# 2015 Taiwan TST Quiz

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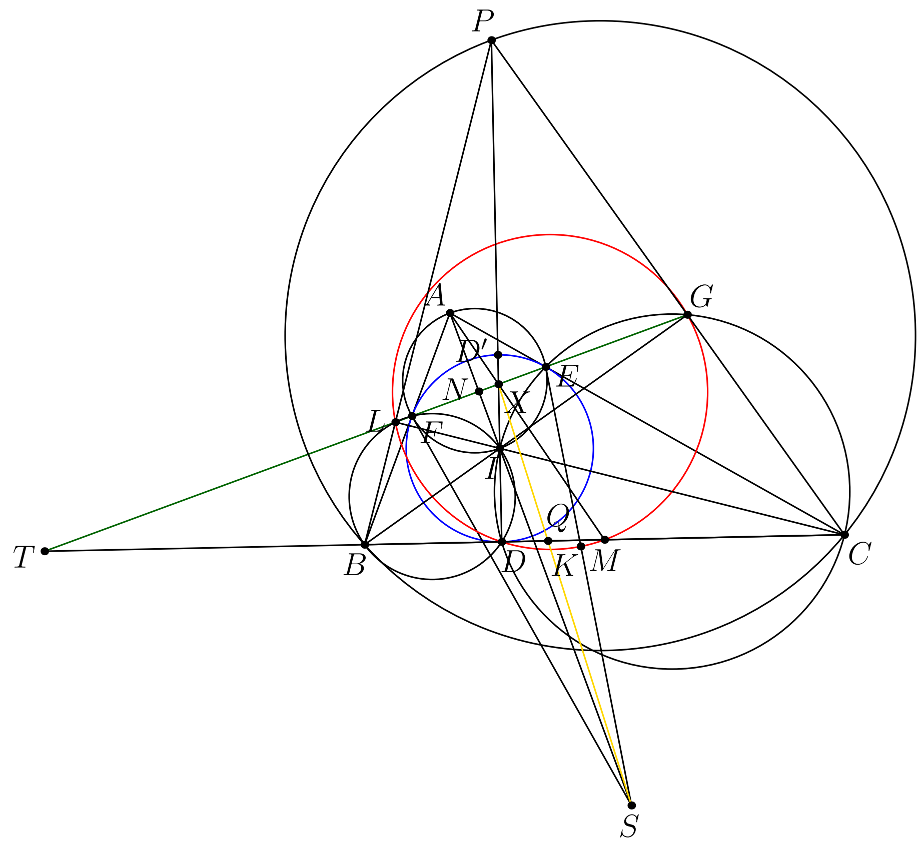
**Problem** (Taiwan TST 2015)**.** In scalene triangle $ABC$ with incenter $I$, the incircle is tangent to sides $CA$ and $AB$ at points $E$ and $F$. The tangents to the circumcircle of $\triangle AEF$ at $E$ and $F$ meet at $S$. Lines $EF$ and $BC$ intersect at $T$. Prove that the circle with diameter $\overline{ST}$ is orthogonal to the nine-point circle of triangle $BIC$.




**¶ Main Idea** Use well known lemmas(Iran Lemma, Incircle concurrency) to get a grasp of the general picture, and use harmonics and projective geometry to simplify the condition and prove a simplified statement.

**¶ Solution(harmonic)** Note, by the Self-Polar Orthogonality lemma, that it suffices to prove that $S$ and $T$ lie on each other's polar wrt the Nine Point circle of $\triangle BIC$. It is well known that $AM$, $EF$, and $DI$ concur at a point, say $X$. For the naming of other points, assume the one found in the diagram.

**Claim** — $T, L, F, N, E, G$ are collinear

*Proof.* Note that $DI$, $EF$, and $AM$ concur, and $(LFBDI)$, $(GEIDC)$ are cyclic. By angle chasing,

$$\angle EGI = \angle ECI = \angle ICD = \angle LCB = \angle LGB$$

hence $L, E, G$ are collinear. Similarly, $L, F, G$ are collinear. This achieves the desired result. $\square$

**Claim** — $(T, X, L, G) = -1$

*Proof.* By Menelaus-Ceva on $GLBC$, $(T, D, B, C) = -1$. Projecting from $A$,

$$-1 = (T, D, B, C) \stackrel{A}{=} (T, X, L, G)$$

$\square$

**Claim** — $(T, Q = SX \cap BC, D, M) = -1$

*Proof.* $-1 = (A, I, N, S) \stackrel{X}{=} (M, D, T, Q)$ $\square$

By our second and third claim, $S$ lies on the polar of $T$, as desired.