A Nozzle Path Planner for 3D Printing Applications: Theoretical Analyzes

The cost for a transition in the NPPP represents the time required by the nozzle to traverse that transition. The total time required to complete a transition $e = (i, j)$ is given by

$$t_T(e) = t_M(e) + t_R(e), \tag{1}$$

where $t_M(e)$ is the nozzle motion time, and $t_R(e)$ is the retraction time of the 3D printer. The definition of retraction will be provided shortly.

1) Motion Model: In this work, a motion model similar to that in [1] is utilized. Here, a_1 and a_2 represent the acceleration and deceleration values of the nozzle, respectively. The maximum velocity of the nozzle is denoted as v_{max}. When the nozzle traverses a transition, it tries to maximize its velocity to minimize the time spent on traversing it, then decelerates and stops at the other extremity of the transition. The minimum distance d_{min} required for nozzle to accelerate to its maximum velocity and then decelerate to stop exactly at the other extremity of the transition is therefore calculated as

$$d_{\text{min}} = \frac{v_{\text{max}}^2}{2}\left(\frac{1}{a_1} - \frac{1}{a_2}\right). \tag{2}$$

An illustration of a motion profile is given in Fig. 1. If $d(e) < d_{\text{min}}$, where $d(e)$ denotes the length of a transition e, the printing nozzle decelerates before reaching its maximum velocity v_{max} as shown in Fig. 1 from time t_1 to t_2. If $d(e) \geq d_{\text{min}}$, the nozzle accelerates to its maximum velocity, maintains a constant velocity for a certain period, and then decelerates to stop at the other extremity of the transition as shown in Fig. 1 from time t_3 to t_4. When using the above motion model,

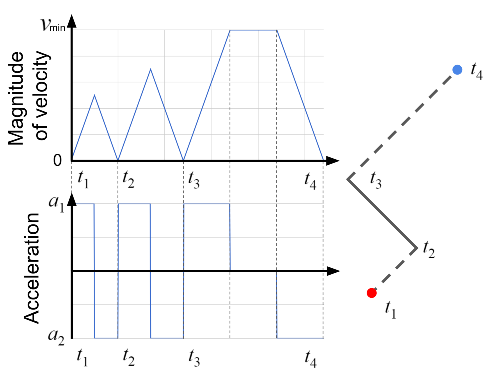

Fig. 1: The velocity profile and the acceleration profile of a printing nozzle when traversing a print segment as shown on the right. Red and blue dots represent start and end locations, respectively. Solid and dotted lines represent transitions and the print segment, respectively.

the time required by the nozzle to traverse a transition e is generalized as

$$t_M(e) = \begin{cases} \dfrac{2\sqrt{d(e) \times d_{\text{min}}}}{v_{\text{max}}} & \text{if } d(e) < d_{\text{min}}, \\ \dfrac{d(e) + d_{\text{min}}}{v_{\text{max}}} & \text{otherwise.} \end{cases} \tag{3}$$

It is observed that $t_M(e)$ strictly increases and continues for all $d(e) \geq 0$.

2) Retraction: Retractions are referring to a series of operations, including retracting filament back into the feeder hole, lowering the print bed, moving the nozzle through a transition, elevating the print bed to its original level, and restoring the retracted filament. The objective of these operations is to avoid unwanted molten filament dripping from the nozzle when it moves between print segments. Retractions are conducted when a nozzle traverses across dissected parts or passes through the exterior of a part. A *dissected part* is an isolated group of printed segments in a layer. A dissected part is usually bounded by a set of connected print segments known as the shell. The time spent on a retraction when traversing a transition e is denoted as

$$t_R(e) = \begin{cases} \tau & \text{if a retraction is required,} \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where τ is the required time for conducting the series of operations mentioned above. Here, τ is a constant which is mainly based on the hardware specifications of the 3D printer and the preferences of users. In this work, the term *transitions with retraction* denotes transitions across dissected parts or the exterior of a part, in which a retraction is involved.

In a nozzle path planning problem, transitions $E_t = E \setminus E_r$ are classified exclusively into one of the following three subsets of E according to the following conditions.

$$E_A = \{e\colon e \in E \setminus E_r,\ t_R(e) \neq 0\},$$
$$E_B = \{e\colon e \in E \setminus E_r,\ t_R(e) = 0,\ d(e) > 0\},$$
$$E_C = \{e\colon e \in E \setminus E_r,\ t_R(e) = 0,\ d(e) = 0\}.$$

Here, $|E_A|$, $|E_B|$, and $|E_C|$ represent the number of transitions in the subset E_A, E_B, and E_C respectively. It is given that $t_D(d)$ represents the time cost required by a printing nozzle to traverse a transition with length d.

Lemma 1.

$$t_D(d_1 + d_2) \leq t_D(d_1) + t_D(d_2),$$

for all $d_1 \geq 0$ and $d_2 \geq 0$.

Proof.

