



**ELECTRICAL & COMPUTER
ENGINEERING**
TEXAS A&M UNIVERSITY

Pre-Lab 1: First Order Circuits

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ECEN 325 -501

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Calculations:

1. Solution:

$$H_{LP}(s) = \frac{V_{LP}}{V_i}(s) = K_L \frac{1}{1 + \frac{s}{\omega_L}}$$

$$H_{HP}(s) = \frac{V_{HP}}{V_i}(s) = K_H \frac{s}{s + \omega_H}$$

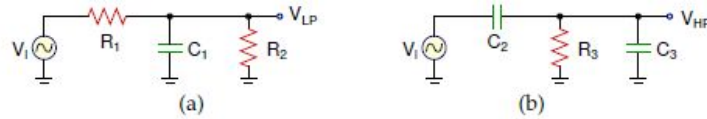


Figure 3: First order (a) lowpass filter (b) highpass filter

A capacitor in the s-domain (Laplace Transform) = $\frac{1}{sC}$

Figure (a): First we take the parallel combination of C_1 & R_2

$$C_1 \parallel R_2 = \frac{\frac{1}{sC_1} \cdot R_2}{\frac{1}{sC_1} + R_2} = \frac{R_2}{1 + sC_1R_2}$$

$$\text{Voltage Division: } V_{LP} = \frac{\left(\frac{1}{sC_1} \parallel R_2\right) \cdot V_i}{\left(\frac{1}{sC_1} \parallel R_2\right) + R_1}$$

$$\begin{aligned} H_{LP}(s) &= \frac{V_{LP}}{V_i} = \frac{\left(\frac{1}{sC_1} \parallel R_2\right)}{\left(\frac{1}{sC_1} \parallel R_2\right) + R_1} \\ &= \frac{\frac{R_2}{1 + sC_1R_2}}{\frac{R_2}{1 + sC_1R_2} + R_1} = \frac{R_2}{R_2 + R_1(1 + sC_1R_2)} = \frac{R_2}{R_2 + R_1 + sC_1R_1R_2} \end{aligned}$$

$$H_{LP}(s) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{1 + \frac{R_1 \cdot R_2 \cdot s C_1}{R_1 + R_2}}, \quad \left(H_{LP}(S) = K_L \cdot \frac{1}{1 + \frac{s}{\omega_L}} \right)$$

$$K_L = \frac{R_2}{R_1 + R_2}$$

$$\omega_L = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

Figure (b): Taking a KCL at V_{HP} .

$$\frac{V_{HP}(s) - V_i(s)}{\frac{1}{sC_2}} + \frac{V_{HP}(s)}{\frac{1}{sC_3}} + \frac{V_{HP}(s)}{R_3} = 0,$$

$$\frac{R_3 \cdot s(C_2 + C_3) + 1}{R_3} \cdot V_{HP}(s) = C_2 \cdot sV_i(s)$$

$$H_{HP}(s) = \frac{V_{HP}(s)}{V_i(s)} = \frac{R_3 \cdot sC_2}{R_3 \cdot s(C_2 + C_3) + 1}$$

$$= \frac{C_2}{C_2 + C_3} \cdot \frac{s}{s + \frac{1}{R_3(C_2 + C_3)}}$$

$$K_H = \frac{C_2}{C_2 + C_3}$$

$$\omega_H = \frac{1}{R_3(C_2 + C_3)}$$

2. Solution

$$f_L = f_L = 5kHz$$

$$K_L = K_H = 0.5$$

$$K_L = 0.5 = \frac{R_2}{R_1 + R_2},$$

$$0.5 R_1 + 0.5 R_2 = R_2,$$

$$0.5 R_1 = 0.5 R_2,$$

$$R_1 = R_2$$

$$f_L = \frac{\omega_L}{2\pi} = 5kHz,$$

$$\omega_L = \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C_1},$$

$$\frac{\frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C_1}}{2\pi} = 5kHz,$$

$$5kHz \cdot 2\pi = \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C_1}, \left(\& R_1 = R_2 \right)$$

$$\frac{1}{R \cdot C_1} = 5kHz \cdot \pi,$$

$$RC_1 = 63.66 \cdot 10^{-6}, \text{ assuming } C_1 \text{ to be } 1\mu F \left(10^{-6} F \right)$$

$$R_1 = 63.66 \Omega$$

Computed values are:

$$C_1 = C_2 = C_3 = 1\mu F$$

$$R_1 = R_2 = 63.66\Omega$$

$$R_3 = 15.915 \Omega$$

3. Solution:

$$K_H = 0.5,$$

$$\frac{C_2}{C_2 + C_3} = 0.5,$$

$$\dots C_2 = C_3$$

$$f_H = \frac{\omega_H}{2\pi} = 5kHz,$$

$$\omega_H = \frac{1}{R_3(C_2 + C_3)},$$

$$\frac{\frac{1}{R_3(C_2 + C_3)}}{2\pi} = 5kHz, \left(\& C_2 = C_3 \right)$$

$$\frac{1}{2 \cdot R_3 C_2} = 5kHz \cdot 2\pi,$$

$$R_3 C = \text{assuming } C_1 \text{ to be } 1\mu F \left(10^{-6} F \right)$$

$$R_3 = 15.915 \Omega$$

$$H_{LP}(s) = \frac{63.66\Omega}{63.66\Omega + 63.66\Omega} \cdot \frac{1}{1 + \frac{s}{\frac{63.66\Omega + 63.66\Omega}{(63.66\Omega)(63.66\Omega)(10^{-6})}}}$$

