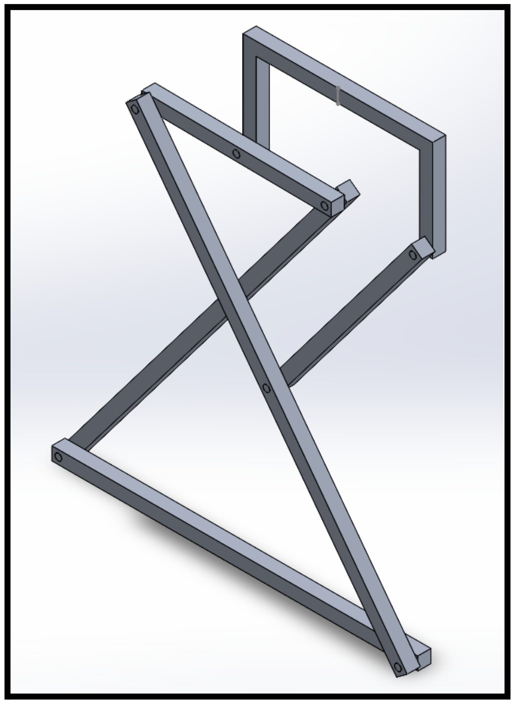
Final Group Project

Design of a Horizontal-platform Mechanism



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MAE 162A Introduction to Mechanisms & Mechanical Systems

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Table of Contents

I. Abstract

Mechanisms have transformed the world and help us innovate within existing spaces of technology. An example of such a mechanism is a table that can translate horizontally to expand from its original applications. The goal of this project is to design a horizontal-platform mechanism that can achieve horizontal translational motion within a specific range of oscillation. The design of this mechanism utilizes the Chebyshev linkage as the straight line mechanism to achieve approximate pure translational motion from a rotational motion input. Another kinematic chain loop is introduced to keep the platform approximately parallel with respect to the horizontal throughout the entire motion. A MATLAB script is written to calculate the required length ratio of the links in the mechanism and to simulate the motion of the mechanism given the angle input of the input link. Based on the simulation, the resulting motion regarding the mechanism can be calculated and compared with the required design specifications. Overall, our horizontal-platform mechanism can successfully travel the minimum 850-mm horizontal distance while fitting inside a 1050-mm by 850-mm box in at least one orientation. Furthermore, the maximum vertical displacement during oscillation does not exceed 4-mm, and the maximum angle of the platform, with respect to the horizontal, does not exceed more than 1°. Given the dimensions and material of the mechanism, as well as the input motion, the total required torque to actuate the mechanism is also calculated. In summary, our mechanism satisfies all the requirements from specifications and is expected to work in reality, based upon the following simulations.

II. Introduction

Mechanisms have been the core of many technologies that have transformed humanity. Humans have learned to utilize mechanisms to achieve different goals that would be hard or impossible to achieve otherwise. Essentially, the purpose of mechanisms is to help humans achieve motions and works that would be otherwise inefficient or impossible to carry out. An everyday example of this application of mechanisms can be found in transportation. With an understanding and effective application of mechanisms, humans were able to invent various modes of transportation, from the common horse carriage developed centuries ago, to automobiles, airplanes, and trains which we rely upon today. Some transportation technologies even allow us to explore the far reaches of our planet, and beyond. All of these modes of transportation would not be possible without mechanisms.

Mechanisms also allow humans to carry heavy weight that would otherwise be inefficient to move. For example, tower cranes can bring heavy materials to higher places which make building skyscrapers possible. These are just some early purposes that pushed early inventors to design new mechanisms. However, the purpose of mechanisms have expanded to satisfy every need of humanity. Clocks and watches use systems of gears to achieve accurate time keeping. Assembly robotic arms have made manufacturing processes that require high precision possible, such as manufacturing of chips and other electronic components, while other robotic arms accelerate the assembly line processes that produce goods at a much cheaper price. Every piece of technology that we currently own and use has some correlation with the application of mechanisms.

As a result of all of these examples and more, it is not unreasonable to say that mechanisms are at the core of human technology, and that importance cannot be understated. Without mechanisms, it would not have been possible for human society to develop to what it is today. As such, there is great value in the study and analysis of mechanisms. Through a greater understanding of the fundamentals of mechanisms, we strive to apply our knowledge in the creation of further advancements in mechanism design, as a means of further advancing technology to a higher level.

The report is organized as follows: The theory on different mechanisms and the Chebyshev linkage needed to achieve a nearly linear motion from rotational input. This is followed by a brief history section that explains the importance of mechanisms, the role they played in the industrialization of the world and more examples of straight line linkages. The methodology section goes more into the specifications of the project (space requirements, vertical and horizontal movement, and maximum angle of the platform) as well as choosing the Chebyshev linkage as the model for our design. This section also lays out the design process for the final model: Vector loops, MATLAB simulation of the defined vector loops and finally a physical model in SOLIDWORKS. The analysis section further goes through the vector loop calculations and equations with the full hand calculations included in the appendix. The results section starts with the final dimensions for each link length, a figure of the completed SOLIDWORKS model and max and min values for the angle of link 2. This is followed by graphs of vertical, horizontal and angular displacement of link 6 and a plot of the required torque to run the mechanism for a couple cycles. Lastly the conclusion section sums up the findings, difficulties and the importance of completing this model.

III. Theory

The Reuleaux Definition describes a mechanism as an assemblage of resistant bodies (links) connected by movable joints, to form a closed kinematic chain with one link fixed and having the purpose of transforming motion (Moon, 2003).

