

$$1. \dot{y} + y = u(t)$$

$$sY + Y = \frac{1}{s}$$

$$Y(s+1) = \frac{1}{s}$$

$$Y = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A=1, B=-1$$

$$\begin{array}{c} \nearrow \\ \text{set } s=0 \end{array} \quad \begin{array}{c} \nearrow \\ \text{set } s=-1 \end{array}$$

$$Y = \frac{1}{s} - \frac{1}{s+1}$$

$$y(t) = u(t) - e^{-t}u(t)$$

$$y(t) = (1 - e^{-t})u(t)$$

$$\overline{sY + Y = X} \quad \frac{\overline{Y}}{\overline{X}} = \frac{1}{s+1}$$

$$\operatorname{Re}\{s\} > -1$$

$$2. \lim_{s \rightarrow 0} \frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{K_I}{s} * H}{1 + \frac{K_I}{s} * H}$$

$$\text{L'Hôpital's rule} \rightarrow \lim_{s \rightarrow 0} \frac{-\frac{K_I}{s^2} \cdot H}{-\frac{K_I}{s^2} \cdot H} = 1$$

$$B) \frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH} \quad \text{where } H = \frac{1/\tau}{s + 1/\tau} = \frac{1}{\tau s + 1}$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{K_I}{\tau s^2 + s}}{1 + \frac{K_I}{\tau s^2 + s}} = \frac{K_I}{\tau s^2 + s + K_I}$$

↑  
zeros on s

Set denom = 0;

$$0 = \tau s^2 + s + K_I$$

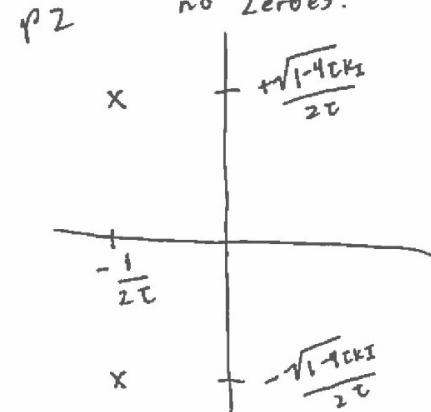
$$s = \frac{-1 \pm \sqrt{1 - 4\tau K_I}}{2\tau}$$

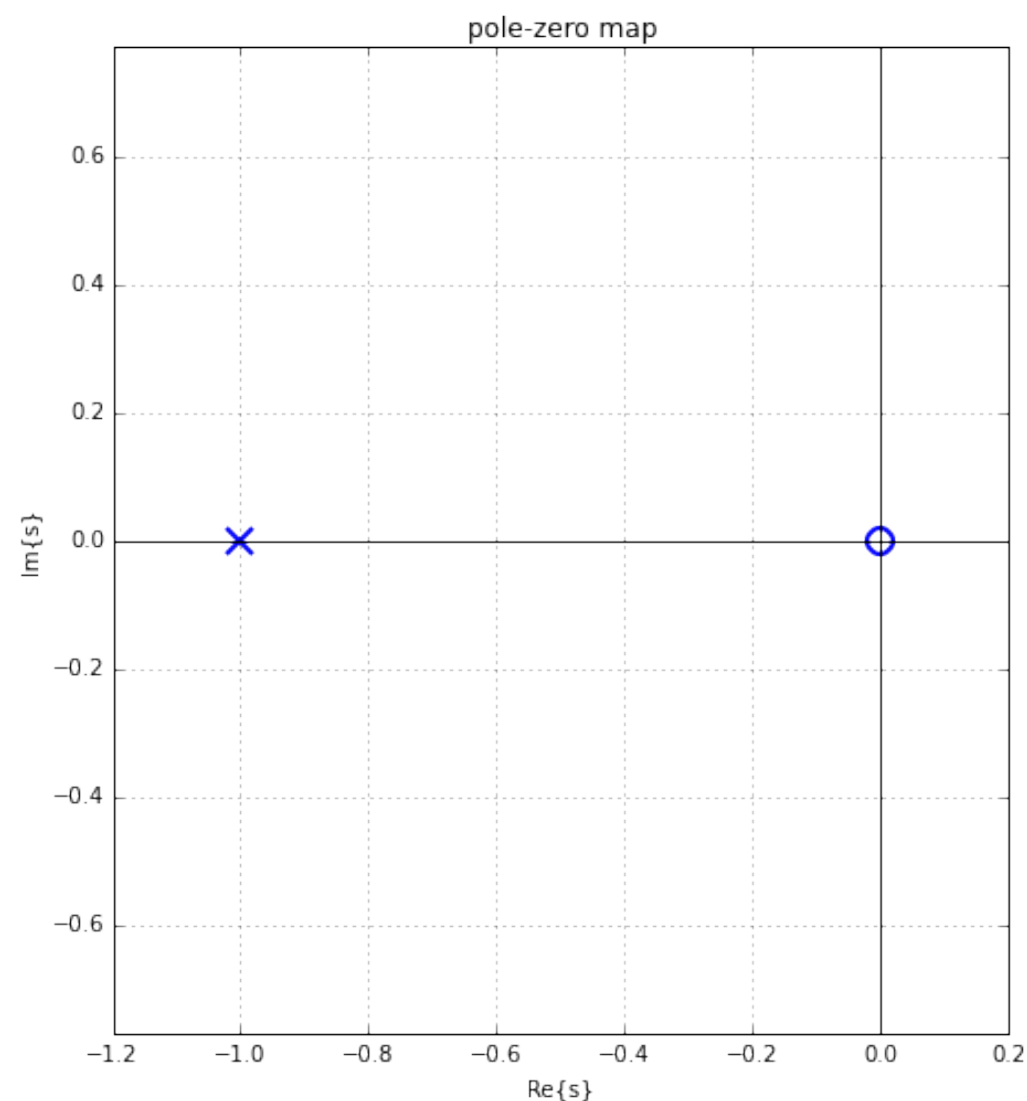
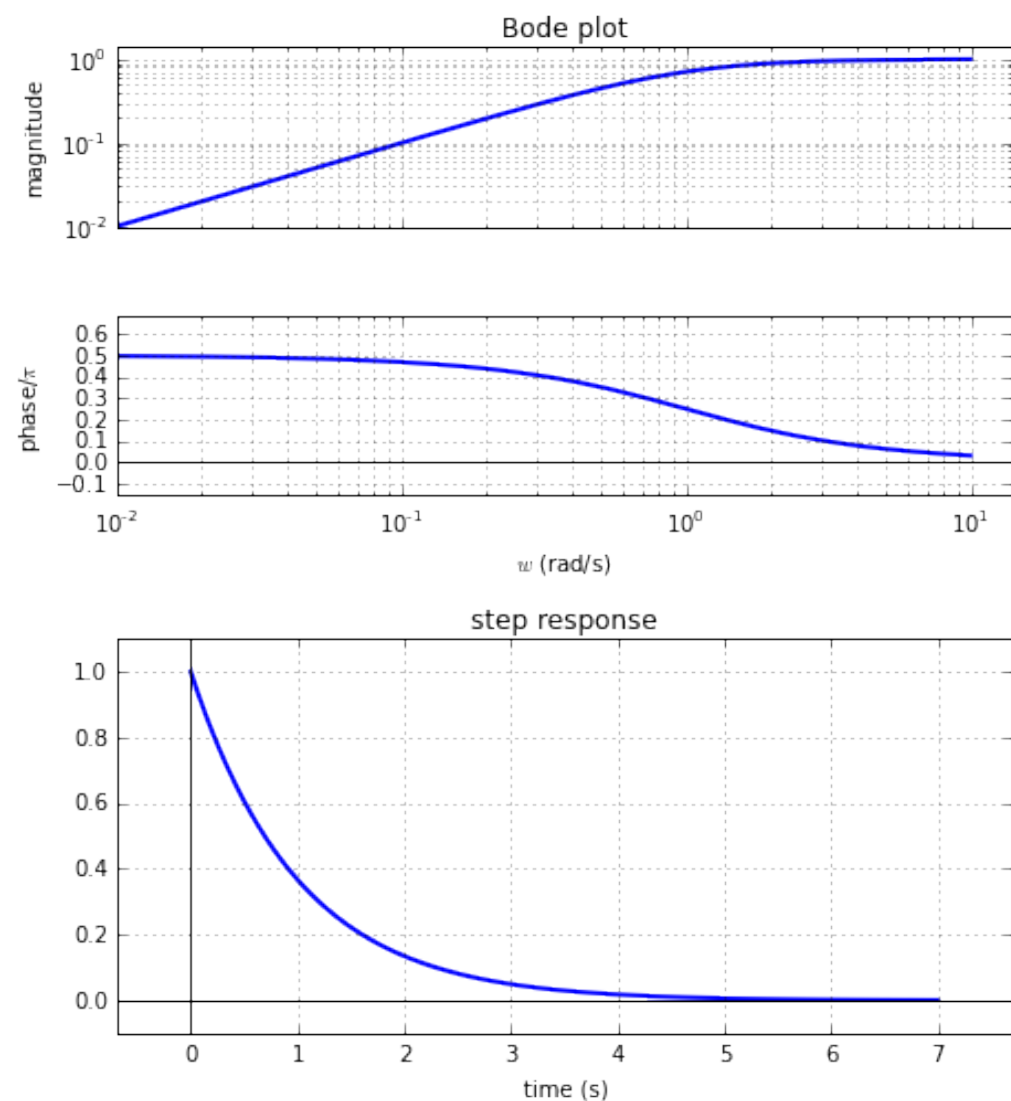
$$4) B: \frac{K_P}{s^2 - 0.01s + 1 + K_P}$$

