANSYS 1999*). The arch members, cross girders, and bracing members were modeled by two-node beam elements. All suspenders were modeled by the truss elements. The slab of the bridge was modeled as shell elements. In order to simulate the behavior of connection between the cross girder and bridge deck in the transverse direction of the bridge, spring element is used. The value of spring stiffness is assumed as 500,000 N/m based on the previous experience on a similar bridge. Totally 3,120 nodes, 3,446 elements, and 14,060 active degrees of freedom were recognized in the FE model as shown in Fig. 4.

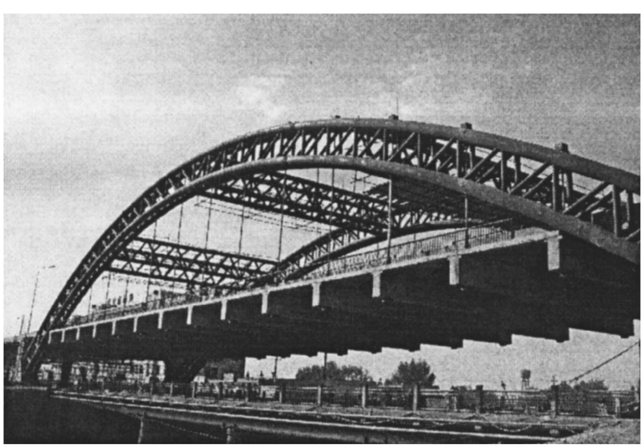

Fig. 3. Beichuan River concrete-filled steel tubular arch bridge

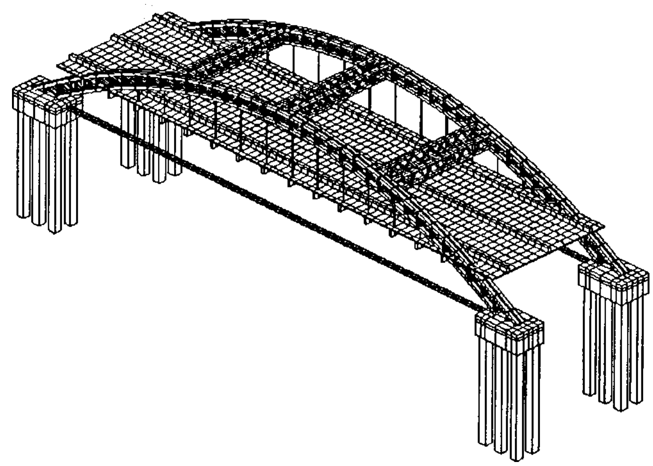

Fig. 4. Three-dimensional finite element model of the bridge

Ambient Vibration Testing and Modal Parameter Identification

On-site dynamic testing of a structure provides an accurate and reliable description of its correct dynamic characteristics. Compared with traditional forced vibration testing, the ambient vibration testing using natural or environmental vibrations induced by traffic, winds, and pedestrians is more challenging to the dynamic testing of bridges. Ambient vibration tests have an advantage of being inexpensive since no equipment is needed to excite the bridge. It corresponds to the real operating condition of the bridge. The service state need not have to be interrupted to use this technique. However, relatively long records of response measurements are required and the signal levels are considerably low in ambient vibration testing.

Just prior to officially opening, the field dynamic testing on the Beichuan River arch bridge was carried out using the method of ambient vibration. The equipment used for the tests included accelerometers, signal cables, and a 32-channel data acquisition system with signal amplifier and conditioner. The force-balance (891-IV type) accelerometers and INV306 data acquisition system were used. Accelerometers convert the ambient vibration responses into electrical signals. Cables are used to transmit these signals from sensors to the signal conditioner. A signal conditioner unit is used to improve the quality of the signals by removing undesired frequency contents (filtering) and amplifying the signals. The amplified and filtered analog signals are converted to digital data using an analog-to-digital (A/D) converter. The signals converted to digital form are stored on the hard disk of the data acquisition computer.