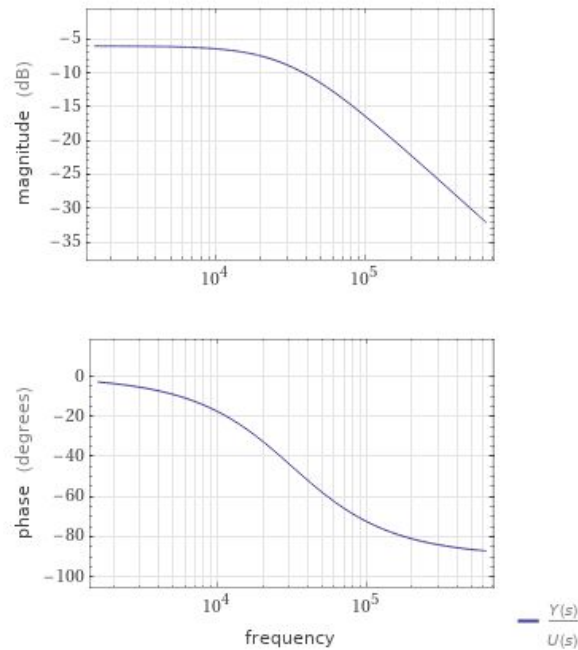
$$= \frac{0.5}{1 + 3.18 \cdot 10^{-5} \cdot s}$$

Input interpretation:

Bode plot

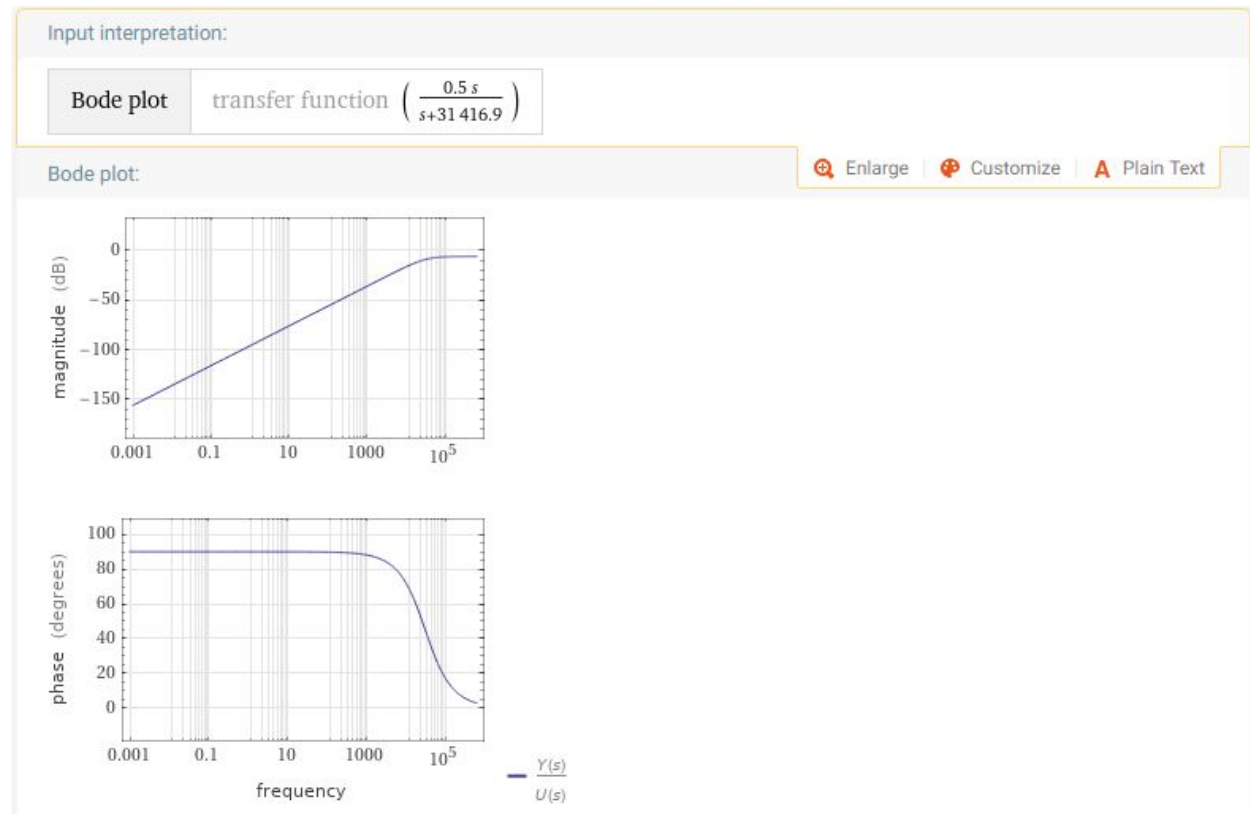
transfer function $\left(\frac{0.5}{1 + 3.18 \cdot 10^{-5} s} \right)$