$$C: \frac{K_I}{s^2 - 0.01s^2 + s + K_I}$$

$$D: \frac{K_D \cdot s}{s^2 + (K - 0.01)s + 1}$$

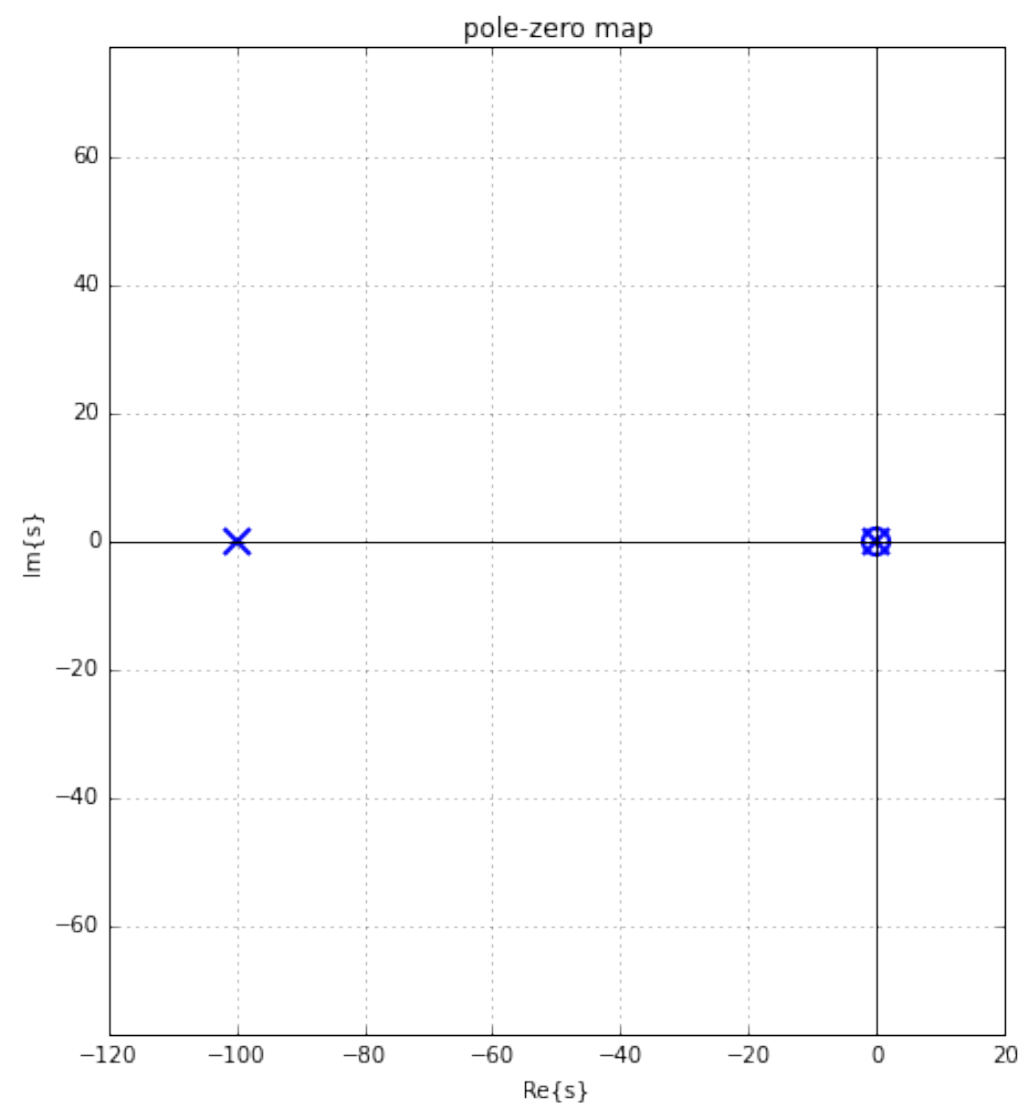
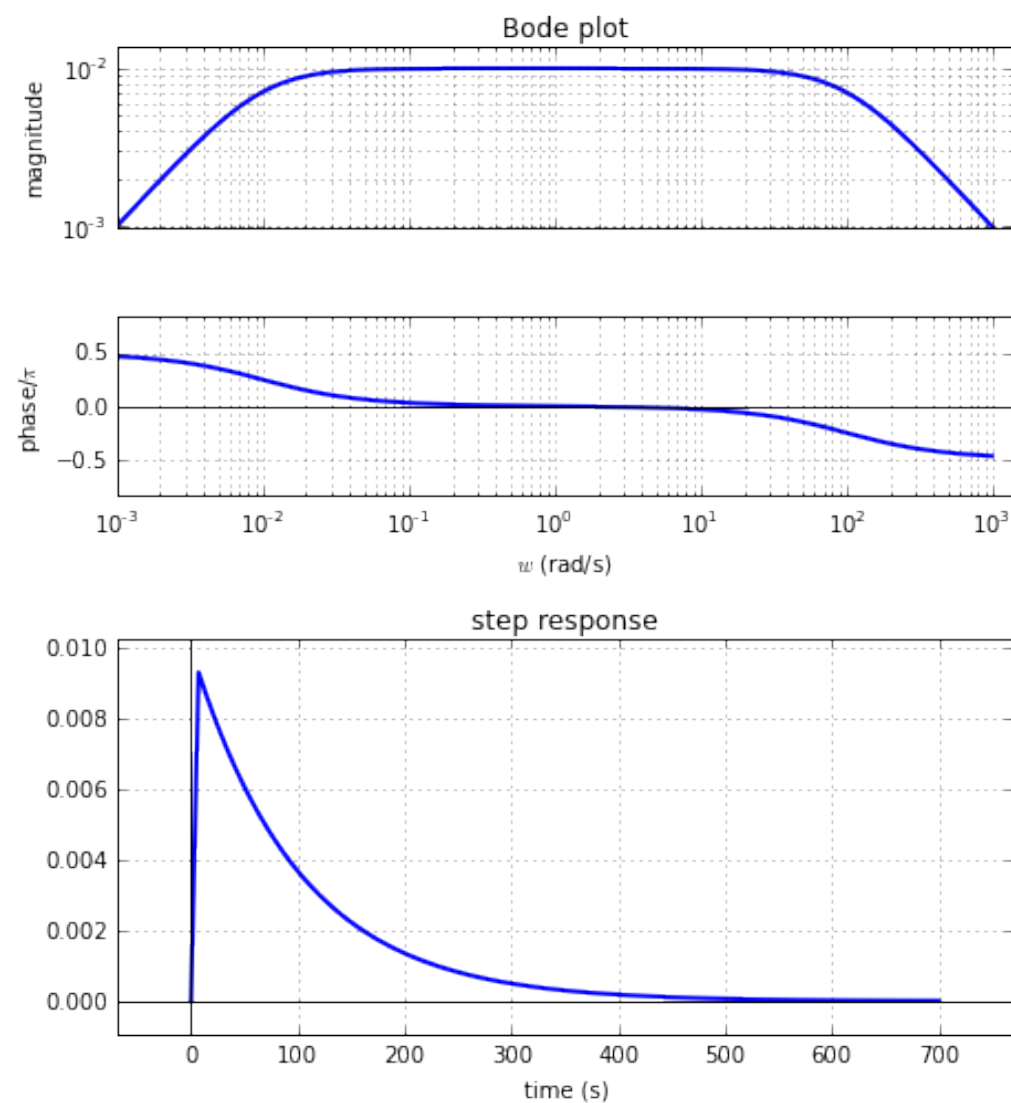
No 's' in numerator, no zeroes.





