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Damping estimation using free decays and ambient vibration tests

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ABSTRACT

This paper studies the quality of modal damping ratios estimates based on ambient and free vibration tests using both numerical simulations and data collected on large Civil Engineering structures where both tests were performed. The simulated data allowed to study the influence of factors like non-proportional damping or proximity of natural frequencies on the quality of the estimates and also to illustrate the influence of the identification algorithms parameters on the accuracy of the results. The analysis of data collected on a cable-stayed bridge, on the suspended roof of a stadium and on a footbridge permitted to compare the results of both testing approaches in real applications where some factors that cannot be realistically included in the numerical simulations can play an important role. The processing of ambient vibration responses is performed with two output-only identification approaches: frequency domain decomposition and stochastic subspace identification methods. The numerically simulated and the measured free decays were analyzed with a simple method based on filters and fitting of exponential decays and also with the use of subspace models.

1. Introduction

Modal damping ratios are very difficult to predict at the design stage, but have a strong influence on the dynamic structural behaviour, the amplitude of vibrations at resonance being inversely proportional to these coefficients. Therefore, in structures subjected to relevant dynamic excitations, it is important to perform dynamic tests after construction to check the assumptions adopted during the design and eventually implement solutions to reduce the vibrations.

The experimental identification of modal damping ratios can be based on forced vibration, free vibration or ambient vibration tests (AVT).

Forced vibration tests are accurate but require very heavy and expensive equipments, like servo-hydraulic shakers or eccentric mass vibrators. Therefore, because of their costs and the needed logistics, these tests are, more and more, less used. Besides that, the results are always influenced by the uncontrolled ambient excitation and the estimated values are associated with levels of vibration that are higher than the ones observed during the normal use of the structure.

Ambient vibration tests have the strong advantage of being very practical and economical, as they use the freely available ambient excitation. Furthermore, the data are collected during the normal use of the structure and consequently the identified modal parameters are associated with realistic vibration levels. However, although modern output-only modal identification techniques, in conjunction with the use of appropriate state-of-the-art vibration measurement equipment, can provide very accurate estimates of natural frequencies and mode shapes, it is usually observed that the

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corresponding damping estimates present a significant scatter. That is why it is still very common to perform complementary free vibration tests (FVT), when the accurate identification of modal damping ratios is required.

Free vibration tests, based on the sudden release of a suspended mass or the sudden cut of a tensioned cable, are less practical than ambient vibration tests, but considerably simpler than forced vibration tests. In this type of tests, the modal parameters are also derived from vibrations levels that are higher than the service ones.

During the last few years, the Laboratory of Vibrations and Structural Monitoring (ViBest, www.fe.up.pt/vibest) of FEUP has been involved in the performance and/or signal processing of a considerable number of dynamic tests in large Civil Engineering structures (e.g. "Vasco da Gama" bridge, Braga Sports Stadium suspended roof and Coimbra footbridge), collecting very complete and high-quality databases that enable comparative studies of the application of alternative methods of modal identification.

In this context, this paper starts with a brief introduction about the practical issues and identification algorithms of the experimental techniques more commonly used for damping estimation of Civil Engineering structures, namely ambient vibration tests and free vibration tests. Then, a simulation study is performed to illustrate the influence of the identification algorithms parameters on the accuracy of the results and to study the influence of factors like non-proportional damping or the proximity of natural frequencies on the quality of the estimates provided by the two considered tests. Finally, the results provided by the ambient and free vibration tests developed on three large structures are presented, in order to evaluate the level of accuracy that modern output-only modal identification techniques can provide for damping ratios estimates.

2. Experimental techniques for damping estimation

2.1. Ambient vibration tests

During ambient vibration tests, the accelerations of structures excited by ambient loads are measured. Accordingly, the traffic over the bridge and the wind are welcome, to increase the signal intensity of the measured time series. Still, the levels of excitation are generally low, and so, the used accelerometers have to be very sensitive. Furthermore, the test of slender structures demands the use of transducers that should present a linear response from very low frequencies. Besides that, when the size of the structure is considerable, the use of wireless systems duly synchronized by GPS is advantageous.

All the applications presented in Section 4 were based on the use of seismographs that contain internal tri-axial force-balance accelerometers (linear response from 0 to 50 Hz), a battery, an A/D converter with 18-bit that ensures a good resolution, a memory card to store the time series and the GPS to continuously update the internal clock using information provided by the satellites, in order to have a perfect synchronization between acquisition units, without using cables between them.

The recorded data are then processed using output-only identification tools. Nowadays, there are several robust methods, working in time or frequency domains, which are already implemented in user-friendly software [1,2]. A review of the most commonly used methods in civil applications can be found in Ref. [3].

In the present work, the modal parameters identification is based on the enhanced frequency domain decomposition method (EFDD) and on stochastic subspace identification methods. The first one is a non-parametric frequency domain method, whereas the second ones are parametric time domain methods.

2.1.1. Enhanced frequency domain decomposition method

The first step of this method consists on the construction of a spectral matrix of the ambient responses, with one row for each measurement point and one column for each point elected as reference. Therefore, the columns contain the cross-spectra relating the structural response at all measurement points with the corresponding response at each reference point. The elements of these matrices are estimated using the Welch procedure [4] by dividing the available time series in segments of a certain length, eventually considering an overlap between segments (usually 50%) and adopting a window to reduce the leakage (usually a Hanning window).

It can be shown [5] that, under some assumptions (white noise excitation, low damping and orthogonal mode shapes for close modes), the singular values (SV) of the spectral matrix are auto-spectral density functions of single degree of freedom systems with the same frequency and damping as the structure vibration modes.

Auto-correlation functions, associated with the different modes of the structure, can be calculated by applying an inverse fast Fourier transform to the auto-spectral density functions. From these functions, it is straightforward to identify the modal damping ratios and obtain enhanced estimates of the natural frequencies. These frequencies are evaluated looking at the time intervals between zero crossings. The modal damping ratios are estimated adjusting an exponential decay to the relative maxima of the auto-correlation functions. Mode shapes are identified from the singular vectors of the spectral matrix evaluated at the identified resonance frequencies and associated with the singular values that contain the peaks.

Fig. 1 illustrates the application of the EFDD to simulated data of a model with two degrees of freedom described in Section 3 (model SM1). The selection of the auto-spectra is based on the modal assurance coefficient (MAC) relating the singular vectors (equivalent to the mode shapes) of the peaks with the ones of the points in their neighbourhood, the points

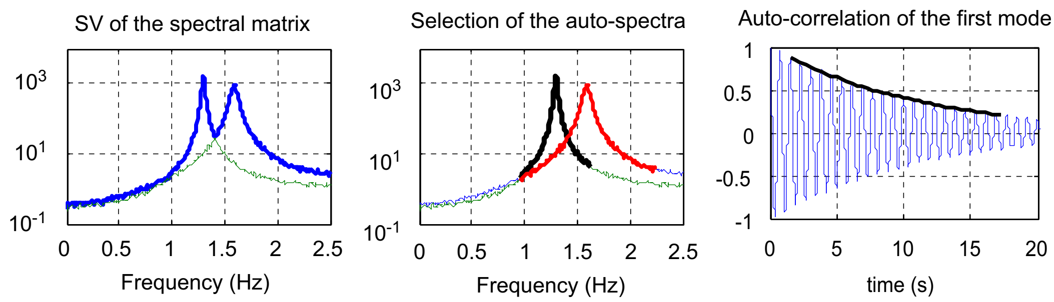

Fig. 1. Steps of the EFDD method.

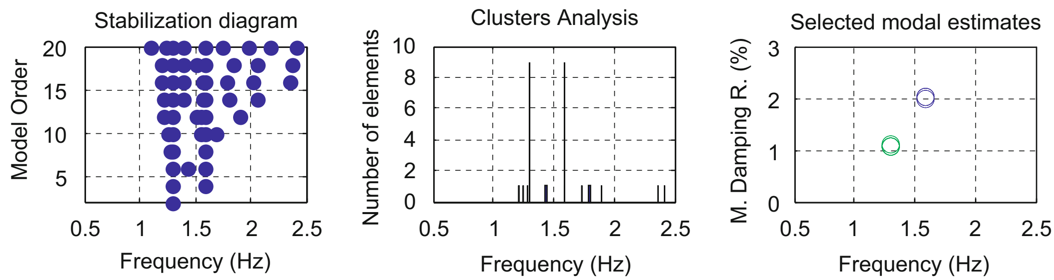

Fig. 2. Steps of the post-processing after the application of the SSI methods.

with a MAC higher than 0.6 having been selected in this case. The fit of the exponential decay to the relative extremes of the auto-correlation is performed after the application of an interpolation algorithm (Matlab command – *spline*) to increase the resolution and consequently increase the accuracy on the extremes identification. In this example, only the extremes with amplitude lower than 90% of the maximum and higher than 20% of the maximum are used in the fitting.