Measurement points were chosen to both sides of the bridge at a location near the joint of the suspenders and deck. As a result, a total of 32 locations (16 points per side) were selected. The accelerometers were installed on the surface of the bridge in the vertical and transverse directions. Four test setups for vertical measurements and four test setups for transverse measurements were conceived to cover the planned testing locations of the

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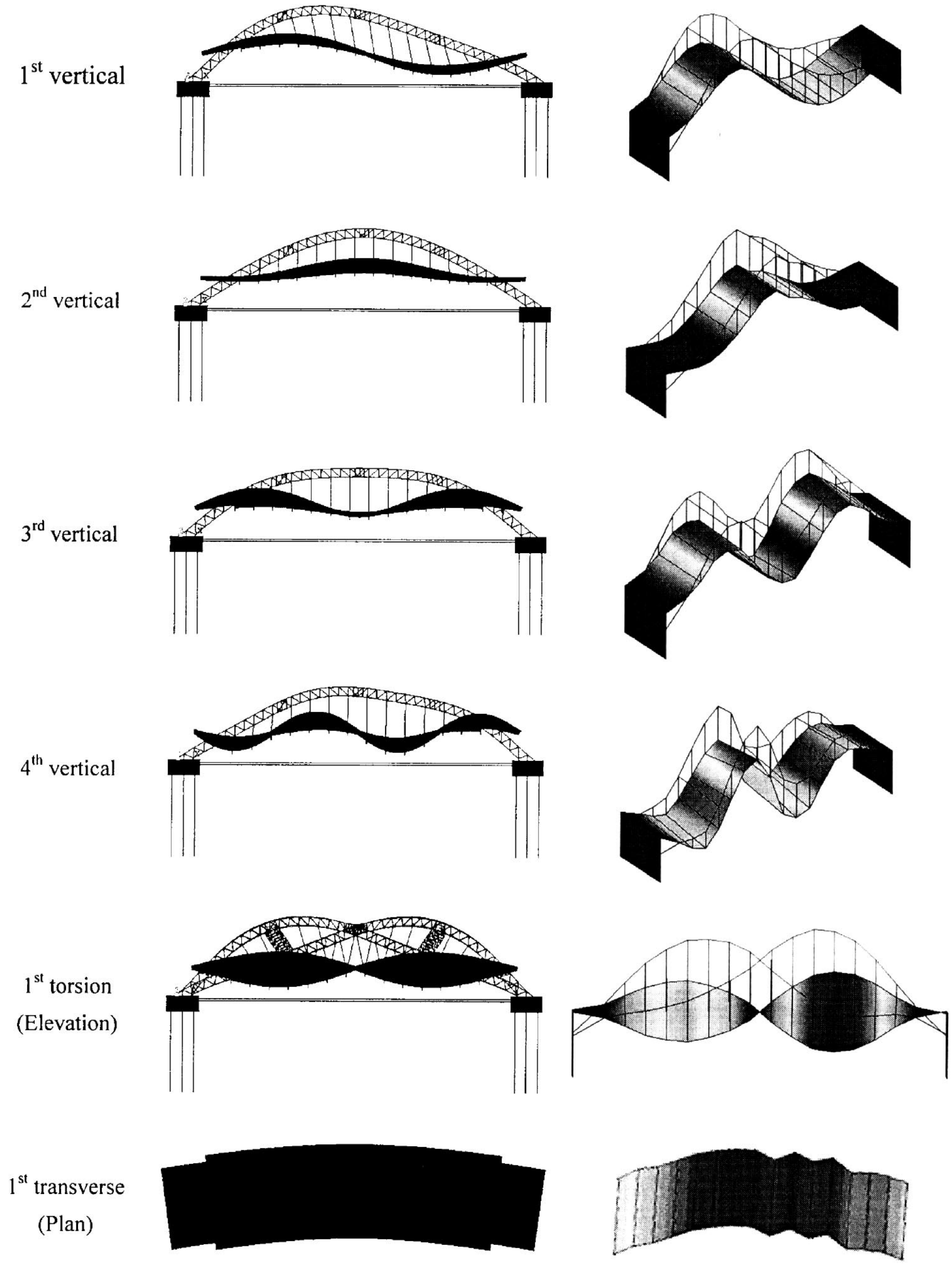

Fig. 5. Six mode shapes obtained from finite element analysis and tests

bridge. One reference location was selected near one side of the abutments for each setup. Each setup consisted of eight moveable accelerometers and one fixed reference accelerometer. The sampling frequencies on site for vertical data and transverse data are 80 and 200 Hz, respectively and corresponding recording time is 15 and 20 min, respectively. During all tests, normal traffic was simulated by using a truck to go back and forth in a random manner, not in a controlled way.