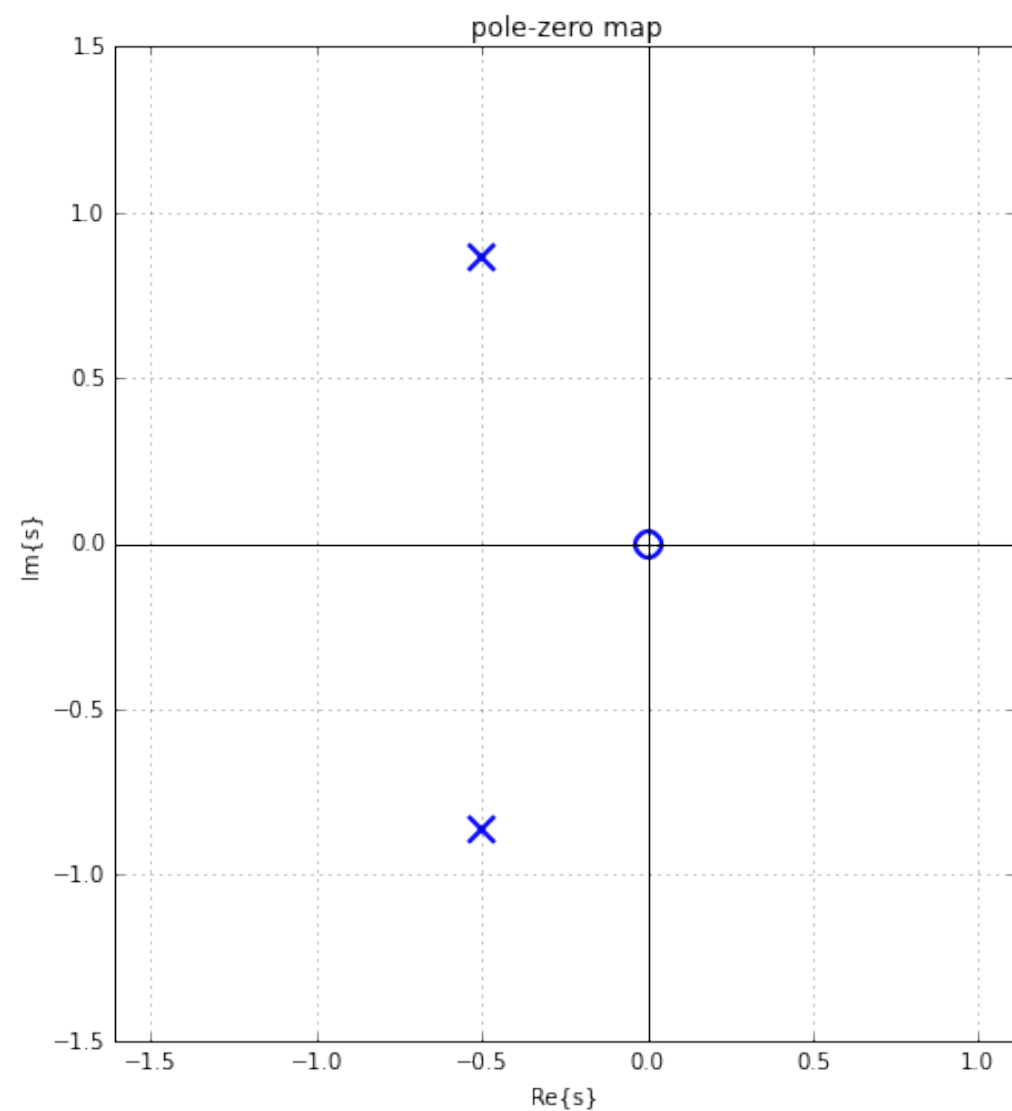
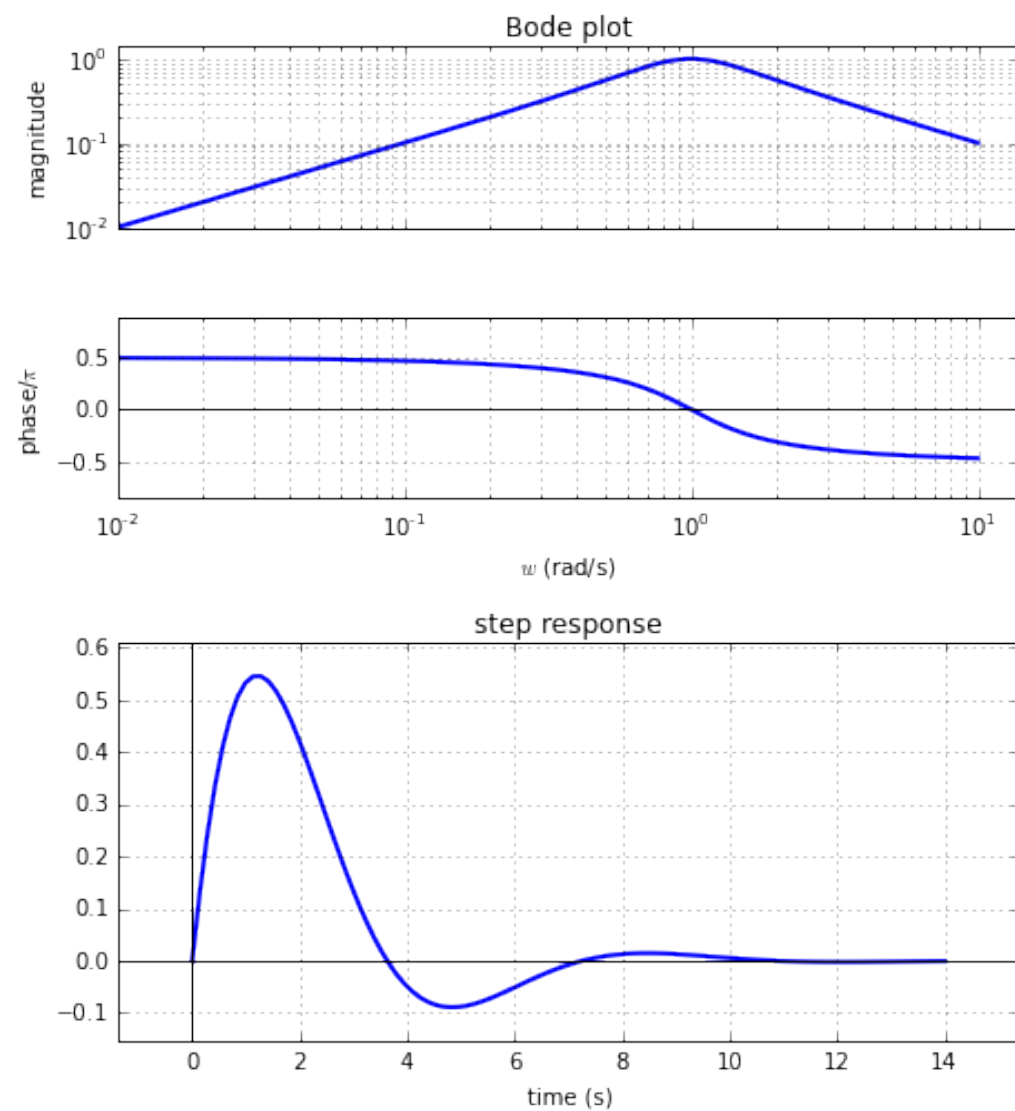
Step response jumps immediately to 1 when step inserted. This gives preference to high frequencies, as high frequency oscillations will switch before the step response can decay. Thus the bode plot shows a high pass nature. The single pole and single zero mean that for a first order input, the output can only be first order. The pole is only real, reflecting the lack of oscillation, and it is negative, showing that the system will decay to zero rather than resonant.

