2019 ELMO SL G1

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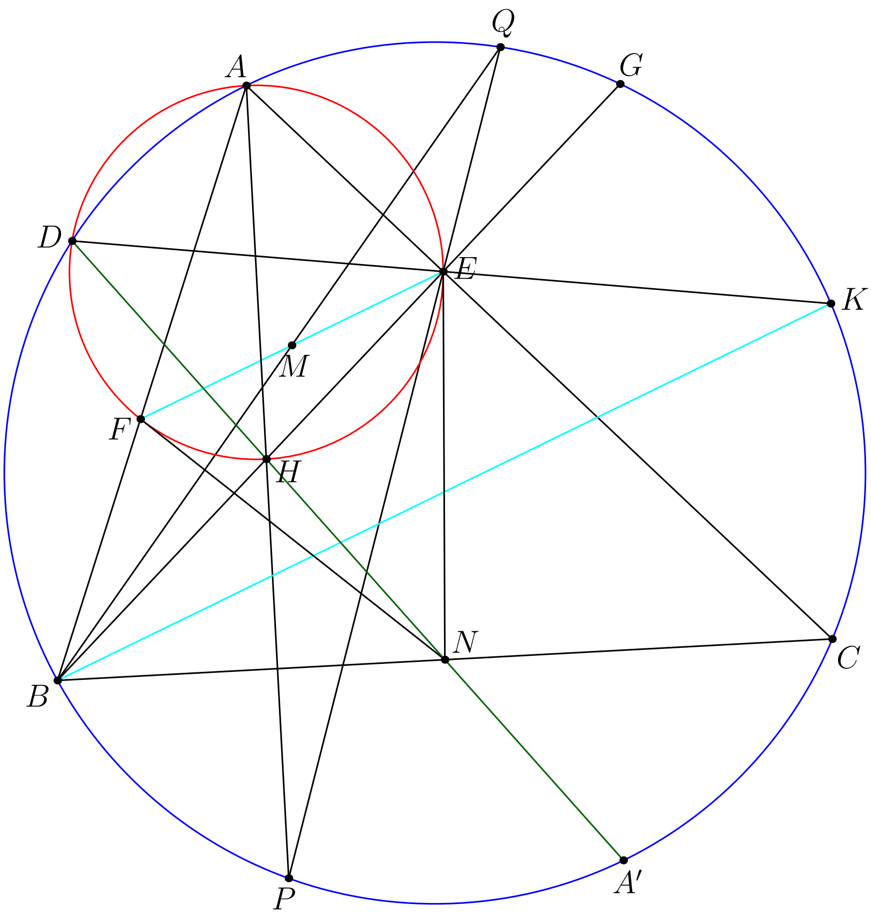
Problem (ELMO SL 2019/G1)**.** Let ABC be an acute triangle with orthocenter H and circumcircle Γ. Let BH intersect AC at E, and let CH intersect AB at F. Let AH intersect Γ again at $P \neq A$. Let PE intersect Γ again at $Q \neq P$. Prove that BQ bisects segment $\overline{EF}$.



¶ Main Idea The main idea is to project around and notice some nice occurences(concyclity, collinearity, harmonicity). One possible solution is to consider the parallel line through B to EF, and using harmonic cross ratios.

¶ Solution 1(Harmonic)

Claim — $DBPC$ is harmonic

Proof. By spiral similarity, it suffices to prove that $DFHE$ is harmonic. It then suffices, noting that the tangents at E and F intersect at the midpoint of BC, to prove that D, H, N are collinear. This follows by noting that the reflection of H over BC, A', is in fact the antipode of A wrt (ABC). In particular, $90 = \angle ADH = \angle ADA'$. □

Claim — $EF \parallel BK$

Proof. $\angle DEF = \angle DAF = \angle DAB = \angle DKB$ □

Claim — BQ bisects EF

Proof.

$$-1 = (D, P, B, C) \stackrel{E}{=} (K, Q, G, A) \stackrel{B}{=} (\infty, M, E, F)$$

Hence $M = BQ \cap EF$ is the midpoint of EF. □

¶ Other Methods

- Similarity
- Pascal's theorem
- Complex bashing
- Bary bashing
- Trig bashing