Bode plot:

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$$H_{HP}(s) = \frac{10^{-6}}{10^{-6} + 10^{-6}} \cdot \frac{s}{s + \frac{1}{15.915 \cdot (10^{-6} + 10^{-6})}}$$

$$= \frac{0.5s}{s + 31416.9}$$



4. Solution

$$\frac{V_{LP}(s)}{V_i(s)} = \frac{15708.5}{s + 31416.9}, \quad s = j\omega$$

$$V_i(t) = 0.4\sin(2\pi 4000t); \quad \omega = 2\pi \cdot 4000$$

$$V_{LP}(j\omega) = \frac{15708.5}{j(2\pi \cdot 4000) + 31416.9} V_i(j\omega)$$

$$V_{LP}(t) = (0.39044\angle -38.6589) \cdot (0.4\angle 0) =$$

$$V_{LP}(t) = 0.156\sin(2\pi \cdot 4000t - 38.6589) \text{ V}$$

$$\frac{V_{HP}(s)}{V_i(s)} = \frac{0.5s}{s + 31416.9}, \quad s = j\omega$$

$$V_{HP}(j\omega) = \frac{j0.5(2\pi 4000)}{j(2\pi 4000) + 31416.9} V_i(j\omega)$$

$$V_{HP}(t) = (0.312\angle 51.34) (0.4\angle 0)$$

$$V_{HP}(t) = 0.125\sin(2\pi 4000t + 51.34) \text{ V}$$

(* Note that the solution for the low-pass and high-pass output voltages should have a degree symbol in the parentheses to indicate phase shift)

5. Solution

$$V_{LP}(j\omega) = \frac{15708.5}{j(2\pi \cdot 6000) + 31416.9} \cdot V_i(j\omega),$$