Problem 3



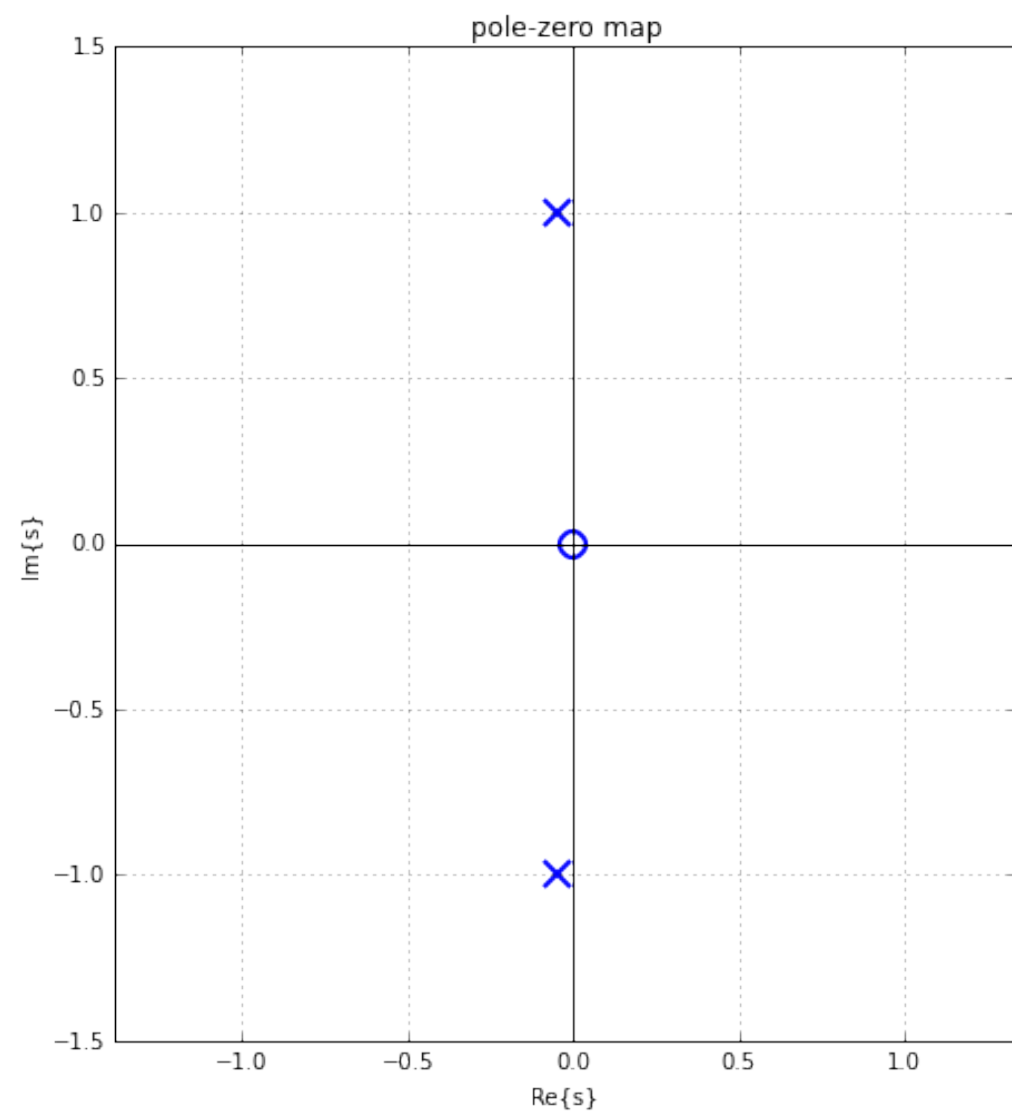
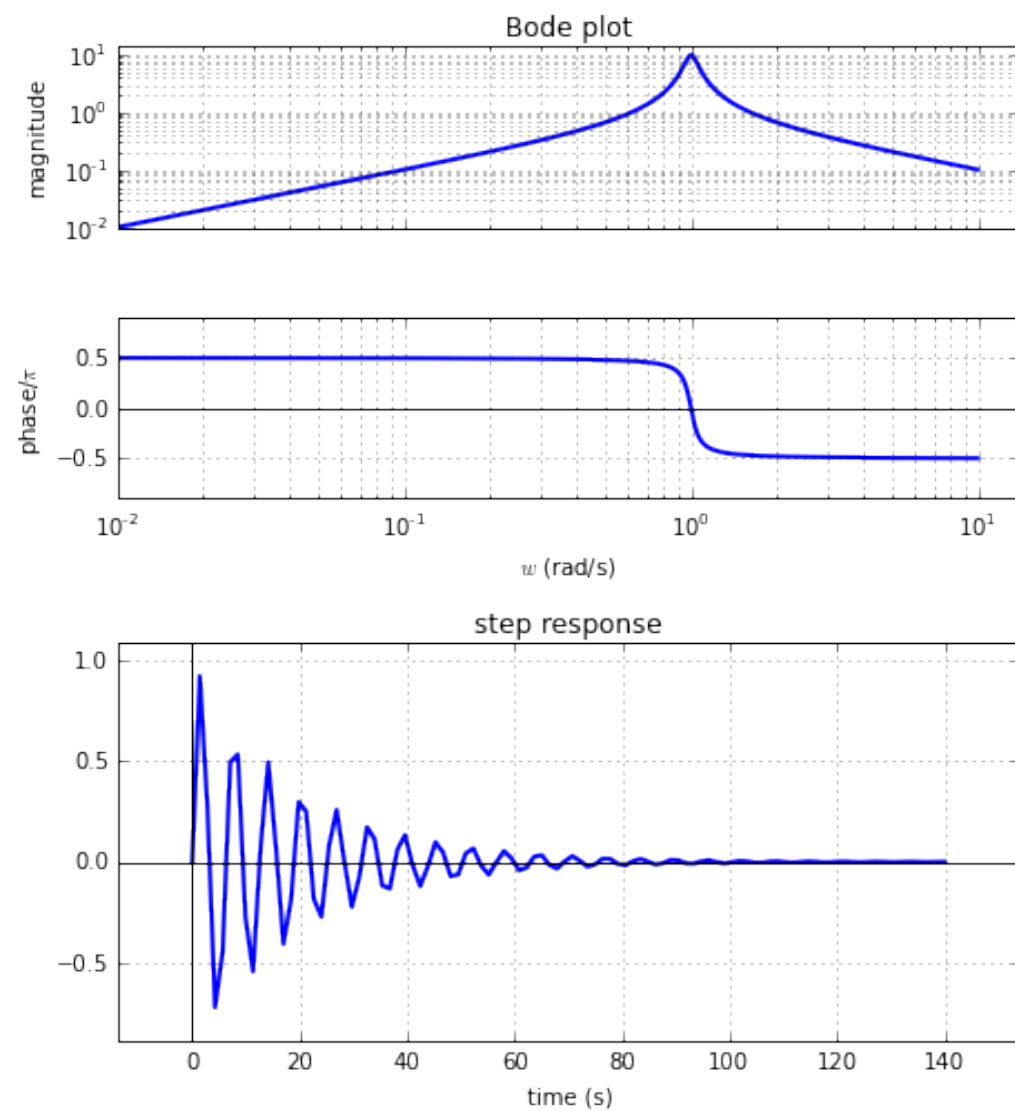
Unlike the first system, this takes a small amount of time to ramp to 1, meaning that frequencies faster than it can reach one will be attenuated. Thus, this system is bandpass. The presence of two poles means it is second order, and the large negative magnitude of the pole with a negative real part means that the system will decay to zero.