In pursuit of a more mathematically rigorous understanding of mechanisms, individual links can be considered to have algebraic relations with coordinates of links, or specific points on those links. As such, if the position of some coplanar points of the linkage are fixed, then the trajectories of the other moving points on the linkage are algebraic curves. From the fundamental basis of these algebraic relations and relative trajectories of defined points, the task of achieving some specific output through precision of mechanism design and controlled input can begin to take shape (“Math”, 2022).

While mechanisms defined by the Reuleaux Definition can take on a wide range of shapes and forms, a select few are capable of translating rotational input to approximate linear motion. One such mechanism, which is capable of translating rotational motion into approximate linear motion, is the Chebyshev linkage mechanism:

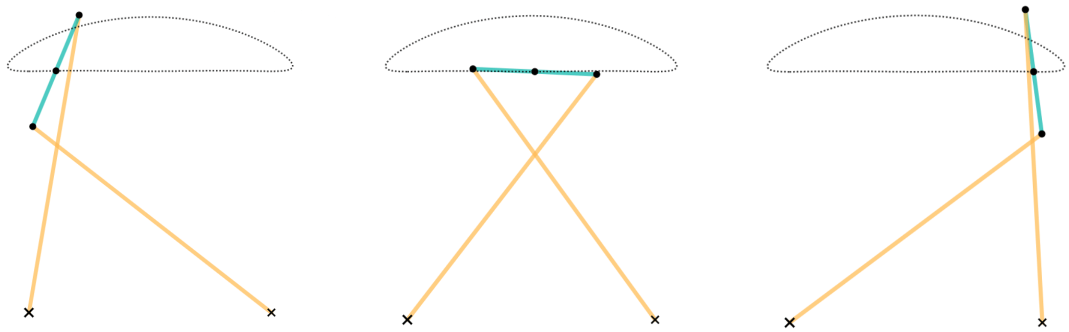


Figure 1: Sequential positions of a Chebyshev linkage mechanism.

As shown in Figure 1, the Chebyshev linkage is capable of outputting a nearly straight-line trace within a constrained range of rotational input angles, in which the input angle is the angle of rotation of either of the two longer links. The output trace of such a mechanism is *nearly* linear in that the vertical deviation, while ideally small, is a non-zero value ("Math", 2022).

IV. History

The development of mechanisms has facilitated the advancement of human technology as a means of enabling humanity to achieve tasks which humans would otherwise have a hard time with. One common desirable output from a mechanism is pure translational motion from other types of input such as rotational motion. While the conversion of rotational to translational motion seems quite simple as a concept, mechanisms that are able to achieve it precisely and effectively were developed only within the last few centuries.

One common mechanism used to convert rotational to translational motion is the cam-follower mechanism. Essentially, the cam, which is an irregular-shaped link, rotates and pushes a sliding follower to achieve translational motion. This mechanism was developed as early as 600 BCE in China as a trigger mechanism for the crossbow, and was later implemented in further applications in the Tang dynasty. Today, while the cam-follower mechanism is able to achieve some degree of translational motion, the distance of travel is greatly limited and remains unsuitable for most wider applications. Thus, different mechanism designs which can convert rotational to translational motion are required.

As the result of rapid development that came about during the Industrial Revolution, many engineers began to develop new mechanism designs which were capable of converting and producing different types of motion. It was in this era that straight line mechanisms were first invented and implemented.

The first such mechanism was the Watt's linkage, developed by James Watt in 1784. The Watt's linkage could be powered by a continuously rotating source while drawing out a path of oscillation in which a portion of that path was an approximately straight line. While this path was not perfectly straight, the tolerance was small enough to be useful in many applications that required translational motion, such as automobile suspension. Many similar mechanisms were developed following the Watt's linkage, which include the Roberts linkage and Chebyshev linkage, which stands as the foundation of the horizontal platform mechanism developed for this project. As was the case with previous linkage designs, the approximate straight line output would prove unsuitable for more sophisticated mechanisms, which ultimately led engineers to develop perfect straight-line linkages.

Perfect straight-line linkages, unlike approximate straight-line linkages, are capable of outputting perfectly straight lines of motion without a nonlinear section. As a result, perfect straight-line linkages are more useful in applications where only translational motion is allowed. The first perfect straight-line mechanism, called a Sarrus mechanism, was introduced by Pierre Federic Sarrus in 1853, but this mechanism is not a planar mechanism. In 1864, the Peaucellier-Lipkin linkage was introduced as the first planar perfect straight-line linkage. Many other designs soon followed that could also achieve translational motion from different rotational inputs. With the implementation of brilliant designs such as these, more sophisticated machines could be built to further shape the world that we live in today.

V. Methodology

For the task of designing a horizontal-platform mechanism which is capable of maintaining an approximate linear and horizontal path of travel, the fundamentals of the Chebyshev linkage will be employed. Per design specifications, the path and orientation of the platform (link 6), along a restricted range of rotational input angle (θ_2) , must be as follows:

1. Minimum 850mm of horizontal travel (x-axis).

2. Vertical translation of the platform center of mass < 4mm (y-axis).
3. Angular deviation of the platform < 1° from the horizontal.
4. The entire mechanism must fit within a rectangular footprint defined as 850mm wide (x-direction) and 1050mm tall (y-direction) in at least one position.

