MACM316 CA7 – Ngawang Kyirong – 301312227

Approximations to: $\boldsymbol{I = \int_0^1 \frac{\sin(x^{-1}\log(x))}{x} dx}$.

Table 1: $I \approx Q_n = \sum_{i=0}^{n} I_i$, $I_i = \int_{a_{i+1}}^{a_i} f(x)dx$, $a_0 = 1$. a_n are roots to f(x).

Table 2: : $I \approx \widehat{Q_n} = Q_n - \frac{(Q_{n+1}-Q_n)^2}{Q_{n+2}-2Q_n+Q_n}$

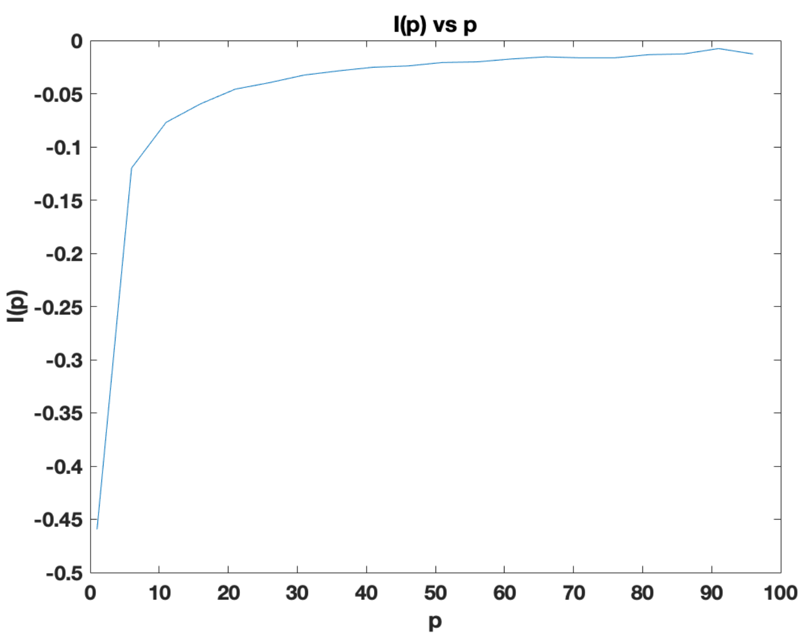
n	$I \approx Q_n =$
50	-0.464289002017376
100	-0.461917922130111
150	-0.461094455803492
200	-0.460674560911444
250	-0.460419485047569
300	-0.460247926419383
350	-0.460124553884726
400	-0.460031522253631
450	-0.459958839353907
500	-0.459900471625607

Table 1

n	$I \approx \widehat{Q_n} =$
50	-0.459361333772362
100	-0.459360970845284
150	-0.459360930686411
200	-0.459360920518306
250	-0.459360916813490
300	-0.459360915150366
350	-0.459360914294815
400	-0.459360913810249
450	-0.459360913515406
500	-0.459360913325759

Table 2

* For the approximations in Table 1, roughly **4** digits of I are being accurately computed. As we can see from n=450 to n=500, the first 4 digits of the approximation are staying the same (-0.4599). Therefore, due to the stability and consistency of the first 4 digits, we can state that our approximation can compute roughly 4 digits accurately.
* For the approximations in Table 2, roughly **9** digits of I are being accurately computed. As we can see from n=400 to n=400, the first 9 digits of the approximation are equal (-0.459360913). Therefore, due to the stability and consistency of the first 9 digits, we can state that our approximation can compute roughly 9 digits accurately.



* The plot of I(p) vs p shows an asymptotic graph where I(p) is growing and approaching 0 as p is increasing. We can compare and see that our original value of p=1 is much smaller than p=100.

```
format long
f = @(x) power(x,-1) .* sin(power(x,-1) .* log(x));
n = 50:50:500;
Q_n = [];


% part 1

%{
for j = 1:length(n)
    a = [];
    n(j)
    for i = 1:(n(j) + 1)
        b = fzero(@(x) x*exp(x)-i*pi,0);
        a = [a exp(-b)];
    end
    q = integral(f, a(1), 1);
    for p = 1:(n(j))
        q = q + integral(f, a(p + 1), a(p));
    end
    q
    Q_n = [Q_n, q];
end
%-----------------------------------------------------------------------

% part 2

Q_n_hat = [];
for j = 1:length(n)
    a = [];
    n(j)
    for i = 1:(n(j) + 1)
        b = fzero(@(x) x*exp(x)-i*pi,0);
        a = [a exp(-b)];
    end
    q = integral(f, a(1), 1);
    for p = 1:(n(j))
        q = q + integral(f, a(p + 1), a(p));
    end
    a_1=[];
    for i = 1:(n(j) + 2)
        b = fzero(@(x) x*exp(x)-i*pi,0);
        a_1 = [a_1 exp(-b)];
    end
    q_n_1 = integral(f, a_1(1), 1);
    for p = 1:(n(j) + 1)
        q_n_1 = q_n_1 + integral(f, a_1(p + 1), a_1(p));
    end
    a_2 = [];
    for i = 1:(n(j) + 3)
       b = fzero(@(x) x*exp(x)-i*pi,0);
       a_2 = [a_2 exp(-b)];
    end

    q_n_2 = integral(f, a_2(1), 1);
```

```
    for p = 1:(n(j) + 2)
        q_n_2 = q_n_2 + integral(f, a_2(p + 1), a_2(p));
    end

    q_hat = q - ((power(q_n_1 - q, 2)) / (q_n_2 - (2*q_n_1) + q))
    Q_n_hat = [Q_n_hat q_hat];
end
%}
%-----------------------------------------------------------------------

% part 3

I = @(s) power(x,-1) .* sin(power(x,-s) .* log(x));

Q_n_hat = [];
for s = 1:5:100
    I = @(x) power(x,-1) .* sin(power(x,-s) .* log(x));
    a = [];
    for i = 1:(500 + 1)
        b = fzero(@(x) (x)*exp(x*s)-i*pi,0);
        a = [a exp(-b)];
    end
    q = integral(f, a(1), 1);
    for p = 1:(500)
        q = q + integral(I, a(p + 1), a(p));
    end
    a_1=[];
    for i = 1:(500 + 2)
        b = fzero(@(x) (x)*exp(x*s)-i*pi,0);
        a_1 = [a_1 exp(-b)];
    end
    q_n_1 = integral(f, a_1(1), 1);
    for p = 1:(500 + 1)
        q_n_1 = q_n_1 + integral(I, a_1(p + 1), a_1(p));
    end
    a_2 = [];
    for i = 1:(500 + 3)
       b = fzero(@(x) (x*s)*exp(x*s)-i*pi,0);
       a_2 = [a_2 exp(-b)];
    end

    q_n_2 = integral(f, a_2(1), 1);
    for p = 1:(500 + 2)
        q_n_2 = q_n_2 + integral(I, a_2(p + 1), a_2(p));
    end

    q_hat = q - ((power(q_n_1 - q, 2)) / (q_n_2 - (2*q_n_1) + q));
    Q_n_hat = [Q_n_hat q_hat];
end

plot(1:5:100, Q_n_hat)
xlabel('p')
ylabel('I(p)')
title('I(p) vs p')
```