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PS07

1. a. $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$

b. $C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k}{T}} dt$

choose y as the height of an impulse, and τ as its width

$C_k = \frac{1}{T}\int_{-\tau/2}^{\tau/2} y\, e^{-j\frac{2\pi k}{T}t} dt$

$= \frac{y}{T}\left(\frac{-T}{j\pi 2k} e^{-j\frac{2\pi k t}{T}}\right)\Big|_{-\tau/2}^{\tau/2}$

$= \frac{y}{T}\frac{T}{k\pi}\sin\left(\frac{k\pi\tau}{T}\right)$

$= \frac{y}{T}\cdot\tau \frac{\sin\left(\frac{k\pi\tau}{T}\right)}{\frac{k\pi\tau}{T}}$ sinc

$= \frac{y\cdot\tau}{T}\,\text{sinc}\left(\frac{k\pi\tau}{T}\right)$

b/c choose $\frac{1}{\tau}$ as the height of y b/c $\int_{-\tau/2}^{\tau/2} y\, d\tau$ must be 1 for the delta function.

$= \frac{1}{T}\,\text{sinc}\left(\frac{k\pi\tau}{T}\right)$

$\lim_{\tau\to 0}\frac{1}{T}\,\text{sinc}\left(\frac{k\pi\tau}{T}\right) = \frac{1}{T}$

Take the limit b/c the δ is infinitely thin.

$C_k = \frac{1}{T}$

c) $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$, $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 k t}$

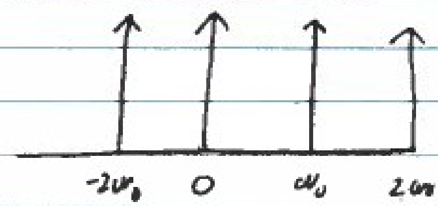
$X(\omega) = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} C_k e^{j\omega_0 k t} e^{-j\omega t} dt$; $\int_{-\infty}^{\infty} e^{j\omega_0 t}\cdot e^{-j\omega t} dt = 2\pi\,\delta(\omega - \omega_0)$

$X(\omega) = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} C_k \cdot 2\pi\,\delta(\omega - \omega_0 k)\, dt$

d) $C_k = \frac{1}{T}$

$P(\omega) = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} \omega_0 \cdot 2\pi\,\delta(\omega - \omega_0 k)\, dt$

e)



Raise T spreads $p(t)$ and contracts $P(\omega)$.

2. a $H(\omega) = \begin{cases} 1 & -\omega_c < \omega < \omega_c \\ 0 & \text{else} \end{cases}$

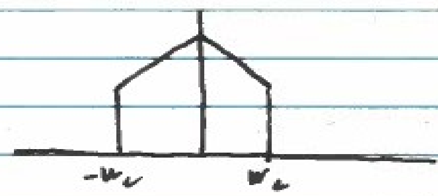
$h(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} H(\omega) e^{j\omega t} d\omega$

$h(t) = \frac{1}{2\pi}\frac{1}{jt} e^{j\omega t}\Big|_{-\omega_c}^{\omega_c}$

$h(t) = \frac{1}{2\pi j t}\left(e^{j\omega_c t} - e^{-j\omega_c t}\right)$

$h(t) = \sin(\omega_c t) / \pi t$

b.



c. It passes all frequencies $-\omega_c < f < +\omega_c$ w/ unity and is zero (fully attenuated) elsewhere. Since LTI systems cannot produce frequencies, all regions where $H(\omega)$ is 0 will remain zero when $H(\omega)$ is multiplied with any signal $X(\omega)$.

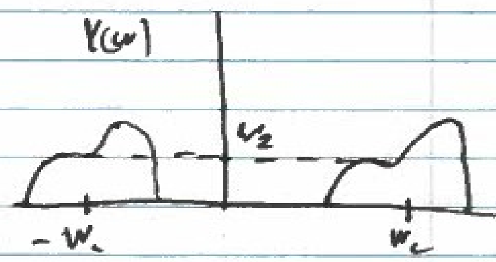
d. see graphs.