### Problem 3



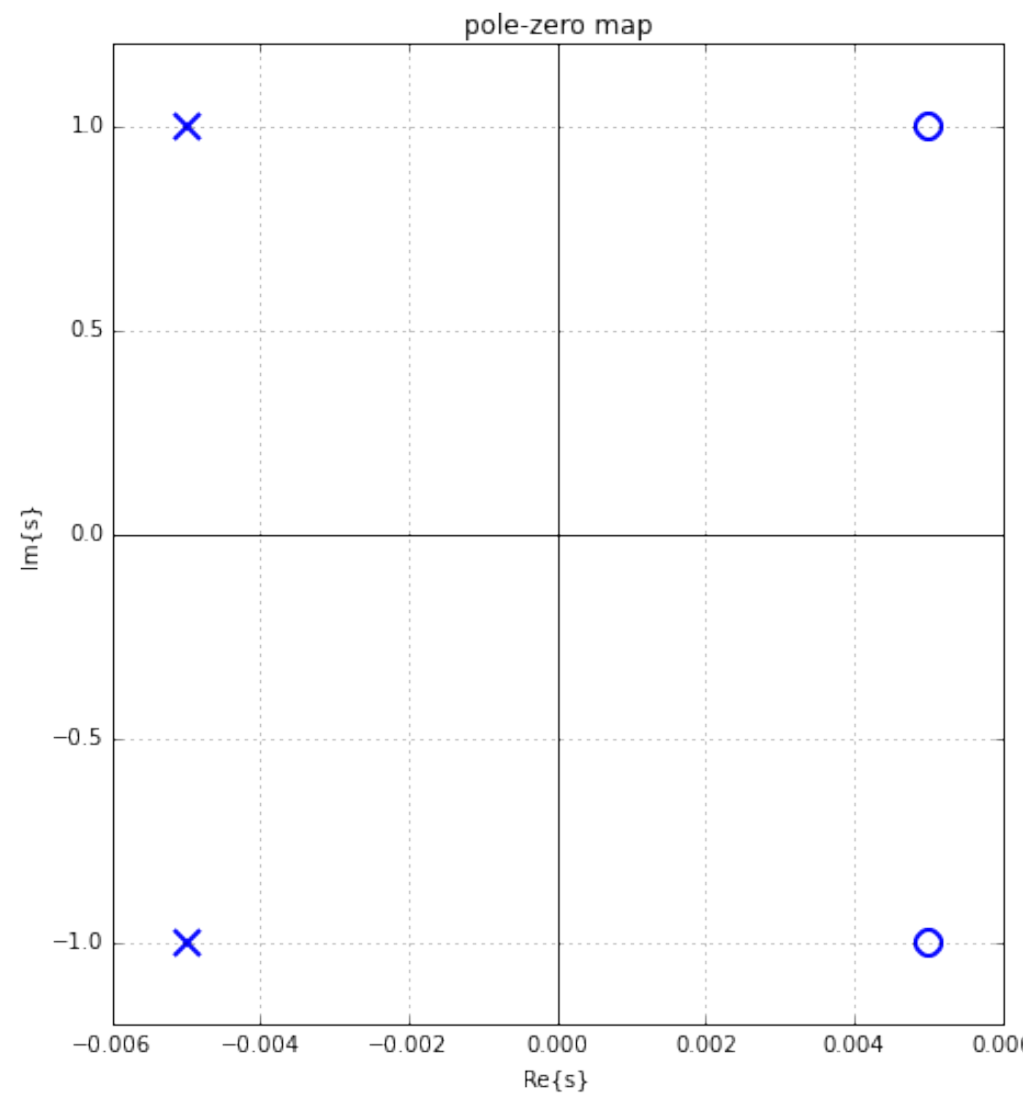
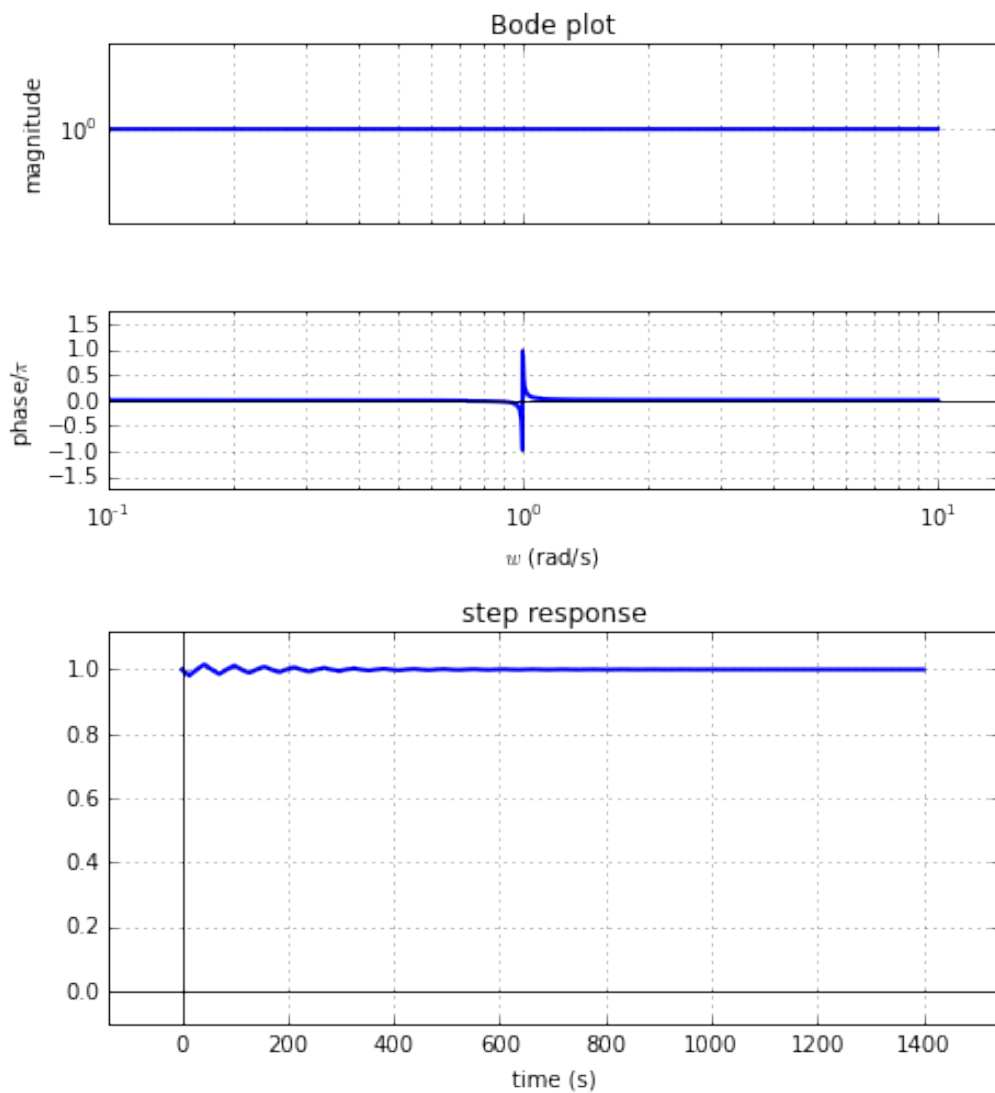
This system responds to the step by ramping up and then oscillating while decaying. Because it takes time to reach 1, but also time to decay, it must be bandpass, reflected in the bode plot. Two poles indicate second order behavior, and the imaginary magnitude accounts for the oscillatory nature. The real part of the poles is zero, so the system decays to zero.