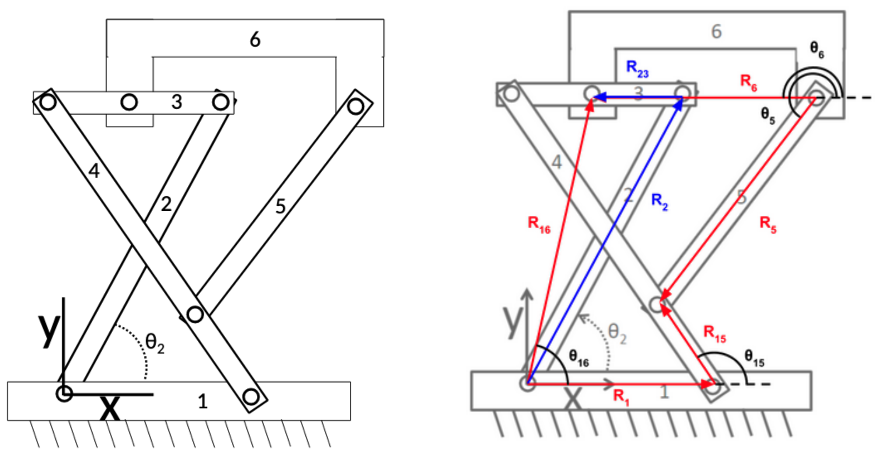


Figure 2: Template horizontal-platform mechanism design (left) and vector loop overlay (right).

As is the case in designing any mechanism in which precision of input and output is a requirement, the task of determining precise dimensions of interconnected links is of paramount importance. For this, a pair of vector loops, which completely describe the geometry of the template mechanism in terms of known and unknown values, are defined. Next, with respect to the above design specifications, a series of vector loop simulations are constructed within MATLAB from which unknown dimensions and respective angles can be obtained. Lastly, after verifying the design and performance of our model through a series of analytical graphs, individual links are constructed and assembled in SOLIDWORKS.

VI. Analysis

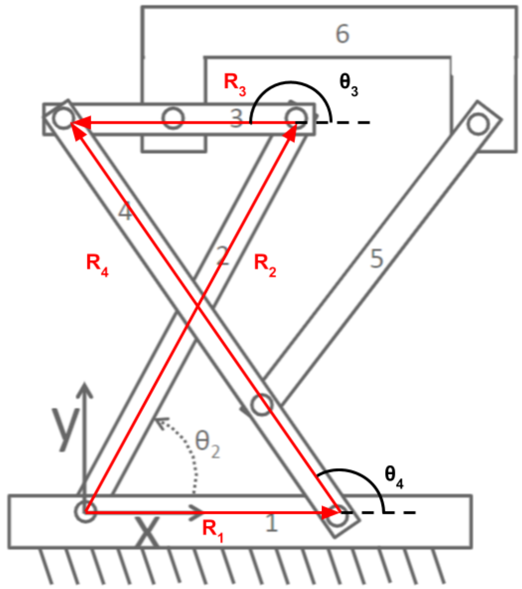


Figure 3a: Vector loop 1 of mechanism

Vector loop equations:

✓I ✓? ✓? ✓✓

$$R_2 + R_3 - R_4 - R_1 = 0 \tag{1}$$

Splitting equation (1) into its components,

x-component: $r_2 cos\theta_2 + r_3 cos\theta_3 - r_4 cos\theta_4 - r_1 = 0$ (2)

y-component: $r_2 sin\theta_2 + r_3 sin\theta_3 - r_4 sin\theta_4 = 0$ (3)

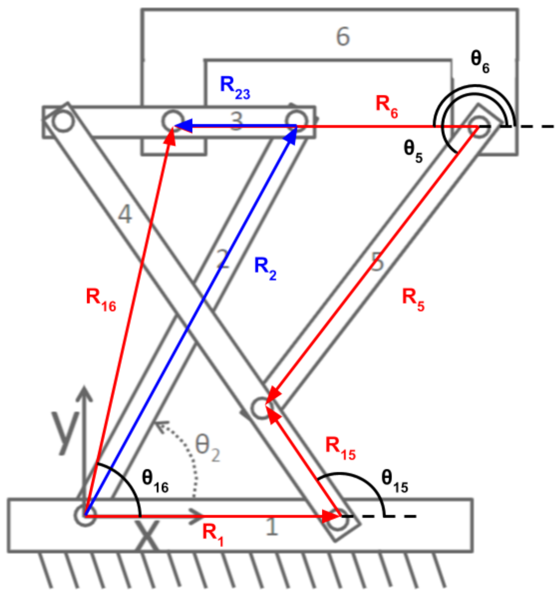


Figure 3b: Vector loop 2 of mechanism

Vector loop equations:

CC ✓? ✓? ✓C ✓✓

$$R_{16} - R_6 + R_5 - R_{15} - R_1 = 0 \tag{4}$$

Constraints:

CC ✓✓ CC

$$R_{16} = R_2 + R_{23} \tag{5}$$

CC ✓✓

$$R_{23} = \frac{1}{2}R_3 \tag{6}$$

$$\theta_{15} = \theta_4 \tag{7}$$

Substituting constraints (5) and (6) into equation (4),

$$R_2 + \frac{1}{2}R_3 - R_6 + R_5 - R_{15} - R_1 = 0 \tag{8}$$

Splitting equation (8) into its components and substituting in constraint (7), x-component: $r_2 cos\theta_2 + \frac{1}{2} r_3 cos\theta_3 - r_6 cos\theta_6 + r_5 cos\theta_5 - r_{15} cos\theta_4 - r_1 = 0$ (9)

y-component: $r_2 sin\theta_2 + \frac{1}{2} r_3 sin\theta_3 - r_6 sin\theta_6 + r_5 sin\theta_5 - r_{15} sin\theta_4 = 0$ (10)

Given the input θ_2, the rest of the unknown lengths and angles can be calculated using Newton-Raphson method within the MATLAB scripts. Since none of the lengths are given, an iterative process combined with some educated guesses can be used to find the ratio of length between the links before scaling the lengths to comply with the project requirements.