$$V_i(t) = 0.3\sin(2\pi \cdot 6000t) = 0.3\angle 0$$

$$V_{LP}(t) = (0.32\angle -50.1936) (0.3\angle 0)$$

$$V_{LP}(t) = 0.96\sin(2\pi \cdot 6000t - 50.1936^\circ)$$

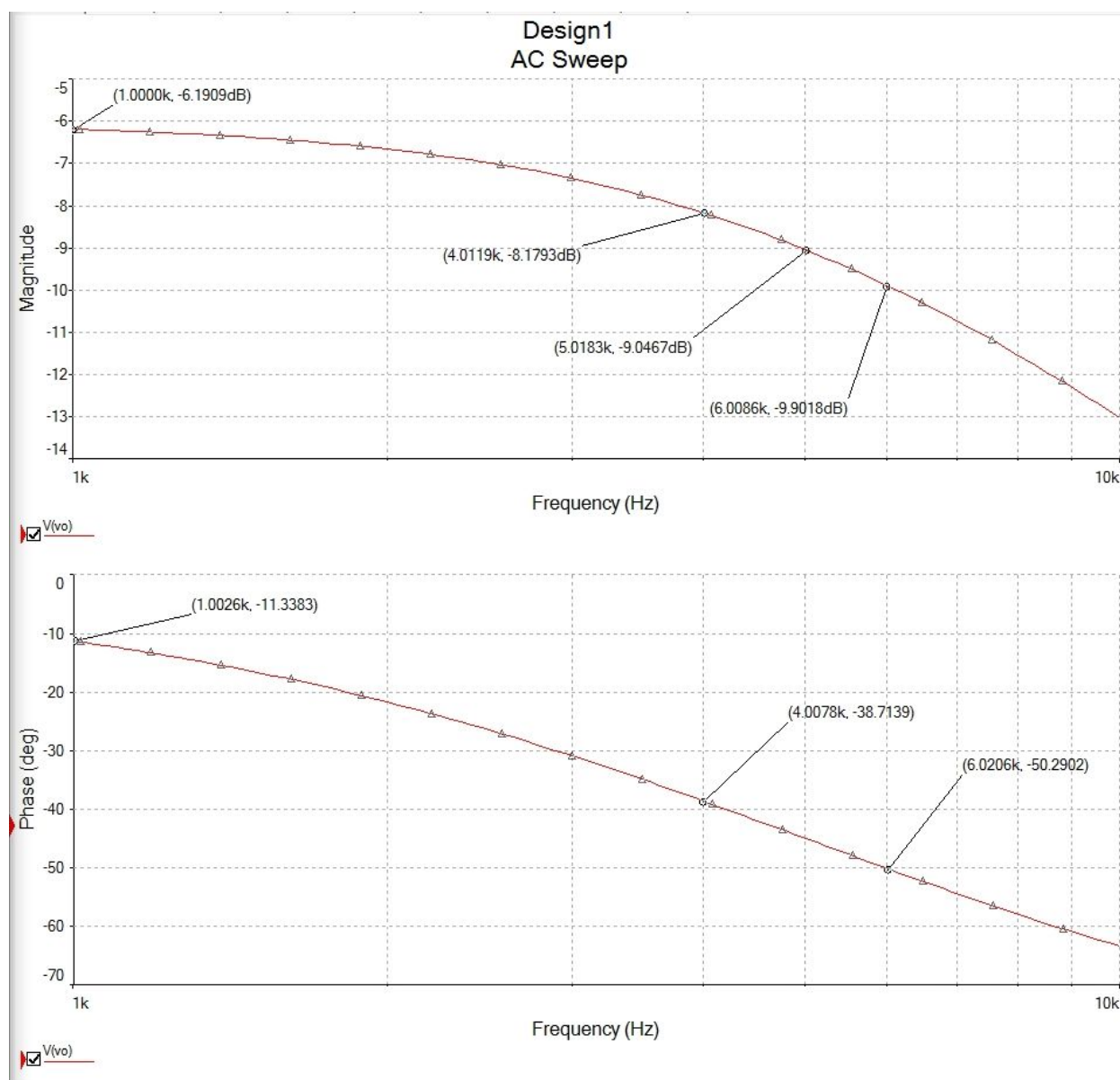
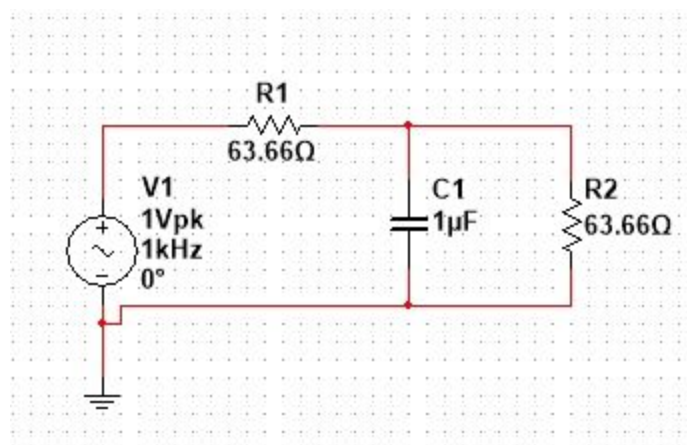
$$V_{HP}(j\omega) = \frac{j0.5(2\pi 6000)}{j(2\pi 6000) + 31416.9} \cdot V_i(j\omega)$$