Case 1: $d_1 \geq d_{\text{min}}$, $d_2 < d_{\text{min}}$, and $d_1 + d_2 \geq d_{\text{min}}$.

$$\begin{aligned}
t_D(d_1 + d_2) &= \frac{d_1 + d_2 + d_{\text{min}}}{v_{\text{max}}}. \\
t_D(d_1) + t_D(d_2) &= \frac{d_1 + d_{\text{min}}}{v_{\text{max}}} + \frac{2\sqrt{d_2 d_{\text{min}}}}{v_{\text{max}}} \\
&\geq \frac{d_1 + d_{\text{min}} + 2d_2}{v_{\text{max}}} \\
&\geq t_D(d_1 + d_2).
\end{aligned}$$

Case 2: $d_1 \geq d_{\text{min}}$, $d_2 \geq d_{\text{min}}$, and $d_1 + d_2 \geq d_{\text{min}}$.

$$\begin{aligned}
t_D(d_1 + d_2) &= \frac{d_1 + d_2 + d_{\text{min}}}{v_{\text{max}}}. \\
t_D(d_1) + t_D(d_2) &= \frac{d_1 + d_2 + 2d_{\text{min}}}{v_{\text{max}}} \\
&\geq t_D(d_1 + d_2).
\end{aligned}$$

Case 3: $d_1 < d_{\text{min}}$, $d_2 < d_{\text{min}}$, and $d_1 + d_2 < d_{\text{min}}$.

$$\begin{aligned}
t_D(d_1) + t_D(d_2) &= \frac{2\sqrt{d_{\text{min}}}(\sqrt{d_1} + \sqrt{d_2})}{v_{\text{max}}}. \\
t_D(d_1 + d_2) &= \frac{2\sqrt{d_{\text{min}}}\sqrt{d_1 + d_2}}{v_{\text{max}}} \\
&= \frac{2\sqrt{d_{\text{min}}}\sqrt{(\sqrt{d_1} + \sqrt{d_2})^2 - 2\sqrt{d_1 d_2}}}{v_{\text{max}}} \\
&\leq t_D(d_1) + t_D(d_2).
\end{aligned}$$

Case 4: $d_1 < d_{\text{min}}$, $d_2 < d_{\text{min}}$, and $d_1 + d_2 \geq d_{\text{min}}$.

$$\begin{aligned}
t_D(d_1 + d_2) &= \frac{(d_1 + d_2) + d_{\text{min}}}{v_{\text{max}}}. \\
t_D(d_1) + t_D(d_2) &= \frac{2\sqrt{d_1 d_m}}{v_{\text{max}}} + \frac{2\sqrt{d_2 d_m}}{v_{\text{max}}} \\
&\geq \frac{2\sqrt{d_1 d_1}}{v_{\text{max}}} + \frac{2\sqrt{d_2 d_2}}{v_{\text{max}}} \\
&= \frac{d_1 + d_2 + (d_1 + d_2)}{v_{\text{max}}} \\
&\geq t_D(d_1 + d_2).
\end{aligned}$$

The Lemma 1. holds for all $d_1 \geq 0$ and $d_2 \geq 0$. ■

Lemma 2.

$$t_D\Big(\sum_{i=1}^{k}(d_i)\Big) \leq \sum_{i=1}^{k} t_D(d_i),$$

where $d_i \geq 0 \ \forall i \in [1, k]$ and $k \geq 2$.

Proof. The proof follows simply by using Lemma 1., but is omitted for clarity. ■

Theorem 1. The combinations of one E_B component and $(k-1)$ E_C components do not deliver improvement to the solution.

Proof. When apply a combination that selected one E_B transition and $(k-1)$ E_C components to a solution by using k-opt. The improvement can be indicated as

$$\Big(t_T(b) + \sum_{i=1}^{k-1} t_T(c_i)\Big) - \Big(\sum_{i=1}^{k} t_T(d_i)\Big), \tag{5}$$

where b is the length of the selected E_B transition and c_i is the length of the i-th selected E_C transitions. Here, d_i is the length of the i-th transitions that established after updating the solution using that combination. Since the cost of any transition in E_C is zero and any transition in E_B or E_C does not associated with retraction, (5) can be rewritten as

$$\Big(t_D(b)\Big) - \Big(\sum_{i=1}^{k} t_T(d_i)\Big), \tag{6}$$

Furthermore, by using triangle inequality, it is known that

$$b \leq \sum_{i=0}^{k} d_i.$$

Since $t_D(d)$ is strictly increasing and continuous for $d \geq 0$, it can be shown that

$$\Big(t_D(b)\Big) \leq t_D\Big(\sum_{i=1}^{k}(d_i)\Big).$$

With Lemma 2.,

$$\Big(t_D(b)\Big) \leq t_D\Big(\sum_{i=1}^{k}(d_i)\Big) \leq \sum_{i=1}^{k} t_D(d_i) \leq \sum_{i=1}^{k} t_T(d_i). \tag{7}$$

By considering (6) and (7),

$$\Big(t_D(b)\Big) - \Big(\sum_{i=1}^{k} t_T(d_i)\Big) \leq 0.$$

Theorem 1 holds for all $k \geq 2$. ■

I. Theorem 2

Theorem 2. Combinations with only E_C components do not deliver improvement to the solution.

Proof. It is known that for all combinations with only E_C components, the improvement can be calculated as

$$\begin{aligned}
\sum_{i=1}^{k} t_T(c_i) - \Big(\sum_{i=1}^{k} t_T(d_i)\Big) &= -\Big(\sum_{i=1}^{k} t_T(d_i)\Big) \\
&\leq 0
\end{aligned}$$

Theorem 2 holds for all $k \geq 2$. ■

References

[1] B. Thompson and H.-S. Yoon, "Efficient path planning algorithm for additive manufacturing systems," *IEEE Transactions on Components, Packaging and Manufacturing Technology*, vol. 4, DOI 10.1109/TCPMT.2014.2338791, no. 9, pp. 1555–1563, 2014.