VII. Results

The results of the MATLAB script gave us the necessary data to design a functional mechanism in Solidworks. After creating an initial working prototype, we were able to narrow down the specifications of each link, and create the final assembly, which is shown in Figure 4 below.

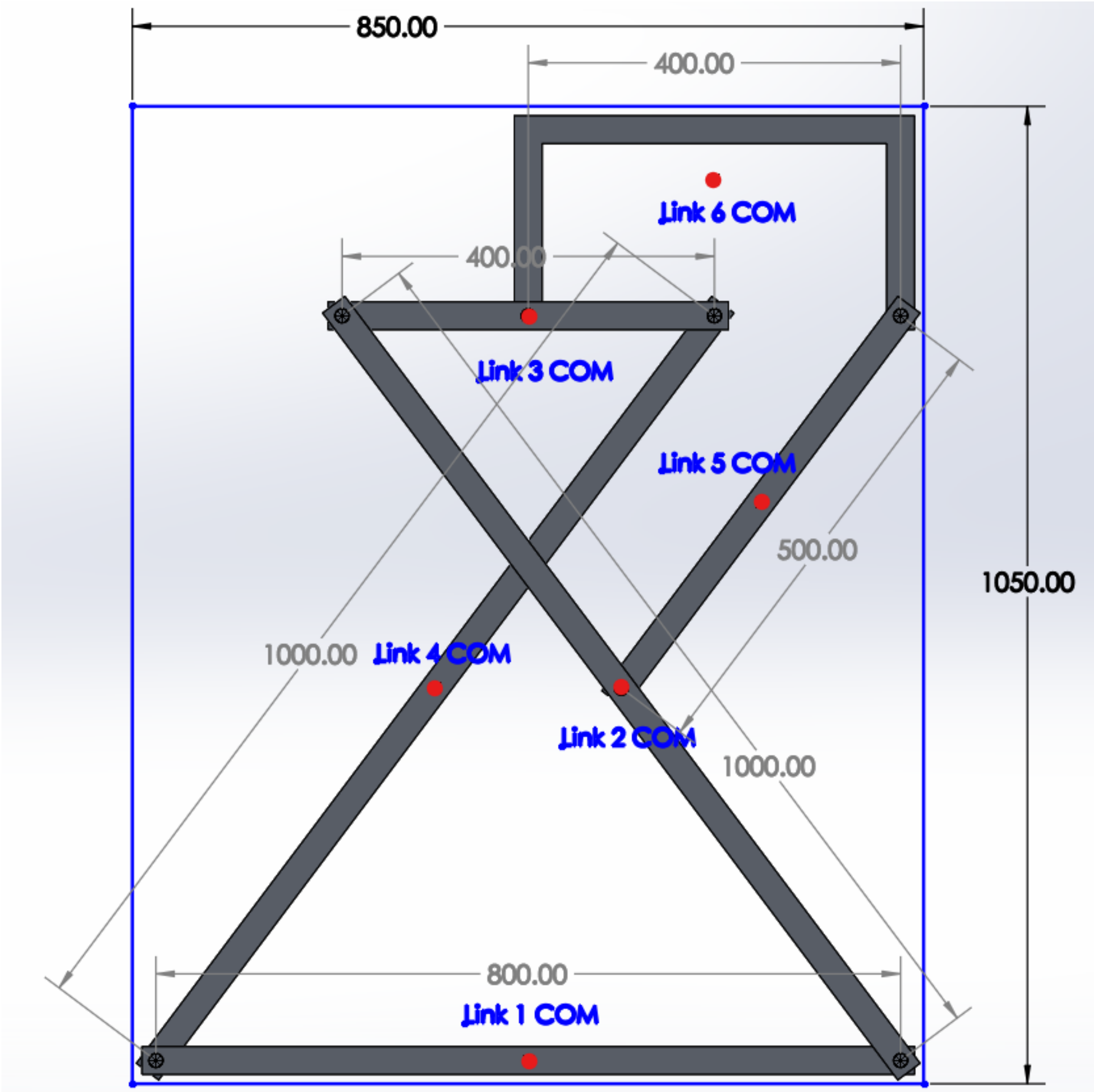


Figure 4: Mechanism within rectangular footprint labeled with lengths and centers of mass (850mm by 1050mm) (Note: all dimensions in mm)

In general, the ratios of Chebyshev's straight line mechanism were used to determine the relationships between the first four links, where the ratios between links one, two, and three are 2:5:4. The length of link five was then determined to be half the length of link two, as it would be connected at the midpoint of link two. This would then allow the midpoint of link three to be on the same vertical level as the end of link five, which provides a vertical base for the tabletop. For the tabletop, the horizontal dimension was determined to be the same as the length of link three. The vertical dimension was determined such that the entire mechanism fit within the specified footprint, which was finalized as half the length of link three. So, after choosing link one to be 800 mm, the other link dimensions were able to be calculated. The final dimensions of each link are as follows:

- Link 1: 800 mm
- Link 2: 1000 mm
- Link 3: 400 mm
- Link 4: 1000 mm
- Link 5: 500 mm
- Link 6: 400 mm (x-direction) by 200 mm (y-direction)

These dimensions align with the functional specifications outlined in the project guidelines. Additionally, the entire mechanism fits within the desired 850 mm by 1050 mm footprint, as shown in Figure 4, and the platform (link 6) is able to move 850 mm in horizontal motion with less than 1° of angular displacement, and less than 4 mm of vertical displacement along its path of travel.