2.1.2. *Stochastic subspace identification methods*

The stochastic subspace identification methods perform the identification of modal parameters using a stochastic state-space model that, in its discrete form and assuming the excitation as a white noise, is represented by the following equations:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{1}$$

where x_k is the discrete-time state vector at time instant k, y_k is the vector with the sampled outputs, A is the discrete state matrix, C is the discrete output matrix and w_k, v_k are vectors that represent the noise due to disturbances and modelling inaccuracies and the measurement noise due to sensor inaccuracy.

Identification of matrices A and C is performed either directly from the measured time series (data-driven stochastic subspace identification – SSI-DATA) or from the correlations of the time series (covariance-driven stochastic subspace identification – SSI-COV). Both algorithms are based on the properties of stochastic systems [6] and involve a singular value decomposition and the resolution of a least-squares equation [7].

After the identification of the state-space model, modal parameters are extracted from matrices A and C [7]. It is worth noting that the identification of the state-space model requires the definition of the order of the model. However, for real Civil Engineering structures, it is not possible to predict the order of the model that better fits the experimental data and more realistically characterizes the dynamic behaviour of the structure. The most appropriate way to overcome this difficulty is to estimate the modal parameters using models with an order within an interval previously defined in a conservative way. The identified modal parameters are then represented in a stabilization diagram (Fig. 2). This diagram allows the distinction of parameters that are stable for models of increasing orders (points over the same vertical alignment), and these are the ones with structural significance. The others are just associated with numerical modes, which are important to model the noise that exists always in measured data.

However, the stabilization diagram by itself does not solve the problem of the modal parameters identification; it is just a graphical tool to help on the manual selection of the mode estimates that are more likely to represent the structure physical modes. In the context of the simulation studies presented in Section 3, which comprehend the analysis of a huge number of datasets, it is crucial to develop tools to perform the identification without any user interaction. In this work, a procedure that was specifically developed to integrate a continuous monitoring system of a bridge is used. This is based on a hierarchical clustering algorithm that groups all the mode estimates with similar natural frequencies and mode shapes, allowing the automatic selection of the mode estimates associated with the same physical mode. All the details of the algorithm are presented in Ref. [8]. Its practical application is illustrated with Fig. 2 by means of the same dataset that was used in the previous sub-section.

The stabilization diagram represents the natural frequencies values of the mode estimates provided by models of orders between 2 and 20. Then, the outputs of the cluster analysis are characterized by the mean natural frequencies of the modes estimates inside each group and the number of elements of the group. Two groups stand out; these contain the mode estimates that are associated with physical modes (the two modes of the two degree of freedom model with natural frequencies around 1.3 and 1.5 Hz). The elements inside the two selected groups are represented in a natural frequency vs. modal damping ratio diagram. If extreme values of modal damping ratios appear, these are eliminated by an outlier analysis.

2.2. Free vibration tests

The free vibration tests performed in Civil Engineering structures for the identification of modal damping ratios can be of two types: measurement of the free response after the application of a sinusoidal load with an excitation frequency coincident with one of the structure natural frequencies or measurement of the free structural response after the application of an impulse or imposed displacement.

The first type of tests is less practical, since the application of a sinusoidal load requires the use of special equipment, it takes long time involving a separate test for each mode, and previous knowledge of the natural frequencies and mode shapes is needed, as the sinusoidal load must be applied with the correct frequency at the anti-node of the mode that is being excited. Even so, these tests were recently performed on the suspended roof of a stadium and results are presented in Section 4.1. The decays measured after the application of the force should contain only the contribution of a single mode, the modal damping ratio being then directly estimated by fitting an exponential function to the relative maxima of the recorded decay.

On the other hand, the impulse tests are more convenient, because they can rely on the sudden release of a mass previously suspended from the structure (no special mechanical equipment is needed) and with just one test it is possible to identify the modal damping ratios of several modes. Of course that the prior identification of the mode shapes is useful, as the mass must be hanged from a point where the most important modes have significant modal components.

The most traditional procedure to analyse the recorded free decays consists of applying several band-pass filters to isolate the contributions of the most important modes. As the resulting sets of filtered data should only contain the contribution of a single mode, the modal damping ratio is directly estimated by fitting exponential decays to the relative maxima of these time series. This approach faces difficulties to isolate the contribution of modes with close natural frequencies.

As alternative to this rather simple procedure, the output-only SSI-COV method can be adapted to extract the modal parameters from free decays measured after the application of an impulse, imposed displacements or sinusoidal loads. The observed free decays are closely related with the correlations of the responses associated with a white noise excitation. Consequently, these measured free decays can be used as input of the SSI-COV method, taking the place of the correlation functions calculated from the ambient responses. However, in this case, instead of basing the identification on the correlations between all the measured responses and some responses elected as references evaluated at predefined time lags, the algorithm is applied to a time-dependent matrix with a single column containing the free decays measured at the instrumented points (yd_i, where i represents the time instant and yd is an $l \times 1$ matrix, l being the number of measured responses).

This approach overcomes the limitations of the traditional procedure, due to the fact that, in this method, a model with non-proportional damping is fitted to the data, allowing the identification of closely spaced modes. Furthermore, with this technique, after the identification of the modal properties, it is possible to decompose the measured free decays in modal decays using the well-known decomposition of the output correlation matrix [6]:

$$R_i = CA^{i-1}G \tag{2}$$

where R is a matrix containing the auto- and cross-correlations at the time instant i, A and C are the matrices already defined in Eq. (1) and G is the "next state-output" covariance matrix (expected values of the product of x_{k+1} by y_k transposed, both defined in Eq. (1)). If the correlation matrix is replaced by the measured free decays (yd_i) and A substituted by its modal decomposition, the following expression is obtained:

$$yd_i = C\Psi\Lambda^{i-1}\Psi^{-1}G \tag{3}$$

Ψ contains in its columns the mode shapes, Λ is a diagonal matrix, whose elements are equal to $e^{\lambda_k \Delta t}$. Δt is the time interval between each sample and λ_k, for $k = 1,\ldots,$ model order, are the eigenvalues of the state-space model that are related with the natural frequencies and modal damping ratios of the tested structure. The contribution of a specific mode for the measured decays can be obtained by the use of expression (3), considering in the diagonal matrix only the two eigenvalues (complex conjugate pairs) that are associated with that mode.

This feature is important to check the results and to evaluate the importance of each mode in the measured response. The damping estimates of the less excited modes are expected to be less reliable.

3. Analysis of simulated data

3.1. Description of the simulated models

The simulation of dynamic tests was based on a model with two degrees of freedom that is characterized in Fig. 3 by the stiffness (K), mass (M) and damping (C) matrices and by the corresponding modal parameters: natural frequencies (f_i), modal damping ratios (ξ_i) and mode shapes (ϕ_i). The damping is viscous (damping forces proportional to velocity) and proportional (the damping matrix is a linear combination of the stiffness and mass matrices). The system matrices were defined aiming to achieve modal parameters with values of the same order of magnitude of the ones usually found in Civil Engineering structures.

To test the accuracy of the used techniques under different situations, five variant models were created introducing changes on the matrices of the base model (called SM1). These are characterized in Table 1, which contains the theoretical values of natural frequencies and modal damping ratios and a measure of the non-proportionality of the damping matrices. The studied models differ on the proximity of the two natural frequencies (SM – well-separated modes; CM – closely spaced modes) and on the type of damping (proportional or non-proportional). This last aspect has influence on the characteristics of the mode shapes, which are real if damping is proportional and can become complex if damping is not proportional (the complexity increases with the proximity of the natural frequencies of two closely spaced modes). If damping is proportional, then the damping matrix in the modal space is diagonal. Thus, in this work the damping non-proportionality is quantified by the ratio between the sum of the absolute values of the off-diagonal elements and the sum of the absolute values of the diagonal elements of the modal coordinate damping matrix.

The ambient vibration tests were replicated adopting as inputs for the models time series with normally distributed random numbers. For the simulation of free decays after the application of an impulse, an initial displacement in a degree of freedom where both modes have significant modal ordinates was imposed. The free decays after the application of a sinusoidal load were obtained by considering as input, at one of the model degrees of freedom, a function with two segments: first segment is a sinusoid with a frequency equal to one of the model natural frequencies; second segment is composed by zeros and has the length of the desired free decay. A sampling frequency of 5 Hz was adopted in all the simulations. The responses were evaluated using a discrete-time state-space model following the methodology described by Juang [9].

