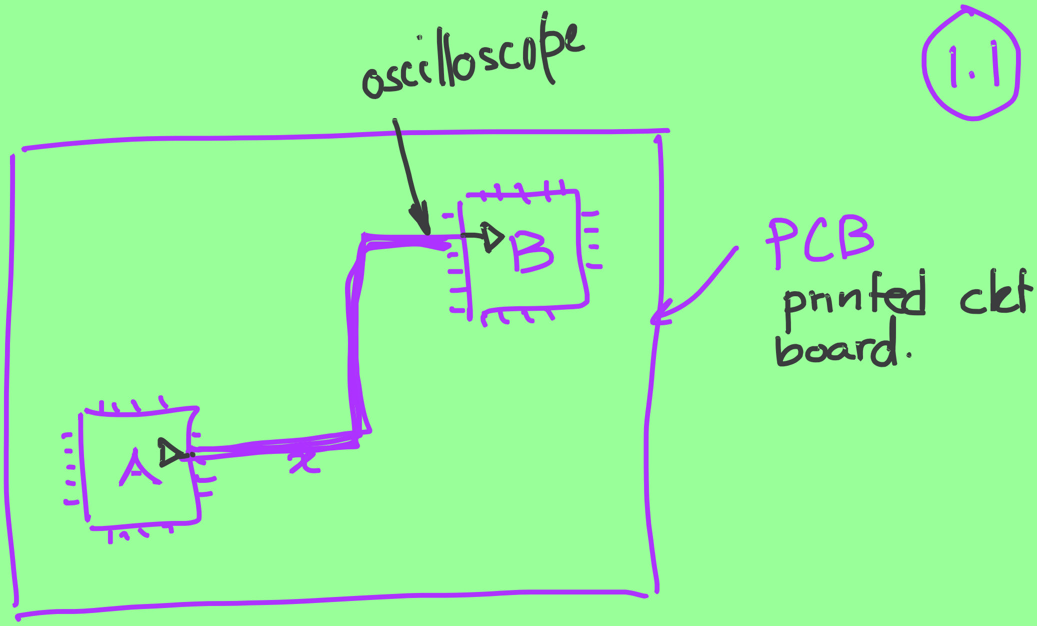
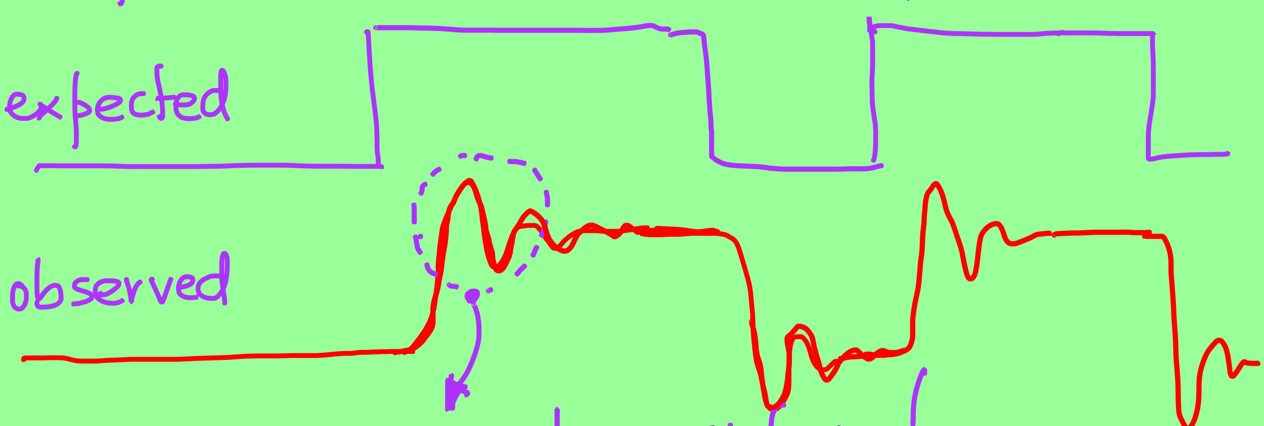
Transmission Lines

- Outline
  - When to Invoke a Transmission Line Model
  - Equations for Propagation
  - Reflections

1.1



- chip A drives a signal $\underline{x}$ to chip B
- I probe signal x at chip B.



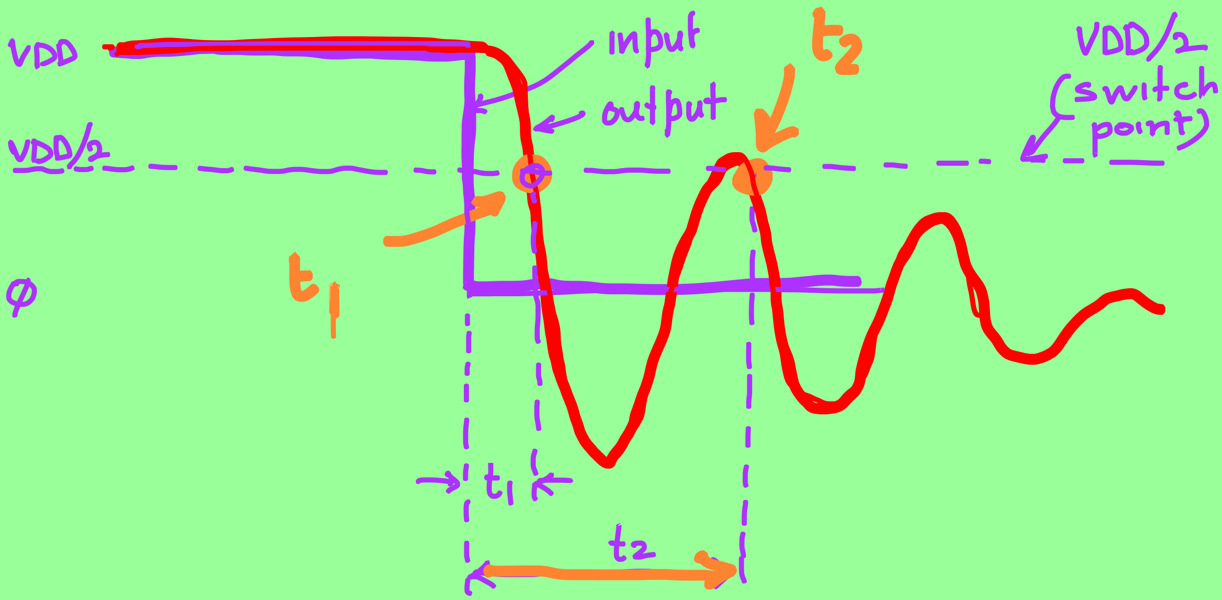
→ wiggly waveform!
(technically "ringing")

→ Is it a problem? (A: yes)

→ I want a math model for ringing to understand it and avoid it!