To accomplish the 850 mm of horizontal displacement, the input link (link 2) is acted upon by a torque. This causes the link to rotate, which the mechanism translates into the horizontal motion of the platform link. At different positions along its path of travel, the desktop link corresponds to a different angle of link 2. At the furthest to the right horizontal position, the angular position of link 2 is $\theta_{2,min}$, which is equal to 37°. At the furthest to the left horizontal position, the angular position of link 2 is $\theta_{2,max}$, which is equal to 97°.

The next task we accomplished was to graph the horizontal displacement of the platform link as a function of the input link's angular displacement. The goal of this step was to prove that the mechanism could move 850 mm horizontally through the use of an input torque on Link 2. Below is the graph created by the MATLAB script.

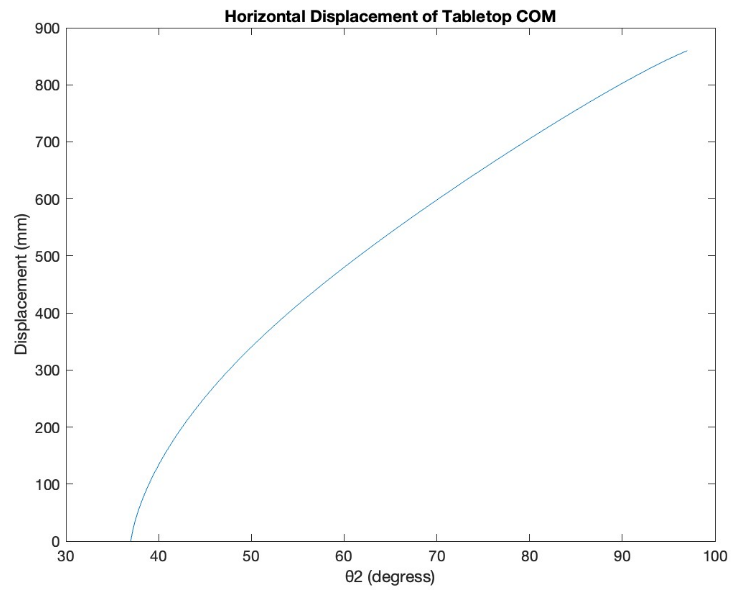


Figure 5: Horizontal Displacement of Platform Graph

Figure 5 shows a fairly linear relationship between the displacement of the platform and the angular position of link 2. As the angle of link 2 ranges from 37° to 97°, the position of the Center of Mass of the platform travels from 0 mm to 850 mm. From 37° to around 45°, the horizontal displacement of the platform increases at a faster rate, and afterwards increases mostly linearly with respect to the angular position of link 2 until its maximum angular displacement of 97°.

Next, we graphed the vertical displacement of the platform's center of mass. Per the given functional specifications, the maximum vertical displacement of the center of mass of the platform must not exceed 4 mm. The goal with this graph was to show that within the range of angular displacement of link 2, the vertical displacement would be less than 4 mm. Below is the graph the MATLAB simulation produced.

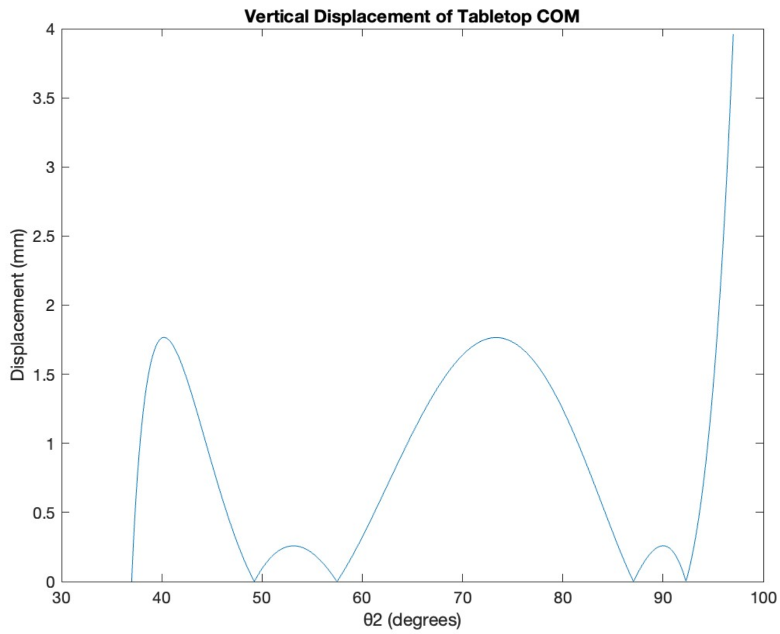


Figure 6: Vertical Displacement of platform Graph

Figure 6 shows a sinusoidal-esque graph. As the angular position of link 2 increases from 37° to 97°, the vertical displacement of the platform varies. Local maximums of the sinusoidal function are at around 1.75 mm and 0.25 mm. The displacement increases to 1.75 mm, then decreases back to 0 mm, and then increases to 0.25 mm before decreasing to 0 mm again. This repeats, and then increases rapidly to 4 mm at 97°. The cause for this is likely because of the shape of the mechanism and how it works to translate angular motion into linear displacement. Since the links are rotating, it makes sense for results to relate, even tangentially, to circular motion, which is described by sinusoidal graphs.