Equation of motion: $M \cdot \ddot{u} + C \cdot \dot{u} + K \cdot u = f(t)$

$\ddot{u}$, $\dot{u}$ and u - vectors with the acceleration, velocity and displacements of the model

$f(t)$ - vector with the forces applied to the model (kN)

$$K = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} (kN/m); \quad M = \begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1.25 \end{bmatrix} (ton); \quad C = \begin{bmatrix} 0.3275 & -0.0725 \\ -0.0725 & 0.3275 \end{bmatrix} (kN.s/m)$$

$f_1 = 1.2995$ Hz $\quad \xi_1 = 1.0410$ % $\quad \phi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$f_2 = 1.5915$ Hz $\quad \xi_2 = 2.0000$ %

Fig. 3. Reference model (SM1) characteristics.

Table 1
Characteristics of the simulated models (f – natural frequency, ξ – modal damping ratio).

Model	Characterization	Measure of damp. non-proportionality	Mode 1		Mode 2	
			f (Hz)	ξ (%)	f (Hz)	ξ (%)
SM1	No modal complexity	0.0	1.2995	1.0410	1.5915	2.0000
SM2	Some modal complexity	0.50	1.2998	1.0398	1.5911	2.0013
SM3	Strong modal complexity	1.00	1.3009	1.0360	1.5898	2.0053
CM1	No modal complexity	0.00	1.5720	1.0124	1.5915	2.0050
CM2	Some modal complexity	0.50	1.5721	1.0160	1.5915	2.0014
CM3	Strong modal complexity	0.95	1.5717	1.0115	1.5926	2.0070

3.2. Ambient vibration tests simulation

Firstly, the influence of the adopted time length and of the selected algorithms parameters in the estimated modal damping ratios was studied using model SM1 (well-separated modes and proportional damping). In order to study the bias and the variance of the estimates, for each scenario, 100 simulations were performed. The time length of the output time series was modified from 5 to 80 minutes. The simulated output were corrupted with noise, which was simulated by normally distributed random numbers with a standard deviation equal to 10% the standard deviation of the simulated outputs (this percentage of noise is quite conservative in case of well-conducted ambient vibration tests).

In the case of the EFFD method, the parameter that has a stronger influence on the results is the number of points considered in the time segments used for the spectra calculation (Welch method). Therefore, this method was tested using five alternative time segment lengths. Table 2 characterizes the 44 considered scenarios (named from 1a to 5i). Scenario 5a was not considered because the length of each time segment is higher than the total available time (the sampling frequency is 5 Hz, so 2048 points correspond to 409.6 s or 6.83 min).

The huge number of datasets to be analysed (44×100) forced the use of a procedure without any user interaction. The spectral peaks were automatically selected and then the selection of the auto-spectra and the exponential decay fitting were performed using the same parameters that were used in the example presented in Section 2.1.1. Complementary identifications with small variations on these parameters proved that, in the processing of simulated data, they do not affect significantly the results.

Fig. 4 presents the results of the 100 simulations performed for scenario 4i, demonstrating that the variance of the modal damping ratios estimates is much higher than the variance of the natural frequencies estimates.

Fig. 5 synthesizes the results obtained with the EFDD method, representing the mean and the standard deviation values of the results obtained with the 100 simulations performed for all the scenarios characterized in Table 2.

It is clear that, if the number of points is not well chosen then the estimates can present high bias (more than 100%). In practice, it is possible to check if the damping estimate is likely to be biased looking at the obtained auto-correlation. If the correlation is far from vanishing for maximum time lag, this indicates that damping is biased by leakage introduced in the estimation of the spectral density functions [10]. This is proven with the scheme presented in Fig. 6 that shows the theoretical modal decay of the first mode and the limits of the auto-correlation functions calculated using the time segment lengths defined in Table 2. The first two vertical lines are clearly before the end of the decay, consequently the corresponding estimates are biased (-●- and -x- lines of the top-left plot of Fig. 5). Therefore, the correct choice of the time segment length depends on the natural frequency and modal damping ratios of the mode under analysis. Lower natural frequencies and higher modal damping ratios demand longer time segments (as also stems from the comparison of the bias of modes 1 and 2 estimates using 256 points).

Table 2
Scenarios for the application of the EFDD method.

No. of points of the time segments	Total time length (min)								
	5	10	20	30	40	50	60	70	80
128 (-●-)[a]	1a	1b	1c	1d	1e	1f	1g	1h	1i
256 (-x-)	2a	2b	2c	2d	2e	2f	2g	2h	2i
512 (-○-)	3a	3b	3c	3d	3e	3f	3g	3h	3i
1024 (-□-)	4a	4b	4c	4d	4e	4f	4g	4h	4i
2048 (-◊-)	–	5b	5c	5d	5e	5f	5g	5h	5i

[a] Symbols used in the graphics of Fig. 5.

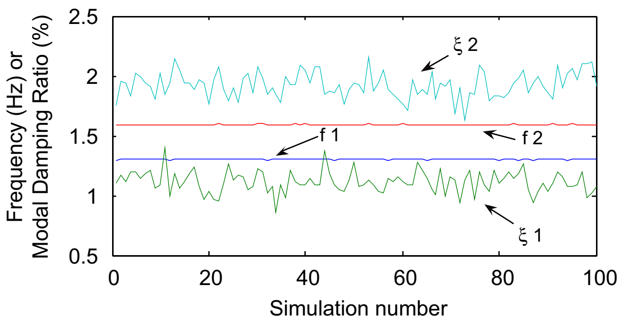

Fig. 4. Modal parameters estimated for the 100 simulations performed for scenario 4i.

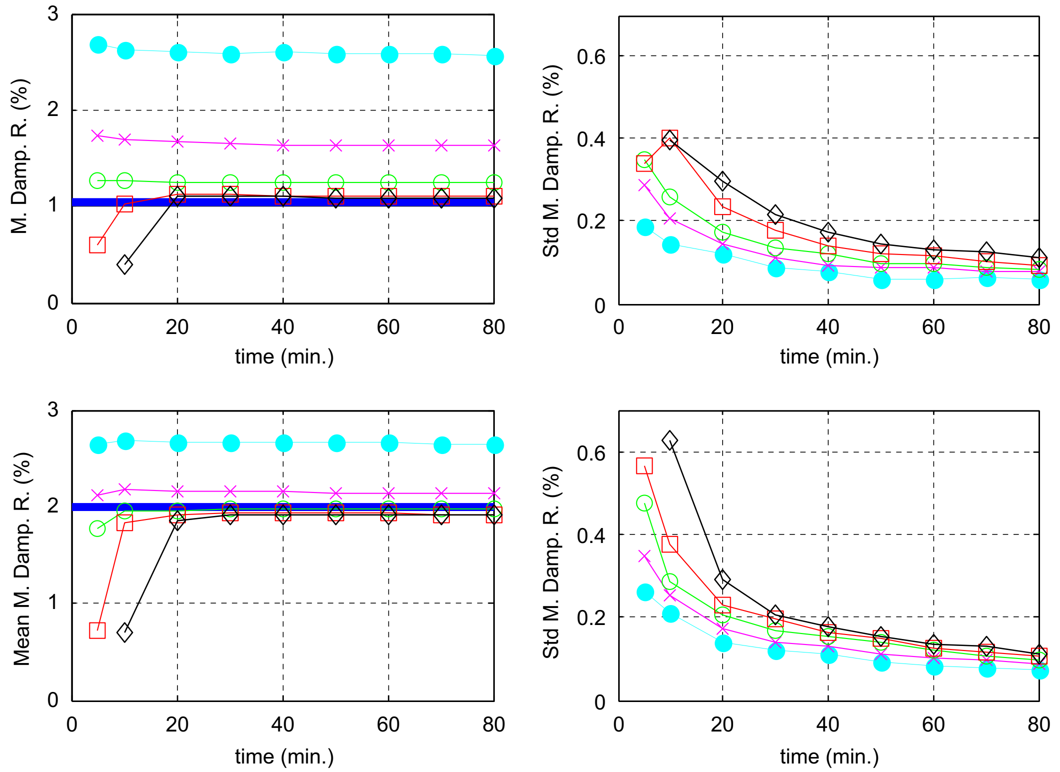

Fig. 5. Mean values and standard deviations of the estimates obtained with the EFDD method for the 2 modes of model SM1 using the parameters characterized in Table 2 (mode 1 at the top, theoretical values represented by a horizontal solid line, symbols of the lines defined in Table 2).

