2006 ISL G6

Kim Yong Joon

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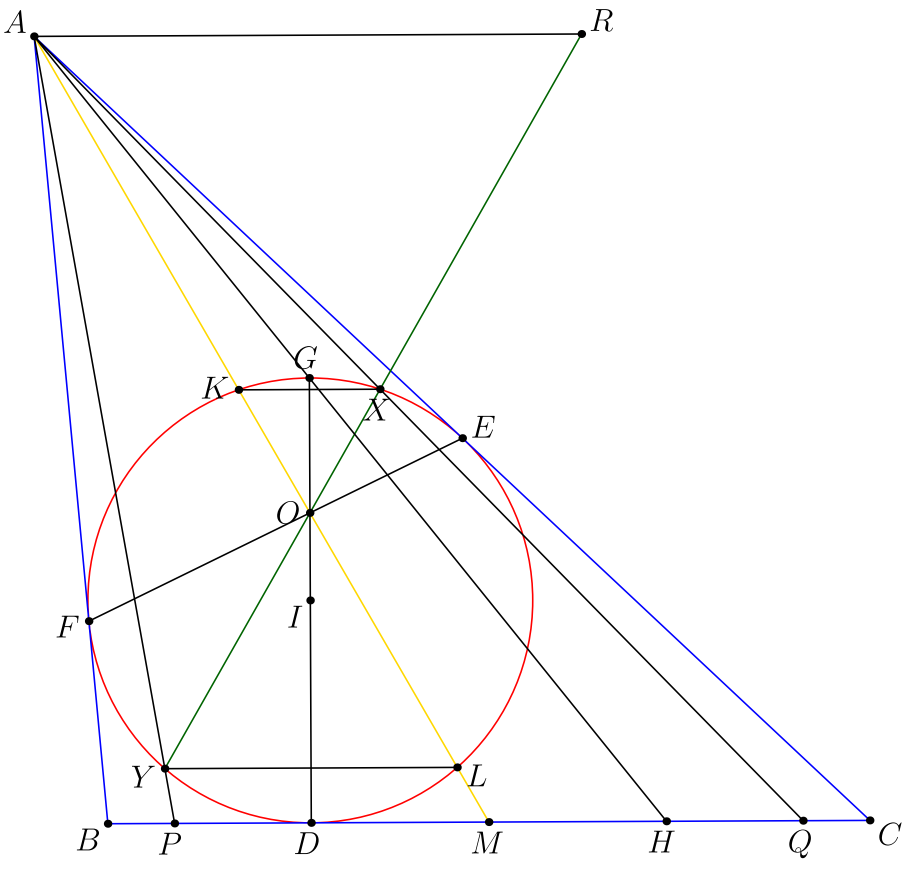
Problem (ISL 2006/G6). Let ABC be a triangle, and M the midpoint of its side BC. Let γ be the incircle of triangle ABC. The median AM of triangle ABC intersects the incircle γ at two points K and L. Let the lines passing through K and L, parallel to $\overline{BC}$, intersect the incircle γ again in two points X and Y. Let the lines AX and AY intersect BC again at the points P and Q. Prove that $BP = CQ$.



¶ Main Idea Project (P, Q, M, ∞) around and get insights. Also, exploit the symmetrical condition induced by the parallel condition wrt (I).

¶ Solution(projective) As always, let D, E, F be the conventional intouch points. It is well known that EF, AM, and DI concur at a point, say O. Projecting through A, we have

$$-1 = (P, Q, M, \infty) \stackrel{A}{=} (Y, X, O, R),$$

where R is the intersection of XY and the line through A parallel to BC. So it suffices to prove that $(Y, X, O, R) = -1$. However, this is evident, as, by symmetry,

$$-1 = (A, O, K, L) = (R, O, X, Y).$$

¶ Other methods Some common methods to tackle the problem is using **Trig Bashing** and **Length Chasing**. Ratio chasing works perfectly for this problem, because, in a way, projective manipulations are a consequence of them. There also exists solutions that use **Bary Bashing** and advanced projective geometry(**DDIT**)