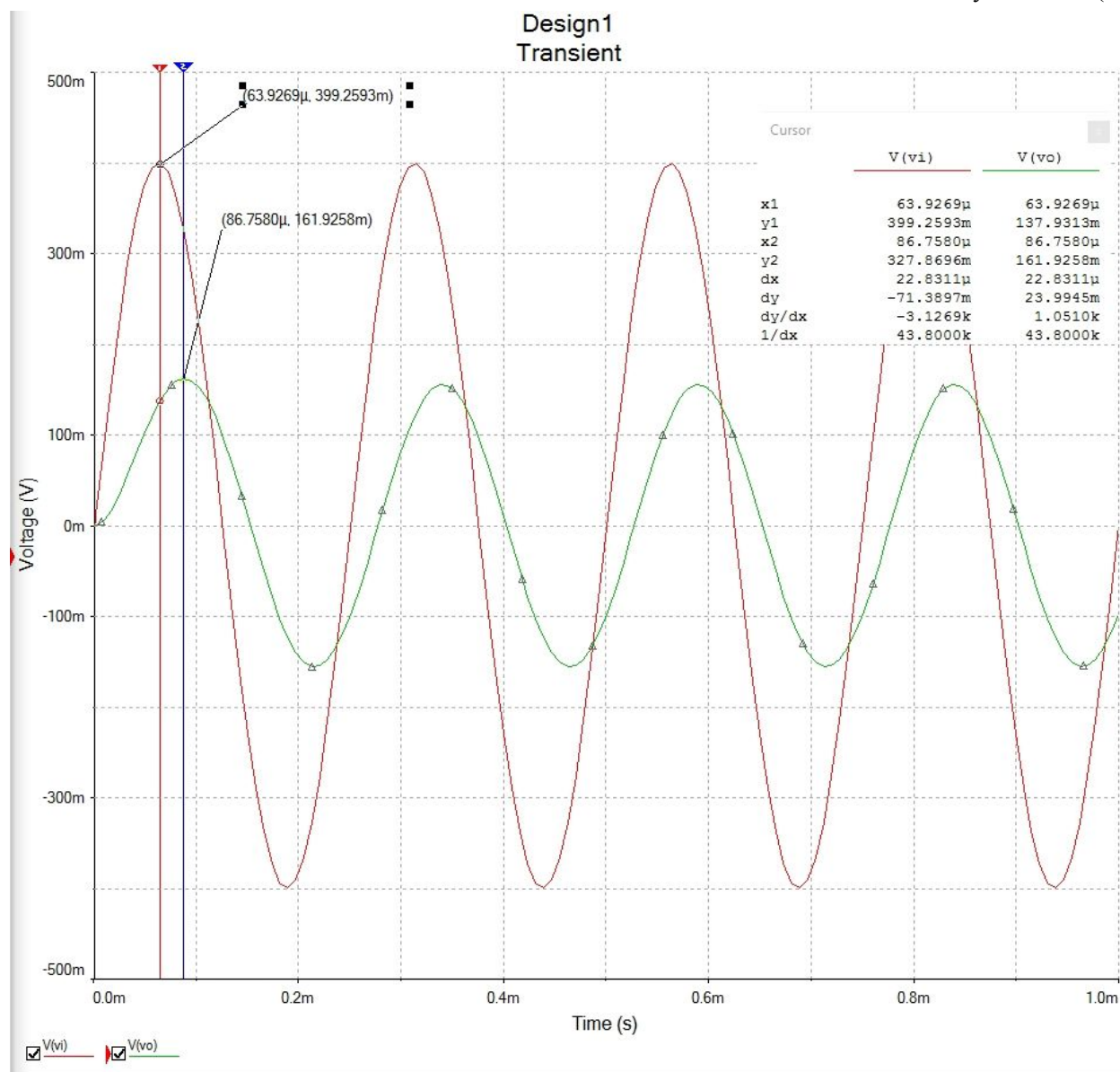
$$= (0.384\angle 39.8) (0.3\angle 0)$$

$$= 0.1152\angle 39.8 \text{ V}$$

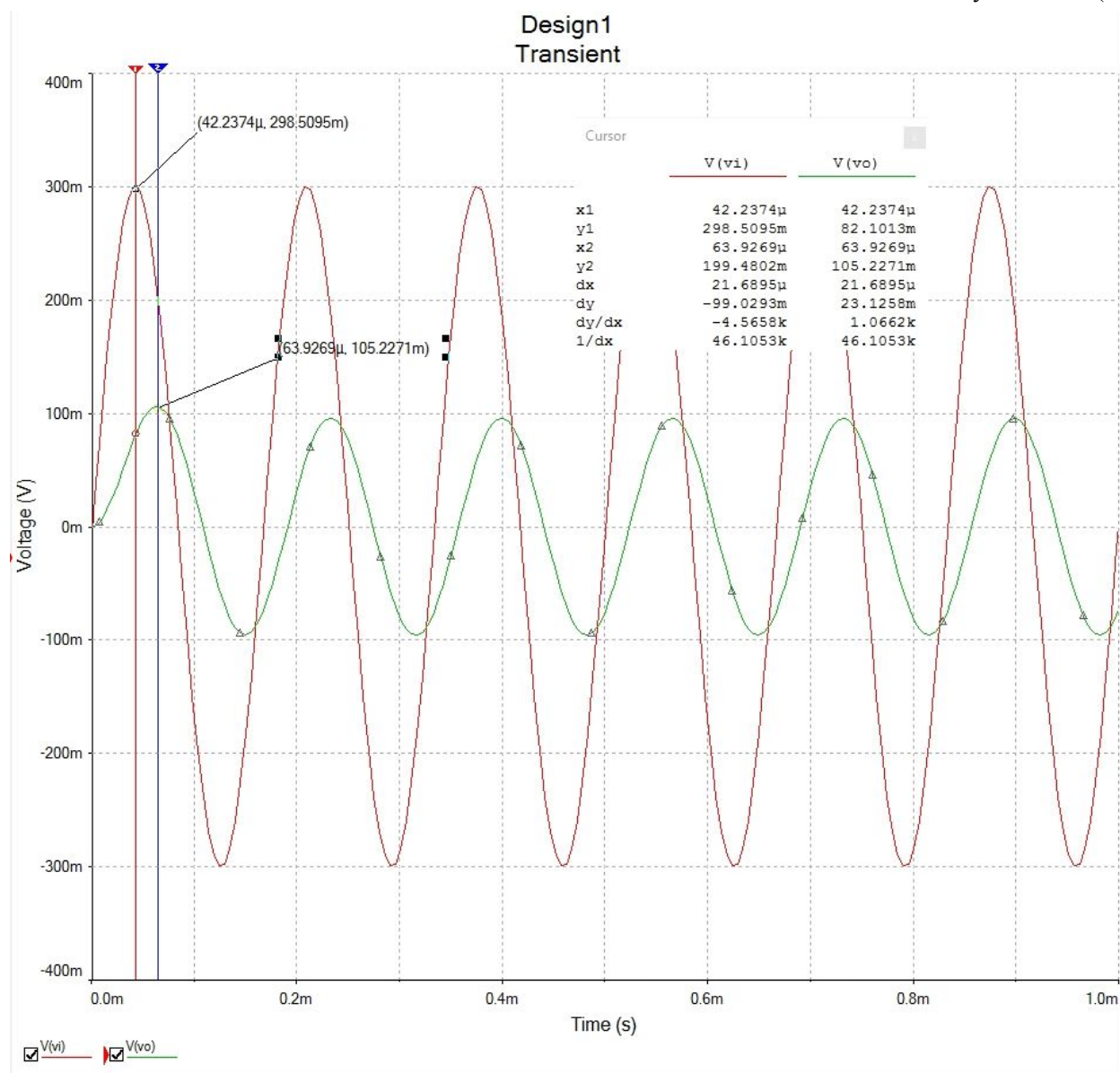
$$V_{HP}(t) = 0.115\sin(2\pi \cdot 6000t + 39.8^\circ) \text{ V}$$

