2017 ISL G4

Kim Yong Joon

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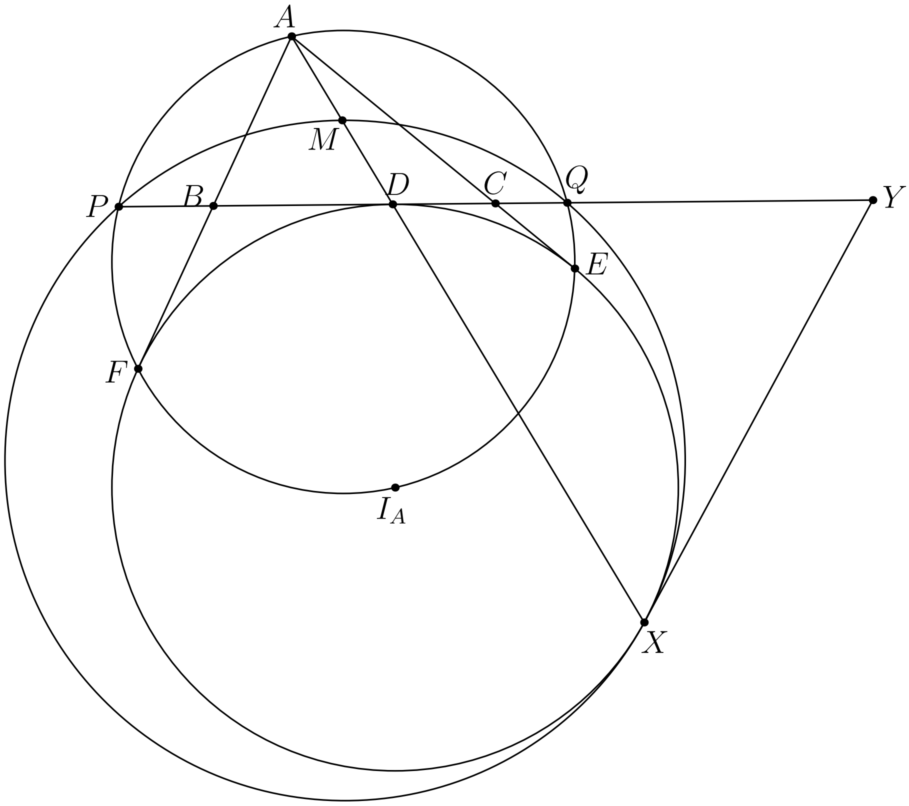
Problem (2017 ISL G4)**.** Let ABC be a triangle and let ω be the A-excircle, tangent to BC, CA, and AB at D, E, F. The circumcircle of AEF intersects line BC at P and Q. Let M be the midpoint of AD. Prove that the circumcircle of MPQ is tangent to ω.



¶ Solution The solution to this problem is actually a lot simpler than you think. I would consider it very easy for a G4.

The main idea is to identify the tangency point, note that it actually lies on AD, and use power of a point to prove the concylcity and finish off with radixal axes/centers.

In particular,

Claim — $X \in (MPQ)$

Proof. By power of a point, $PD \cdot DQ = AD \cdot DY$
$\Rightarrow PD \cdot DQ = \frac{1}{2}AD \cdot 2DY \Rightarrow PD \cdot DQ = MD \cdot DX$ □

Note that it now suffices to prove that DFEX is harmonic, by the radical center theorem. This is evident as EF is the polar of A and $X \in AD$.