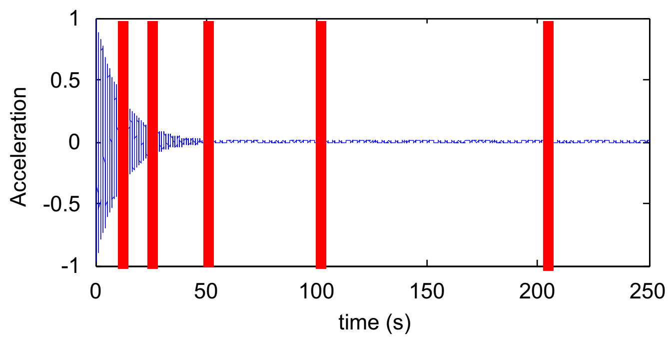

Fig. 6. Theoretical modal decay of the first mode and limits of the auto-correlations function calculated using the time segment lengths defined in Table 2.

The plots of Fig. 5 also show that, for the model under analysis, total time length has to be longer than 20 min. Increasing it further, increases the number of performed averages and therefore, the standard deviation is reduced. However, it is relevant to observe that even using time series longer that 1 h the standard deviation is still quite high (around 0.1%), as is also evident in the scatter of the estimates presented in Fig. 4. An additional set of simulations using the parameters of scenario 4i applied to 100 simulations of structural responses without the inclusion of noise showed that even in this ideal case the standard deviations keep values around 0.1% (relative standard deviation around 10% for the 1st mode). Therefore, this uncertainty cannot be reduced increasing the number of averages or improving the quality of the signals.

Regarding the stochastic subspace identification methods, only the SSI-COV method was considered in the present simulations study, since the numerical behaviour of the SSI-DATA is analogous, as already proven in other similar studies [7].

For the SSI-COV method, the parameter that has more relevant influence on the results is the number of points of the correlation functions. Therefore, 45 scenarios were considered by the combination of five different values for this method parameter and the same nine total time length values adopted in the study of the EFDD method. These are characterized in Table 3.

Table 3
Scenarios for the application of the SSI-COV method.

Number of points of the correlation functions	Total time length (min)								
	5	10	20	30	40	50	60	70	80
10 (-●-)[a]	1a	1b	1c	1d	1e	1f	1g	1h	1i
20 (-×-)	2a	2b	2c	2d	2e	2f	2g	2h	2i
40 (-○-)	3a	3b	3c	3d	3e	3f	3g	3h	3i
60 (-□-)	4a	4b	4c	4d	4e	4f	4g	4h	4i
80 (-◊-)	5a	5b	5c	5d	5e	5f	5g	5h	5i

[a] Symbols used in the graphics of Fig. 7.

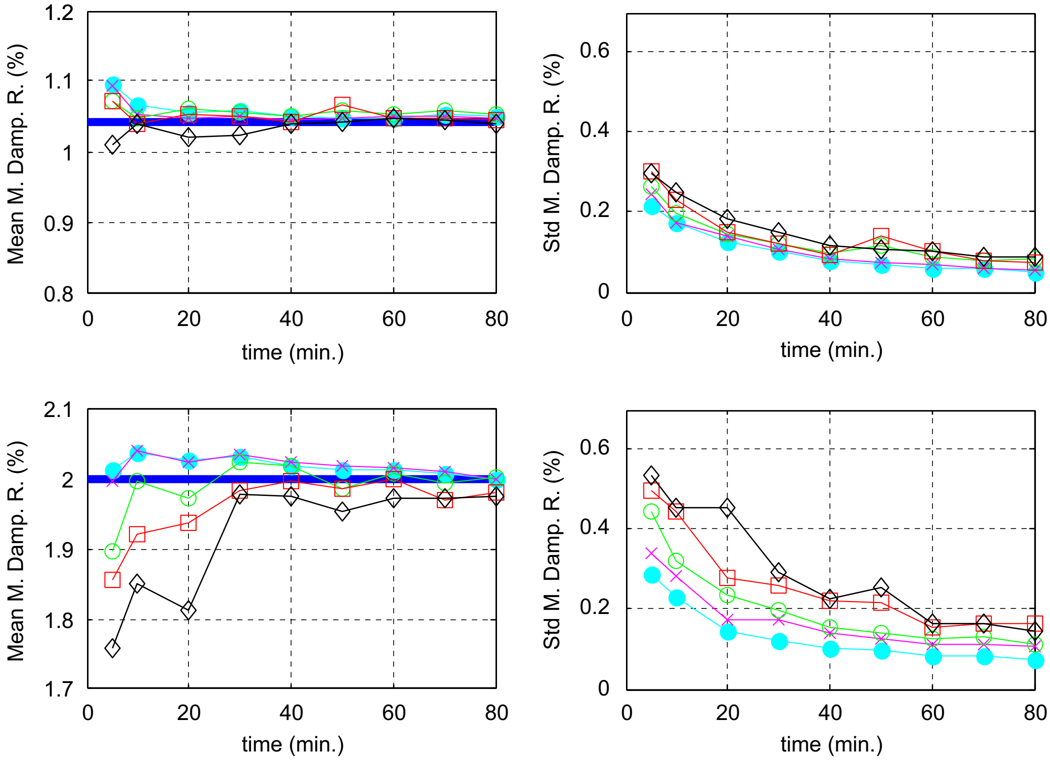

Fig. 7. Mean values and standard deviations of the estimates obtained with the SSI-COV method for the 2 modes of model SM1 using the parameters characterized in Table 3 (mode 1 at the top, theoretical values represented by a horizontal solid line, symbols of the lines defined in Table 3).

The number of points of the correlation functions limits the maximum order of the stochastic state-space models (maximum order = number of points of the correlations/2 × number of measured responses). Therefore, for the scenarios of the first line of Table 3, models of orders between 2 and 10 were used. For the other scenarios, the maximum model order was limited to 20 (already a very conservative limit taking into account that in the absence of noise and numerical inaccuracies, a model with two degrees of freedom is represented by a state-space model of order four). In the clusters analysis, the mode estimates with a distance (sum of the relative natural frequencies difference with one minus the MAC between the corresponding mode shapes [8]) inferior to 0.001 were considered inside the same group.

The results are presented in Fig. 7 using the same type of plots that were used in Fig. 5. It is evident that, in this example, the SSI-COV method could provide good estimates using just 5 min of data (one quarter the time that is needed by the EFDD method). Another advantage is that the results are less dependent on the method main parameters (the vertical scales of the plots with the mean values shown in Fig. 7 are much more detailed than their counterparts presented in Fig. 5). Still, a good selection of the length of the correlations can improve the results, especially if short time series are used. The obtained results show that an increase in the length does not improve the estimates. This is understandable, because the first points of the correlations present richer information, since the amplitudes are higher and therefore the noise effect is smaller. The latter points also tend to contain very few information of the modes with higher damping.

The minimum number of correlation points must be chosen taking into account that the correlations should contain more than one complete cycle of the largest structure period of vibration and that the model order limit depends on this number (this order must be considerably higher than two times the number of expected modes in the frequency range of analysis).

With respect to the standard deviations of the estimates, the plots of Fig. 7 show that they decrease with the increasing of the used time length, reaching minimum values around 0.10%. Therefore, the scatter of the estimates provided by this method is similar to the observed with the EFDD. Additional simulations have also demonstrated that it is not reduced if the artificial noise is not considered.

Then, both identification methods were applied to ambient responses simulated for the six models described in Table 1. In these analyses, time segments with 20 min were used and the noise/signal ratio of 10% was kept. Both methods were applied using parameters that proved to provide good results for model SM1: EFFD using 1024 points spectra (scenario 4c) and SSI-COV adopting correlation with 20 points (scenario 2c). For each model, 100 groups of responses were generated and processed without any user interaction.

Fig. 8 presents the results obtained for the modal damping ratios, representing the outputs of the identification algorithms by the mean ($\times$ marks) and the standard deviation (amplitude of the error bars) values associated with the 100 performed identifications. The SSI-COV method provided, for the six models, bias free estimates (average values almost coincident with the theoretical values) with standard deviations varying from 0.11% to 0.37%, the values associated with model CM3 (closely spaced modes and strong modal complexity) being slightly higher than the ones found for the remaining models. The EFDD method led to less good results for model CM2 and extremely biased estimates for modal damping ratios of the second mode of model CM3. Due to the coexistence of closely spaced modes and non-proportional damping, the mode shapes of model CM3 are not geometrically orthogonal. This is one of the assumptions of the EFDD method [5] that fails and may be a possible justification for the observed bad results.

It is relevant to point out that the procedures used to perform the modal identification automatically in some setups failed to provide acceptable estimates (defined as the estimates with an error on the natural frequency lower than 1%). The success rate observed for each model is quantified in Table 4. As expected, the number of unsuccessful identifications is higher when closely spaced modes are present and the EFDD method is less robust. The high coupling of the two modes of model CM3 justifies the inability of the EFDD method to individualize the contribution of the second mode in approximately one-half of the setups.