1.2

Why is ringing a problem?



Without ringing → delay is t_1
With ringing → delay is $t_2 > t_1$ [NOT GOOD!]

How to a) model ringing in spice?
b) model ringing analytically so we can control it?

1.3

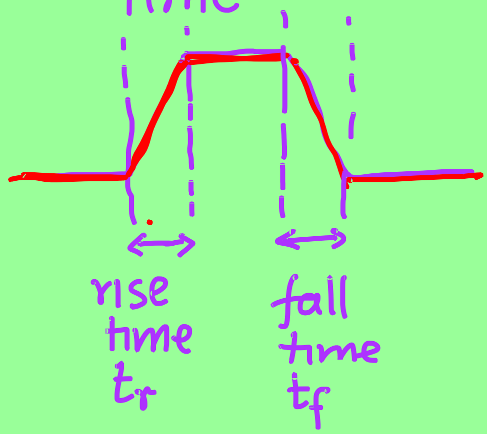
a) How to model ringing in spice?

driver rise/fall time

time of flight ←

$$TOF = \ell / u = \frac{\text{distance}}{\text{speed}}$$

[speed of light is 3×10^8 m/s ≡ 1ft/ns]



If in doubt distribute

if $t_r (\simeq t_f) \ll TOF$

we need to model ringing (wire is treated as a transmission line) "distributed"

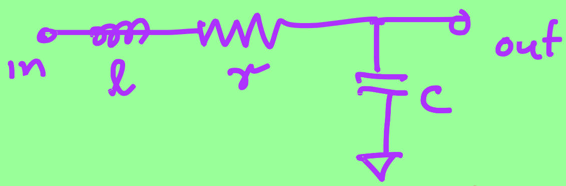
else

we dont need to model ringing. Wire is not treated as a transmission line "lumped"

1.4

Lumped (no transmission line)

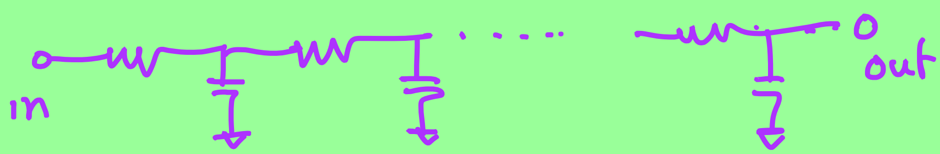
l, r, c are in a single "lump"



[Delay ~ RC] (estimate)

Distributed (transmission line)

l, r, c are "spread out" over the length of the wire

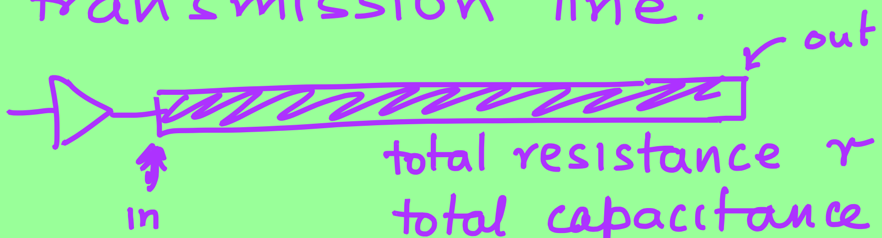


lots of little resistance/capacitance whose total is r/c

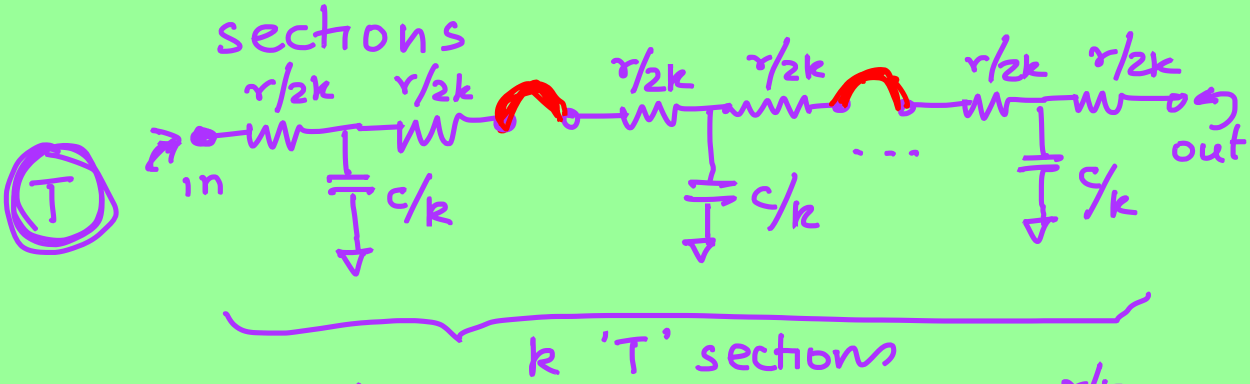
[Delay ~ $RC/2$] (estimate)

