MACM 316 – Computing Assignment 8

- Read the *Guidelines for Assignments* first.
- Write a one-page PDF report summarizing your finding. This report must also include all your figures.
- Submit the one-page PDF report using Crowdmark and add your Matlab code to it (as extra pages). You must use the link sent by Crowdmark to your SFU email address. Follow the instructions on the Crowdmark to upload your file. If the report is hand-written then use the CamScanner app with your cellphone to scan the report and every page of the code. You will lose marks for poor quality pictures.
- You must acknowledge any collaborations/assistance from fellow students, discussion forums, TAs, instructors, etc.

Numerical Solution of Kepler's problem

One of science's great achievements was the discovery of Kepler's laws for planetary motion; in particular, that the planets follow closed elliptical orbits around the sun. In this assignment you will compare two different numerical methods for solving Kepler's problem for the motion of a simple solar system consisting of two planets (the two-body problem).

For a system of two planets, we may assume one is fixed at the origin with the motion of the other planet being in a 2D plane. Let

$$\mathbf{q}(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}, \qquad \mathbf{p}(t) = \begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix},$$

be the position and momentum vectors of the moving planet, respectively. Kepler's laws give the following ordinary differential equation for $\mathbf{q}(t)$ and $\mathbf{p}(t)$:

$$\mathbf{q}'(t) = \mathbf{p}(t), \qquad \mathbf{p}'(t) = -\frac{1}{(q_1(t)^2 + q_2(t)^2)^{3/2}}\mathbf{q}(t). \tag{1}$$

a) Write a code that implements Euler's method for this problem for time $0 \le t \le T = 200$ and stepsize $h = 0.0005$. Use the initial conditions

$$q_1(0) = 1 - e, \quad q_2(0) = 0, \quad p_1(0) = 0, \quad p_2(0) = \sqrt{\frac{1+e}{1-e}}, \qquad e = 0.6.$$

Plot your output in the q_1-q_2 plane, i.e. plot the approximate position of the moving planet at time t_n for $n = 0, 1, \ldots, N$, where $N = \lceil T/h \rceil$. Briefly describe the *qualitative* behaviour of the numerical solution.

b) The ODE (1) has several conserved quantities, including the angular momentum $A(t)$ and Hamiltonian $H(t)$, defined by

$$A(t) = q_1(t)p_2(t) - q_2(t)p_1(t), \qquad H(t) = \frac{1}{2}(p_1(t)^2 + p_2(t)^2) - \frac{1}{\sqrt{q_1(t)^2 + q_2(t)^2}}.$$

Compute and plot these quantities for your numerical solution. Does your numerical solution also conserve the angular momentum and Hamiltonian? If not, briefly comment on their behaviour for large t.

c) For systems such as (1), an alternative to the standard Euler's method is the so-called symplectic Euler method

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\mathbf{p}_n, \qquad \mathbf{p}_{n+1} = \mathbf{p}_n - \frac{h}{(q_{n+1,1}^2 + q_{n+1,2}^2)^{3/2}}\mathbf{q}_{n+1}.$$

Here $q_{n+1,1}$ and $q_{n+1,2}$ are the components of the vector $\mathbf{q}_{n+1}$. Implement this method and compare it with the standard Euler's method. Plot and describe the behaviour of the numerical solution, and also the angular momentum and Hamiltonian.

Which of the two methods would you choose to compute planetary orbits? Explain your reasoning.