### Problem 3



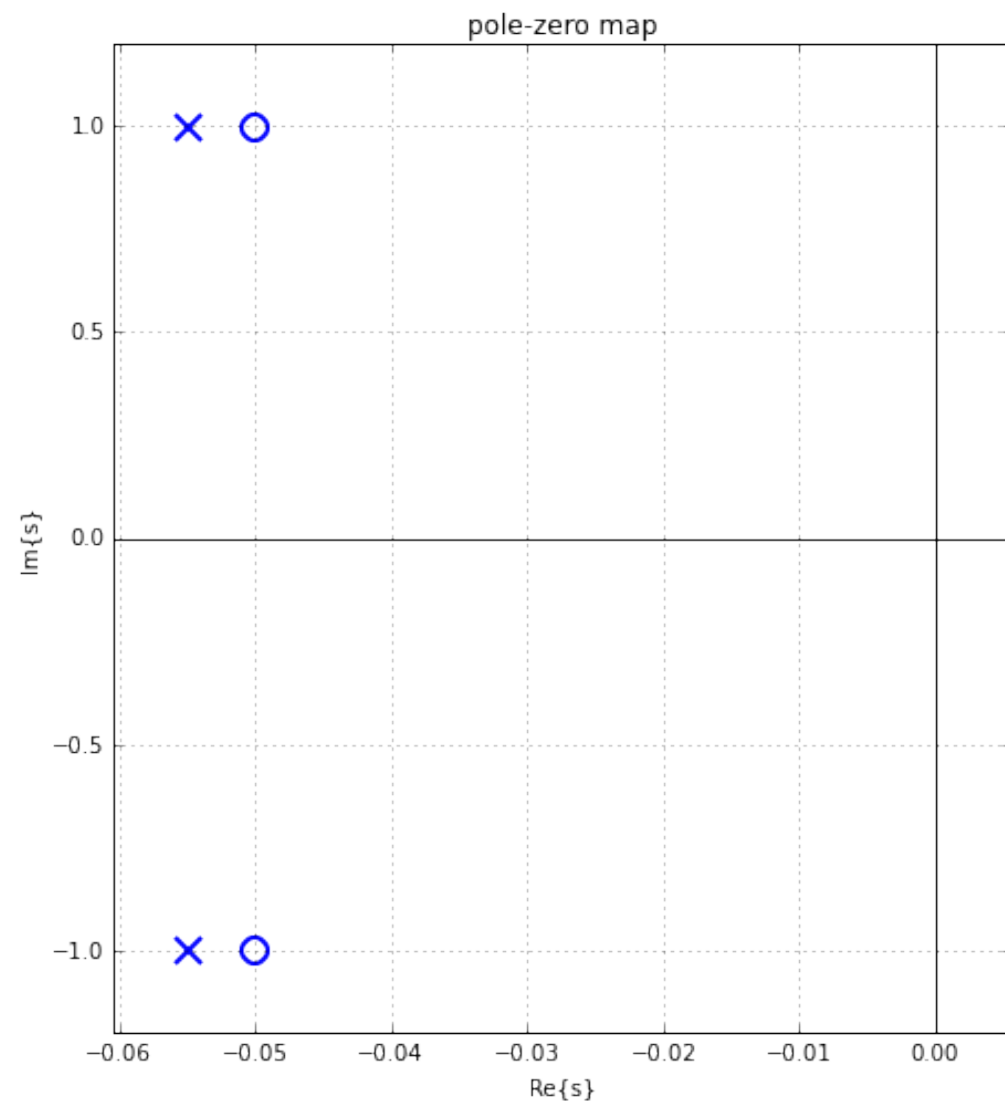
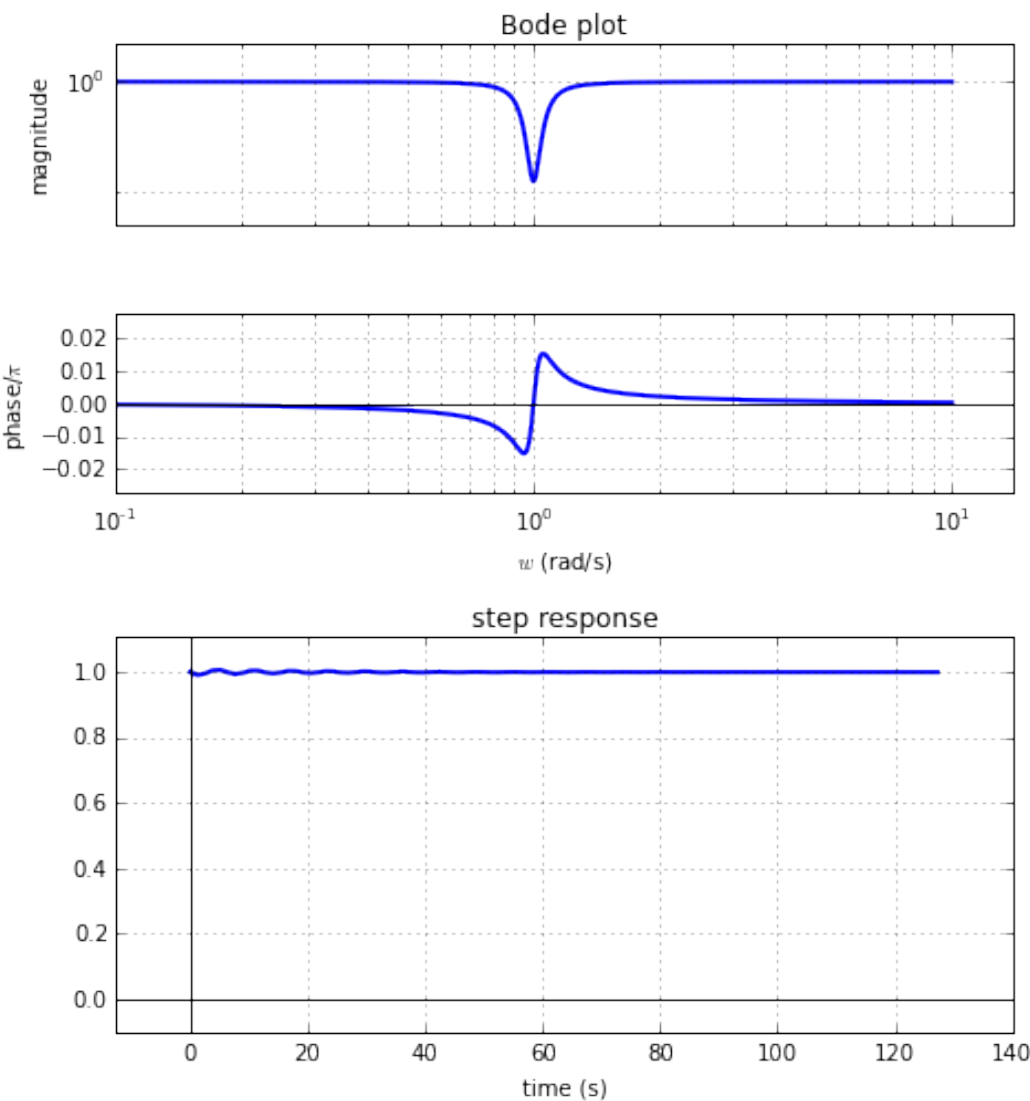
This system is comparable to the previous one in pretty much every way, except that the smaller negative magnitude of the poles (which is exactly a tenth) means that the exponential will decay more slowly, in fact, ten times as slowly. Similar complex magnitudes indicate a similar oscillation frequency and two poles mean second order behavior.

### Problem 3



The system has two poles in the 2<sup>nd</sup> and 3<sup>rd</sup> quadrants, and two zeroes in the 1<sup>st</sup> and 4<sup>th</sup> quadrants. The presence of complex magnitudes indicates oscillation, and negative real parts of the poles indicates decay over time to a constant value. The magnitudes of the zeroes may indicate the DC bias in the system as well? The bode plot of this system is flat, indicating that it pretty much passes all frequencies untouched.

