Packet 1: Basic Probability Concepts

Student Learning Outcomes:

- Define probability and statistics and how they are used in inference.
- Create Venn Diagrams to visualize sets.
- Learn basic vocabulary and properties of probability.
- Be able to use counting rules to find the number of possible outcomes of an experiment.
- Understand and utilize conditional probability rules.
- Determine if events are independent.
- Utilize the law of total probability and Bayes' Theorem.
- Understand how independence and dependence affect probability problems.

Probability and Statistics: An Overview

Probability (the course) is about creating appropriate mathematical models to quantify uncertainty in real life.

Example:

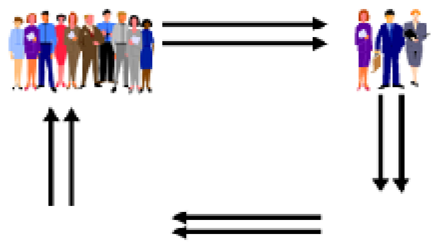
Note:

Statistics is science of making informed decisions using data.

Example:

Note:

In this class you will learn some mathematical probability models that are commonly used in statistics.



Sample Space, Events, and Probability

____________________: Any action or process whose outcome is subject to uncertainty.

Example:

Example:

Example:

____________________: The set of all possible outcomes of an experiment

Notation:

Example: (Flipping a coin)

Example: (Taking 3 free throws)

Note:

Example: (Breaking Strength of Cotton Thread)

______________________: Any collection of outcomes from the sample space.

Notation:

- ______________________: Consists of exactly one outcome.

  Example: (Rolling a die)

  Example: (Taking 3 free throws)

- ______________________: Consists of more than one outcome.

  Example: (Taking 3 free throws)

______________________ of event A: Set of all outcomes in S that are not in A.

Notation:

Example: (Taking 3 free throws)

______________________ of two events A and B: Event consisting of all outcomes that are either in A or in B or in both (i.e. in at least one event).

Notation:

Example: (Rolling a die) Let A = $\{1, 2, 3\}$ and B = $\{1, 3, 5\}$.

Example: (Taking 3 free throws) Let A and B be defined as above.

________________________ of two events A and B: Event consisting of all outcomes that are in both A and B.

Notation:

Example: (Rolling a die) Let A = $\{1, 2, 3\}$ and B = $\{1, 3, 5\}$.

Example: (Taking 3 free throws) Let A and B be defined as on the previous page.

________________________________: A and B are said to be mutually exclusive (or disjoint) if their intersection is empty.

Distributive Laws

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DeMorgan's Law

- $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
- $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

______________________: A precise measure of the chance that a particular event will occur

Properties of Probability:

1.

2.

3.

Note: Together, these imply that property (3) also holds for a finite collection of disjoint events.

Propositions:

1. For any event A:

2. For any event A:

3. For any two events A and B:

Counting Rules

Counting rules are ways for us to be able to determine the total number of possible outcomes in an experiment. Some counting rules include the *multiplication principle, combinations* and *permutations*.

Multiplication Rule

Suppose an experiment has "a" different outcomes and for each of these outcomes, a second experiment has "b" different outcomes. The combined set of the two experiments has _______ possible outcomes.

> *Example:* If you can choose one of 3 ice cream flavors: chocolate, strawberry, and vanilla, and one of 4 toppings: caramel, nuts, cookies, or candy, how many different choices do you have?

> *Example:* If you can choose between putting your ice cream and topping in a cup or a cone, how many different choices do you have?

Combinations

Another useful way to count the number of experimental outcomes is the counting rule for combinations. This rule applies when we are sampling ______________________ from a larger group ______________________ and *the order in which items are selected does not matter*!

The number of combinations of n objects taken r at a time is: $$nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$ where $n! = n(n-1)(n-2)...(2)(1)$

Example: Our class has _____ students enrolled. Suppose I need two students to aid in a demonstration. How many possible pairs of students could be chosen from our class?

Example: Suppose you draw 5 cards at random from a standard deck of 52 cards that has been well-shuffled. How many possible sets of 5 cards exist?

Permutations

Note that when counting outcomes with combinations, the order of the sampled objects did not matter. If *the order of the objects matters*, then the counting rule for permutations is used.

The number of permutations of n objects taken r at a time is given by $$nPr = r!\binom{n}{r} = \frac{n!}{(n-r)!}$$

Example: How many ways are there to select a president, a vice president, a secretary and a treasurer from a club of 10 members? Which counting rule was used for this example and why?

Example: What is the probability of getting a flush in a 5 card hand?

