Pre-Lab 2: Second Order Circuits

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ECEN 325 -501
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Calculations:

1. **Solution**:

$$H_{LP}(s) = \frac{V_{LP}}{V_i}(s) \qquad H_{HP}(s) = \frac{V_{HP}}{V_i}(s) \qquad H_{BP}(s) = \frac{V_{BP}}{V_i}(s)$$

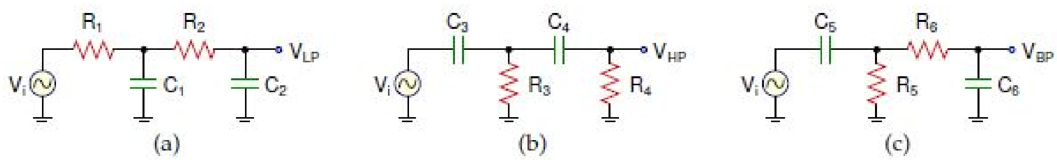
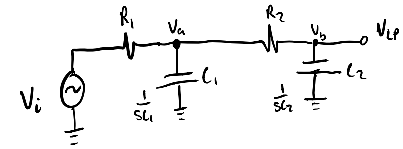


Figure 1: Second order (a) lowpass filter (b) highpass filter (c) bandpass filter

1.

a)



Using velocity division

$$V_a = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} V_i \quad -(1)$$

$$V_{LP} = V_b = \frac{\frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} V_a \quad -(2)$$

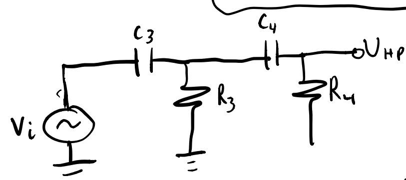
from (1) & (2)

$$V_{LP} = \frac{\frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} \times \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} V_i$$

$$\frac{V_{LP}(s)}{V_i(s)} = H_{LP}(s) = \frac{1}{1 + sR_2C_2} \times \frac{1}{1 + sR_1C_1}$$

$$\boxed{H_{LP}(s) = \frac{\frac{1}{R_1R_2C_1C_2}}{\left(s + \frac{1}{R_1C_1}\right)\left(s + \frac{1}{R_2C_2}\right)}}$$

b)



Using Voltage division

$$V_c = \frac{R_3}{R_3 + \frac{1}{sC_3}} V_i(s) \quad -(3)$$

$$V_{HP} = \frac{R_4}{R_4 + \frac{1}{sC_4}} V_c \quad -(4)$$

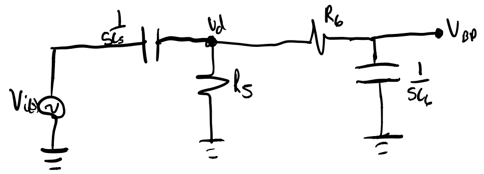
from (3) & (4)

$$V_{HP} = \frac{R_4}{R_4 + \frac{1}{sC_4}} \times \frac{R_3}{R_3 + \frac{1}{sC_3}} V_i(s)$$

$$\frac{V_{HP}(s)}{V_i(s)} = H_{HP}(s) = \frac{R_3R_4}{\left(R_4 + \frac{1}{sC_4}\right)\left(R_3 + \frac{1}{sC_3}\right)}$$

$$\boxed{H_{HP}(s) = \frac{s^2}{\left(s + \frac{1}{R_3C_3}\right)\left(s + \frac{1}{R_4C_4}\right)}}$$

c)



using Voltage division

$$V_d = \frac{R_5}{R_5 + \frac{1}{sC_5}} V_i(s) \quad — (5)$$

$$V_{BP}(s) = \frac{\frac{1}{sC_6}}{R_6 + \frac{1}{sC_6}} V_d \quad — (6)$$

from (5)+(6)

$$V_{BP}(s) = \frac{1}{sR_6C_6+1} \times \frac{R_5}{R_5 + \frac{1}{sC_5}} V_i(s)$$

$$\frac{V_{BP}(s)}{V_i(s)} = \frac{s}{R_6C_6\left(s+\frac{1}{R_6C_6}\right)\left(s+\frac{1}{R_5C_5}\right)}$$

$$\boxed{H_{BP}(s) = \frac{\frac{s}{R_6C_6}}{\left(s+\frac{1}{R_6C_6}\right)\left(s+\frac{1}{R_5C_5}\right)}}$$

