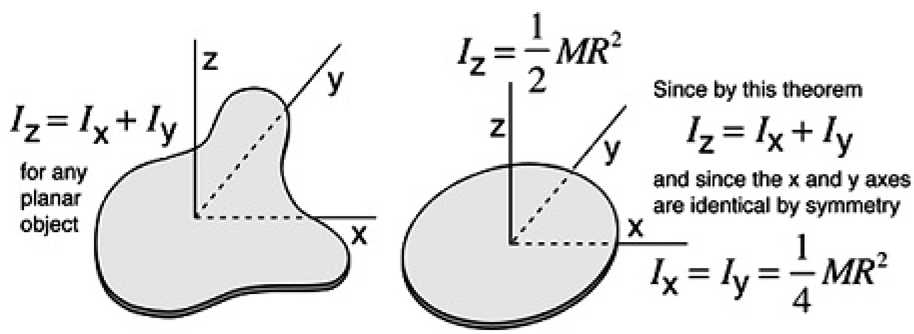
Circular Motion

Basics

Angular Momentum $= L = mvr = m\omega r^2$
Centripetal Force $= \frac{mv^2}{r} = m\omega^2 r$

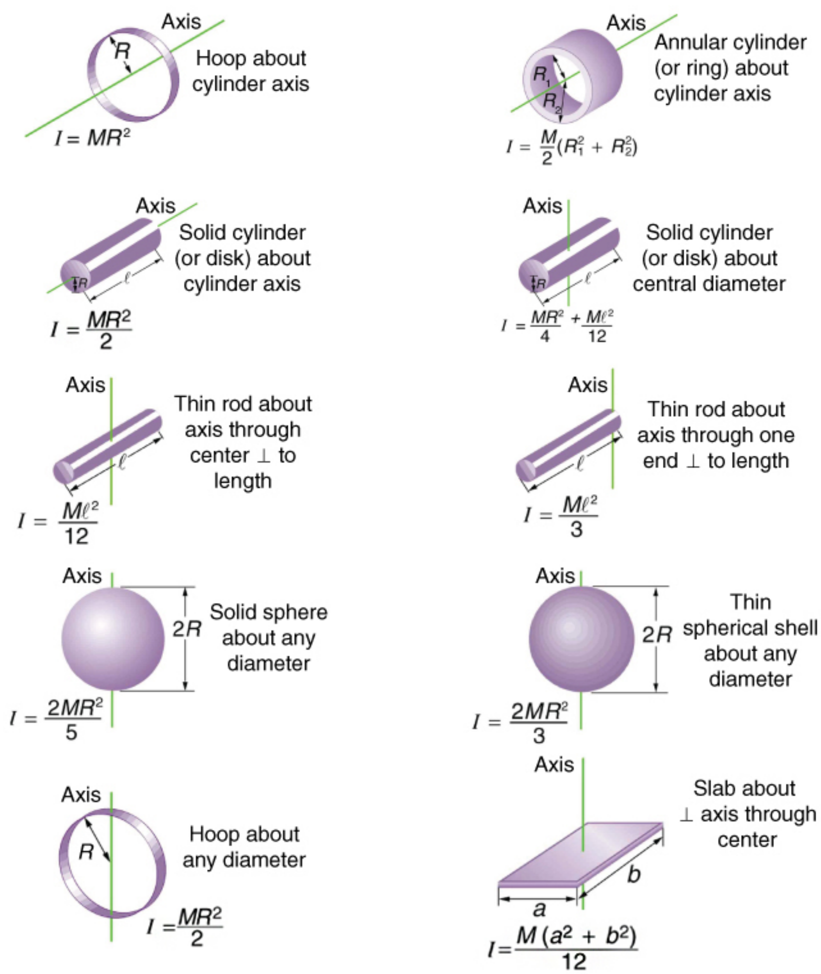
Perpendicular Axis Theorem



Parallel Axis Theorem

$I_f = I_i + mr^2$

Common Moment of Inertias



Simple Harmonic Motion

$d = a\sin(\omega t)$
$f = \frac{1}{T}$
$f = \frac{\omega}{2\pi}$

GENERAL SHM

If displacement x causes acceleration a, then the period of oscillations is

$T = 2\pi\sqrt{\frac{x}{a}}$

$(a = -\omega^2 x)$

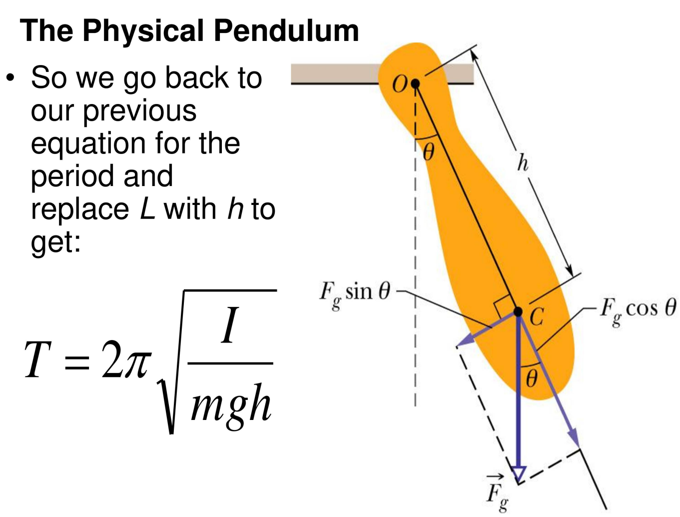
Spring

$T = 2\pi\sqrt{\frac{m}{k}}$
$\omega = \sqrt{\frac{k}{m}}$

Simple Pendulum

$T = 2\pi\sqrt{\frac{l}{g}}$
$\omega = \sqrt{\frac{g}{l}}$

Physical Pendulum



Error Propagation

$\Delta(Cx) = C\Delta x$
$\Delta(x+y) = \Delta(x-y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
$\Delta(xy) = xy * \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$

