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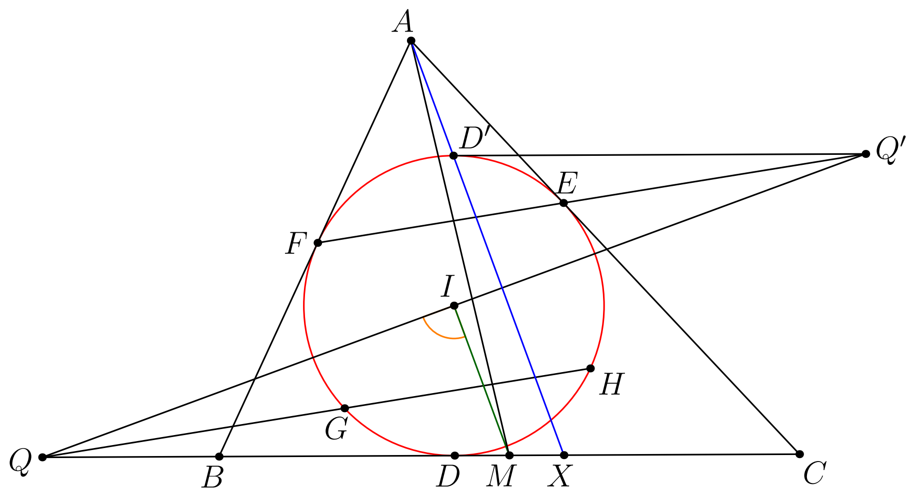
Problem (Taiwan TST 2014/1J/3)**.** In $\triangle ABC$ with incenter I, the incircle is tangent to $\overline{CA}$, $\overline{AB}$ at E, F. The reflections of E, F across I are G, H. Let Q be the intersection of $\overline{GH}$ and $\overline{BC}$, and let M be the midpoint of $\overline{BC}$. Prove that $\overline{IQ}$ and $\overline{IM}$ are perpendicular.



¶ Main Idea Show the perpendicuarlity using the polar fact, and prove another parallel claim using common incenter configurations.

¶ Solution By symmetry, let Q' be the reflection of Q over I. It follows that Q' passes through the tangent to D'.

> **Claim —** AD' is the polar of Q' wrt (I)

Proof. By La-Hire's theore, A lies on the polar of Q', as $Q' \in EF$, and EF is in fact the polar of A. Moreover, $Q' \in D'D'$, so AD' is indeed the polar of Q' $\square$

Note that it suffices to prove that $AD' \parallel IM$, as $AD' \perp Q'I$ by the polar condition.

> **Claim —** $AD' \parallel IM$

Proof. The intersection of AD' and BC, X, is in fact the A-extouch point, so M is the midpoint of DX. This implies that $AD' \parallel IM$ by midline. $\square$