Multisim:

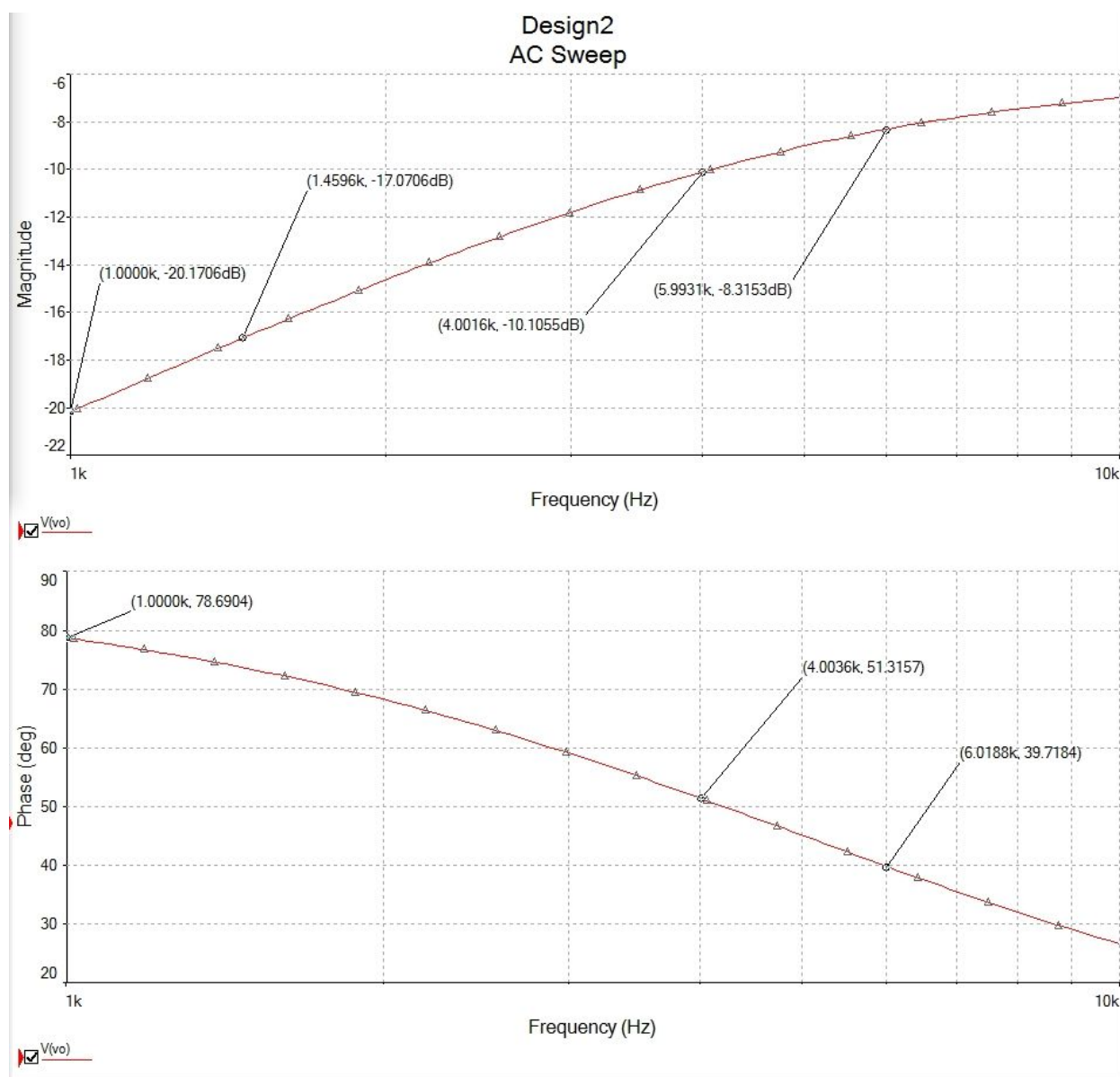
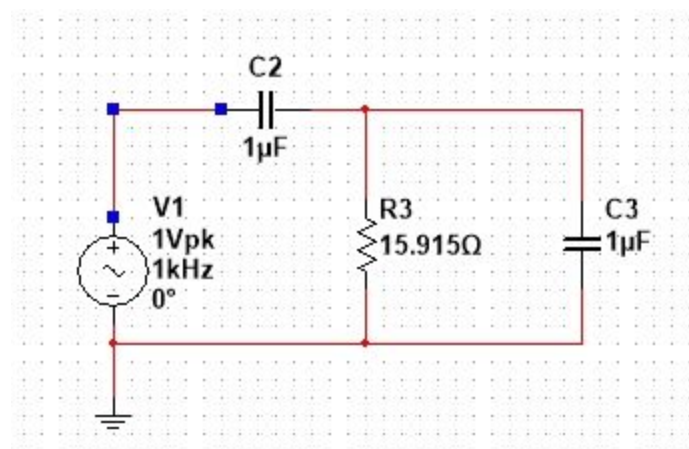


