KYJ Problem 1

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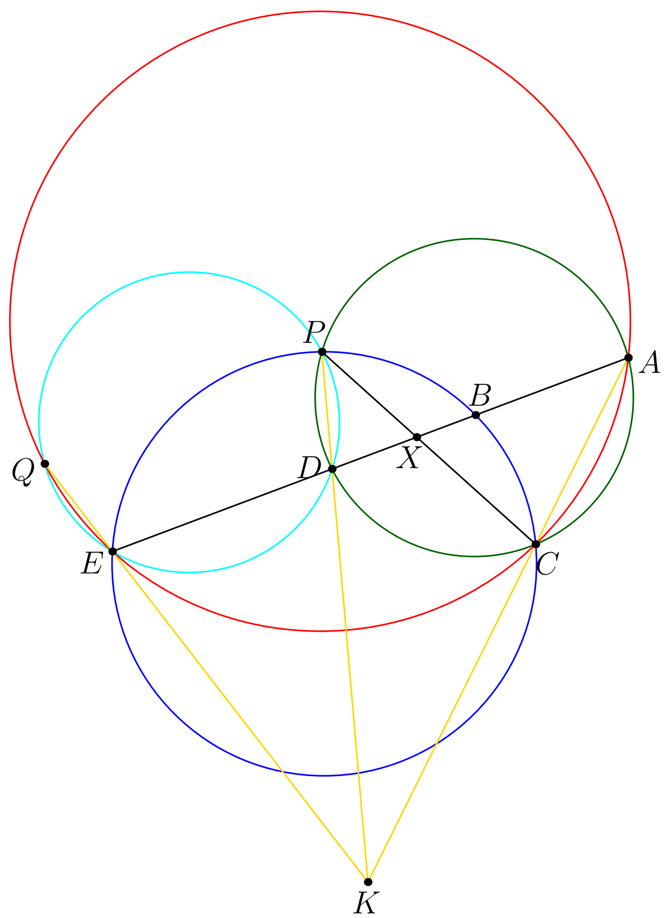
Problem (KYJ Problem 1)**.** For triangle ABC, Let D, E be points on ray AB such that $AD < AE$. Let $P \neq C$ be the intersection of the circumcircles of EBC and ADC. Let Q be a point on ray CD such that $\angle EQD = \angle ACB$. Prove that if QE, PD, and AC are concurrent, then $QECA$ is concyclic.



¶ Main Idea Angle chase and notice nice concyclicities, and finish off by rephrasing the statement using radical center theorem.

¶ Solution

Claim — $QEDP$ is cyclic.

Proof.

$$\begin{aligned}\angle EPD &= \angle EPC - \angle DPC \\ &= \angle EBC - \angle DAC \\ &= \angle DBC - \angle BAC \\ &= 180 - \angle ABC - \angle BAC \\ &= \angle BCA = \angle BQD\end{aligned}$$

$\square$

Let the concurrence point be K. Then by power of a point, we have

- $KE \times KQ = KD \times KP$
- $KD \times KP = KC \times KA$

$$\implies KE \times KQ = KC \times KA$$

and by converse of power of a point, we have that $QECA$ is cyclic. $\blacksquare$