$\Delta(k = \frac{xy}{z}) = k\sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2}$

$\Delta(xyz) = xyz * \sqrt{(\frac{\Delta x}{x})^2 + (\frac{\Delta y}{y})^2 + (\frac{\Delta z}{z})^2 ...}$

$\Delta(x^m y^n) = x^m y^n * \sqrt{(\frac{m\Delta x}{x})^2 + (\frac{n\Delta y}{y})^2 ...}$

$\Delta(\frac{4x+y}{5}) = \frac{\sqrt{(4\Delta x)^2+(\Delta y)^2}}{5}$

Collisions

Perfectly Elastic Collisions

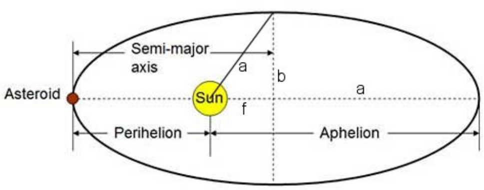
- $v_2' = \frac{2m_1}{m_1+m_2}v_1 - \frac{m_1-m_2}{m_1+m_2}v_2$
- $v_1' = \frac{m_1-m_2}{m_1+m_2}v_1 + \frac{2m_1}{m_1+m_2}v_2$

Perfectly Elastic Collisions (still object)

- $v_2' = \frac{2m_1}{m_1+m_2}v_1$
- $v_1' = \frac{m_1-m_2}{m_1+m_2}v_1$

Gravity/ Orbit

Vis-Viva Equation



2019.8. To solve this problem it is useful to know that if an object is at distance r from the center of the Earth and moves with speed v tangentially to the Earth then:

- For $0 < v < \sqrt{\frac{GM}{r}}$, its orbit is elliptical and r is the maximum distance from the Earth (apogee)
- For $v = \sqrt{\frac{GM}{r}}$, its orbit is circular with radius r
- For $\sqrt{\frac{GM}{r}} < v < \sqrt{\frac{2GM}{r}}$, its orbit is elliptical and r is the minimum distance from the Earth (perigee)
- For $v = \sqrt{\frac{2GM}{r}}$ (the escape speed), its orbit is parabolic
- For $v > \sqrt{\frac{2GM}{r}}$, its orbit is hyperbolic

$v^2 = GM(\frac{2}{r} - \frac{1}{a})$

* a is the length of the semi-major axis (a > 0 for ellipses, a = ∞ or 1/a = 0 for parabolas, and a < 0 for hyperbolas)

Kepler's Laws

$$m\frac{v^2}{r} = \frac{GmM_{Sun}}{r^2}$$

Applying Newton's 2nd Law for the case of circular motion, the required centripetal force is supplied by gravity.

$$v^2 = \frac{GM_{Sun}}{r}$$

$$v = \sqrt{\frac{GM_{Sun}}{r}} = \frac{2\pi r}{T}$$

The application of Newton's 2nd law gives you the velocity. For a circular orbit, the period T can be found from the orbit velocity.

$$\frac{T}{2\pi r} = \sqrt{\frac{r}{GM_{Sun}}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{Sun}}$$

Solving for the period gives you Kepler's Law of Periods for the special case of a circular orbit.

The expressions for velocity and period are seen to follow from Newton's 2nd law and the law of gravity.

$$v = \sqrt{\frac{GM_{Sun}}{r}} \qquad T = \frac{2\pi r^{3/2}}{\sqrt{GM_{Sun}}}$$

$$T^2 = \frac{4\pi^2}{GM}a^3$$

can be expressed as simply

$$T^2 = a^3$$

If expressed in the following units:

- T Earth years
- a Astronomical units AU (a = 1 AU for Earth)
- M Solar masses $M_\odot$

Then $\frac{4\pi^2}{G} = 1$

Energy

Power $= P = \frac{W}{\Delta t} = Fv$
Impulse $= J = F\Delta t = m\Delta v$

Kinematics

1. $v = v_0 + at$
2. $\Delta x = \frac{(v+v_0)}{2}t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v = v_0 + at$
5. $v^2 = v_0^2 + 2a\Delta x$

Launching Ball from Ground

Range $= \frac{2v^2 \sin\theta \cos\theta}{g} = \frac{v^2 \sin(2\theta)}{g}$

Launching ball from cliff

Optimal angle $h = \frac{v^2}{g}\left(\frac{1}{2\sin^2\theta} - 1\right)$
$\Rightarrow \sin\theta = \left(2\left(\frac{gh}{v^2} + 1\right)\right)^{-1/2}$

Helpful Math

Sums of series

Sum of Geometric $= a_1(\frac{1-r^n}{1-r})$
Sum of Infinite Geometric $= \frac{a_1}{1-r}$

Trig

Double Angle Formulas	Half Angle Formulas
$\sin 2\theta = 2\sin\theta\cos\theta$	$\sin^2\theta = \frac{1-\cos 2\theta}{2}$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$	$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
$= 2\cos^2\theta - 1$	$\cos^2\theta = \frac{1+\cos 2\theta}{2}$
$= 1 - 2\sin^2\theta$	$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$	$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$