Ambient vibration measurements do not lend to FRFs or IRFs calculations because the input excitations are not measured. Two complementary modal parameter identification techniques are implemented here. They are the rather simple peak picking (PP) method in frequency-domain and the more advanced stochastic subspace identification (SSI) method in time-domain. The data processing and modal parameter identification were carried out by *MACEC* (De Roeck and Peeters 1999).

Model Correlation

The mode shapes obtained from the initial FE model of the arch bridge are paired with those identified from field ambient vibration measurements as shown in Fig. 5. It is clearly seen from the visual inspection of mode shapes that good mode shapes of the bridge were extracted by the stochastic subspace identification from ambient vibration output-only data and they are paired well in each considered modes. It should be kept in mind that usually only limited number of modes could be excited using ambient vibration in practice and quite likely the spatial resolution of these modes could be poor. Considering this fact, in this work, the first six modes of frequencies up to 3.86 HZ are considered for updating purposes.

To evaluate the correlation of mode shapes, the modal assurance criteria (MAC) is widely used since it is easy to apply and does not need an estimation of the system matrices. The initial FE

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Table 3. Modal Parameters of Arch Bridge before Updating

Nature of modes	Initial FE (Hz)	Test results (Hz)	Frequency errors (%)	MAC values (%)
1st vertical	1.743	2.002	−12.93	93.0
2nd vertical	2.210	2.511	−11.98	96.0
1st torsion	2.391	2.827	−15.42	96.8
1st transverse	2.669	2.780	−3.99	62.1
3rd vertical	2.778	3.473	−20.01	75.1
4th vertical	3.541	3.864	−8.35	79.6

and experimental modal properties of the bridge are compared in Table 3 and shown in Fig. 6. Table 3 demonstrates that the frequencies correlation is not so good with the maximum error of 20% in the third bending mode. Fig. 6(a) clearly shows the pairing of frequencies between the initial FE model and tests emphasizing errors as a departure from a diagonal line with a unit slope. It can be seen that the point representing the third bending mode with a maximum difference has the largest departure from diagonal line, whereas the first transverse mode with a least error of 3.99% is near a diagonal line showing well matching of that frequency.

However, Table 3 shows that the correlation of mode shapes expressed by MAC values seems good except for the first transverse mode shape which is only 62.1%. Fig. 6(b) presents a plot of the MAC matrix that illustrates the orthogonal conditions between all combinations of analytical and experimental mode shapes. For well-paired modes, the MAC values are high and off diagonal values have the magnitudes near zero.

Selection of Parameters for Finite Element Updating

The choice of parameters is an important step in model updating. The crucial step is how many parameters should be selected and which parameters from many possible parameters are used in the FE updating. If too many parameters are included in the FE updating, the problem may appear ill-conditioned [Eq. (13*a*)] because only a few modes are correctly recognized in the ambient vibration testing. Sensitivity analysis is carried out to see the sensitivity of parameters to various modes of interest.

To perform sensitivity analysis, it is better to start from all possible parameters (Brownjohn and Xia 2000) and then identify the most sensitive and nonsensitive parameters to response. The possible parameters for the bridge structure may include Young modulus of elasticity and mass density of reinforced concrete components, cross-sectional area and inertia moment of beam elements, thickness of deck elements, and boundary conditions. In this case study of one span arch bridge, the boundary conditions are not very complicated. The abutments rest on stone strata through piles. The two ends of the deck are simply rested on the piers of two sides. So it is not taken as an updating parameter. Out of possible parameters, the eigenvalue sensitivity analysis with respect to initial estimation of parameters is performed for 15 influential parameters as shown in Fig. 7 and further the 10 most sensitive and logical parameters listed in Table 4 are selected for updating purposes. Only the sensitivity criterion is not enough to select the updating parameters for real structures. Parameters chosen should have physical meaning and they should be able to model the errors in the finite element model. If the selection of updating parameters is purely based on the sensitivity analysis, the updated model may have no physical meaning. It can be seen from Fig. 7 that the mass density of concrete-filled steel tubular arch ribs, deck thickness, deck mass density, and other selected parameters are very sensitive to most of the modes considered, whereas the parameters like the inertia moment of arch ribs and the sectional area of cable connecting two abutments are not so sensitive.