3.3. Free vibration tests simulation

The two types of free vibration tests usually performed in Civil Engineering structures are studied in this work. Initially, the free decays observed after the application of a sinusoidal load in resonance were analysed. Then, the quality of the estimates extracted from free decays measured after the application of an impulse were evaluated. In these cases, the structure excitation and response are deterministic. Therefore, only one simulation was performed for each scenario.

For the simulation of both tests, noise was added to all the simulated responses. However, in this type of tests, the noise-to-signal ratio is very small when compared with the observed in ambient vibration tests, because the level of vibration is much higher. So, it was adopted a very conservative ratio of 1%, taking into account that the accelerations induced during the free decays should be at least higher than 10 times the responses measured during ambient vibration tests. Its effect on the results was found to be negligible.

In the sinusoidal load test, the traditional analysis, described in Section 2.2 (fitting of an exponential decay), assumes that the decay has only the contribution of a single mode. It is well known that this assumption is close to reality when damping is small and proportional and the modes are well separated. The first graphic of Fig. 9 presents the responses simulated for model SM1 when the first mode is excited in resonance. The second plot shows the exponential function that

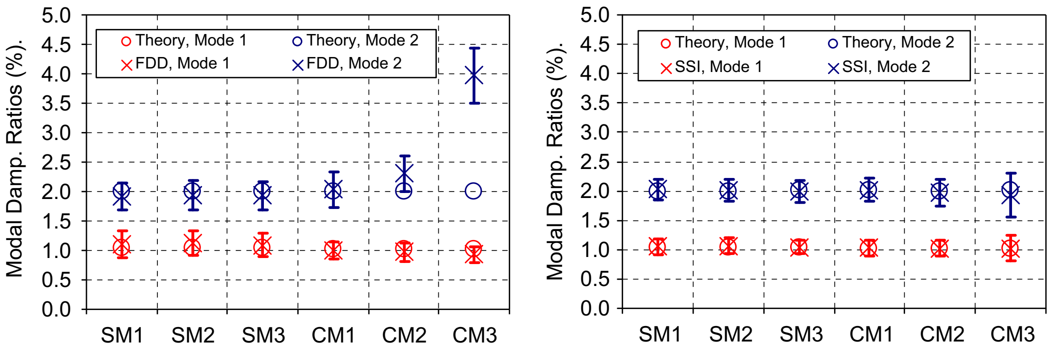

Fig. 8. Modal damping ratios estimates provided the EFDD and SSI-COV methods for the six studied models.

Table 4
Percentage of successful identifications.

	EFDD		SSI	
	Mode 1	Mode 2	Mode 1	Mode 2
SM1	98	99	100	100
SM2	96	99	100	99
SM3	99	100	99	100
CM1	100	76	98	78
CM2	97	73	100	82
CM3	92	45	98	77

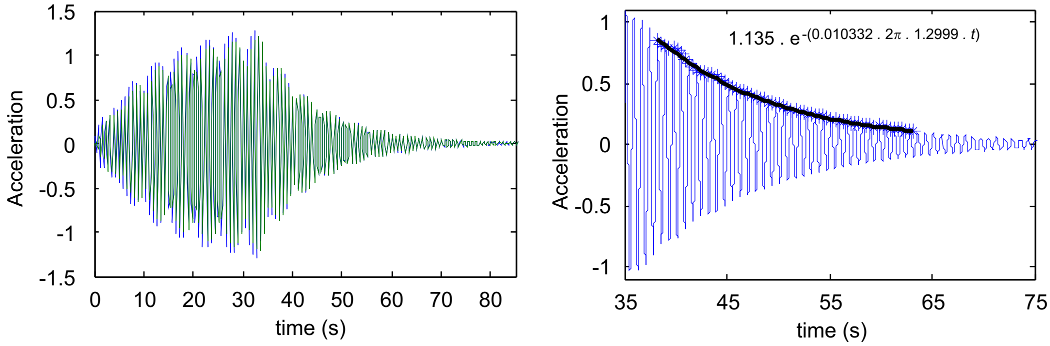

Fig. 9. Simulated response of model SM1 during and after the application of sinusoidal load with a frequency of 1.2995 Hz (the response of the two degrees of freedom is almost coincident) and fitted exponential function: $y = Ae^{-\xi 2\pi f t}$ (ξ – modal damping ratio, f – natural frequency, t – time).

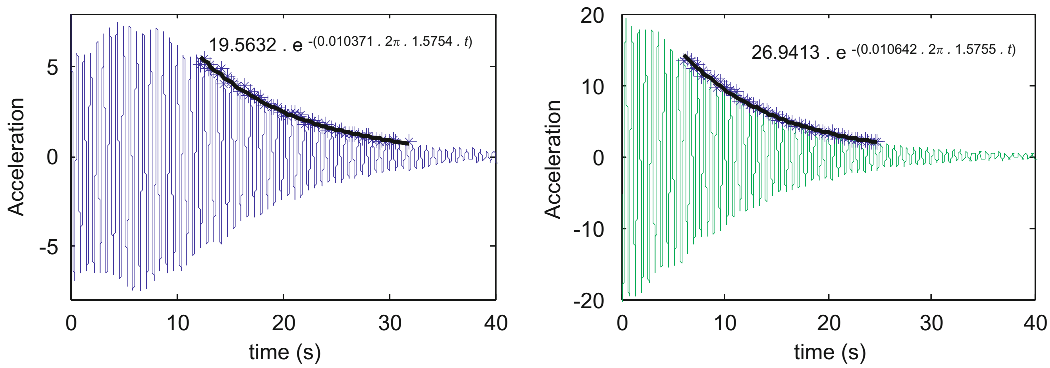

Fig. 10. Simulated response for the 2 degrees of freedom of model CM3 (1st – left, 2nd – right) after the application of sinusoidal load with a frequency of 1.5926 Hz and fitted exponential functions.

was fitted to the extremes of a spline previously adjusted to the simulated time series of the first degree of freedom. The fitting was preformed for amplitudes between 0.9 and 0.2 of the maximum acceleration. It shows that for this model the previously referred assumption is valid, as the estimated modal damping ratio (1.037%) is very close to the theoretical value.

Fig. 10 exemplifies the exponential fitting applied to the responses of the two degrees of freedom of model CM3, after the application of a sinusoidal load with the frequency of its second mode. In this case, it was not possible to get a decay with just the contribution of the second mode, because there is a strong coupling of the two modes of the model. At the beginning of the decay, the response is driven by both modes, but at the end, the first one becomes dominant, as it has lower damping. Therefore, the modal damping ratio provided by the exponential fitting gives a wrong estimate for the damping of the second mode.

Table 5 resumes the results provided by the exponential fitting applied to the simulations of the free decay, when trying to have just one dominant mode. The presented modal damping ratios are the average of the two values that resulted from the analysis of the responses simulated for the two degrees of freedom of each model. As expected, the presented results show that this traditional procedure produced good results only for the models without closely spaced modes.

Table 5
Modal damping ratios (%) estimated from free decays after the application of sinusoidal loads in resonance.

	Exp. fitting		SSI	
	Mode 1	Mode 2	Mode 1	Mode 2
SM1	1.0282	1.9184	1.0408	2.0005
SM2	1.0309	1.9130	1.0375	2.0123
SM3	1.0305	1.9325	1.0357	2.0073
CM1	1.0827	1.1498	0.9742	1.9503
CM2	1.1648	2.0154	0.9484	2.1020
CM3	0.9648	1.0507	0.9990	2.0330

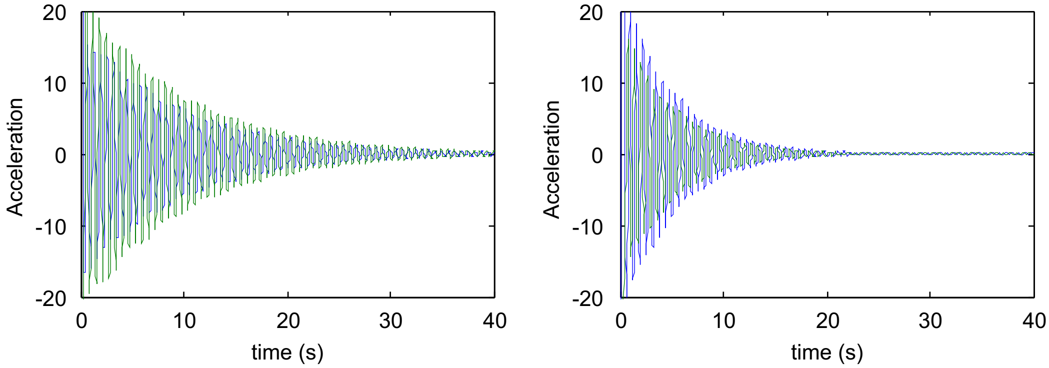

Fig. 11. Modal decays (mode 1 – left; mode 2 – right), provided by the SSI-COV method, of model CM3 after the application of sinusoidal load with a frequency of 1.5926 Hz.

