Homework 2

Name: Kyla Jones

Your solutions must be submitted on a scanned copy of this template.

1. (7.5 points) For each of the following, determine which named discrete distribution should be used, including the appropriate parameter values and support.

   a. The manager at Eternal Youth is trying to determine how many employees need to be present when the store opens on Saturday mornings, so she is modeling the number of customers that enter the store during the first hour the store is open, Z. She knows from extensive data collection that the mean and variance for the number of customers that enter the store during the first hour on Saturday mornings are both 48.

   $Z =$ No. of customers that enter the store during the first hour the store is open

   $$\mathbb{E}(Z) = 48$$
   $$\mathbb{V}(Z) = 48$$
   $$Z \sim \text{Poisson}(\lambda = 48) \quad z = 0, 1, 2, ...$$

   b. AJ is practicing shooting free throws. On average he makes about 60% of his shots. His sister challenges him to make 3 free throws and counts the number of shots, Y, that it takes him to make them.

   $p = 0.60$

   $Y = \{$Number of trials for $r = 3$ successes to occur$\}$

   $$Y \sim \text{NegativeBinomial}(r = 3, p = 0.60) \quad y = 3, 4, ...$$

   c. Suppose a book has 200 pages and 20 of those pages contain an error. An editor will go through and randomly select 40 pages of the book to check for errors. As part of the editing process, she will count the number of pages in her sample of 40 that contain an error, X.

   $N = 200$ objects (pages)

   $r = 20$ successes (errors)

   $n = 40$ independent trials (pages)

   $X = \{$Number of successes (errors)$\}$

   $$X \sim \text{Hypergeometric}(n = 40, r = 20, N = 200), \quad x = 0, 1, ..., 20$$

   d. A submarine's probability of sinking an enemy ship with any firing of its torpedos is 0.8. Let X be the number of torpedos needed until sinking the enemy ship.

   $p = 0.8$ chance of sinking an enemy ship

   $X = \{$Number of torpedos needed until sinking the enemy ship$\}$

   $$X \sim \text{Geometric}(p = 0.8), \quad x = 1, 2, 3, ...$$

   e. A production plant produces thousands of parts per day. On average 1% of these parts will be defective. A random sample of 50 parts is taken for quality control purposes and the number of defective parts Y, is recorded. You may assume that since a random sample was taken that the indpendence assumption is met.

   $p = 0.01$ chances that one part is defective (success)

   $n = 50$ random sample (number of trials)

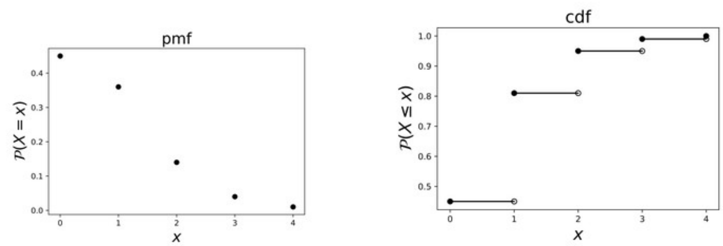
   $Y = \{$Number of defective parts (successes)$\}$

   $$Y \sim \text{Binomial}(n = 50, p = 0.01) \quad y = 0, 1, ..., 50$$

2. Let X be the number of major hurricanes that make landfall in the United States in a season. The pmf of X can be reasonably modeled by:

$$p(x) = \begin{cases} .45, & x = 0 \\ .36, & x = 1 \\ .14, & x = 2 \\ .04, & x = 3 \\ .01, & x = 4 \end{cases}$$

a. (3 points) Draw the pmf and cdf of X. Please see code at end



b. (1.5 points) What is the expected number of major hurricanes that will make landfall in a season? That is, what is E(X)?

$$\begin{aligned} \mathbb{E}(X) &= \sum_{i=0}^{4} x_i p(x_i) \\ &= (0)(0.45) + (1)(0.36) + (2)(0.14) + (3)(0.04) + (4)(0.01) \\ &= 0.8 \end{aligned}$$

c. (2 points) What is the variance for the number of major hurricanes that will make landfall in a season? That is, what is Var(X)?

$$\begin{aligned} \mathbb{E}(X^2) &= \sum_{i=0}^{4} x_i^2 p(x_i) \\ &= (0)^2(0.45) + (1)^2(0.36) + (2)^2(0.14) + (3)^2(0.04) + (4)^2(0.01) \\ &= 1.44 \end{aligned}$$

$$\begin{aligned} \mathbb{V}(X) &= \mathbb{E}(X^2) - \mathbb{E}(X)^2 \\ &= 1.44 - (0.80)^2 \\ &= 0.80 \end{aligned}$$

d. (2 points) Total cost (in billions of dollars) for hurricane relief efforts in a season, paid out by the government, can be reasonablly modeled by $C(X) = 8X + 2.8$. What are the expected value ($E[C(X)]$) and variance ($Var[C(X)]$) of the cost to the government in a season?

$$\begin{aligned} \mathbb{E}(C(X)) &= \mathbb{E}(8X + 2.8) \\ &= 8\mathbb{E}(X) + \mathbb{E}(2.8) \\ &= 8(0.8) + 2.8 \\ &= 6.4 + 2.8 \\ &= 9.2 \end{aligned}$$

$$\begin{aligned} \mathbb{E}(C^2(X)) &= \mathbb{E}(64X^2 + 44.8X + 7.84) \\ &= 64\mathbb{E}(X^2) + 44.8\mathbb{E}(X) + 7.84 \\ &= (64)(1.44) + (44.8)(0.8) + 7.84 \\ &= 135.8 \end{aligned}$$

$$\mathbb{V}(C(X)) = \mathbb{E}(C^2(X)) - \mathbb{E}(C(X))^2 = 135.8 - 9.2^2 = 51.2$$

e. (2 points) Each year the government needs to budget for hurricane relief efforts. To be sure that they set aside enough money to be effective in disaster relief, they estimate that they should budget 2 standard deviations more than the expected value. How much money should the government put aside?