The parameters which are sensitive to certain modes can be effectively updated with these sensitive modes while neglecting the others which are not affected (Bohle and Fritzen 2003). Spring stiffness in the transverse direction is selected as an updating parameter to update it separately with respect to transverse mode only. The initial values of the parameters listed in Table 4 are taken from the design blueprints and related codes. The elastic modulus of the arch is obtained by considering the transformed contribution from the steel tubes. The values of elastic modulus of cross girder and bridge deck are taken from code (CECS 28:90 1990) corresponding to their grade of concrete.

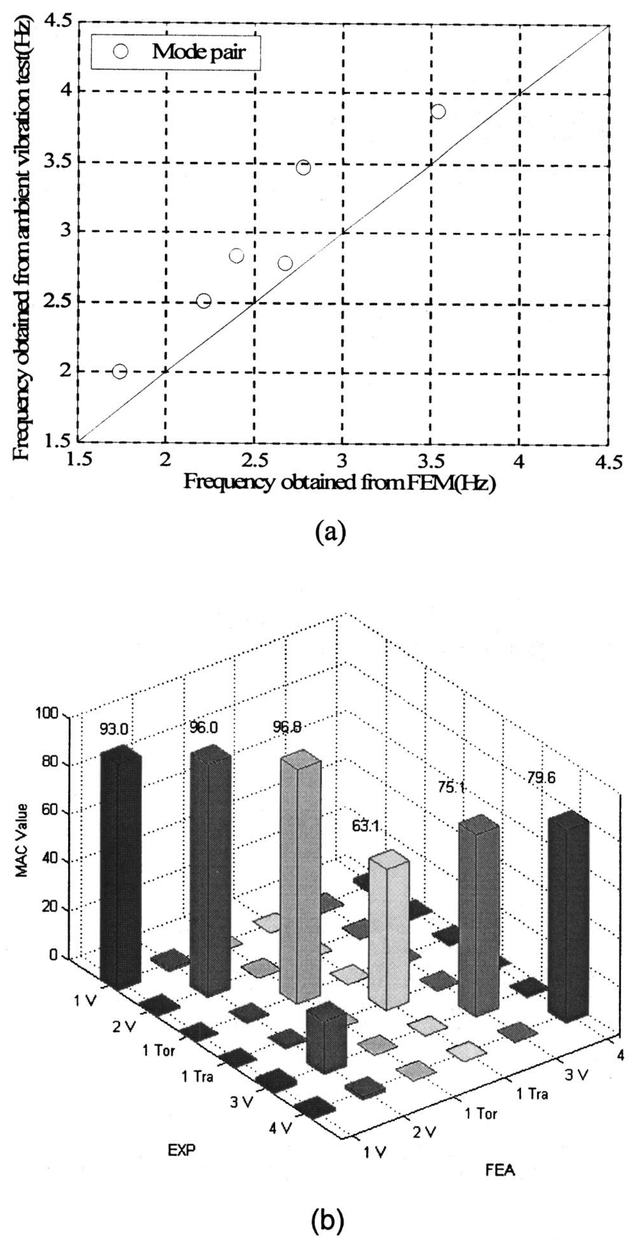

Fig. 6. Frequency and modal assurance criterion (MAC) correlation of arch bridge before updating