Another specification in the project was to ensure that the angular rotation of the platform was less than 1°, with respect to the horizontal. This meant that, throughout the translational motion of the platform by the angular rotation of link 2, the platform must remain approximately parallel to the ground. Below is the graph of the MATLAB simulation produced.

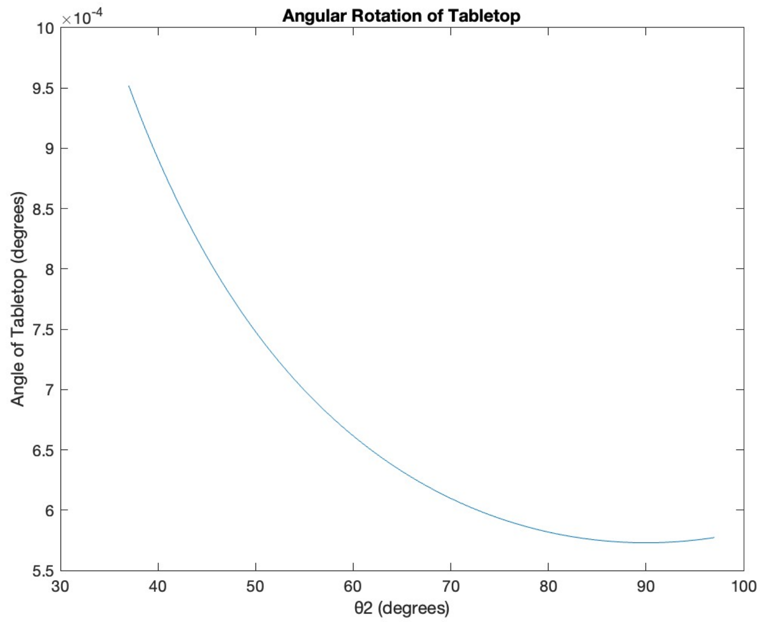


Figure 7: Angular Displacement of Platform Graph

Figure 7 shows half of a fairly parabolic shape over the input angular displacement of link 2 from 37° to 97°. As the angle of link 2 increases, the angular displacement of the platform decreases until it reaches the bottom of the parabola at about 90°. After 90°, the angular displacement begins to increase again slightly before reaching the maximum angular displacement of link 2 at 97°. It is important to note that the places of maximum angular displacement of the platform are also the extremes of the angle of Link 2, meaning simply that at $\theta_{2,min}$ and $\theta_{2,max}$, the angular displacements of the platform are at local maximums. The maximums are 9.5×10^{-4}° at 37° input, and 5.75×10^{-4}° at 97° input.

The final task to complete was to find the required torque to drive link 2 to power the mechanism through three full cycles. The goal of this graph was to show the maximum and minimum values of the torque with respect to time.

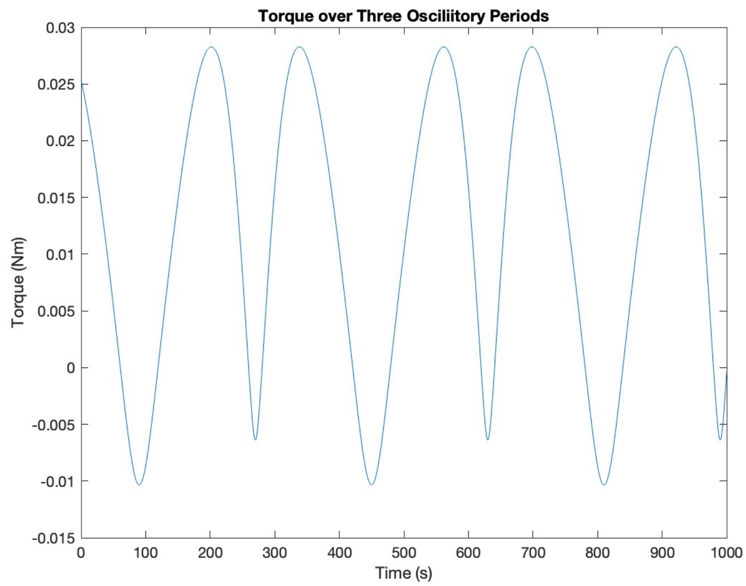


Figure 8: Torque over Three Oscillations Graph

Figure 8 shows the sinusoidal torque graph with respect to time. The motor in this configuration would need to be capable of both positive and negative rotation due to the fact that the input link does not complete a full rotation, and only oscillates between 37° and 97°. Therefore, as the graph shows, the torque switches between positive and negative values. Additionally, the graph shows a sinusoidal relationship, which makes sense for something that spins, like a motor attached to a rotating link. There are local maximums at $0.029\ Nm$ and local minimums at $-\ 0.01\ Nm$ and $-\ 0.0075\ Nm$, which corresponds to when the motor is exerting maximum torque, rotating in either the positive or negative direction. Interestingly, it takes less force (and torque) to spin the links in the negative direction than the positive direction.

