2018 APMO P1

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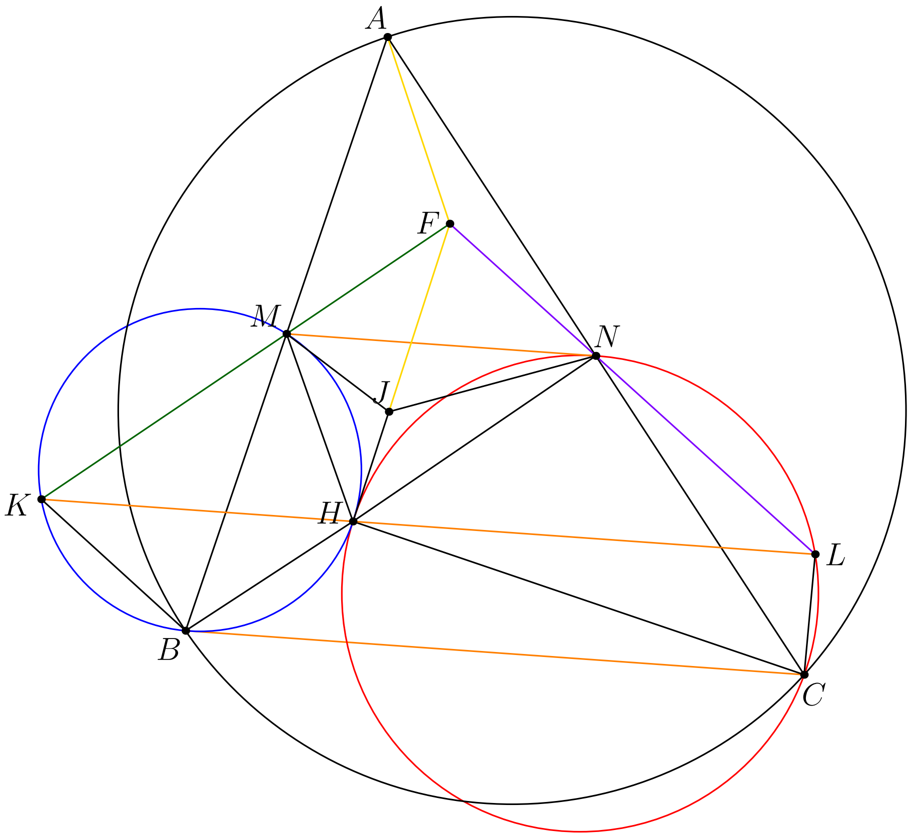
Problem (APMO 2018/1)**.** Let H be the orthocenter of the triangle ABC. Let M and N be the midpoints of the sides AB and AC, respectively. Assume that H lies inside the quadrilateral $BMNC$ and that the circumcircles of triangles BMH and CNH are tangent to each other. The line through H parallel to BC intersects the circumcircles of the triangles BMH and CNH in the points K and L, respectively. Let F be the intersection point of MK and NL and let J be the incenter of triangle MHN. Prove that $FJ = FA$.



¶ Main Idea Use **Incenter-Excenter**, by proving that F is the circumcenter of (AMN) and $AMNJN$ is cyclic.

¶ Solution

Claim — F is the circumcenter of the circle (AMN).

Proof. Easy angle chasing. □

Claim — $AMJN$ is cyclic.

Proof. Easy angle chasing. □