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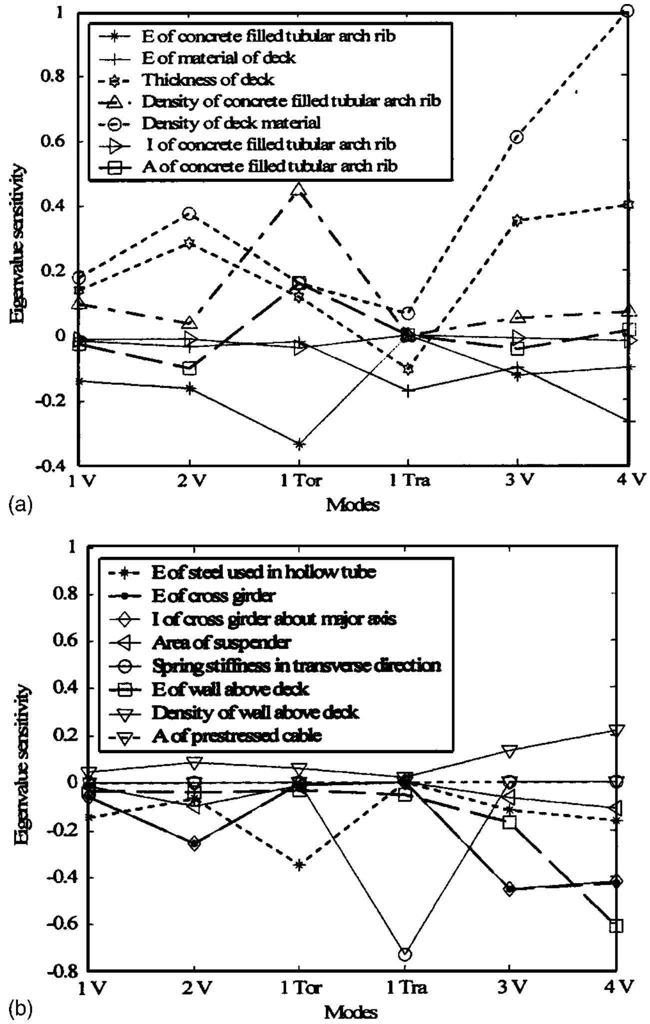

Fig. 7. Eigenvalues sensitivity of arch bridge to potential parameters

Finite Element Model Updating

To carry out updating of parameters, the nature and number of mode shapes to be used are first confirmed. Then, an objective function and state variables are defined. In this second case study of a real bridge, the objective function considering the frequency residual, MAC related function, and modal flexibility residuals shown in Eq. (10) and state variables defined by Eqs. (11) and (12) are implemented. The weighting matrix should be chosen in the objective function to reflect the relative accuracy among the measured modes. The experimental eigenfrequencies are in general identified more accurately than mode shapes. An appropriate weighting is therefore necessary. Appropriate weights can be identified in an iterative way, for example, if for the obtained results, the eigenfrequencies correspond fully but the mode shapes show a significant discrepancy, it can be assumed that too much weight is given to the eigenfrequency residuals. Typically, the frequencies of the lower few modes are measured more accurately than those of the higher modes. If each natural frequency is weighted equally in absolute terms, the algorithm will effectively weight the higher frequency more. By assigning proper values for α_i, the difference between analytical and the measured eigenvalues of the lower modes can be further minimized. In this work, based on the iterative procedures, α_i values corresponding to the first four modes are set to be five times larger than the remaining modes and weighing factors for mode shape residuals are not applied.

Although it is very hard to estimate the variation bound of the parameter during updating, it is assumed according to some engineering judgement. In the studies of Zhang et al. (2000, 2001), the maximum variation of ±40% is given for some uncertain parameters. In this work, the variation ±20% is allowed for the thickness of deck and ±30% for all other remaining parameters. Similarly, suitable tolerances for the objective function, updating parameters, as well as state variables are confirmed and at last the number of iterations to complete the optimization is defined. These values depend on the nature of the problems, so there is no fixed and fast rule to set the magnitude of these values.

An iterative procedure for model tuning was then carried out. One important issue to be aware of is that one has to be able to pair the mode shapes in each iteration. This is done in this paper with the help of MAC criterion between FE mode shapes and experimental mode shapes. The selected updating parameters were estimated during an iterative process. The tuning process is over when the tolerances were achieved or a predefined number of iterations was reached. For subproblem approximation, the optimizer initially generates random designs to establish the state variable and objective function approximations. The convergence may be slow due to random designs. It is necessary sometimes to speed up convergence by providing more than one feasible starting design. It can be simply achieved by running a number of random design tools and discarding all infeasible designs. Compared to the subproblem approximation method, the first order method is seen to be more computationally demanding and more accurate. However, the high accuracy does not always guarantee the best solution. Here are some situations to be watched.