It should be mention that, the average values obtained for the second mode of model CM2 are close to the theoretical value just by accident; because, the results obtained for each degree of freedom response are quite far from the real values.

As alternative to this rather simple procedure, the adaptation of the SSI-COV method described in Section 2.1 can be used. The results shown in the last two columns of Table 5 demonstrate that this methodology delivered excellent results for all the considered models. Furthermore, with this technique it was possible to decompose the measured responses into the contribution of the two modes. In the case of the models with separate modes, the response contains almost only the contribution of the mode that was deliberately excited. On the contrary, in the case of the models with closely spaced modes, it is impossible to excite just one mode and consequently the obtained responses are far from being modal decays. Fig. 11 shows the modal decays that contribute to the responses represented in Fig. 10, confirming what had been previously anticipated: at the beginning both modes have similar energy (even exciting the structure in resonance with the second mode) and then the second mode vanishes and the first one becomes dominant. These results are also important to show that, even for the scenario where there is a strong coupling of two modes, this technique can separate the modal contributions, overcoming the limitation of the filters that are useless in this situation.

This methodology can also be applied to free decays measured after the application of impulses. In this case, with just one impulse, all the modes with significant modal ordinates at the degree of freedom where the impulse is applied are excited. In this case, the application of the exponential fitting has to be preceded by the application of filters to try the separation of the modes involved in the response, which are obviously not effective in the presence of closely spaced modes. As the fragilities of this simple procedure have already been stressed, the simulated impulse responses are only processed by the SSI-COV method.

Fig. 12 presents the stabilization diagrams associated with the analysis of the data of models SM1 and CM3, showing for both cases two clear vertical alignments of stable poles that were successfully detected by the already described cluster analysis. The final estimates, displayed in Table 6, present differences with respect to the theoretical values (Table 1) always inferior to 0.1% that confirm the good performance of the applied method.

4. Full-scale tests

The previously presented numerical study is now completed with the characterization of modal damping ratios estimates obtained from full-scale tests performed on three large Civil Engineering structures.

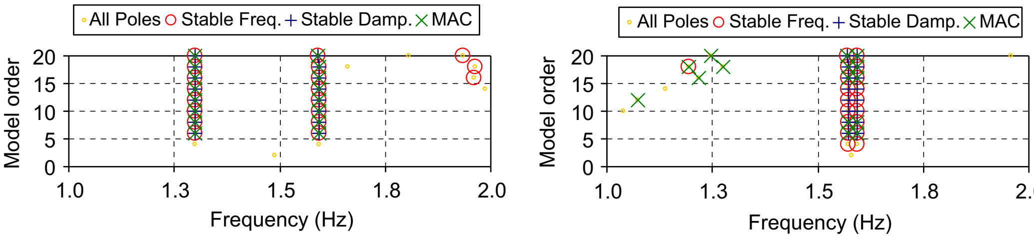

Fig. 12. Stabilization diagrams of the application of the SSI-COV algorithm to the free decays of models SM1 (left) and CM3 (right).

Table 6
Modal damping ratios (%) estimated from the free decays associated with the application of an impulse.

	SSI	
	Mode 1	Mode 2
SM1	1.0457	2.0119
SM2	1.0322	2.001
SM3	1.0414	2.0105
CM1	0.9347	2.0225
CM2	1.0121	1.9511
CM3	1.0272	1.9723

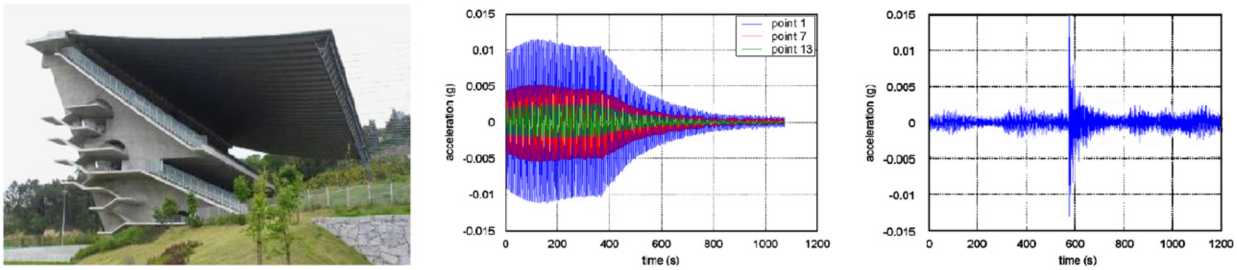

Fig. 13. Braga Stadium (view of the east side and records of measured decays in the roof after the application of a sinusoidal load in resonance and after the application of an impulse).

4.1. Stadium suspended roof

The Braga Stadium was one of the Stadia constructed in Portugal for the 2004 European Championship (Fig. 13). The most outstanding element of the structure is its very flexible suspended roof, which is formed by pairs of full locked coil cables spaced 3.75 m apart from each other, supporting two concrete slabs over the stands of the stadium. The cables span is 202 m and the slabs length is 57.3 m; therefore, the remaining 88.4 m of the central part are free. The slabs have a thickness of 0.245 m.

The necessity of analysing the susceptibility of the suspended roof to buffeting effects required the accurate experimental identification of modal damping ratios. Taking that in mind, three different types of dynamic tests were performed: an ambient vibration test that allowed the identification of the natural frequencies, modes shapes and modal damping ratios, forced vibration tests with the application of sinusoidal loads and free vibration tests after the application of an impulse for the identification of the modal damping ratios. In Ref. [11], the dynamic tests and the obtained results are extensively described. The roof structure is characterized by low natural frequencies (the first 10 modes have natural frequencies between 0.28 and 0.74 Hz) and closely spaced modes.

Fig. 14 summarizes the results of all used methodologies in terms of modal damping ratios. The ambient vibration test comprehended the performance of 28 setups with duration of 16 min. Each setup was processed independently with the SSI-COV method, and therefore, besides the average values, it is possible to obtain a rough characterization of the estimates variability. These are characterized in the plot by means of the error bars (the distance between the mark ♦, indicating an average value, and the horizontal ticks is equal to the standard deviation). The standard deviations present an average value (involving the first 10 modes) of 0.21%, which corresponds to a mean relative standard deviation (standard deviation of each mode estimate divided by its average damping value) of 52.6%.

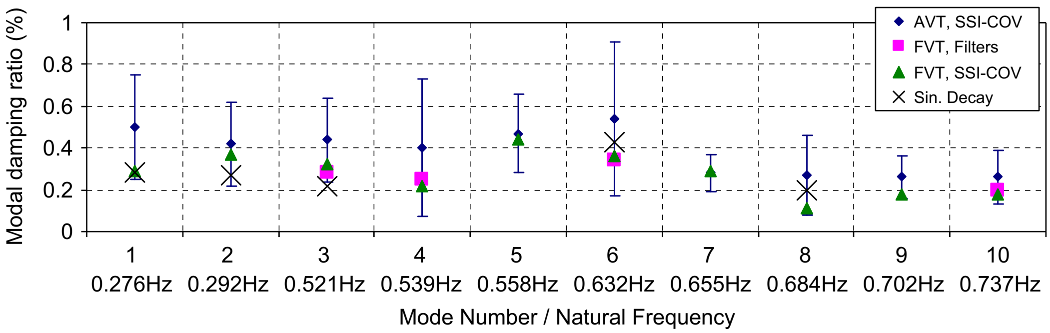

Fig. 14. Modal damping ratios of the stadium suspended roof obtained from the ambient vibration test (AVT), from the free vibration test (FVT) and from the tests with sinusoidal excitation (Sin. Decay).

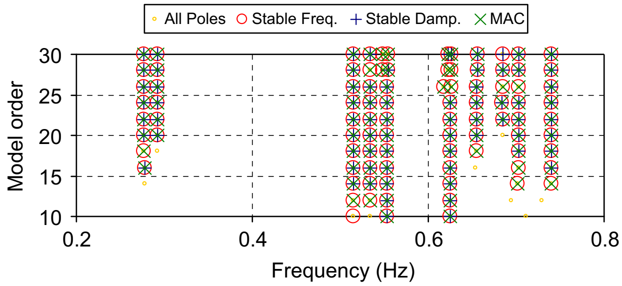

Fig. 15. Stabilization diagram associated with the application of the SSI-COV method to the free decays measured after the application of an impulse in the stadium suspended roof.

