2012 AIME II 15

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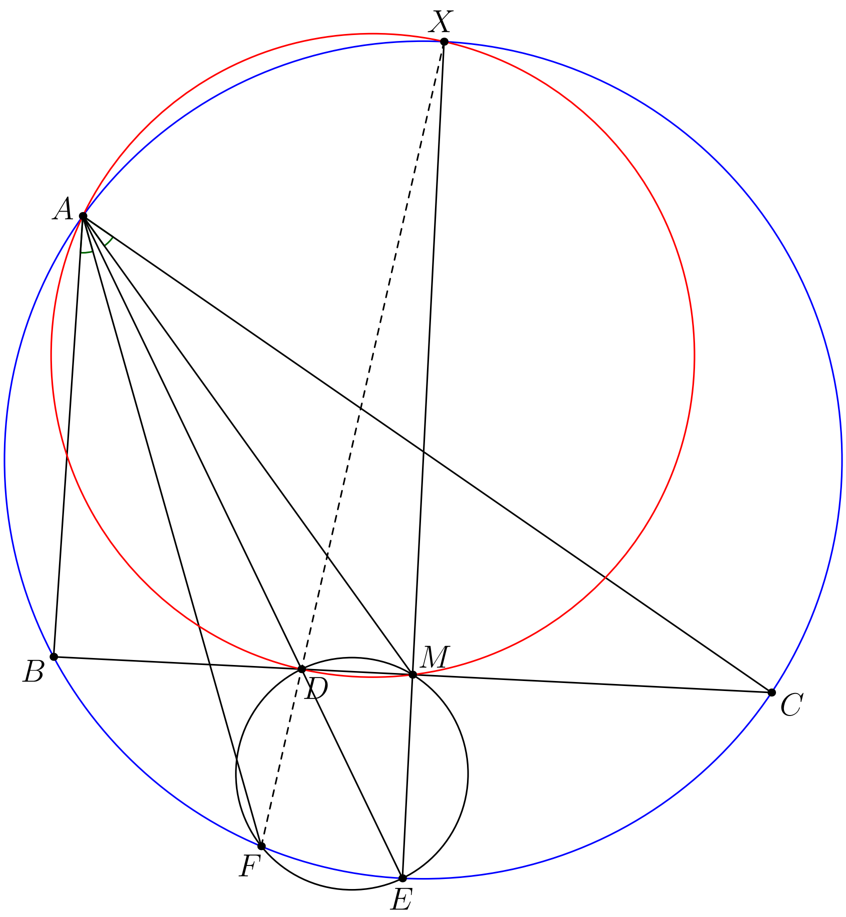
25 August 2025

Problem (2012 AIME II #15). Triangle ABC is inscribed in circle ω with $AB = 5$, $BC = 7$, and $AC = 3$. The bisector of angle A meets side BC at D and circle ω at a second point E. Let γ be the circle with diameter DE. Circles ω and γ meet at E and a second point F. Then $AF^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



¶ Main Idea AF is in fact the A-**symmedian** wrt $\triangle ABC$.

¶ Method 1 $\sqrt{bc}$ invert wrt (ABC)(perform inversion centered at A with radius $\sqrt{AB \cdot AC}$ and reflect about the angle bisector of $\angle BAC$.). This swaps B and C, D and E, and lastly F and M are swapped, because $\angle DFE = 90$ so F is sent to a point on BC

such that $\angle D'F'E' = EFD = 90$, which is clearly the midpoint M of BC. This proves that AF is indeed the symmedian.

¶ Method 2 It suffices to prove that X, D, F are collinear. The rest follows by angle chasing. This is true by angle chasing as well:

Proof. $90 = \angle XFE = \angle EAX = \angle DAX = \angle DMX = \angle DME = \angle DFE$ □

¶ Other Methods Bashing via trigonometry(trig bash), coordinates, barycentrics & complex??(probably not). To note, coordinate bashing was really common for this problem. As well as trig bash, as always(it's an AIME problem!)