

Data Format

Each image

784
28 x 28

Each row is
an image

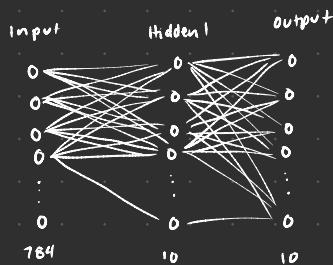
$\begin{bmatrix} x^1 & \dots & x^7 & \dots \\ \vdots & & \vdots & \end{bmatrix}$
60000
784

Transform

Each column
is image

$\begin{bmatrix} x^1 & x^2 & x^3 & \dots \\ \vdots & \vdots & \vdots & \end{bmatrix}$
60000
784

Network Structure



Forward Propagation → Starts with input image, ends with prediction output

input layer

$$A^0 = X \quad (784 \times m)$$

Hidden

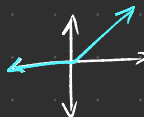
$$Z^1 = W^1 A^0 + b^1$$

10 x m 10 x 784 784 x m 10 x 1 → 10 x m

Activation Function

$$A^1 = g(Z^1) = \text{ReLU}(Z^1)$$

$$\text{ReLU} = \text{Rectified linear unit} = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$



$$Z^2 = W^2 A^1 + b^2$$

$$A^2 = \text{softmax}(Z^2)$$

output

$$\begin{bmatrix} 1.3 \\ 5.1 \\ 2.2 \\ 0.7 \end{bmatrix} \xrightarrow{\text{softmax}} \sum_{j=1}^K \frac{e^{z_j}}{e^{z_1} + e^{z_2} + e^{z_3} + e^{z_4}} \rightarrow \begin{bmatrix} .02 \\ .9 \\ .05 \\ .01 \end{bmatrix}$$

probabilities

Backward Propagation → update weights + biases for next forward prop

Starts with prediction, calculate how far prediction deviated from label

Find out how much w + b contributed from that deviation, and adjust

How much was prediction off by?

$$dz^2 = A^2 - \gamma$$

Prediction label

How did w + b contribute to loss

$$dw^2 = \frac{1}{m} dz^2 A^{1T}$$

derivative of loss function

$$db^2 = \frac{1}{m} \sum dz^2$$

$$dz^1 = W^{2T} dz^2 \cdot g'(z^1)$$

derivative of activation function

$$dw^1 = \frac{1}{m} dz^1 X^T$$

$$db^1 = \frac{1}{m} \sum dz^1$$

Update w + b

α = learning rate multiplier

$$W^1 = W^1 - \alpha dw^1$$

$$b^1 = b^1 - \alpha db^1$$

$$W^2 = W^2 - \alpha dw^2$$

$$b^2 = b^2 - \alpha db^2$$

Back to forward Prop!

Rinse + repeat ALot