3. $h(t) = \cos(\omega t)$

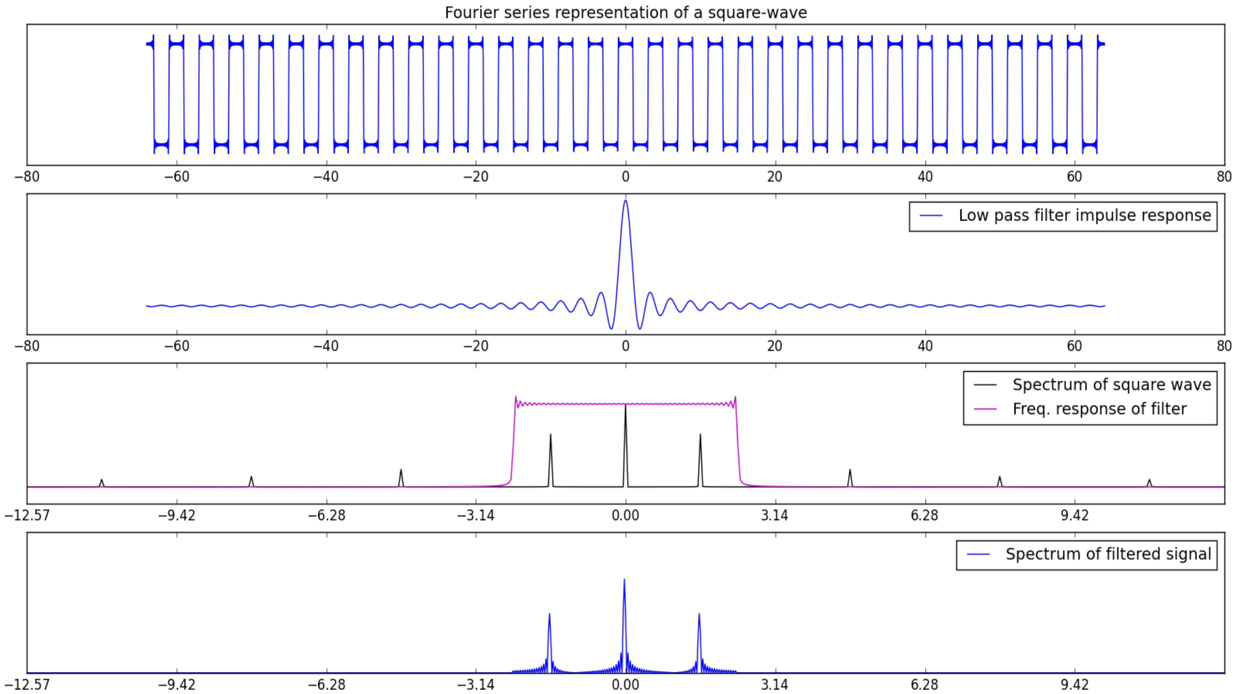
$H(\omega) = \pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)$

$y(t) = x(t)\cdot h(t)$

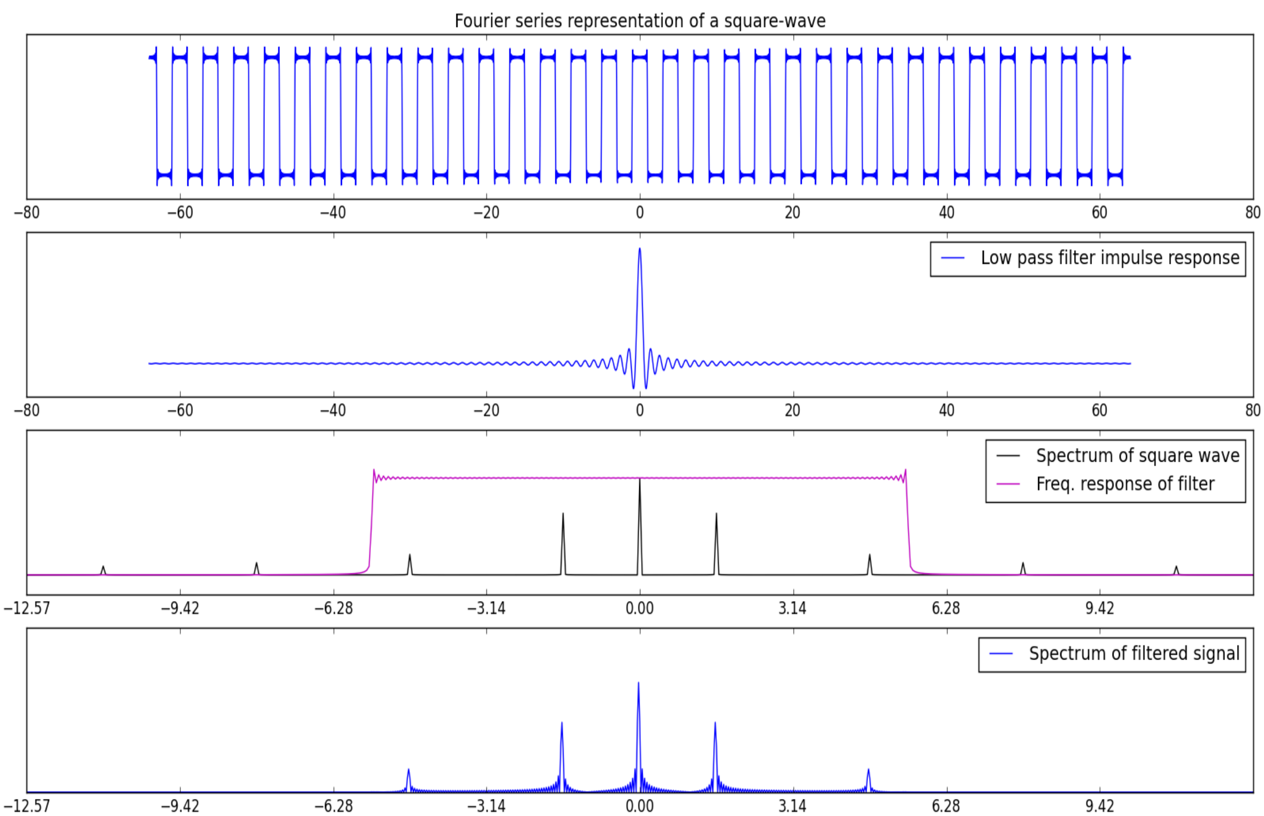
$Y(\omega) = \frac{1}{2\pi}X(\omega) * H(\omega)$



$Y(\omega) = \frac{1}{2\pi} X(\omega) * \left(\pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)\right)$


Fourier series representation of a square-wave
Low pass filter impulse response
Spectrum of square wave
Freq. response of filter
Spectrum of filtered signal

0.75pi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