Homework 1

Name: Kyla Jones

Your solutions must be submitted on a scanned copy of this template.

1. **(3 points) Sketch the regions in the xy-plane corresponding to $A \cup B$ and $A \cap B$ if:**

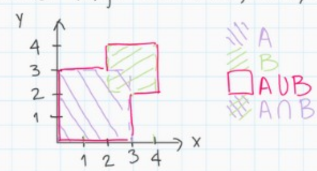
$$A = (x, y): 0 < x < 3, 0 < y < 3$$
$$B = (x, y): 2 < x < 4, 2 < y < 4$$

A: (x,y) : 0 < x < 3, 0<y<3

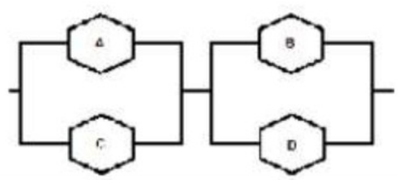
B: (x,y) : 2<x <4, 2<y<4

A∪B : (x,y): 0<x<4, 0<y<4

A∩B : (x,y): 2<x<3, 2<y<3



2. **(3 points) An electronic system has four components divided into two pairs. The two components of each pair are wired in parallel; the two pairs wired in series (see picture). Suppose that the probability each component works is 0.95, and all components work independently of the other components. What is the probability that the system works? Note: if you are unfamiliar with these terms, think of it as "the electricity needs an open path from left to right" in the picture below. A would mean that the electricity can pass through that node and $\bar{A}$ means it cannot.**



The probability that the system works (event W)

$\mathbb{P}(W) = \mathbb{P}((A \cup C) \cap (B \cup D))$

$= \mathbb{P}(A \cup C)\mathbb{P}(B \cup D)$ because the events are disjoint

$\mathbb{P}(A \cup C) = \mathbb{P}(A) + \mathbb{P}(C) - \mathbb{P}(A \cap C)$

$= 2 \cdot 0.95 - 0.95^2$

$= 0.9975$

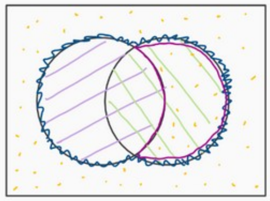
$\mathbb{P}(A \cup D) = \mathbb{P}(A \cup C)$

$= 0.9975$

$\mathbb{P}(W) = 0.9975^2$

$= 0.995$

3. **(3 points) Express $P\left(\overline{A} \cap (A \cup B)\right)$ in terms of $P(A)$, $P(B)$, and $P(A \cap B)$ only. A Venn Diagram may be helpful in solving. Note: you may not need to use all of the terms.**



A
B
$A \cap B$
$A \cup B$ (area enclosed)
$\bar{A}$
$\bar{A} \cap (A \cup B)$ (area enclosed)

$$\mathbb{P}(\bar{A} \cap (A \cup B)) = \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

4. **(3 points) Suppose that two fair dice are tossed. What is the probability that the sum equals 10 given that it exceeds 8? Hint: draw a table that shows all possible outcomes for the sum of two dice.**

4) All possible outcomes for the sum of 2 die:

Event, E_i	Members of E_i	$\lvert E_i \rvert$
2	{1,1}	1
3	{1,2},{2,1}	2
4	{1,3},{3,1},{2,2}	3
5	{1,4},{4,1},{2,3},{3,2}	4
6	{1,5},{5,1},{2,4},{4,2},{3,3}	5
7	{1,6},{6,1},{2,5},{5,2},{3,4},{4,3}	6
8	{2,6},{6,2},{3,5},{5,3},{4,4}	5
9	{3,6},{6,3},{4,5},{5,4}	4
10	{5,5},{6,4},{4,6}	3
11	{5,6},{6,5}	2
12	{6,6}	+ 1
	Cardinality of sample space, $\lvert\Omega\rvert$ =	36

$\mathbb{P}(E_{10} \mid (E_9 \cup E_{10} \cup E_{11} \cup E_{12})) = ?$

let $A = \bigcup_{i=9}^{12} E_i$

$\mathbb{P}(E_{10} \mid A) = \dfrac{\mathbb{P}(E_{10} \cap A)}{\mathbb{P}(A)}$

$$\begin{aligned}\mathbb{P}(A) &= \mathbb{P}(E_9 \cup E_{10} \cup E_{11} \cup E_{12}) \\ &= \mathbb{P}(E_9) + \mathbb{P}(E_{10}) + \mathbb{P}(E_{11}) + \mathbb{P}(E_{12}) \\ &= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} \\ &= \frac{10}{36}\end{aligned}$$

$$\begin{aligned}\mathbb{P}(E_{10} \cap A) &= \mathbb{P}(E_{10} \cap (E_9 \cup E_{10} \cup E_{11} \cup E_{12})) \\ &= \mathbb{P}(E_{10}) \\ &= \frac{3}{36}\end{aligned}$$