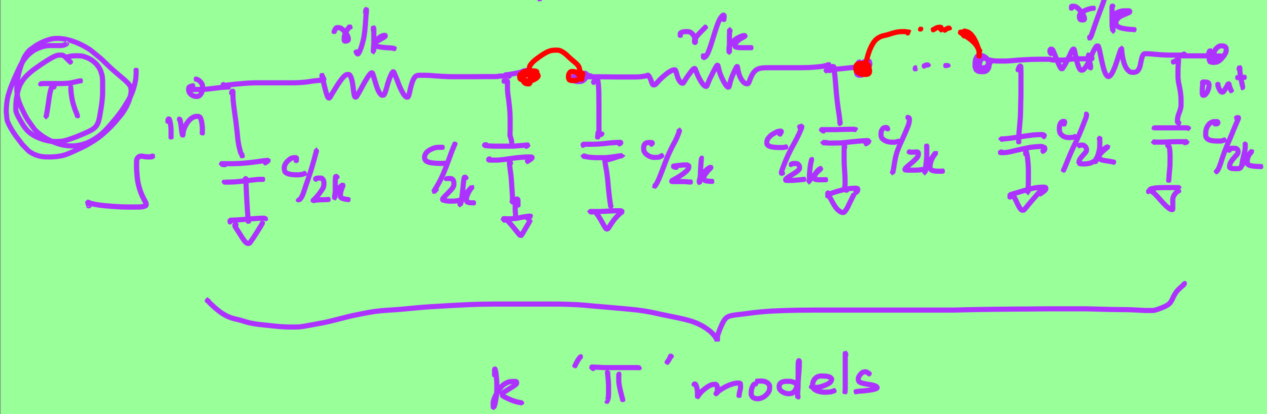
1.5

How to model the wire as a transmission line?



"Distribute" the r, c into T or Π sections





Q: Which one T or Π?

A: either!

How many sections? 1.6

→ choose k experimentally

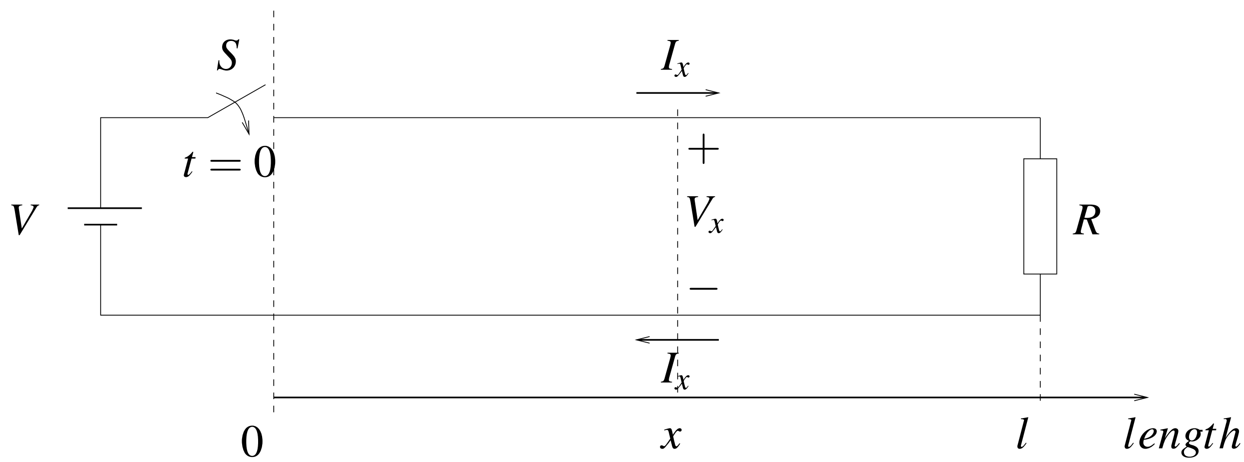
→ Try 1, 2, 3, 4 ... sections and stop at a value ST the delay difference between k-1 and k is within an error tolerance that you want.

When to Invoke a Transmission Line Model

- Suppose we have a wire. When is it to be viewed as a transmission line?
- **Short answer:** When the *time-of-flight* through the wire is comparable or larger than the rise/fall time of the source signal
- Time of flight is known if we know the velocity of wave propagation and the length of the wire. If the velocity of wave propagation is u and the length of the wire is l, then
  $u = \frac{l}{t}$
- What if we need to model a wire as a transmission line?
  - We simply model the R/L/C of the wire in a **distributed** manner instead of a lumped manner.
  - This simply means using several π or T sections as appropriate (depending on the desired accuracy).
- OK, what about the long answer?

Equations of Propagation

- Consider a wire of length l, connected to a voltage source V, and a switch S as shown below. The load resistance R is connected at the end of the wire.
- At time $t=0$, the switch S is closed.