2.

$$H_{LP}(s) = \frac{\frac{1}{R_1R_2C_1C_2}}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}$$

$$H_{LP}(s) = \frac{1}{1+\frac{s}{\frac{1}{R_1C_1}}} \times \frac{1}{1+\frac{s}{\frac{1}{R_2C_2}}}$$

$$\therefore H_{LP}(s) = \frac{1}{1+\frac{s}{2\pi f_1}} \times \frac{1}{1+\frac{s}{2\pi f_2}}$$

compare the eq. $\therefore$ (above)

$$2\pi f_1 = \frac{1}{R_1C_1} \quad \& \quad 2\pi f_2 = \frac{1}{R_2C_2}$$

$$\therefore f_1 = 1\,kHz \quad , \quad f_2 = 10\,kHz$$

$$2\pi = \frac{1}{R_1C_1} \quad , \quad 20\pi = \frac{1}{R_2C_2}$$

$$\frac{20\pi}{2\pi} = \frac{R_1C_1}{R_2C_2}$$

$$\frac{R_1C_1}{R_2C_2} = 10$$

$$\boxed{\frac{R_1}{R_2} = 10\frac{C_2}{C_1}}$$

Let $C = .001\,\mu F = 10^{-9}$

$\therefore\ 2\pi f_1 = \frac{1}{R_1 C_1}$

$2\pi \times 1 \times 10^3 = \frac{1}{R_1 \times .1 \times 10^{-9}}$

$R_1 = \frac{1}{2\pi \times 1 \times 10^3 \times 10^{-9}}$

$R_1 = .0159 \times 10^6\,\Omega$

$\boxed{R_1 = 15.9\,k\Omega}$

$C_2 = .002\,\mu F$

$\therefore\ \frac{R_1}{R_2} = 10\frac{C_2}{C_1}$

$R_2 = \frac{R_1 C_1}{10 C_2} = \frac{15.9 \times .001}{10 \times .002}$

$\boxed{R_2 = 1.59\,k\Omega}$

$H_{HP}(s) = \frac{s}{s + 2\pi f_3} \times \frac{s}{s + 2\pi f_4}$

$H_{HP}(s) = \frac{s}{s + \frac{1}{R_3 C_3}} \times \frac{s}{s + \frac{1}{R_4 C_4}}$

compare

$2\pi f_3 = \frac{1}{R_3 C_3}$ & $2\pi f_4 = \frac{1}{R_4 C_4}$

$\therefore\ f_3 = 1\,kHz$, $f_4 = 10\,kHz$

Let $C_3 = .001\,\mu F$ & $C_4 = .002\,\mu F$

then $\boxed{R_3 = 15.9\,k\Omega}$ & $\boxed{R_4 = 1.59\,k\Omega}$ which is the same as $H_{LP}(s)$

$H_{BP}(s) = \frac{s}{s + 2\pi f_5} \times \frac{1}{1 + \frac{s}{2\pi f_6}}$

$\therefore\ f_5 = 1\,kHz$, $f_6 = 100\,kHz$

Let $C_5 = .002\,\mu F$

$R_5 = \frac{1}{2\pi \times 1 \times 10^3 \times .002\,\mu F}$

$\boxed{R_5 = 1.59\,k\Omega}$

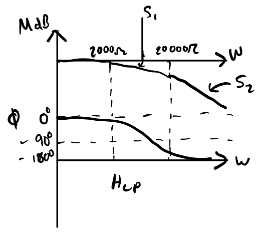
Let $C_6 = .004\,\mu F$

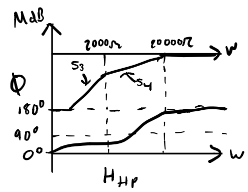
$R_6 = \frac{1}{2\pi C_6 f_6}$

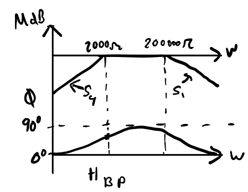
$R_6 = \frac{1}{2\pi \times .004 \times 10^{-6} \times 100 \times 10^3}$

$\boxed{R_6 = .0397\,k\Omega}$

3. Bode Plots







$S_1 = -20\,dB/dec$ $\quad S_2 = -40\,dB/dec$

$S_3 = -40\,dB/dec$ $\quad S_4 = 20\,dB/dec$

4. For calc. the output Voltage @ $w = 2\pi \times 10{,}000 = 20{,}000\pi$, we need to find the magnitude and Phase of each system.