$\mathbb{P}(E_{10} \mid A) = \left(\frac{3}{36}\right)\left(\frac{36}{10}\right) = \frac{3}{10}$

5. **(4 points) In a roll of a pair of fair six-sided dice (one red and one green), let A be the event the red die shows a 3, 4 or 5; let B be the event the green die shows a 1 or 2; and let C be the event the dice total is 7. Show that A, B and C are independent.**

green die

red die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

sum of rolls
sample space Ω
event A
event B
event C

$\mathbb{P}(A) = \frac{3}{6} = \frac{1}{2}$

$\mathbb{P}(B) = \frac{2}{6} = \frac{1}{3}$

$\mathbb{P}(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{6}{36} = \frac{1}{6}$

$\mathbb{P}(C) = \frac{|C|}{|\Omega|} = \frac{6}{36} = \frac{1}{6}$

$\mathbb{P}((A \cap B) \mid C) = \frac{1}{6}$

Events A and B are independent if

$$\begin{aligned}\mathbb{P}(A \cap B) &= \mathbb{P}(A \mid B)\mathbb{P}(B) = \mathbb{P}(A)\mathbb{P}(B) \\ &= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{3}\right) \\ &= \tfrac{1}{6}\end{aligned}$$

∴ Events A and B are independent

$$\begin{aligned}\mathbb{P}((A \cap B) \cap C) &= \mathbb{P}((A \cap B) \mid C)\mathbb{P}(C) \\ &= \left(\tfrac{1}{6}\right)\left(\tfrac{1}{6}\right) \\ &= \tfrac{1}{36}\end{aligned}$$

If A, B, and C are independant, then

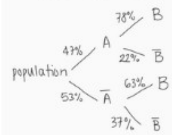
$$\begin{aligned}\mathbb{P}(A \cap B \cap C) &= \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C) \\ &= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{3}\right)\left(\tfrac{1}{6}\right) \\ &= \tfrac{1}{36}\end{aligned}$$

∴ A, B, and C are independent events

6. **(4 points) If men constitute 47% of the population and tell the truth 78% of the time, while women tell the truth 63% of the time, what is the probability that a person selected at random will answer a question truthfully?**

A - person is a woman

B = person tells truth



$$
\begin{aligned}
\mathbb{P}(B) &= \mathbb{P}((A \wedge B) \vee (\bar{A} \wedge B)) \\
&= \mathbb{P}((A \cap B) \cup (\bar{A} \cap B)) \\
&= \mathbb{P}(A \cap B) + \mathbb{P}(\bar{A} \cap B) \text{ because paths are disjoint} \\
&= \mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|\bar{A})\mathbb{P}(\bar{A}) \\
&= (0.78)(0.47) + (0.63)(0.53) \\
&= 0.70
\end{aligned}
$$

The probability that a person selected at random is telling the truth is ~ 70%

7. **(5 points) The crew of the Starship *Enterprise* is considering launching a surprise attack against the Borg in a neutral quadrant. Possible interference by the Klingons though, is causing Captain Picard and Data to reassess their strategy. According to Data's calculations, the probability of the Klingons joining forces with the Borg is 0.2384. Captain Picard feels that the probability of the attack being successful is 0.8 if the Enterprise can catch the Borg alone, but only 0.3 if they have to engage both adversaries. Data claims that the mission would be a tactical misadventure if its probability of success were not at least 0.7306. Should the *Enterprise* attack?**

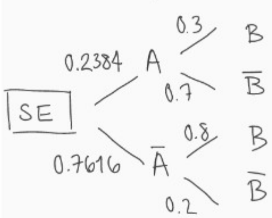
$\mathbb{P}$(Klingons join forces w/ Borg) = 0.2384

$\mathbb{P}$(successful attack w/o Klingons) = 0.8

$\mathbb{P}$(successful attack w/ Klingons) = 0.3

A = Klingons join forces w/ Borg

B = successful attack



$$
\begin{aligned}
\mathbb{P}(B) &= \mathbb{P}((A \wedge B) \vee (\bar{A} \wedge B)) \\
&= \mathbb{P}((A \cap B) \cup (\bar{A} \cap B)) \\
&= \mathbb{P}(A \cap B) + \mathbb{P}(\bar{A} \cap B) \text{ because paths are disjoint} \\
&= \mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|\bar{A})\mathbb{P}(\bar{A}) \\
&= (0.3)(0.2384) + (0.8)(0.7616) \\
&= 0.6808
\end{aligned}
$$

The probability of success is 68.08% so by Data's recommendation, the Starship Enterprise should not attack.