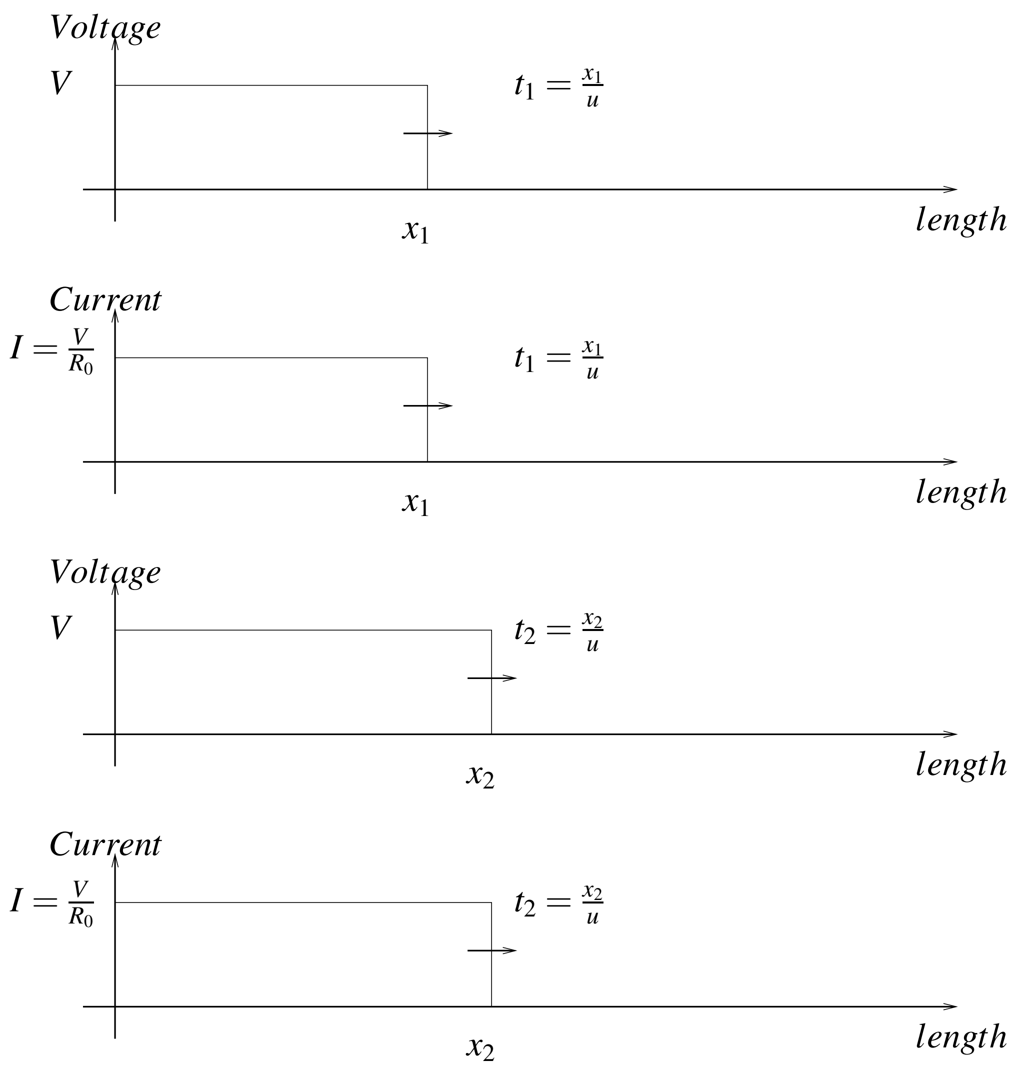


- The effect of closing S is not felt everywhere immediately
- Rather, this effect propagates from the source to load with a **finite** velocity
- Assuming uniform cross section, this velocity is given by $u = \frac{1}{\sqrt{LC}}$. Here $u = \frac{c}{\sqrt{\kappa}}$, where κ is the dielectric constant of the medium in which the wire resides. L

and C are the inductance and capacitance per unit length of wire.

- Note that if L increases, C decreases and vice versa.

- The voltage/current distributions on the wire at times $t_1 = x_1/u$ and $t_2 = x_2/u$ are shown below.



- We introduced a quantity R_0 above. How is it computed and what is it? Lets explain this analytically

- At t_1, the voltage is V for $0 \le x \le x_1$ and 0 for $x_1 < x \le l$

- In other words, a **front** of voltage travels to the right with velocity u.

- Lets say this front moves a distance dx. Then the capacitance to be charged is $C\,dx$ (since C is the capacitance per unit length)

- The charge required is $dQ = VC\,dx$. Hence the current is given by

- $I = \frac{dQ}{dt} = VC\frac{dx}{dt} = VCu = VC\frac{1}{\sqrt{LC}} = V\sqrt{\frac{C}{L}} = \frac{V}{R_0}$

- $R_0 = \sqrt{\frac{L}{C}}$ the **Characteristic Impedance** of the line.

- It is a function of the medium in which the wire resides. For PCBs, R_0 is in the 50-75 Ω range.

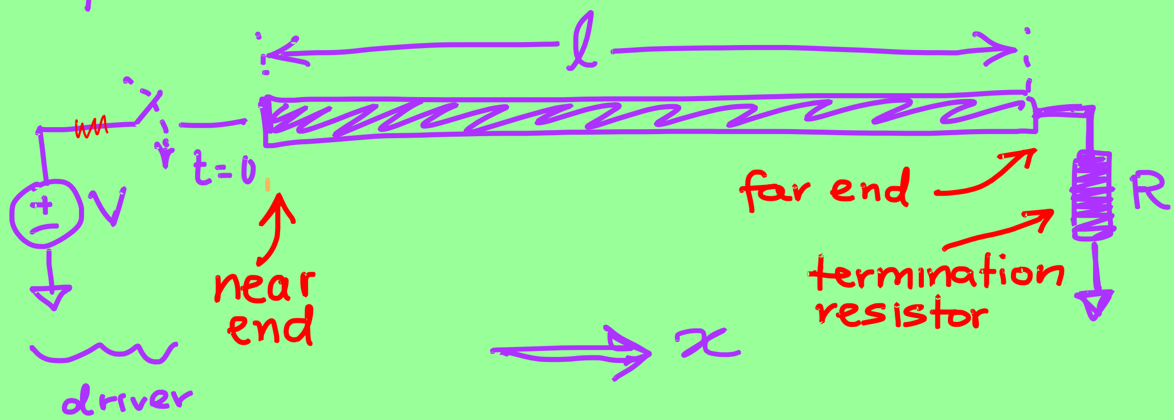
- The expression for I above is the current for $x = 0$ to wherever the front is. To the right of the front, $I = 0$.

- We say that I_x is positive when current flows to the right in the top wire, and to the left in the bottom wire.

- For all x from 0 to the location of the front (call it x), we have $V_x/I_x = R_0$.

- If the source and load were interchanged, we would have $V_x/I_x = -R_0$ based on our sign convention

b) Analytical model for transmission line: 3.1



$$\frac{\text{inductance}}{\text{length}} = L$$

$$\frac{\text{capacitance}}{\text{length}} = C$$

lowercase c, speed of light.

