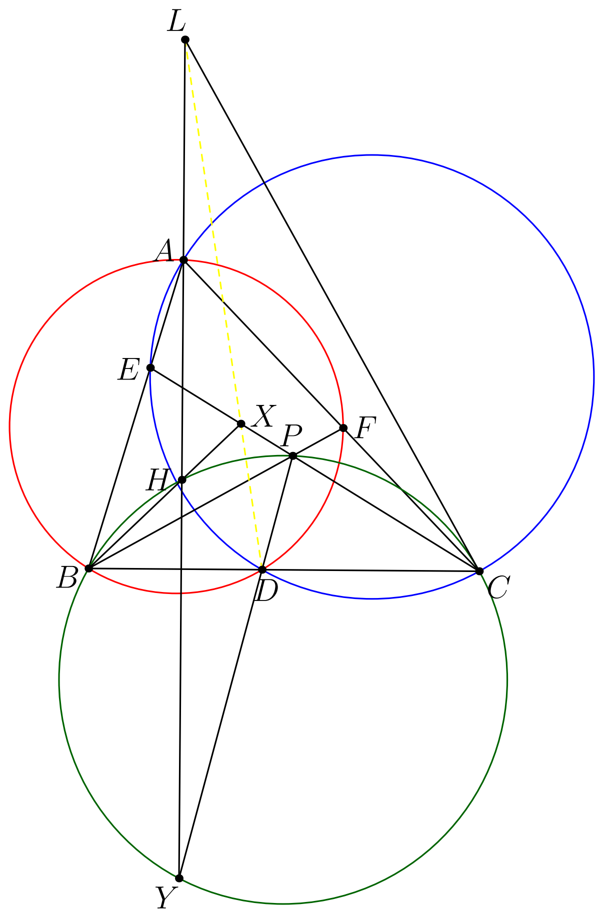
2025 BMO P2

Kim Yong Joon

25 August 2025

Problem (BMO 2025/2)**.** In an acute-triangled $\triangle ABC$, H is its orthocenter and D is an arbitrary point in BC. The points E and F are on segments AB and AC, respectively, such that $ABDF$ and $ACDE$ are cyclic. The segments BF and CE intersect at P. L is a point on AH such that LC is tangent to the circumcircle of $\triangle PBC$ at C. BH and CP intersect at X. Prove that D, X, L are collinear.



¶ Main Idea Use **Pascal's Theorem** to prove the claim. The rest would be very clear, trying to prove some cyclicity claims.

¶ Solution

Claim — Y, the second intersection of AH and (BHC), is the reflection of A over BC.

Proof. $\angle BAC = 180 - \angle BHC = \angle BYC$ □

Claim — $P \in (BHC)$

Proof. $\angle BYP = \angle BYD = \angle BAD = \angle EAD = \angle ECD = \angle PCB$ □

Claim — D, X, L are collinear

Proof. By Pascal's theorem on $CCBHYP$, we achieve the desired collinearity, as desired. □