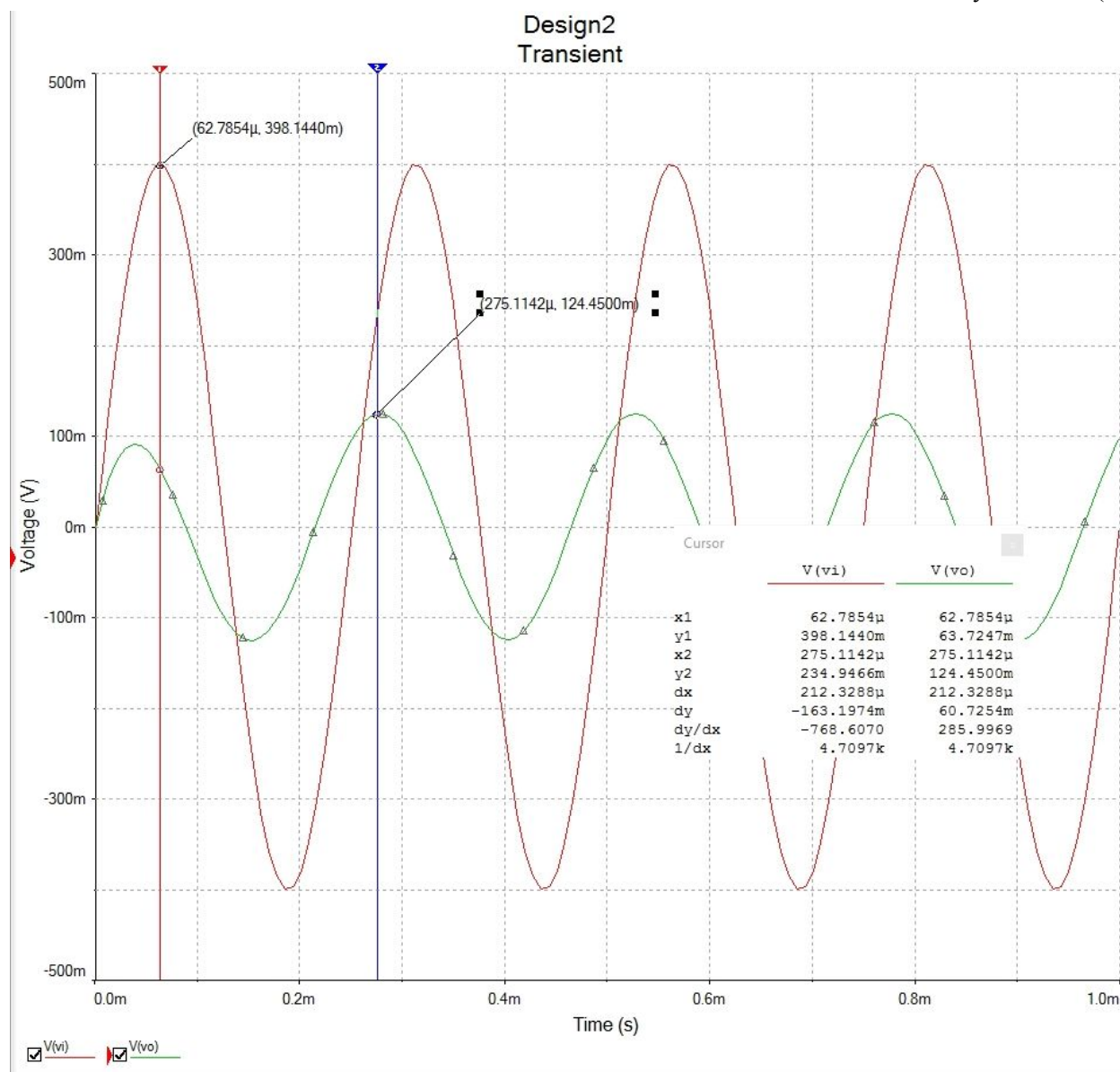


Transient for $V_i(t) = 0.4 \sin(24000t)$

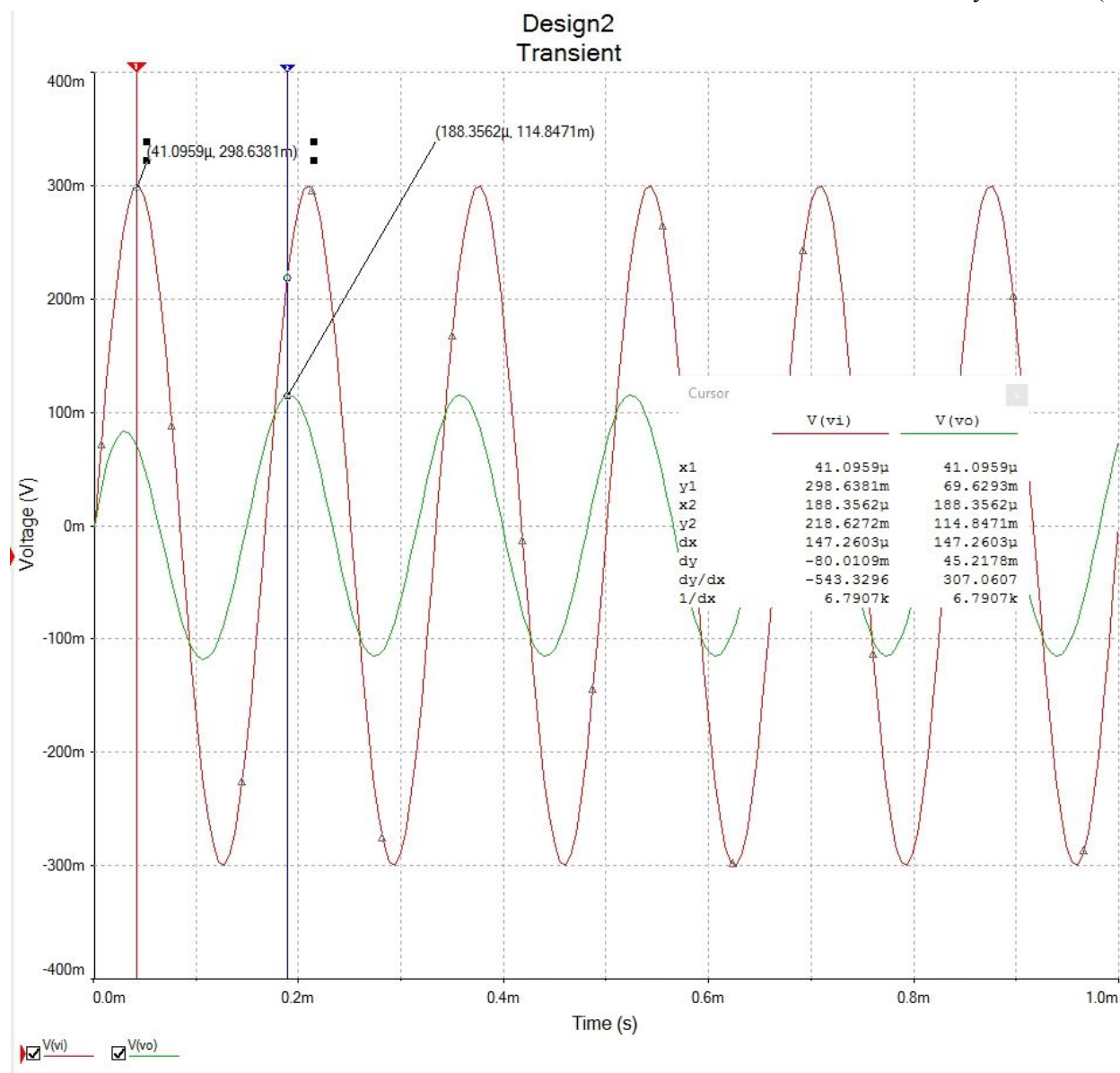


Transient for $V_i(t) = 0.3 \sin(26000t)$.





Transient for $V_i(t) = 0.4 \sin(24000t)$



Transient for $V_i(t) = 0.3 \sin(26000t)$.