$c = 3 \times 10^8$ m/s

$$u = \frac{c}{\sqrt{K}} = \frac{1}{\sqrt{LC}} = \text{speed of wave}$$

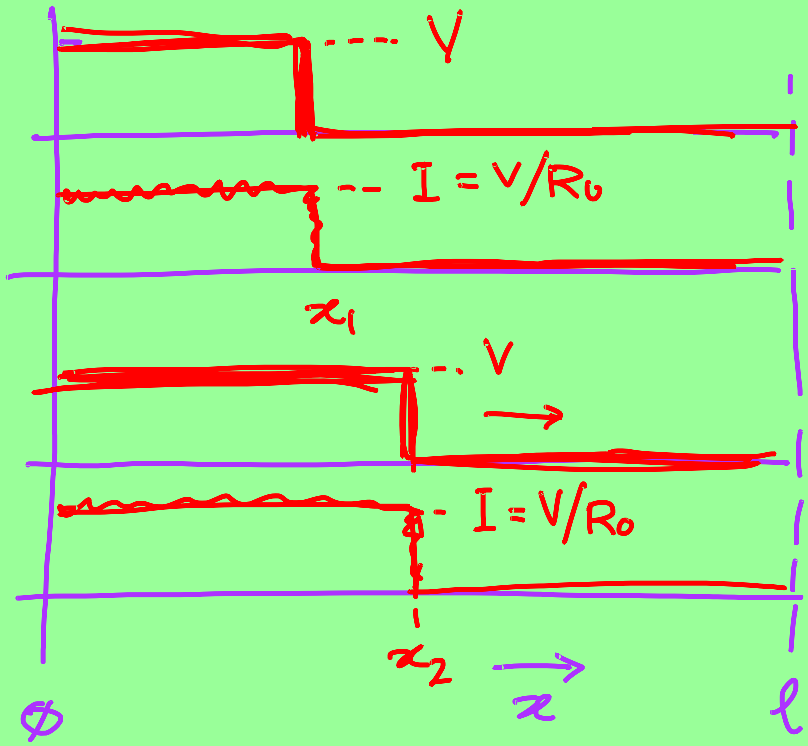
dielectric constant

$K \sim 4$ for most PCB

PCB material is called FR4

3.2

- A wave of voltage (and current) moves from left to right



@ $t_1 = \frac{x_1}{u}$

@ $t_2 = \frac{x_2}{u}$, $t_2 > t_1$

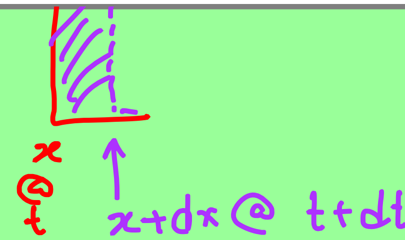
$\frac{V}{I} = R_0$ is a key (and non-intuitive) quantity!

= characteristic impedance of wire

$= Z_0$

3.3

Finding R_o



→ Suppose wave has reached location x. at time t

→ In time dt, it travels an extra distance dx

from 3.1

$$\frac{dx}{dt} = u = \frac{c}{\sqrt{K}} = \frac{1}{\sqrt{LC}}$$

→ In distance dx, the additional capacitance charged is

$$C \cdot dx$$

→ Recall that $(Q = C \cdot V)$

charge = capacitance × voltage

3.4

→ So the electrical charge needed to charge the additional capacitance $C \cdot dx$ from $\emptyset$ to V volts is

$$dQ = (C \cdot dx) \cdot V$$

→ This is done in time dt. So

$$\frac{dQ}{dt} = C \cdot V \cdot \frac{dx}{dt}$$

$$I = C \cdot V \cdot u$$

$$\frac{I}{V} = C \cdot u = C \cdot \frac{1}{\sqrt{LC}} = \frac{1}{R_0}$$

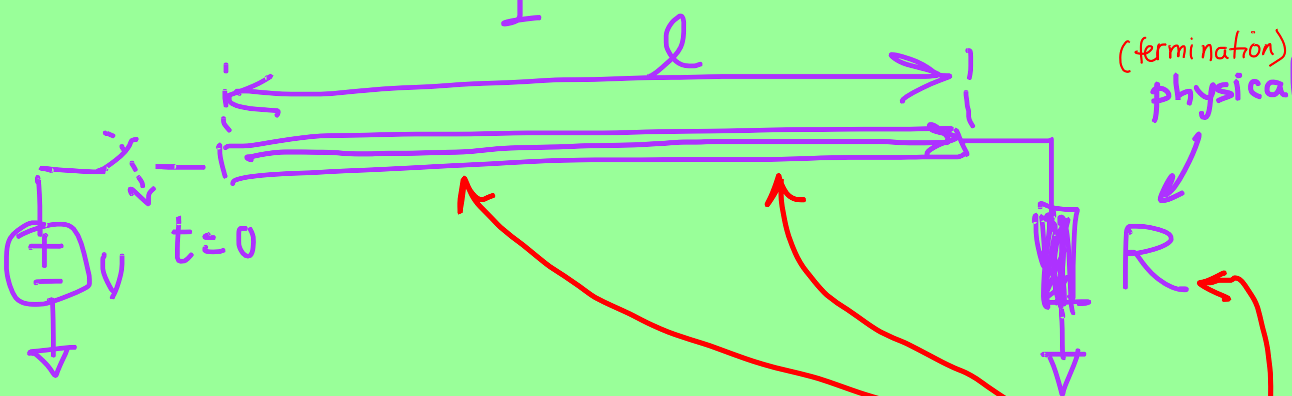
$$R_0 = \sqrt{\frac{L}{C}}$$ (called characteristic impedance)

$$R_0 = 50\text{-}75\,\Omega$$

3.5

R_0 is NOT a physical resistance!! It just "springs into life" when there is a wave of V & I traveling in the wire, and it relates the V wave magnitude to the I wave magnitude

$$R_0 = \frac{V}{I}$$



So when the wave is moving in the wire, $\frac{V}{I} = R_0$

But when the wave reaches the far end, we need $\frac{V}{I} = R$

WHAT IF $R \neq R_0$??