i)

$$|H(j\omega)|_{LP_1} = \frac{1}{\sqrt{1^2 + \left(\frac{10000}{10000}\right)^2}} = .71, \quad \phi_1 = -\tan^{-1}\left(\frac{10}{10}\right) = -.79$$

$$|H(j\omega)|_{LP_2} = \frac{1}{\sqrt{1^2 + \left(\frac{10000}{1000}\right)^2}} = .1, \quad \phi_2 = -\tan^{-1}\left(\frac{10}{1}\right) = -1.47$$

$\therefore\ H(j\omega)_{LP} = .71 \times .1 = \boxed{.071}$

$\phi_{LP} = \phi_1 + \phi_2 = -.79 - 1.47 = \boxed{-2.26}$

$\therefore\ V_{LP}(t) = .071 \times .5 \sin(2\pi 10000t - 2.26)$

$\boxed{= .039 \sin(2\pi 10000t - 2.26)}$

ii)

$$|H(j\omega)|_{HP} = \frac{\frac{10000}{1000}}{\sqrt{1^2 + \left(\frac{10000}{1000}\right)^2}} \times \frac{\frac{10000}{10000}}{\sqrt{1^2 + \left(\frac{10000}{10000}\right)^2}} = .995 \times .71 = \boxed{.7}$$

$\phi_{HP} = \frac{\pi}{2} - \tan^{-1}\left(\frac{10}{1}\right) + \frac{\pi}{2} - \tan^{-1}\left(\frac{10}{10}\right) = \boxed{.89}$

$\therefore\ V_{HP}(t) = .7 \times .5 \sin(2\pi 10000t + .89)$

$\boxed{= .35 \sin(2\pi 10000t + .89)}$

iii)

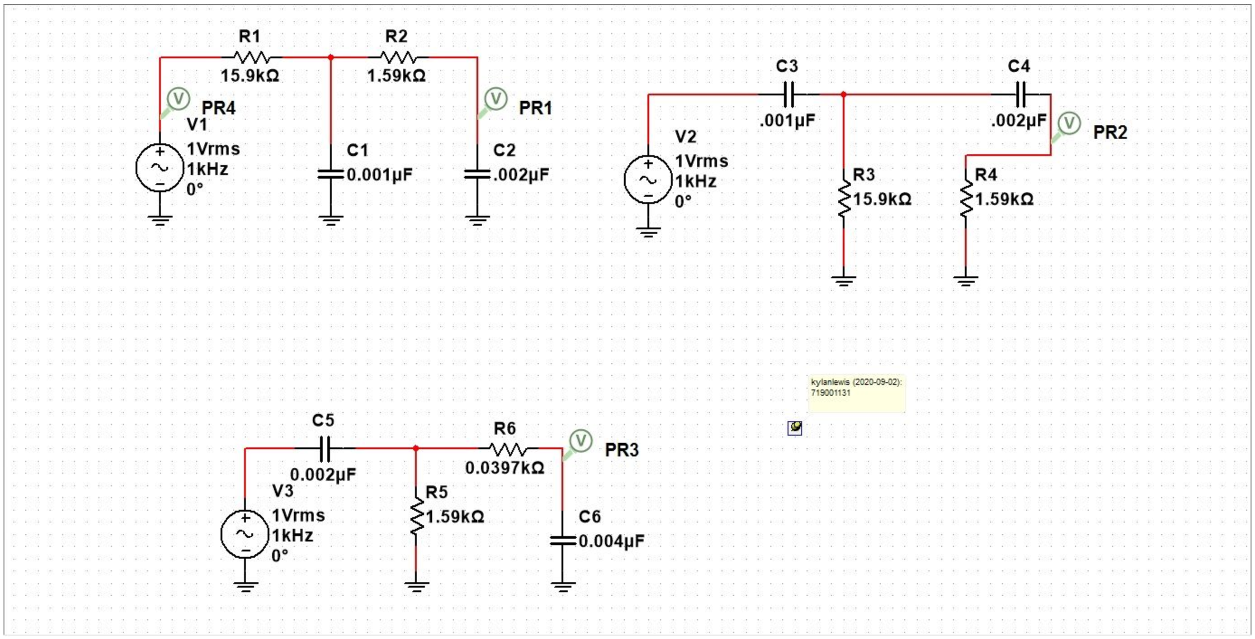
$$|H(j\omega)|_{BP} = \frac{\frac{10000}{1000}}{\sqrt{1 + \left(\frac{10000}{1000}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10000}{10000}\right)^2}} = \boxed{.7}$$