The decays measured after the application of an impulse were processed using the procedure based on band-pass filtering and subsequent fitting of exponential functions and by mean of the SSI-COV method. The use of filters allowed the identification of the modal damping ratios of modes 3, 4, 6 and 10 (FVT, Filters). For the others, it was not possible to get results owing to the closeness of the natural frequencies or due to lack of excitation. The application of the SSI-COV method to the same data (FVT, SSI-COV) made possible the identification of additional damping ratios by the analysis of the stabilization diagram presented in Fig. 15, which contains 10 clear vertical alignments of stable mode estimates. When the comparison is possible, the results of both techniques (FVT, Filters and FVT, SSI-COV) are almost coincident (Fig. 14).

Furthermore, the SSI-COV method delivered the modal decays of the first 10 modes; these are plotted in Fig. 16, considering the decomposition of the accelerations measured simultaneously at 5 points of the roof. It is very interesting to observe that the sum of the calculated modal decays almost coincides with the measured ones. This comparison is performed in Fig. 17 for one of the measured degrees of freedom (located at one corner of the roof).

The forced vibration tests with the application of sinusoidal loads were performed to excite specifically 5 modes, the modal damping ratios estimates being obtained by the more traditional procedure. The identified damping ratios are similar to the ones provided by the impulsive tests. Therefore, the significant amount of work associated with these tests could have been avoided.

The comparison between modal damping ratios identified using artificial and ambient excitation shows some differences, especially for the first mode. However, all the values are within the interval of variation (defined by the standard deviation) presented for the AVT estimates.

4.2. Footbridge

The “Pedro e Inês” footbridge is a new landmark that links both banks of Mondego River at the city of Coimbra, in Portugal. The bridge has a total length of 275 m and is formed by a central parabolic arch that spans 110 m and rises 9 m and two half-parabolic arches in steel supporting with total continuity a composite steel-concrete deck (Fig. 18).

The preliminary dynamic studies of this bridge indicated that it would be prone to vibrations induced by pedestrians, requiring control devices. For the correct final design of the tuned mass dampers, it was crucial to have a good characterization of the dynamic behaviour of the bridge. Therefore, after bridge construction and before the installation of the control devices, dynamic tests were performed. These included an ambient vibration test and several free vibration tests. A detailed description of all the tests, processing procedures and results can be found in [12]. This paper is only focused on the estimation of the modal damping ratios.

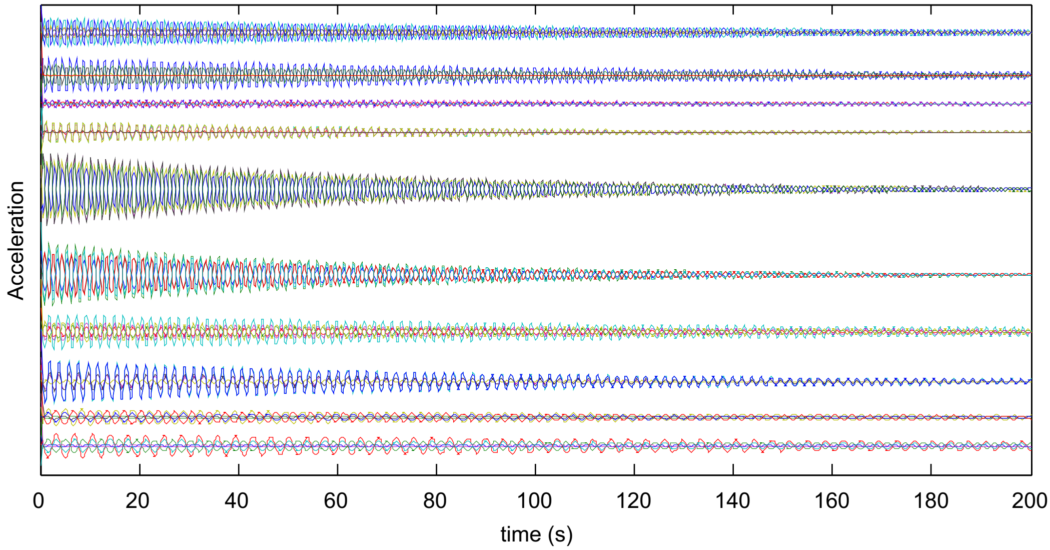

Fig. 16. Decomposition of the measured free decays, after the application of the impulse, in the modal decays of the first 10 modes of the stadium suspended roof.

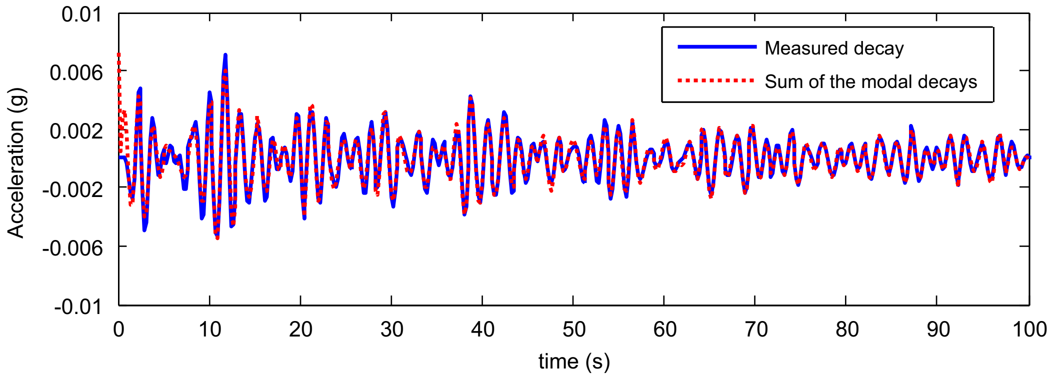

Fig. 17. Comparison between measured free decay and sum of calculated modal decays for one of the instrumented degree of freedom of stadium suspended roof.

Fig. 18. "Pedro e Inês" footbridge (general view and hanging of the mass for application of the impulse).

The ambient vibration test comprehended 19 setups that allowed the measurement of acceleration time series with 16 min at 20 sections. The collected data were processed by the enhanced frequency domain decomposition method and by the data-driven stochastic subspace identification method. Both methods provided estimates of natural frequencies and

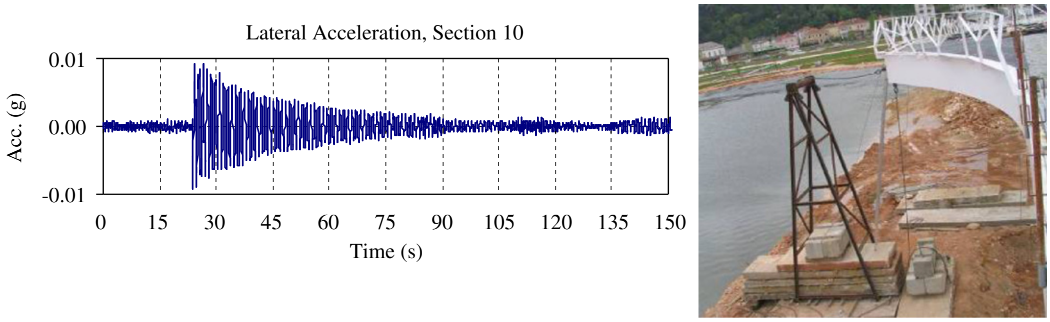

Fig. 19. Free vibration response measured at "Pedro e Inês" footbridge after the application of a lateral impulse and auxiliary structure used for its application.

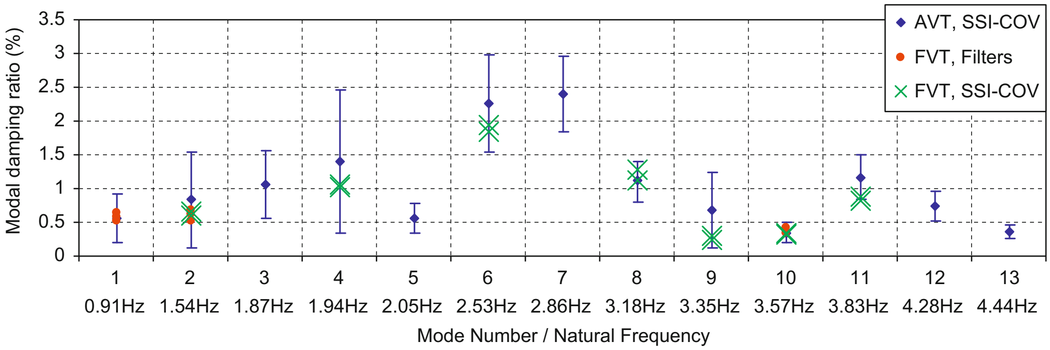

Fig. 20. Modal damping ratios of "Pedro e Inês" footbridge obtained from the ambient vibration test (AVT) and from the free vibration test (FVT).

modal damping ratios for 13 modes in the frequency range 0–5 Hz. In Fig. 20, the results of the SSI-DATA in terms of modal damping ratios are summarized by the average value and by an interval of variation defined by the standard deviation (average standard deviation of 0.45%, average relative standard deviation of 46.5%).