$$\sigma = SD(X) = \sqrt{\mathbb{V}(X)} = 0.90$$

$$\mathbb{E}(X) + 2\sigma = 0.80 + (2)(0.90) = 2.6$$

The government should put aside enough money for 2.6 or 3 hurricanes

f. (3 points) Find the moment generating function of X, then use it to confirm the expected value found in part b

$$\begin{aligned} M_X(t) &= \mathbb{E}(e^{tX}) \\ &= \sum_{x=0}^{4} (e^{tx})p(x) \\ &= (e^{t\cdot 0})(0.45) + (e^{t\cdot 1})(0.36) + (e^{t\cdot 2})(0.14) + (e^{t\cdot 3})(0.04) + (e^{t\cdot 4})(0.01) \\ &= 0.45 + 0.36e^{t} + 0.14e^{2t} + 0.04e^{3t} + 0.01e^{4t} \end{aligned}$$

$$M_X'(t) = 0.36e^{t} + 0.28e^{2t} + 0.12e^{3t} + 0.04e^{4t}$$

$$\mathbb{E}(X) = M_X'(t=0) = 0.36 + 0.28 + 0.12 + 0.04 = 0.8$$

3. Let Y be the number of days a person stays in the hospital after having major surgery. The pmf of Y is given below.

$$P(Y = y) = \begin{cases} \dfrac{6-y}{C} & y = 1,2,3,4,5 \\ 0, & otherwise \end{cases}$$

a. (2 points) Determine the value of C that makes this a valid pmf.

This is a valid pmf if for all values of y the sum of the probabilities = 1 and all values of p(y)≥0

$$\begin{aligned} 1 &= \tfrac{6-1}{C} + \tfrac{6-2}{C} + \tfrac{6-3}{C} + \tfrac{6-4}{C} + \tfrac{6-5}{C} \\ &= \tfrac{5}{C} + \tfrac{4}{C} + \tfrac{3}{C} + \tfrac{2}{C} + \tfrac{1}{C} \\ &= \tfrac{15}{C} \\ C &= 15 \end{aligned}$$

b. (2 points) Suppose an health insurance company will pay $100 for up to three days of hospitalization and $50 per day of hospitalization thereafter. What are the expected value and standard deviation of payment for hospitaliation under this policy?

$$Y = \begin{cases} 1 & p(y) = 5/15 \\ 2 & p(y) = 4/15 \\ 3 & p(y) = 3/15 \\ 4 & p(y) = 2/15 \\ 5 & p(y) = 1/15 \end{cases}$$

$$X = \begin{cases} \$100 & p(y = \{1,2,3\}) = 12/15 \\ \$150 & p(y = 4) = 2/15 \\ \$200 & p(y = 5) = 1/15 \end{cases}$$

$$\begin{aligned} \mathbb{E}(X) &= \sum_{i=1}^{3} x_i p(x_i) \\ &= (\$100)(\tfrac{12}{15}) + (\$150)(\tfrac{2}{15}) + (\$200)(\tfrac{1}{15}) \\ &= \$80 + \$20 + \$13.33 \\ &= \$113.33 \end{aligned}$$

$$\begin{aligned} \mathbb{E}(X^2) &= \sum_{i=1}^{3} x_i^2 p(x_i) \\ &= (\$100)^2(\tfrac{12}{15}) + (\$150)^2(\tfrac{2}{15}) + (\$200)^2(\tfrac{1}{15}) \\ &= 13666.67 \end{aligned}$$

$$\begin{aligned} \mathbb{V}(X) &= \mathbb{E}(X^2) - \mathbb{E}(X)^2 \\ &= 13666.67 - (113.33)^2 \\ &= 822.98 \end{aligned}$$

$$\begin{aligned} SD(X) &= \sqrt{\mathbb{V}(X)} \\ &= \sqrt{822.98} \\ &= \$28.69 \end{aligned}$$

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import numpy as np
import matplotlib.pyplot as plt

x = np.arange(0,5)

p_x = np.array([0.45,0.36,0.14,0.04,0.01])

F_x = 0*p_x

F_x[0] = p_x[0]

for j in np.arange(0,len(p_x)-1):

    F_x[j+1] = p_x[j+1] + F_x[j]

# plot pmf
plt.figure()
plt.plot(x,p_x,'ko')
plt.ylabel(r'$\mathcal{P}(X=x)$', fontsize = 20)
plt.xlabel(r'$x$', fontsize = 20)
plt.title('pmf', fontsize = 20)
plt.xticks(x)
plt.savefig('hw2_q_2_pmf.png', dpi =1200)

# plot cdf
plt.figure()
plt.plot(x,F_x,'ko')
plt.plot(x[1:5],F_x[0:4],'ko', fillstyle = 'none')
plt.plot(x[0:2],np.array([F_x[0],F_x[0]]), 'k')
plt.plot(x[1:3],np.array([F_x[1],F_x[1]]), 'k')
plt.plot(x[2:4],np.array([F_x[2],F_x[2]]), 'k')
plt.plot(x[3:5],np.array([F_x[3],F_x[3]]), 'k')
plt.ylabel(r'$\mathcal{P}(X\leq x)$', fontsize = 20)
plt.xlabel(r'$x$', fontsize = 20)
plt.title('cdf', fontsize = 20)
plt.xticks(x)
plt.savefig('hw2_q_2_cdf.png', dpi =1200)
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