2020 IMO P1

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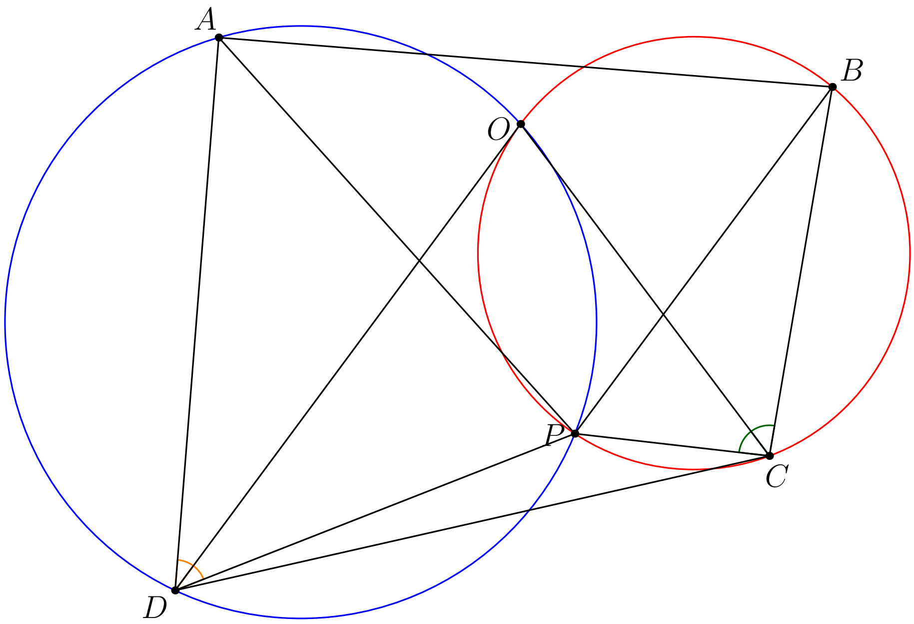
Problem (IMO 2020/1)**.** Consider the convex quadrilateral $ABCD$. The point P is in the interior of $ABCD$. The following ratio equalities hold:

$$\angle PAD : \angle PBA : \angle DPA = 1 : 2 : 3 = \angle CBP : \angle BAP : \angle BPC.$$

Prove that the following three lines meet in a point: the internal bisectors of angles $\angle ADP$ and $\angle PCB$ and the perpendicular bisector of segment AB.



¶ Main Idea Consider the circumcenter of ABP. This is very motivated as our statement talks about the perpendcular bisector of AB. Another motivation too, is that angle bisectors can be transformed to perpendicular bisectors(arc midpoints) when we invoke the circumcircle; this also motivates the circumcenter. Upon doing so, the problem becomes trivial.

¶ Solution Consider the circumcenter of ABP. It is obvious, by angle chasing, that it is cyclic with (PCB) and (ADP), so we're done.