As the identification of accurate modal damping ratios was of utmost importance for the correct design of the control devices, a total of 10 free vibration tests were performed: 3 vertical and 3 lateral impulses applied at mid-span (Fig. 19), 2 vertical impulses applied in the deck span over one of the supports inside the river (Fig. 18) and 2 vertical impulses in a lateral span.

The first eight impulses were applied to excite specific modes (the mass was suspended at the anti-node of the mode). So, it was possible to isolate the contribution of each of those modes in the measured free decays, by application of low-order band-pass filters. The results of this procedure are represented in Fig. 20 by the marks ●. A total of 8 values are represented: three estimates for the first and second modes and two estimates for the 10th mode. It is evident the good agreement between these estimates and the average values provided by the ambient vibration test.

The last two impulses, applied in a lateral span, excited several modes and so the use of filters became inappropriate. Thus, the collected decays were processed by the SSI-COV method, providing the two groups of estimates represented also in Fig. 20 by the marks X. These estimates confirmed the ones provided by the other free decays for the 2nd and 10th modes and it was possible to obtain estimates for more 5 modes that agree quite well with the results of the ambient vibration test.

4.3. *Cable-stayed bridge*

The "Vasco da Gama" bridge, in Lisbon, crosses the Tagus River over a total length of about 12 km. This large structure includes a cable-stayed component (Fig. 21) with a main span of 420 m and 3 lateral spans on each side (62 m+70.6 m+72 m), resulting in a total length of 829.2 m. The height of the two H-shaped towers is 147 m.

At the end of construction, dynamic tests were performed comprising an ambient vibration test and a free vibration test. During the ambient vibration test, the main excitation source was wind, which suffered speed fluctuations between 1 and 22 m/s. A total of 58 points were measured with 25 setups of 16 min. The free vibration test consisted in the measurement of the bridge responses at 3 sections after the sudden release of a 60 ton mass (Fig. 21) previously suspended from the bridge deck. To minimize the effect of undesired aerodynamic damping, this test was performed on a day with low wind speeds (<2.5 m/s).

Fig. 21. "Vasco da Gama" bridge (general view and barge suspended for the application of the impulse).

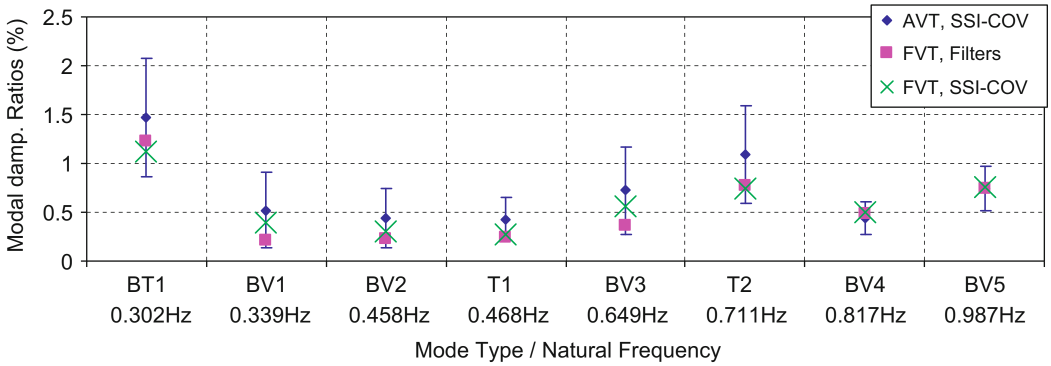

Fig. 22. Modal damping ratios obtained from the ambient vibration test (AVT) and from the free vibration test (FVT) of "Vasco da Gama" cable-stayed bridge.

The identified modal parameters showed that the first mode shape is transversal (BT1) and has a natural frequency of 0.30 Hz. The first vertical bending (BV1) mode has a natural frequency of 0.34 Hz, whereas the first torsion mode (T1) has a natural frequency of 0.47 Hz. Further information about the dynamic tests, the procedures used to process the collected data and the obtained results are presented in references [13,14].

In this paper, only the results related with the modal damping coefficients estimates are detailed. Fig. 22 shows the modal damping ratios estimated using the data collected during the ambient vibration test and during the free vibration test. Like in the other two structures, the results of the AVT have a significant dispersion (average standard deviation of 0.36%, average relative standard deviation of 52%) and the estimates of both procedures of analysis applied to the free decays (filters and SSI-COV) are close to the average values of the AVT.

5. Conclusions

The present paper concerns the experimental estimation of modal damping ratios, comprehending two main sections: one dedicated to numerical simulations and another one with results obtained from full-scale tests of large Civil Engineering structures.

The use of numerical simulations permitted to create models with predefined characteristics, allowing to test the identification techniques under critical situations, like the existence of closely spaced modes and non-proportional damping. Furthermore, the performance of groups of 100 simulations, together with the use of routines that allow the automatic identification of modal parameters, permitted to characterize the variability of the results provided by the tested algorithms.

The use of the enhanced frequency domain decomposition method, which is a very easy to use and fast algorithm to process the data collected during ambient vibration tests, showed that it can provide good results, if the parameters for the spectra estimation are well chosen and if a high coupling between modes is not present. However, the covariance-driven stochastic subspace identification method (SSI-COV) proved to be less dependent of the selected algorithm parameters, enabling the achievement of good results with shorter time series and of unbiased estimates even in presence of highly coupled modes. The application of both methods demonstrated that, even using long time series and suitable algorithm parameters, the estimates present a relative standard deviation around 10%.

The simulation of free vibration tests was important to show the fragilities of a procedure that is still commonly used in the test of Civil Engineering structures: the fitting of an exponential decay to previously filtered decays or to decays measured after the application of a sinusoidal load in resonance. An alternative methodology, based on stochastic subspace models, was described, presenting an excellent performance for all the tested situations. In addition, it was derived an equation that permits the decomposition of the measured free decay into its modal decays.

The second part of the paper, that includes the results of full-scale ambient and free vibration tests performed on three large structures, evaluates the quality of estimates that is possible to achieve on real applications. The SSI-COV method, the one that showed better performance in the simulation studies, is applied to process the ambient responses. The free decays are analysed by the more traditional procedure and also by the SSI-COV method.

The results provided by the ambient vibration tests showed a fairly high scatter, expressed by relative standard deviations around 50%. This stems mainly from two reasons. One is the use of relatively short time series (16 min), taking into account that the first frequency of two of the analysed structures is around 0.3 Hz and that the modal damping ratios are small (generally lower than 1%). The relative standard deviations observed in the simulations when time segments of 20 min were used were around 20%. The other main reason is the non-stationarity of the structures excitation, not reproduced in the simulations. During the ambient vibration tests, sometimes performed during more than one day, significant variations of the intensity and frequency content of the excitation can occur, which can lead to variations of the structural damping, that is commonly dependent on the amplitude of structural oscillations (e.g. existence of friction forces that are only mobilized for certain vibration levels). The variation of wind characteristics during the ambient vibration test can also induce damping variations motivated by the existence of aerodynamic damping, as already observed during the ambient vibration test of a cable-stayed bridge [15].

Besides that, in all the analysed databases the estimates of modal damping ratios obtained from the free decays are within the interval defined by the average values $\pm$ the standard deviation of the estimates of the ambient vibration tests, which leads to an increasing confidence in the results provided by ambient vibration testing.

In the analyses of the free decays, the SSI-COV method and the exponential decay fitting provided similar results when it was possible to individualize the modes, but the former was able to deliver accurate estimates for a higher number of modes. Furthermore, the decomposition of the free decays in modal decays showed good results also on a real example with closely spaced modes.

The results taken from the free decays are more accurate, because the noise influence is smaller (higher amplitudes), there is no need to make assumptions about the loading (in ambient vibration tests it is assumed a stationary white noise) and the influence of wind can be reduced if the impulse is applied with low wind velocities. However, these tests are more difficult to perform than the ambient vibration tests and the estimated values are associated with amplitudes that are higher than the amplitudes observed during the normal use of the structure.

The ambient vibration tests need long periods of acquisition to provide reliable results and so it is more difficult to ensure low wind velocities during the test to reduce the effect of aerodynamic damping. Anyway, they provide reasonable estimates of the modal damping coefficients with very economical and practical procedures. It is important to develop further research in order to better understand all the reasons for the observed dispersion and to find a process to reduce it. The data collected by permanent dynamic monitoring systems that allow the tracking of the damping evolution during long periods under different ambient conditions can be very useful in this context.

Acknowledgments

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