Reflections

- If the length l was infinite, then the front would keep moving to the right.

- If l is finite, then but if the load resistor $R = R_0$, then when the front arrives at the load (at time l/u), currents at all points in the wire and the load are V/R_0, and so nothing further happens.

- It is as though the length l was infinite.

- Now lets suppose $R \neq R_0$. When the front arrives at the load (at time l/u), the current in the wire is $V_x/R_x = R_0$. But **at the load,** the relationship $V_l/R_l = R$ **is required.**

- This causes a discontinuity. Thus a **reflection** develops at the load, which moves to the left.

- The amplitude and polarity of this reflected wave are such that the **total** voltage V_l (i.e. sum of the incident and reflected voltage) is related to I_l by the expression $V_l/I_l = R$

- If the incident voltage was V, the reflected voltage will be ρV where ρ is the **reflection coefficient**

- If the incident current is V/R_0, reflected current is $-\rho V/R_0$ based on our sign convention.

- The ratio of the total voltage to total current is R, therefore
  $R = \frac{V+\rho V}{V/R_0 - \rho V/R_0}$.

- Solving, we get $\rho = \frac{R/R_0 - 1}{R/R_0 + 1}$. This reflection coefficient ranges from -1 to $+1$.
  - When $R = R_0$, then $\rho = 0$. So no reflections are observed (we agreed that should be the case above).
  - When $R = \infty$ (i.e. the wire is **unterminated**), $\rho = 1$
  - When $R = 0$ (i.e. the wire is short-circuited), $\rho = -1$

- There can be several reflections in this manner. We can compute the waveform at l by accounting for such reflections.

Reflections - An Example

- Consider a wire of length l, with $R_0 = 100\Omega$, $R = 900\Omega$ and a source resistance $R_s = 20\Omega$. Also assume the source voltage is $V = 12V$. We want to determine the waveform at the end of the wire

- First we compute the reflection coefficients.
  - $\rho_R = \frac{\frac{900}{100}-1}{\frac{900}{100}+1} = 0.8$
  - $\rho_S = \frac{\frac{20}{100}-1}{\frac{20}{100}+1} = -0.75$

- A 12 V step appears on the line as a step of voltage $V_1 = 12\frac{R_0}{R_0+R_S} = 10\text{V}$

- At time $t = t_d = \frac{l}{u}$, the wave V_1 arrives at the end of the line, where it is reflected as:
  $V_2 = \rho_R \cdot V_1 = 0.8 \cdot 10\text{V} = 8\text{V}$

- Therefore the receiver voltage at this time is $V_1 + V_2 = 18\text{V}$

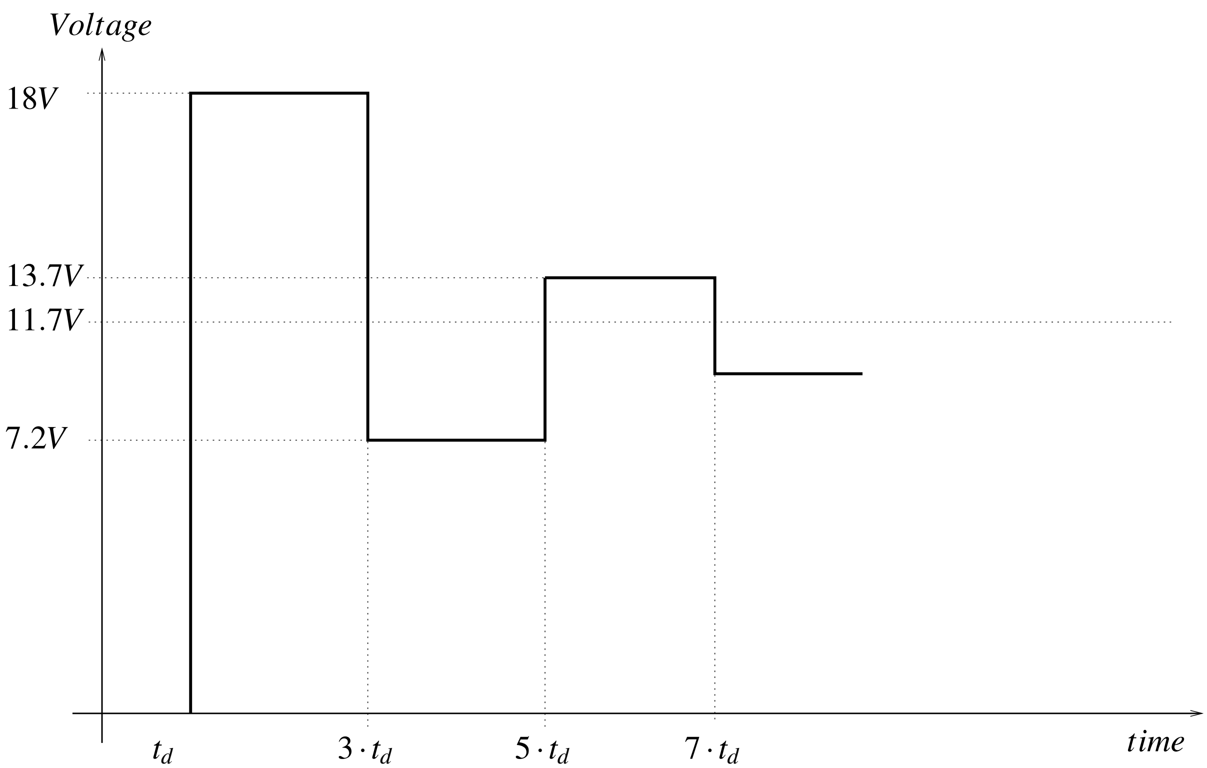
- At time $2 \cdot t_d$, the wave V_2 arrives at the source. It is now reflected as:
  $V_3 = \rho_S \cdot V_2 = -0.75 \cdot 8\text{V} = -6\text{V}$

- At time $3 \cdot t_d$, the wave V_3 arrives at the end of the line. It is now reflected as: $V_4 = \rho_R \cdot V_3 = 0.8 \cdot (-6)$V $= -4.8$V.