VIII. Conclusion

As indicated by the above results, this design fulfills all the requirements of the project and overall is a success. While creating the basic vector loops was relatively simple, the greatest difficulty came from being able to properly move the top table the required distance without rotating it. This was resolved once we found the correlations of the Chebyshev linkage mechanism, which allowed us to translate the rotational motion of the input link to a purely translational motion of a specific point in the third link. Additionally, once we had this information and were able to simulate the individual vector loops in MATLAB, it was difficult to determine exactly how far each mechanism could properly move in relation to the other loop. This was especially true in designing our mechanism such that link 3 would not rotate above the platform during oscillation. We eventually managed to determine a correlation of data that properly aligns with the functional design specifications. Some notable findings include the almost sinusoidal nature of the vertical displacement graph, which we attributed to the translation from rotational to linear motion of link 6, and the fact that it takes lower torque to operate the mechanism in the counterclockwise direction. This lower torque could be because of the tendency for the mechanism to fall counterclockwise due to the center of mass. Another interesting observation was the fact that the horizontal displacement followed an almost linear relationship with input angle in link 2 after about 45° up to 97°. Given more time, we would consider developing design modifications such that the mechanism is more efficient, and with greater tolerances within the given functional requirements.

IX. References

Bryant, John; Sangwin, Chris (2008). *How round is your circle? : where engineering and mathematics meet*. Princeton: Princeton University Press. pp. 33–38, 60–63. ISBN 978-0-691-13118-4. — proof and discussion of Peaucellier–Lipkin linkage, mathematical and real-world mechanical models

Moon, Francis C. "Franz Reuleaux: Contributions to 19th C. Kinematics and Theory of Machines." *Sibley School of Mechanical and Aerospace Engineering; Cornell University*, 2003, pp. 11–15.

Math and Motion: A Look at Chebyshev's Works on Linkages – Bhāvanā. https://bhavana.org.in/math-and-motion-a-look-at-chebyshevs-works-on-linkages/. Accessed 25 Nov. 2022.

Waldron, Kenneth; Kinzel, Gary; Agrawal, Sunil (2016). *Kinematics, Dynamics, and Design of Machinery*. West Sussex, UK: John Wiley & Sons. p. 367. ISBN 9781118933282.

Watt's and Chebyshev's Linkage. https://www.cut-the-knot.org/Curriculum/Geometry/Watt.shtml. Accessed 28 Nov. 2022.

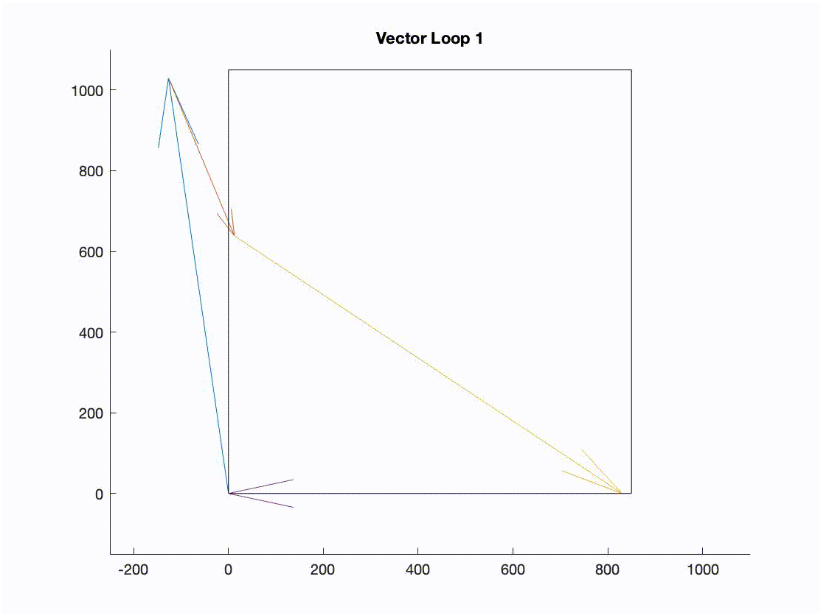
Williams, Robert. *A Bulleted/Pictorial History of Mechanisms and Machines* . Ohio University, 2022, https://www.ohio.edu/mechanical-faculty/williams/html/PDF/HistoryOfMechanisms.pdf.