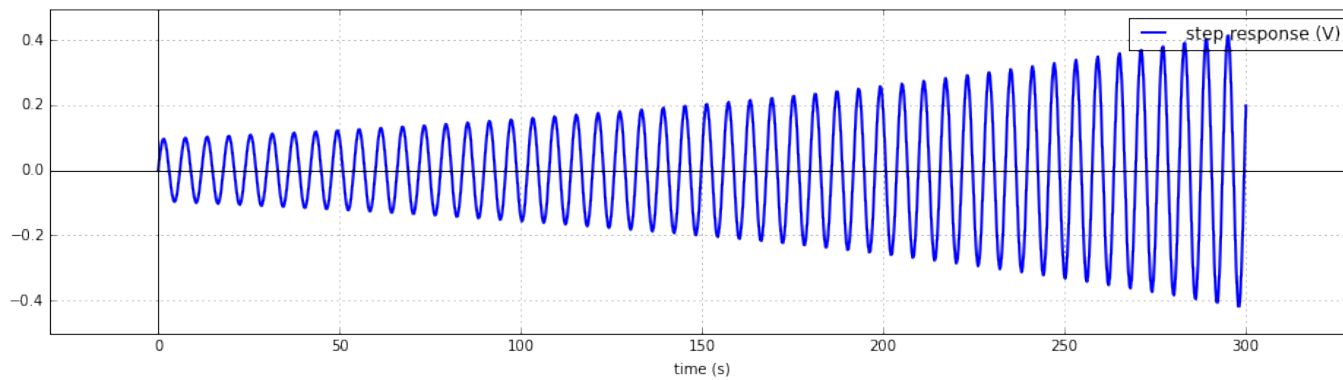
Problem 3



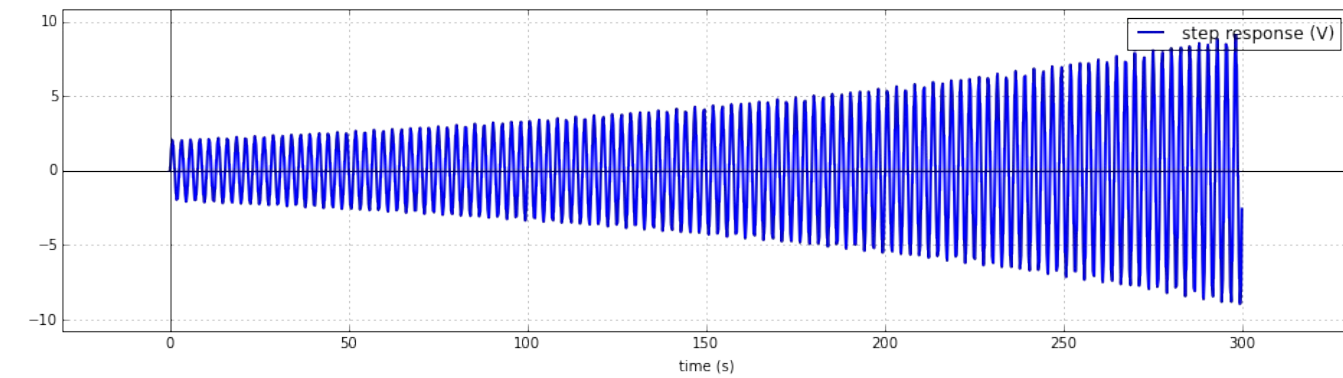
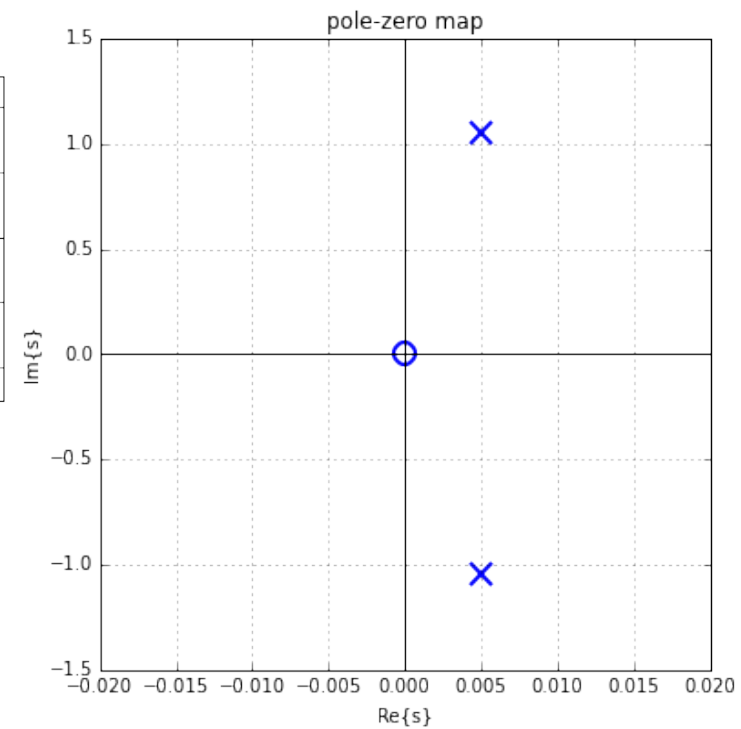
The bode plot of this system is a notch filter its step response looks very similar to the previous system's. It has two poles and two zeroes, as well as complex magnitudes to account for its oscillations.

### Problem 3

# Problem 4: Proportional Control

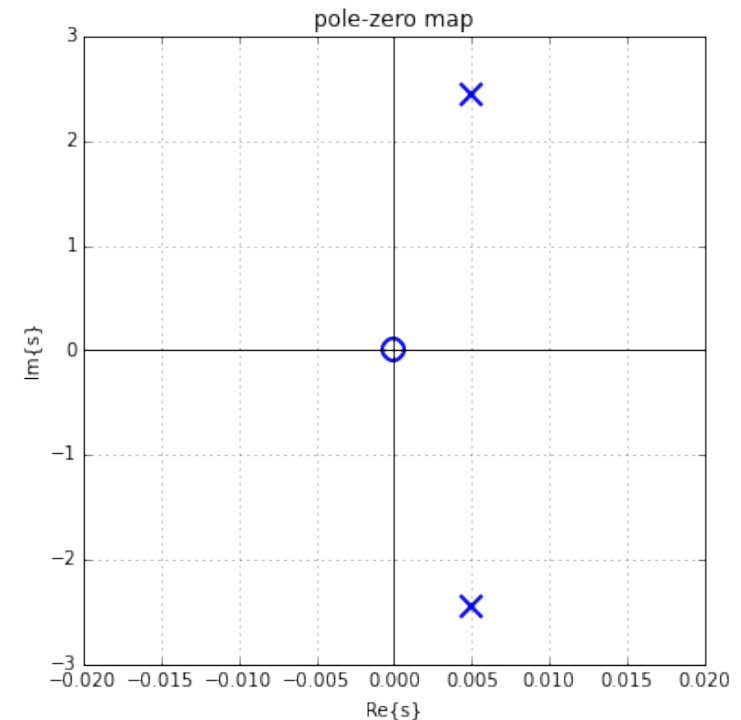


$K_p=0.1$



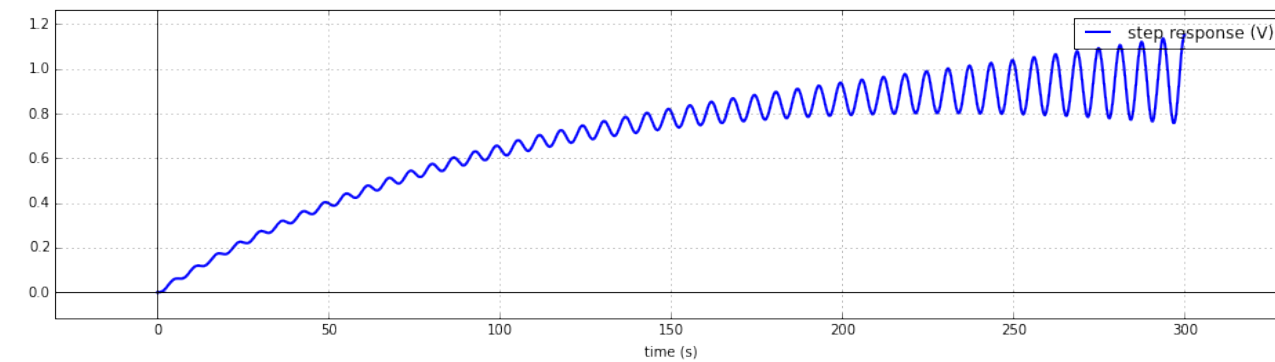
$K_p=5$

It's not possible to stabilize this system with a proportional controller. The positive real magnitude of its poles indicates that it will resonant and “blow up” and it doesn't appear that any value of  $K_p$  can yield poles with negative real magnitudes.