- So the receiver voltage at this time is $V_1 + V_2 + V_3 + V_4 = 7.2$V

- And so on...

- Asymptotically, the voltage at the end of the wire is $V \cdot \frac{R}{R+R_s} = 12 \cdot \frac{900}{920} = 11.739$V The waveform at the end of the wire therefore looks like:



4.1

when the wave reaches the far end.... 2 cases

1. $R = R_0$

$\frac{V}{I}$ along the wire $= R_0$

$\frac{V}{I}$ when wave reaches end $= R$ $(= R_0)$

So its like the wave kept on traveling... crisis averted!

2. $R \neq R_0$

$\frac{V}{I}$ along the wire $= R_0$

$\frac{V}{I}$ when wave reaches end : $R \neq R_0$

CRISIS!!

→ Solution launch a reflected wave back towards the near end.

→ its voltage is $\rho \cdot V$

→ its current is ρI

(but its sign is -ve)

so $\frac{\rho V}{\rho I} = R_0$ as reqd.

So, at far end :

physical (terminating res)

$$\frac{\text{total voltage}}{\text{total current}} = R = \frac{V + \rho V}{I - \rho I}$$

Massage the math: 4.2

$$\boxed{\rho = \frac{R/R_0 - 1}{R/R_0 + 1}}$$; $\rho \equiv$ reflection coefficient

NOTES
- $-1 \le \rho \le 1$
- $R = R_0 \Rightarrow \rho = 0$
- $R = \infty \Rightarrow \rho = 1$ (open ckt termination)
- $R = \emptyset \Rightarrow \underline{\rho = -1}$ (short ckt termination)

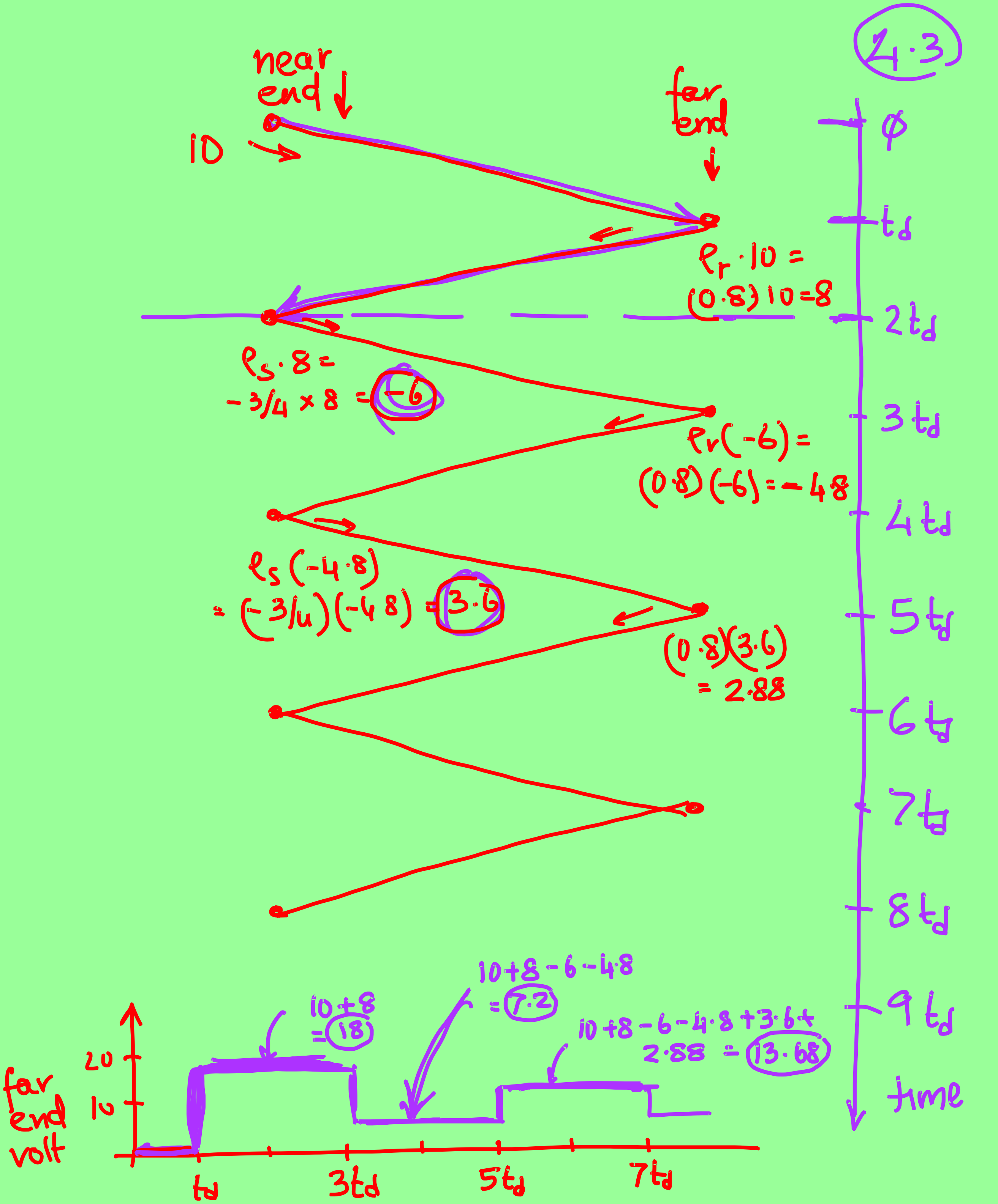
Reflected wave is $\rho \cdot V = \emptyset \cdot V = \emptyset$. (no reflection) (matches intuition of 4.1)

Reflected wave is $\rho V = -V$, so total voltage @ far end is $V + \rho V = V - V = \emptyset$ (matches intuition)

Reflected wave is $\rho V = 1 \cdot V = V$. So total voltage @ far end is $V + V = 2V$. OUCH. Can damage far end chip input circuitry!