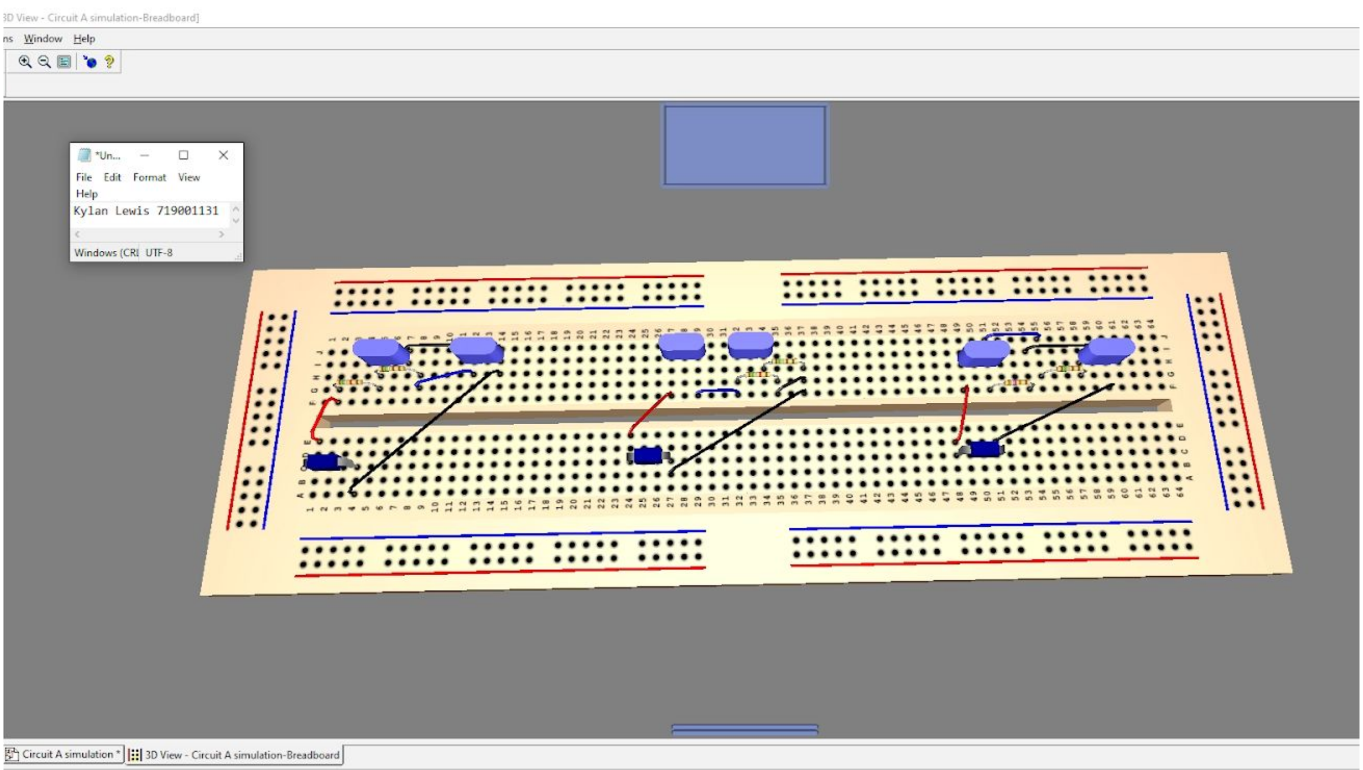
$\phi_{BP} = \frac{\pi}{2} - \tan^{-1}\left(\frac{10}{1}\right) - \tan^{-1}\left(\frac{10}{10}\right) = \boxed{-.68}$

$\therefore\ V_{BP}(t) = .7 \times .5 \sin(2\pi 10000 - .68)$

$\boxed{= .35 \sin(2\pi 10000 - .68)}$

Multisim:


R1
15.9kΩ
R2
1.59kΩ
PR4
V1
1Vrms
1kHz
0°
C1
0.001µF
C2
.002µF
PR1
C3
.001µF
C4
.002µF
PR2
V2
1Vrms
1kHz
0°
R3
15.9kΩ
R4
1.59kΩ
C5
0.002µF
V3
1Vrms
1kHz
0°
R5
1.59kΩ
R6
0.0397kΩ
PR3
C6
0.004µF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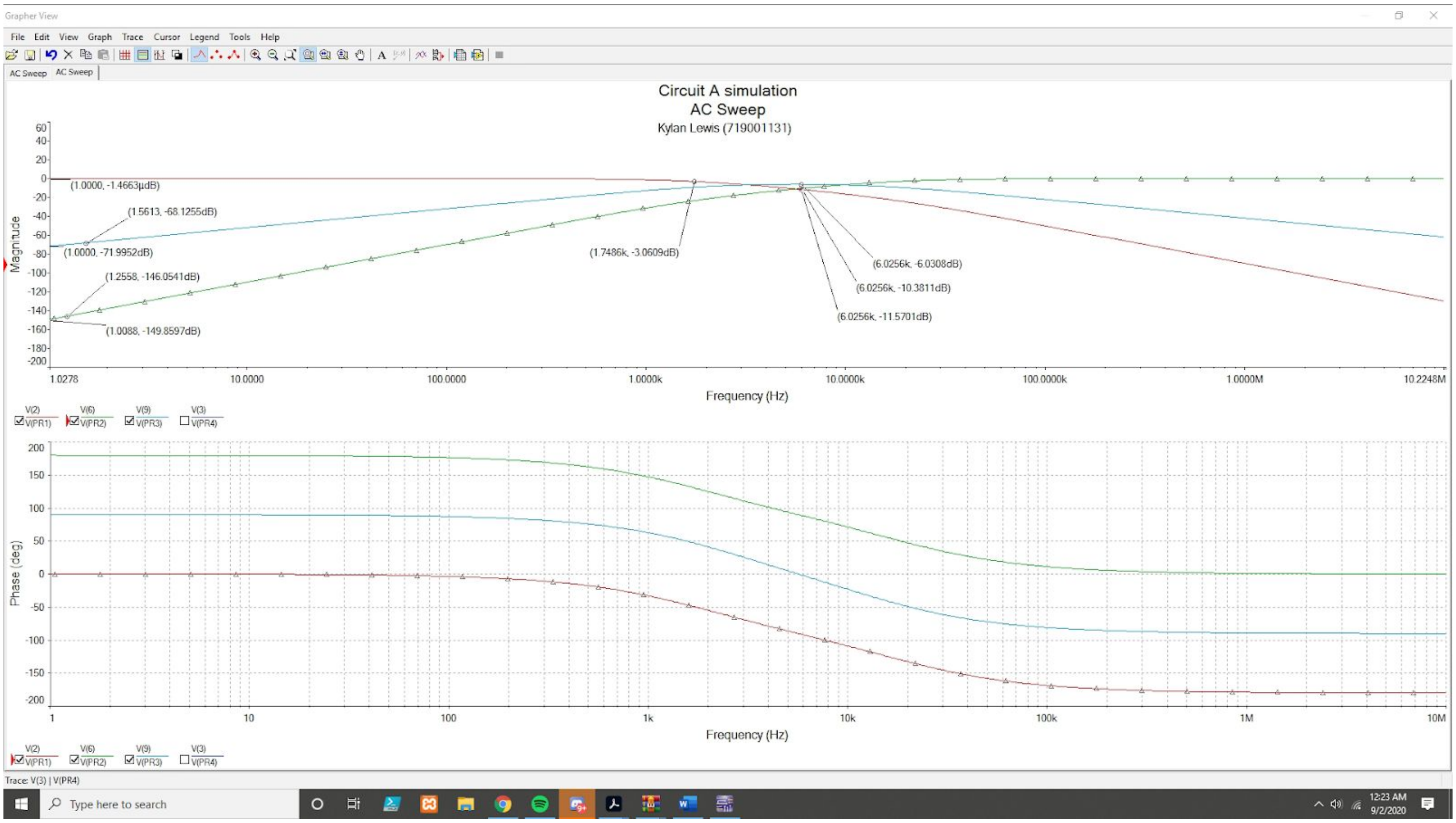
1. Bode Plots of the transfer function



2. Input $V_I(t) = 0.5 * sin(2\pi * 10000t)$