Conditional Probability

Conditional probability is the probability of an event given that we know that another event has already occurred. The given information _______________ the possible number of events in the sample space.

Notation:

Example: Consider an experiment where three names are drawn from a hat. Let A = your name is first and B = your name is one of the three.

Example: Now suppose the experiment is a drug test for a randomly selected individual. Let A = person used the drug and B = person tests positive for the drug.

______________________________________: For any two events A and B with $P(B) > 0$, the conditional probability of A given that B has occurred is:

Example: (Drug Test)

	Test+	Test-	Total
Drug	0.02	0.01	
No Drug	0.01	0.96	
Total			

What is the probability that an individual will test positive given that they have used the drug? (True Positive)

What is the probability that an individual will test negative given that they have not used the drug? (True Negative)

__________________________________: Solving the conditional probability equation for $P(A \cap B)$ gives:

Example: Suppose that when you play a video game for the first time, you have a 0.3 probability of winning. Your probability of winning is higher when you play for a second time, but how much higher depends on your first result. If you win on the first game, then your probability of winning on the second game increases to 0.8. But if you lose on the first game, then your probability of winning on the second game increases only to 0.4.

a. Draw a tree diagram to represent this scenario.

b. What is the probability that you win on both of your first two plays?

c. What is the probability that you win on your second play of the game, regardless of whether or not you won on your first play?

The previous example is an illustration of the ____:

Let $A_1, ., A_k$ be mutually exclusive and exhaustive events. Then for any other event B,

$P(B) =$

This law allows us to calculate probabilities of events by conditioning on other events. Sometimes these conditional probabilities are easier to calculate, meaning it may be easier to find $P(B|A)$ than $P(B)$. Similarly, there are times when it may be easier to calculate $P(B|A)$ than $P(A|B)$. Fortunately, there is another "law" that can be used to help in this situation.

____________________________________: Let $A_1, ., A_k$ be a collection of k mutually exclusive and exhaustive events with prior probabilities $P(A_i)$, $(i = 1, .k)$. Then for any other event B for which the $P(B) > 0$, the posterior probability of A_j given that B has occurred is:

$P(A_j|B) =$

Example: (Video Games) Given that you won on your second play of the game, what is the probability that you won on your first play?

Given that you lost on your second play of the game, what is the probability that you won on your first play?

Example: (Drug test) Drug tests are designed with a known sensitivity. However, those interested in such tests want to know the probability that the person tested actually used the drug given that the test was positive. Using appropriate notation, write out and solve for the probability of interest. Assume the accuracy of the test is 99% and that 0.5% of the population uses the drug.

Independence

Conditional probability tells us about the relationships between events. If $P(A|B)$ is very different than $P(A)$, then knowing B tells us a lot about A. But what if $P(A|B)$ is not very different than $P(A)$? Then knowing B does not give much information about A.

- Two events A and B, are ________________________________ if:

- Otherwise the events are ________________________________.

  Example: Let A = tomorrow's high temperature in Notre Dame and B = tomorrow's high temperature in Paris.

  A and B are ___________________________________ since...

  Let C = tomorrow's high temperature in South Bend. A and C are ___________________________________ since...

Proposition: A and B are independent if and only if:

Example: (Video Game) We could represent the probabilities from the video game example in the table below. Use this to determine if the outcomes of the two turns are independent.

	Win 2nd Turn	Lose 2nd Turn	Total
Win 1st Turn	0.24	0.06	0.3
Lose 1nd Turn	0.28	0.42	0.7
Total	0.52	0.48	1.00

But what if these were the probabilities:

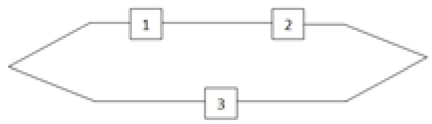
	Win 2nd Turn	Lose 2nd Turn	Total
Win 1st Turn			
Lose 1nd Turn			
Total			

Events $A_1, ., A_n$ are mutually independent if:

$P(A_1 \cap A_2 \cap ... \cap A_k) =$

Example: A multiple choice test has 20 questions, each with four possibilities. If a student has an 80% chance of getting each question right, and the questions are independent, what is the probability that student will get a perfect score? Let A_j = student's answer is right on question j.

Example: Consider the system of components in the figure. Components 1 and 2 are connected in series, so that the subsystem works only if both 1 and 2 work. Component 3 is connected in parallel with this subsystem, so the system works if either the subsystem works or if component 3 works. Assume that the components work independently of each other and the probability a component works is 0.9.



a. What is the probability that all three components work?

b. What is the probability that the system works?