Table 4. Parameters of Arch Bridge before and after Updating

Parameters updated	Initial values	Updated values	Change (%)
Elastic modulus of arch (Pa)	4.56×10^{10}	5.30×10^{10}	16.3
Elastic modulus of cross girders (Pa)	3.45×10^{10}	4.26×10^{10}	23.5
Elastic modulus of deck (Pa)	3.00×10^{10}	3.90×10^{10}	30.0
Moment of inertia of cross girder (m^4)	0.0756	0.0972	28.571
Thickness of bridge deck (m)	0.25	0.246	−1.6
Mass density of arch (kg/m^3)	2,871.0	2,010.0	−30.0
Mass density of deck (kg/m^3)	2,500.0	2,144.0	−14.2
Sectional area of arch (m^2)	0.4311	0.3384	−21.5
Sectional area of suspender (m^2)	0.0025	0.0021	−16.0
Spring stiffness in lateral direction (N/m)	500,000	516,550	3.3

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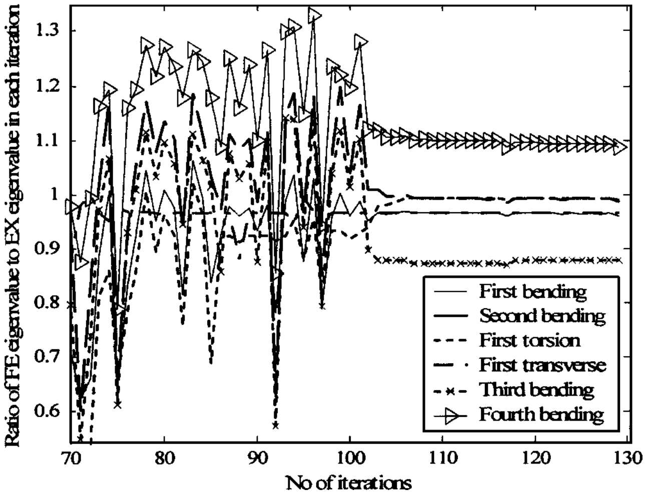

Fig. 8. Convergence of six finite element (FE) eigenvalues during updating of arch bridge

- It is possible for the first order method to converge with an infeasible design. In this case, it has probably found a local minimum, or there is no feasible design space. If this occurs, it may be useful to run a subproblem approximation analysis, which is a better measure of full design spaces. Also, one may try to generate the random designs to locate the feasible design space (if any exists), and then rerun the first order method using a feasible design set as a starting point. So two optimization algorithms can be used complementarily.
- The first order method is more likely to hit a local minimum since it starts from one existing point in the design space and works its way to the minimum. If the starting point is too near to a local minimum, it may find that point instead of the global minimum. If it is suspected that a local minimum has been found, one may try using the subproblem approximation method or random design generation, as described above.
- An objective function tolerance that is too tight may cause a high number of iterations to be performed. Because the method solves the actual finite element representation, it will strive to find an exact solution based on the given tolerance.

In this study, to carry out the optimization more than one feasible starting design was performed to run a number of random designs tools and discard all infeasible designs. The first order optimization was first used until the convergence was achieved. The optimization is then carried out using a subproblem method to see the minimization process which gives some guideline to see whether the first order method traps in local minima or not. The changes of eigenvalues only after 70 iterations are shown in Fig. 8 to show it more clearly. The first 30 iterations correspond to the generation of random sets, which is actually not optimization. Then from 31 iterations, the first order optimization is carried out. Because further iterations after 129 did not yield any progress, it was decided to terminate the optimization after 129 iterations.

The final correlation of frequencies and mode shapes after FE model updating is shown in Table 5. This shows that the differences between FE and experimental frequencies are reduced below 7%. The errors on the first four frequencies fall below 2%, which is a significant improvement compared to the initial FE model result (Table 3). The correlation of mode shapes is also improved as all MAC values are over 80% except for the first transverse mode which also has improvement on the MAC value of 76.6% from initial value of 62.1%. The well pairing of frequencies and MAC indicators are plotted in Fig. 9. It is clearly seen that all pair points are close to the diagonal. Careful inspection of the MAC matrix of Fig. 9(b) shows that there is an improvement on the MAC values since every mode considered has the magnitude more than the initial value shown in Fig. 6(b).

