2024 IMONST P3

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Problem (IMONST 2024/3)**.** Ivan claims that for all positive integers n,

$$\left\lfloor \sqrt[2]{\frac{n}{1^3}} \right\rfloor + \left\lfloor \sqrt[2]{\frac{n}{2^3}} \right\rfloor + \left\lfloor \sqrt[2]{\frac{n}{3^3}} \right\rfloor + \cdots = \left\lfloor \sqrt[3]{\frac{n}{1^2}} \right\rfloor + \left\lfloor \sqrt[3]{\frac{n}{2^2}} \right\rfloor + \left\lfloor \sqrt[3]{\frac{n}{3^2}} \right\rfloor + \cdots$$

Why is he correct?

¶ Main Idea Note that both sides count how many integers satisfy $x^2y^3 \le n$ for some (x, y). We're done because it's just symmetry.