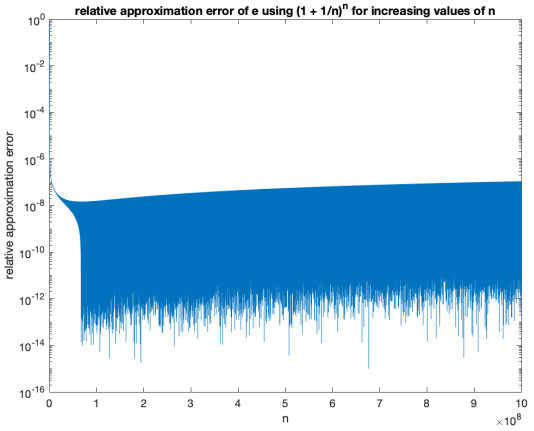
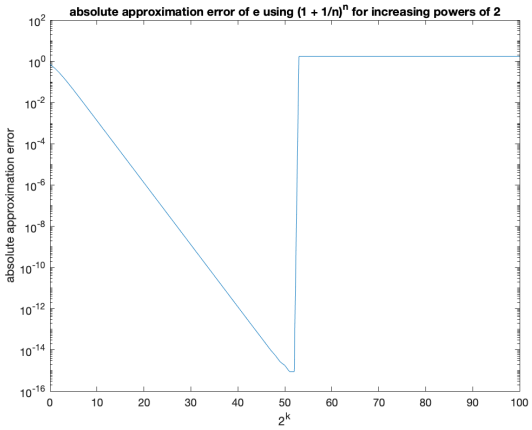
MACM316 ASSIGNMENT 1 – Ngawang Kyirong 301312227

a.)





b.)

- For the **first** figure, the behaviour shown is best described as the bound of the relative approximation error. This bound represents the maximum value the approximation error will take when using $\left(1+\frac{1}{n}\right)^n$ to approximate *e*. The robustness of this algorithm is good for these inputs given that it guarantees an approximation error of at most $\frac{1}{2}B^{1-k}$ even though the graph is consistently fluctuating.

- For the **second** figure, interesting behaviour is produced at around the n=2^{52} mark. Instead of the error decreasing as 2^k increases, the graph spikes back up to a constant value. In terms of robustness, this algorithm is not good as this is an error due to the machine epsilon.

c.)

- For the **first** figure, the graph shows that calculating the approximation error of *e* when using: $\left(1+\frac{1}{n}\right)^n$ will produce a bound of $\frac{1}{2}B^{1-k}$ where *B* is the base and *k* is the precision. In this particular case, the floating-point representation of *e* gives a bounded error of at least $.5 * 10^{-7}$. The effects of cancellation error from the floating-point subtraction of the *e* and the approximation of *e* is also on display which produces the fluctuation.

- For the **second** figure, the graph displays interesting behaviour around the *k=52* mark. The graph shows that when calculating the absolute approximation error of *e* when using: $\left(1+\frac{1}{n}\right)^n$ and $n = 2^{52}$, MATLAB cannot distinguish the values due to it reaching machine epsilon because now $(1+\frac{1}{n})$ is indistinguishable from 1 at $n = 2^{52}$. Hence, $\left(1+\frac{1}{n}\right) = 1$ and the graph remains stuck on the constant $e - 1$.

d.)

- Through testing the algorithm with a variety of inputs for *c* with relative and absolute errors, the results show that if *e* was replaced with e^c for some positive *c* and $e_n^c = \left(1+\frac{c}{n}\right)^n$ then the absolute approximation error will increase a lot as *c* increases. Contrastingly, the relative error will not increase as much due to the division of the magnitude but it will increase.

MATLAB :

```
x1 = linspace(0,power(10,9),65000000);
y1 = arrayfun(@(x) abs((exp(1) - power((1 + (1/x)), x)))/abs(exp(1)), x1);
p1 = semilogy(x1, y1);
xlabel('n');
ylabel('relative approximation error');
title('relative approximation error of e using (1 + 1/n)^n for increasing
values of n');
x2 = power(2, 0:100);
y2 = arrayfun(@(x) abs((exp(1) - power((1 + (1/x)), x))), x2);
p2 = semilogy(log2(x2),y2);
xlabel('2^k');
ylabel('absolute approximation error');
title('absolute approximation error of e using (1 + 1/n)^n for increasing
powers of 2');
```