The changes in selected updating parameters are shown in Table 4. One of the most important issues in FE model updating is

Table 5. Modal Parameters of Arch Bridge after Updating

Nature of modes	Updated Fe (Hz)	Test results (Hz)	Frequency errors (%)	MAC values (%)
1st vertical	1.962	2.002	−1.99	93.7
2nd vertical	2.493	2.511	−0.69	96.5
1st torsion	2.815	2.827	−0.42	97.3
1st transverse	2.770	2.780	−0.359	76.9
3rd vertical	3.256	3.473	−6.23	90.9
4th vertical	4.027	3.864	4.18	80.9

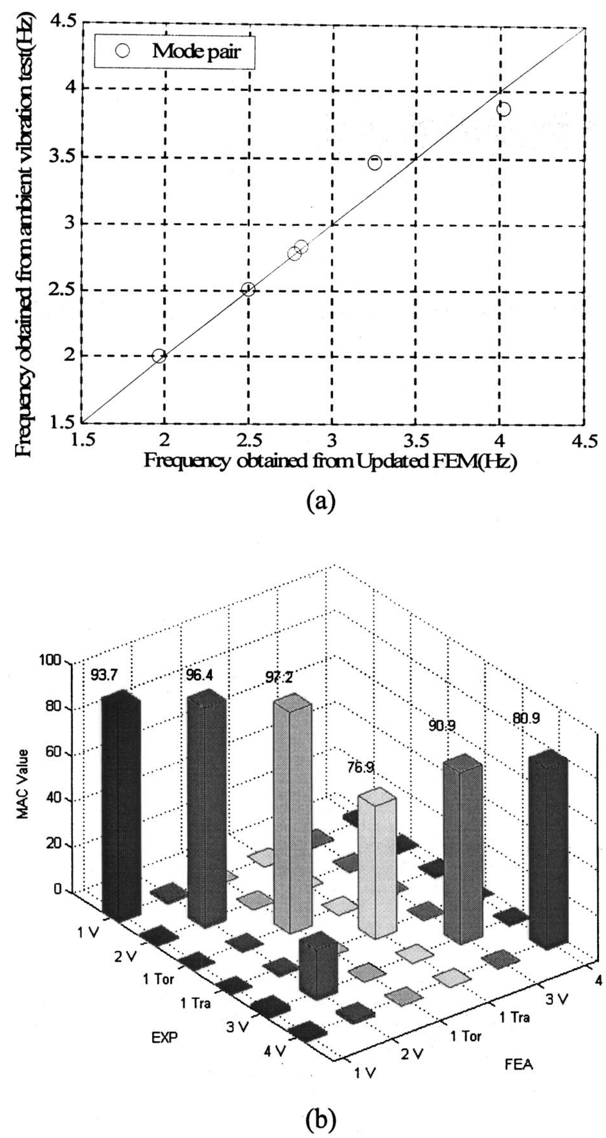

Fig. 9. Frequency and modal assurance criterion (MAC) correlation of arch bridge after updating

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to check the physical meaning of updated parameters against normal practice. In this case study, it is clearly seen from Table 3 that all the test frequencies are more than the values from the initial FE model. In most of the cases, it is observed in Table 4 that there is an increase in value of stiffness related parameters and a decrease in value of mass related parameters after updating, which is as expected. Especially, the updated values of Young's modulus of concrete components are all increased, which are identical to the fact that the dynamic Young's modulus of concrete is larger than the static one. The increase in the value of the inertia moment of cross girders shows that there is good interaction between cross girders and the slab although they are cast separately during construction.

The significant reduction in the value of mass density of the composite arch is found. The fact is that the density of concrete is not always constant. The water cement ratio of concrete mix and many other uncertainties related with concrete may cause the variation of concrete density. It may also be due to the overestimation of the actual mass density of the particular composite arch while using it in the FE model. The initial value of density of the composite arch is taken according to Chinese code (CECS 28:90 1990). Similarly, there is some variation on the values of remaining parameters like sectional area of arch, area of suspender, and value of spring stiffness in the lateral direction, although the deviations are not much.

Conclusions

The following conclusions are drawn from the study.

