2019 USEMO P1

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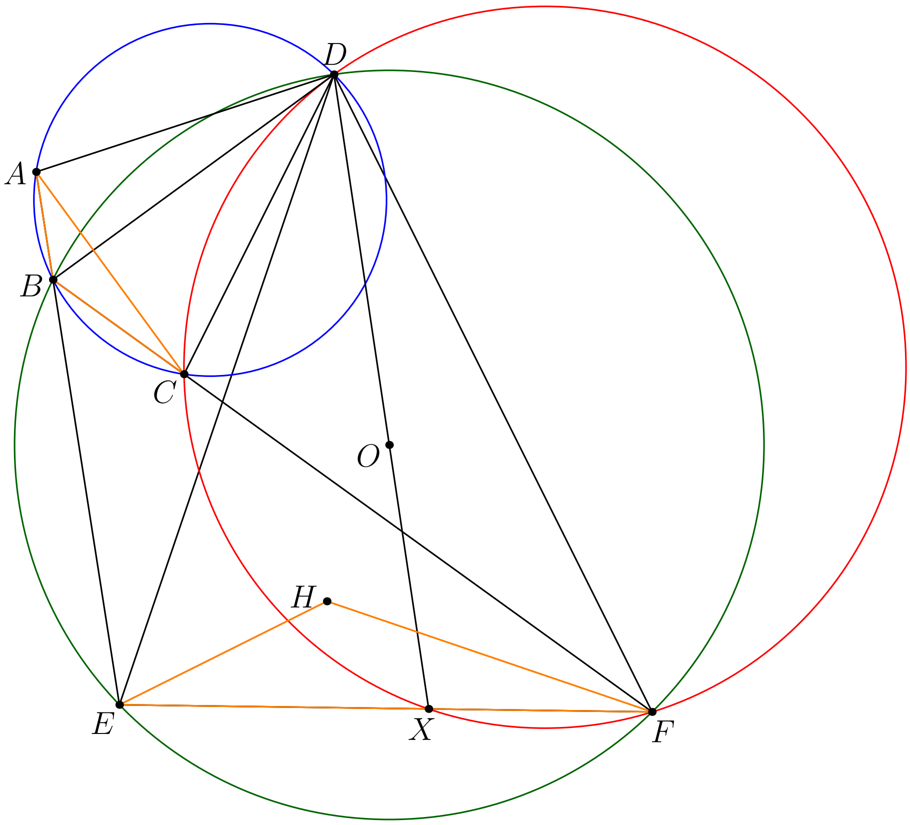
Problem (USEMO 2019/1)**.** Let $ABCD$ be a cyclic quadrilateral. A circle centered at O passes through B and D and meets lines BA and BC again at points E and F (distinct from A, B, C). Let H denote the orthocenter of triangle DEF. Prove that if lines AC, DO, EF are concurrent, then triangles ABC and EHF are similar.



¶ Main Idea Notice the cyclicity of one circle, and finish off with angle chase. The construction of the problem, initially, seems very hard. However, noticing that angle chasing probably works really well for this problem, we deduce the fact that $DCXF$ is cyclic, which makes it very easy to construct from there.

¶ Solution

Claim — $DCXF$ is cyclic

Proof. $\angle DCX = 180 - \angle ACD = 180 - \angle ABD = \angle DFE$ □

Claim — $\angle ABC = \angle EHF$

Proof. $\angle ABC = \angle 180 - \angle EBF = 180 - \angle EDF = \angle EHF$ □

Claim — $\angle BAC = \angle HEF$

Proof.

$$\begin{aligned}
\angle ABC &= \angle BDC \\
&= 180 - \angle DBC - \angle DCB \\
&= 180 - \angle DBF - (180 - \angle DCF) \\
&= \angle DCF - \angle DBF \\
&= \angle DXF - \angle DEF \\
&= 180 - \angle ODF - \angle EFD - \angle DEF \\
&= 180 - (90 - \angle DEF) - \angle EFD - \angle DEF \\
&= 90 - \angle EFD \\
&= \angle HEF
\end{aligned}$$

□