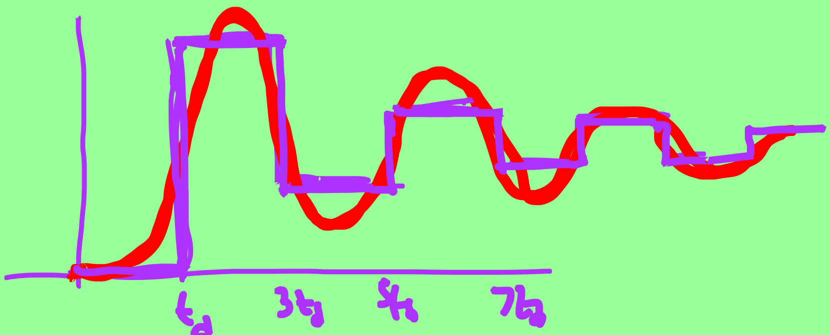
Workspace for 'transmission-lines'

Page 15 (row 3, column 9)



4.4

Far end voltage looks like



* Just like the red waveform in 1.1 and 1.2 !!

* if we want the voltage to never exceed X volts @ far end, just make sure that

$$V + \rho V = X$$

[if $R_0 = 50\Omega$, $V = 2.5V$ and $X = 3V$, then

$$2.5 + \rho(2.5) = 3$$

$$\rho = 0.2 = \frac{R/R_0 - 1}{R/R_0 + 1}$$

calc. R]

4.5

* frequency of ringing is

$$\left(\frac{1}{4t_d}\right) = \frac{1}{4(l/u)}$$

l is wire length
u is speed of wave (see 3.1)

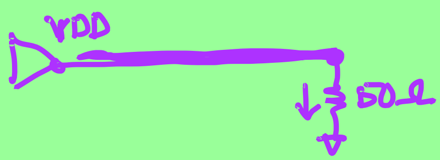
OH .1
11/25/20

For no reflections
(ie no ring-up) } $R = R_0$

Problems w/ $R = R_0$

→ every PCB trace/wire needs a resistor
✓ → expensive ($)
→ expensive (pwr)
✓ → expensive (board area)

→ $R_0 \sim 50\Omega$
$R = 50\Omega$ also



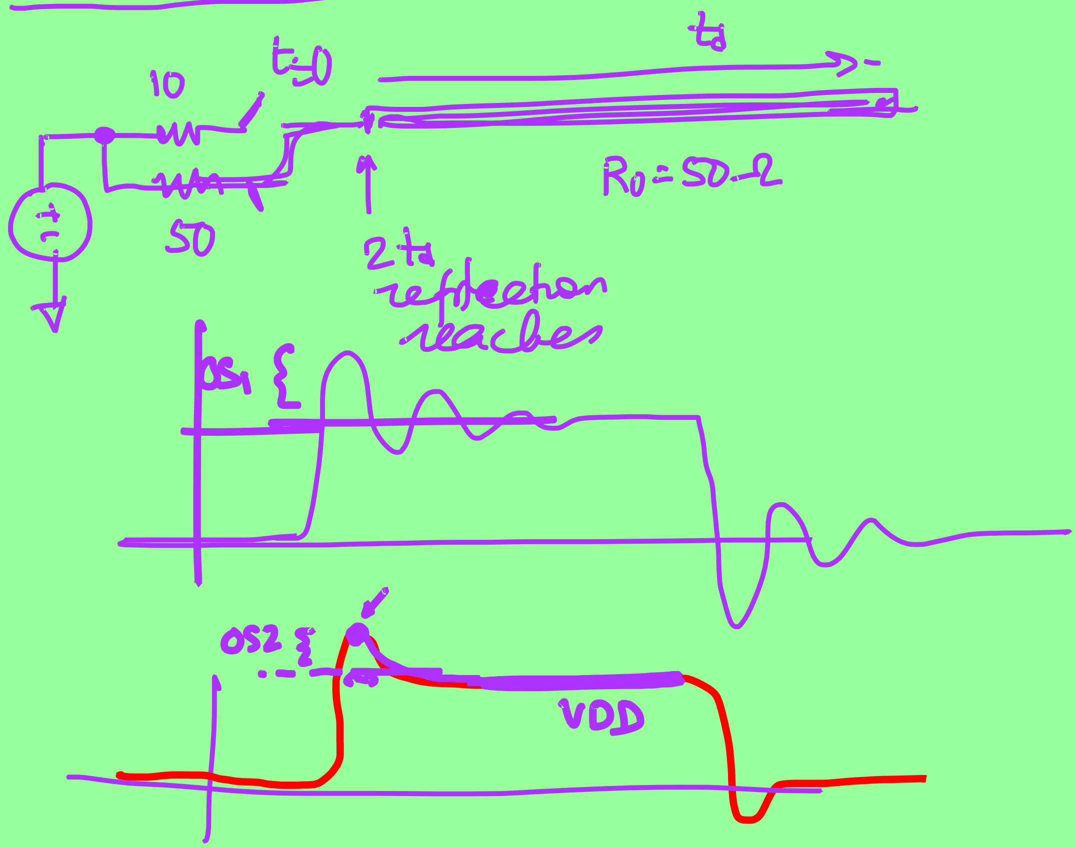
$i = \frac{V}{R} = \frac{VDD}{50} = \frac{2.5V}{50} = 50mA$

$\left(\frac{50}{1000} \times 10000\right) = 500A$

OH.2
11/25/20

$R = 200\,\Omega \quad (R_0 = 50)$

$$\rho = \frac{R/R_0 - 1}{R/R_0 + 1} = \frac{4-1}{4+1} = \frac{3}{5} = .6$$



Workspace for 'transmission-lines'

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