- This paper presented a sensitivity based finite element model updating method for real bridge structures using the test results obtained by the ambient vibration technique. The objective function consisting of a combination of eigenvalue residual, mode shape considered function, and modal flexibility residual is used for FE updating which is the main contribution of this work.
- The objective function that considers frequency residual only, mode shape (MAC) related function only, modal flexibility residual only, and their full combination are studied independently in the first case study of simulated simply supported beam. It is observed that the tuning on modal parameters alone can be achieved even when any one residual explained in this paper is used in the objective function. However, in the damage detection part, the introduction of a modal flexibility residual in the objective function with other residuals considerably improves the detection capability, which supports the fact that the modal flexibility term is sensitive to the local damage. In view of tuning as well as damage detection, the case study demonstrated that the full objective function is effective to the FE model updating in structural dynamics.
- An eigenvalue sensitivity study is feasible to see the effect of various parameters on the concerned modes, according to which the most sensitive parameters can be selected for updating. Only the sensitivity criterion is not enough to select the updating parameters for real structures. Parameters chosen should have physical meaning.
- Since the presented FE model updating method is a sensitivity-based technique, it can be trapped in local minimum. Appropriate initial values for the updating parameters are required. This can be done by creating a random generation of a set of parameters before carrying out real optimization. Two optimization algorithms, subproblem approximation method and first order optimization method, can be used complementarily.
- The updated finite element model of a true concrete-filled steel tubular bridge is able to produce natural frequencies in close agreement with the experiment results with enough improvement on the frequencies and MAC values of the concerned modes and still preserve the physical meaning of updating parameters.
- Successful updating of the real bridge presented in this paper demonstrates that, even for the big model, the cost of calculation is not too high and this method is practical for the daily use of engineers.

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Notation

The following symbols are used in this paper:

A_i, B_j, C_k = state variables related parameters after function approximation available during optimization;
a_0, a_i, b_{ij} = coefficients used in optimization;
$d^{(j)}$ = parameter showing that the direction to the line search parameter s_j corresponds to the minimum value of Q;
e = vector with 1 in its ith component and 0 for all other components;
$F(x, P_k)$ = unconstrained objective function varies with the design variables and parameter P_k;
F_{nxn} = flexibility matrix;
f_i = non-negative function that measures the discrepancy in shapes between the ith experimental mode and the FE mode that has been paired with it;
G, H, W = penalty functions for state variable constraints;
g_i, h_j, w_k = state variables;
I = identity matrix;
i, j, k = indices;
L_1 = lower limit to constrain the MAC;
MAC_i = MAC number for the ith mode pair;
M = global mass matrix;
m = number of mode shapes considered;
m_k = component of diagonal mass matrix;
$m_1+m_2+m_3$ = number of state variables constraints with various upper and lower limit;
N = number of design variables;
n = number of degree of freedom or number of the measurement points;
P_x, P_g, P_h, P_w = penalties applied to the constrained design and state variables;
Q = dimensionless unconstrained objective function;
S_j = sensitivity matrix;
T = superscript showing matrix transpose;
u_i = deflection coefficient at point i;

UL = upper limit whose value can be set as absolute error of *i*th eigenvalue;
u_{aj} = analytical uniform load surface;
u_{ej} = experimental uniform load surface;
X = penalty function used to enforce design variable constraints;
X_j, Z_j = current parameter estimation after j step and the output based on this parameter estimate, respectively;
x = vector of design variables;
x^i, x^j = *i*th and *j*th design variable;
x^* = design point at which Π is stationary;
x_i, x_{li}, x_{ui} = *i*th design variable, lower bound and upper bound, respectively;
Z_m = measured output;
α_i = weighting factor for *i*th eigenvalue;
β_i = weighting factor for *i*th mode shape;
ΔD = forward difference (in percent) step size, taken as 0.2 in this study;
δx = perturbation in the parameter;
δz = error in the measured output;
λ_{ai} = analytical eigenvalue of mode that has been paired with the *i*th experimental mode;
λ_{ei} = *i*th eigenvalue of experiment mode;
τ, ρ_i = objective function and the design variable tolerance;
$\{\varphi_i\}, \{\varphi_j\}$ = any two modal vectors;
ϕ_{ai} = analytical eigenmode that has been paired with the *i*th experimental mode;
ϕ = general representation of mode shape matrix;
ϕ_{ei} = *i*th experimental eigenmode;
Φ_{ij} = normalized i component of the j modal vector;
Φ_{ik} = *i*th coefficient of the $n \times 1$ unit mass normalized modal vector for mode k;
ω_k^2 = square of the *k*th circular frequency;
$1/\omega^2$ = diagonal matrix with ascending square of natural frequencies;
Π, Π_1, Π_2, Π_3 = objective function to be minimized; and
Π_0 = reference objective function value that is selected from the current group of design sets.

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