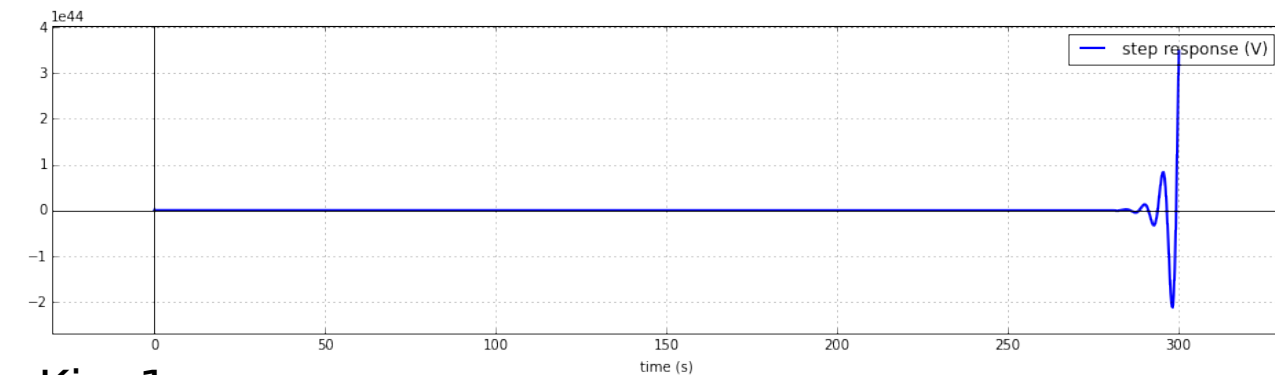




# Problem 4: Integral Control

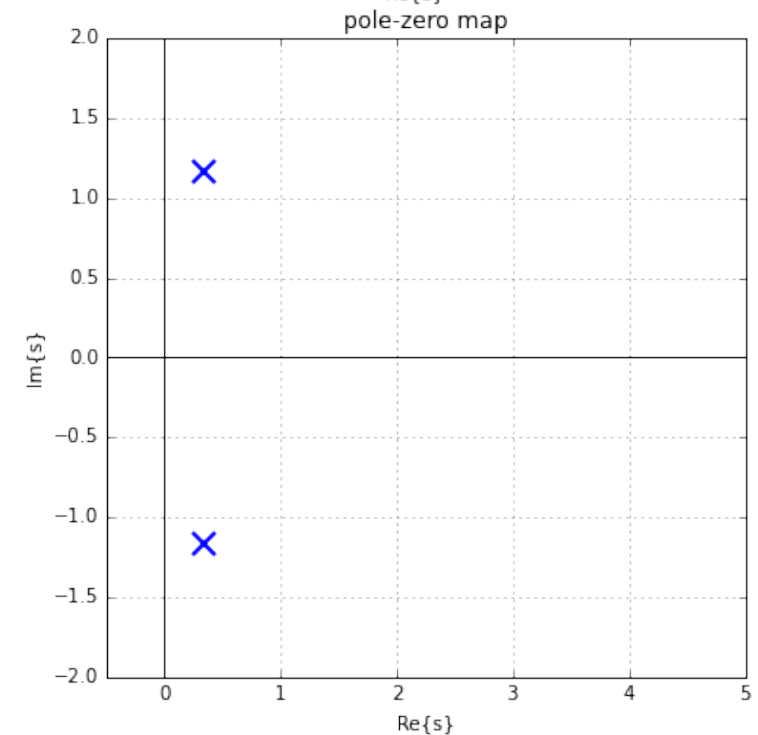
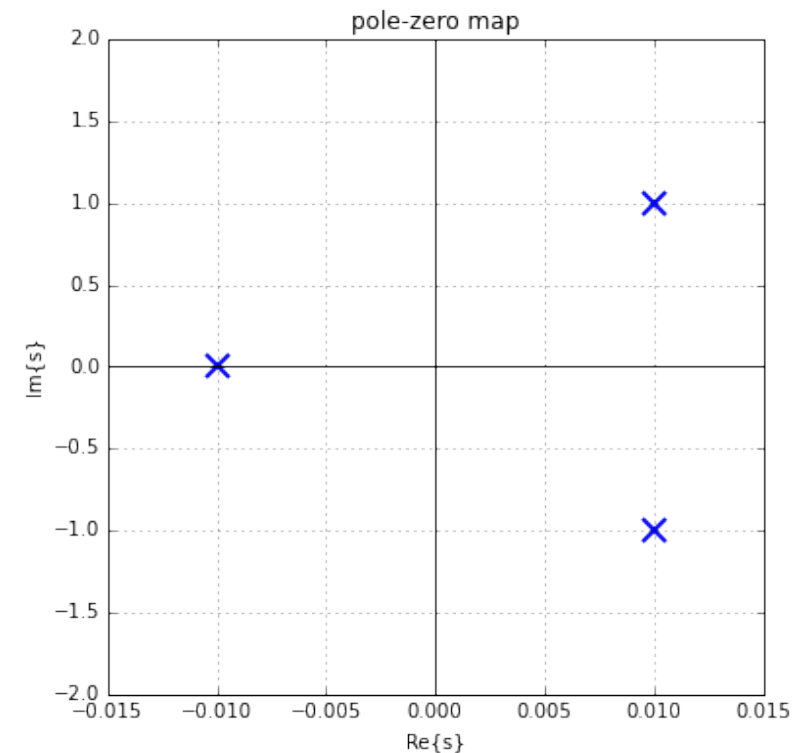


$K_i = 0.01$

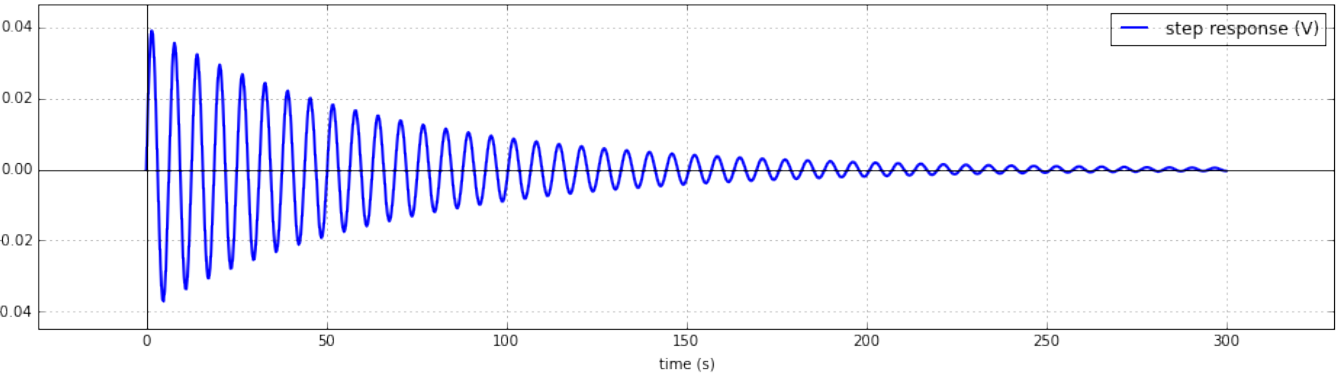


$K_i = 1$

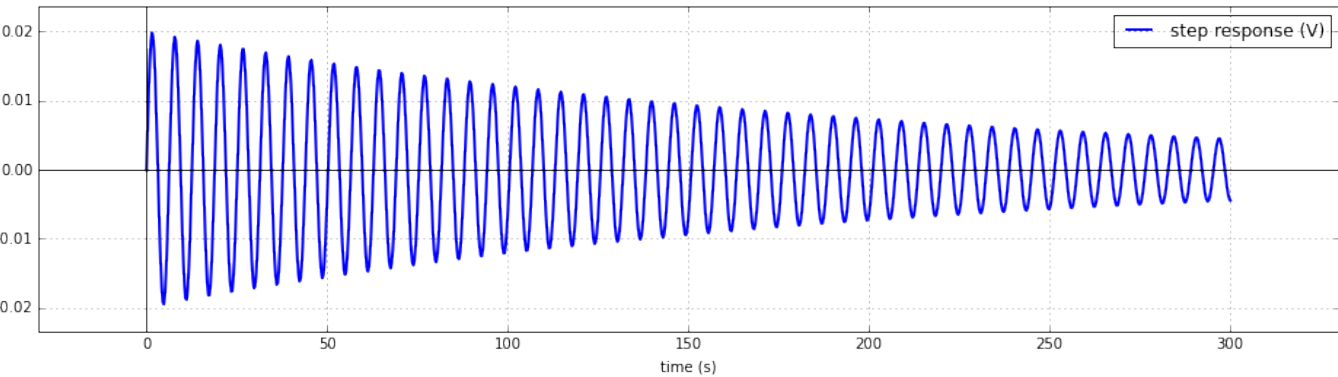
It's not possible to stabilize this system with an integral controller. The three poles in  $K_i = 0.01$  manifest interestingly: The one on the left describes the behavior of increasing DC bias, while the two on the right with imaginary magnitudes describe the resonating sinusoid on the rising DC bias! There doesn't appear to be a way to make ALL poles negative, which would be necessary to stabilize this thing.



# Problem 4: Derivative Control



$K_d=0.05$



$K_d=0.02$

Derivative Control. System stabilizes for  $K_d>0.01$ , during which the poles all have negative real magnitudes.