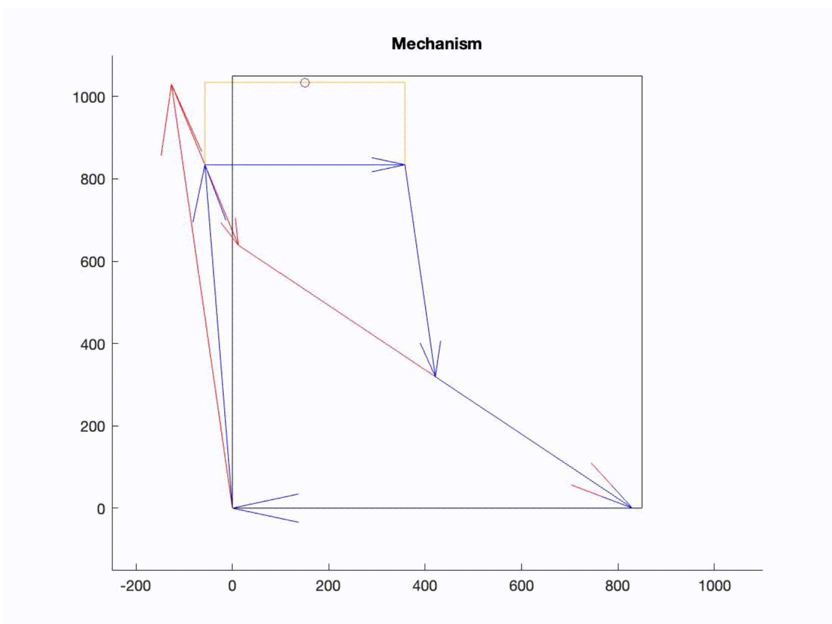
X. Team Member Contributions

Name	Contribution(s)
Fernando Burciaga	Part of Introduction and Part of Conclusion
Aidan Kelly	Maximum/Minimum $\theta 2$, Conclusion
Colston Leonard	Hand calculations to double check, Final CAD model assembly
Jefferson Glickstein	Results section, Preliminary CAD model
Paige Kim	Vector loop diagrams and derivation of vector loop equations, Analysis
Justin Lo	Analysis
Ky Heon	Matlab Simulation and Mechanism Analysis
Andrew Kunkler	Report Outline. Theory & Methodology sections + References
Dan Nguyen	Worked with Colston on hand calcs and CAD model, Table of Contents.
Benny Hsu	Abstract, Part of Introduction, History sections + References
John Lee	Secondary CAD model, Results Section, Determined Link Dimensions

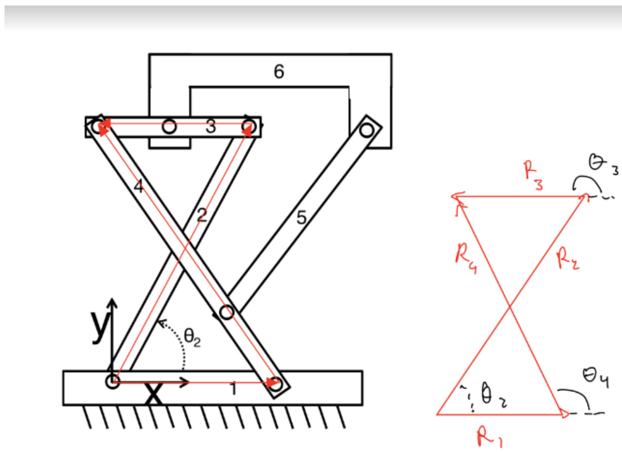
IX. Appendix

MATLAB Simulations





Hand Calculations

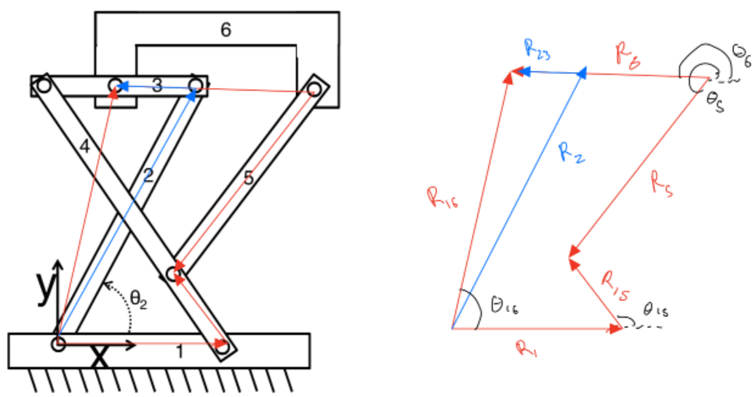


$\theta_1 = 0$

$$\overset{\surd\surd}{R_2} + \overset{\surd ?}{R_3} - \overset{\surd ?}{R_4} - \overset{\surd\surd}{R_1} = 0$$

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$



$$\overset{cc}{R_{16}} - \overset{\surd ?}{R_6} + \overset{\surd ?}{R_5} - \overset{\surd c}{R_{15}} - \overset{\surd\surd}{R_1} = 0$$

$$R_{23} = \frac{1}{2} \overset{\surd\surd}{R_3} \longrightarrow R_{23} = \begin{bmatrix} \frac{1}{2} r_3 \cos\theta_3 \\ \frac{1}{2} r_3 \sin\theta_3 \end{bmatrix}$$

$$R_{16} = \overset{\surd\surd}{R_2} + \overset{cc}{R_{23}} \rightarrow R_{16} = \begin{bmatrix} r_2 \cos\theta_2 + \frac{1}{2} r_3 \cos\theta_3 \\ r_2 \sin\theta_2 + \frac{1}{2} r_3 \sin\theta_3 \end{bmatrix}$$

$$\theta_{15} = \theta_4$$

$$\left[r_2 \cos\theta_2 + \tfrac{1}{2} r_3 \cos\theta_3\right] - r_6 \cos\theta_6 + r_5 \cos\theta_5 - r_{15} \cos\theta_4 - r_1 = 0$$

$$\left[r_2 \sin\theta_2 + \tfrac{1}{2} r_3 \sin\theta_3\right] - r_6 \sin\theta_6 + r_5 \sin\theta_6 - r_{15} \sin\theta_4 = 0$$

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

$$\left[r_2 \cos\theta_2 + \tfrac{1}{2} r_3 \cos\theta_3\right] - r_6 \cos\theta_6 + r_5 \cos\theta_5 - r_{15} \cos\theta_4 - r_1 = 0$$

$$\left[r_2 \sin\theta_2 + \tfrac{1}{2} r_3 \sin\theta_3\right] - r_6 \sin\theta_6 + r_5 \sin\theta_6 - r_{15} \sin\theta_4 = 0$$

Knowns	Unknowns
θ_2 : Input	θ_3
	θ_4
	θ_5
	θ_6

Need to find

$r_1, r_2, r_3, r_4, r_5, r_{15}, r_6$ that fit specs