Notes on Inequalities and Embedding

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1 Introduction

In this note, we always denote $x = (x_1, x_2, \ldots, x_n)$ to be a point in $\mathbb{R}^n$.

2 Definition

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2.1 Definitions

3 Inequalities

4 Sobolev Embedding

In this section, we deal with embeddings of diverse Sobolev spaces into others. Given a Sobolev space, it automatically belongs to certain other space, depending on the relationship between the integrability p and the dimension n. There are three cases:

$$p \in [1, n),$$
$$p = n,$$
$$p \in (n, \infty].$$

In particular, the second case $p = n$ is called the *borderline case*.

4.1 The case $p \in [1, n)$

Definition 4.1. If $p \in [1, n)$, the *Sobolev conjugate* p^* of p is defined by

$$p^* := \frac{np}{n-p}.$$

Remark 4.2. A simple calculation shows the following:

(i) $p^* > p$,

(ii) $\dfrac{1}{p^*} = \dfrac{1}{p} - \dfrac{1}{n}$.

$\Omega \subset \mathbb{R}^n$ to be open and

Theorem 4.3 (Hypothesis)**.** . *The following are equivalent:*

(i) *text*.

(ii) *text*.

(iii) *text*.

Lemma 4.4 (Hypothesis)**.** . *The following hold:*

(a) *text*

(b) *text*

(c) *text*

$$= \begin{cases} , & \text{if} \\ , & \text{if} \\ , & \text{if} \end{cases}$$

then

1. the direct scattery problem is to determine u^s from u^i;
2. the inverse scattery problem is to determine the nature of inhomogeneity to reconstruct the differential equation and/or its domain from a knowledge of the asymptotic behavior u^s.