Sharygin 2013

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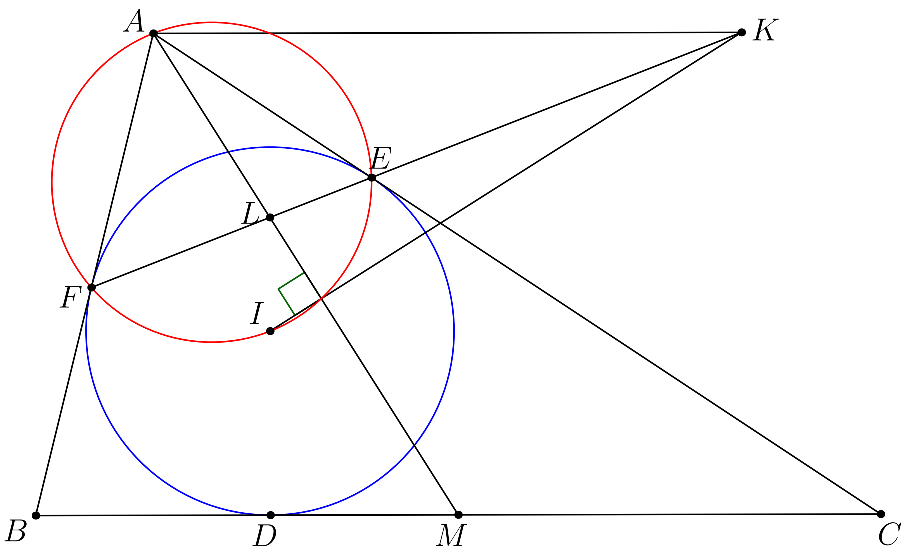
Problem (Sharygin 2013)**.** Define $\triangle ABC$ and its intouch points D, E, and F. Let the line through I perpendicular to the A-median intersect EF at a point K. Prove that AK is parallel to BC.



¶ Main Idea Use projective geometry, harmonics, and pole-polar to use the midpoint condition(+point at infinity) and perpendicularity(right angles and bisectors OR pole-polar)

¶ Solution Let AM intersect EF at L. Redefine K to be the intersection of the parallel line to BC through A and EF. Note that it suffices to prove that $IK \perp AM$. Now we project (B, C, M, ∞). In particular,
$-1 = (B, C, M, \infty) \overset{A}{=} (F, E, L, K)$
This in fact implies that L lies on the polar of K. Moreover, because $A \in EF$, A also lies on the polar of K. This implies that AL is the polar of K, or that $AL = AM \perp IK$, and we're done.