Python N-body simulation of solar system dynamics

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Solar system dynamics computational lab

with a minimum of prerequisites

- familiarity with typical topics for introductory course, e.g.
  - constant acceleration kinematic equations
  - Newton's second law
  - Newton's law of universal gravitation
  - uniform circular motion
  - conservation of energy
- does not use calculus
  - though can be helpful in shedding light on why Runge-Kutta is more accurate than constant acceleration kinematics
- no programming experience assumed
  - not a programming course
  - Python tutorial 'pre-lab' covers relevant functions
  - infrastructure, initial data, output, etc. are all already set up for students in code which is provided

Materials can be found at `https://github.com/kyle-slinker/teaching-resources`.

Building a physical model in a computational environment

- description of two particles' interaction in terms of binding energy and resulting motion
  - $U = -G\dfrac{m_1 m_2}{|\vec{p}_1 - \vec{p}_2|}$
  - $\vec{a}_1 = -G\dfrac{m_2}{|\vec{p}_1 - \vec{p}_2|^3}\left(\vec{p}_1 - \vec{p}_2\right)$

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```
#description of function get_potential:
# - find the gravitational binding energy between object 1 and object 2
#inputs:
# - positions of object 1 and object 2; p1 and p2; each a 3x1 numpy array
# - the masses of object 1 and object 2; m1 and m2; each a single number
# - Newton's gravitation constant; g; single number
#output:
# - gravitational potential energy between object 1 and object 2; retval; single number

def get_potential(p1,p2,m1,m2,g):
    if np.array_equal(p1,p2):
        retval=0
    else:
        ################################
        #Write your code between here...




        retval=
        #...and here.
        ################################
    return retval
```

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```python
#description of function get_acceleration:
# - find the acceleration of object 1 caused by object 2
#inputs:
# - positions of object 1 and object 2; p1 and p2; each a 3x1 numpy array
# - the mass of object 2; m2; single number
# - Newton's gravitation constant; g; single number
#output:
# - the acceleration of object 1; retval; 3x1 numpy array

def get_acceleration(p1,p2,m2,g):
    if np.array_equal(p1,p2):
        retval=np.array([0,0,0])
    else:
        ################################
        #Write your code between here...




        retval=
        #...and here.
        ################################
    return retval
```

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- constant acceleration motion
  - $x_f = x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$
  - $v_f = v_i + a\Delta t$

```
x[:,:,-1]+=v[:,:,-1]*dt+0.5*a*dt2
v[:,:,-1]+=a*dt
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- constant acceleration motion
  - $x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$
  - $v_f = v_i + a\Delta t$
- better approximation: fourth order Runge-Kutta

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- for one thing, exposure to useful tools such as Python and Runge-Kutta

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- without simulation, physics is somewhat constrained
- Kepler's law is almost a one-liner

$$\frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = m\frac{v^2}{r} = ma = \sum F = G\frac{mM}{r^2}$$

$$r^3 = \frac{GM}{4\pi^2}T^2$$

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  - limits on usability of Kepler's law
  - 3-body effects
  - stability vs chaos

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- with simulation, dynamics become much more interesting
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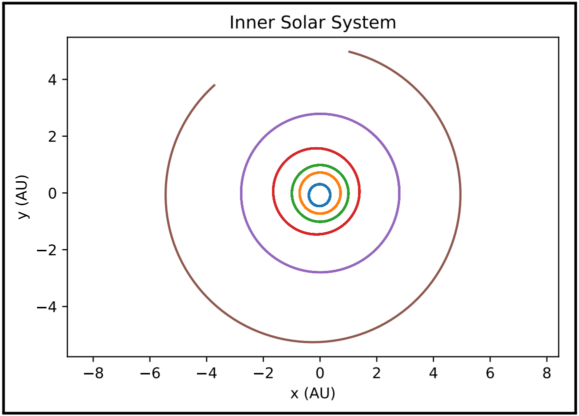
  - stability vs chaos
- new tool means interesting new considerations in assessing accuracy

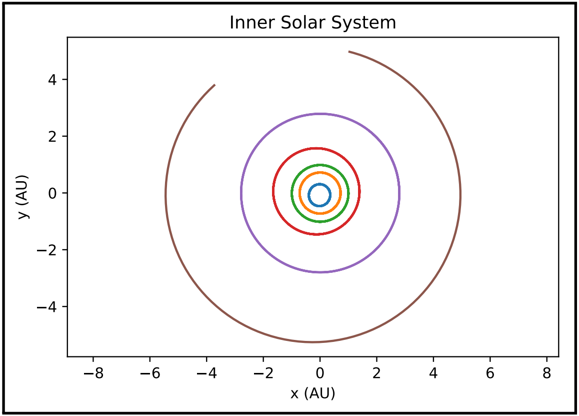
Applying Kepler's law within a simulation

- add dwarf-planet Ceres to simulation of planets
- Kepler's law useful, but with context!



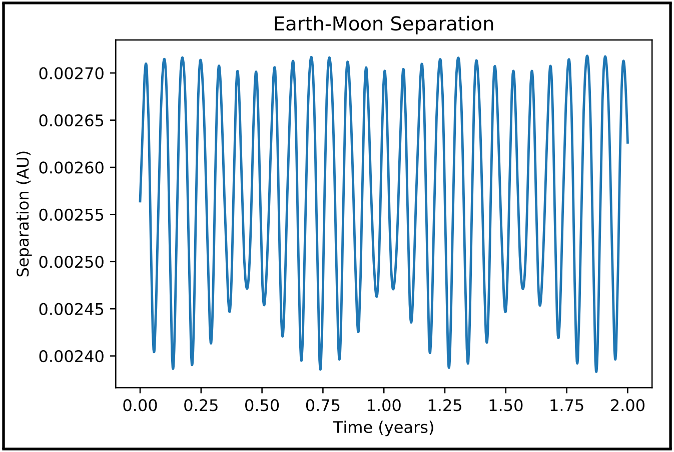
Applying Kepler's law within a simulation

- add dwarf-planet Ceres to simulation of planets
- Kepler's law useful, but with context!
- other approximations required to construct model
  - circular
  - equatorial
  - starting phase
  - perturbations from other planets negligible



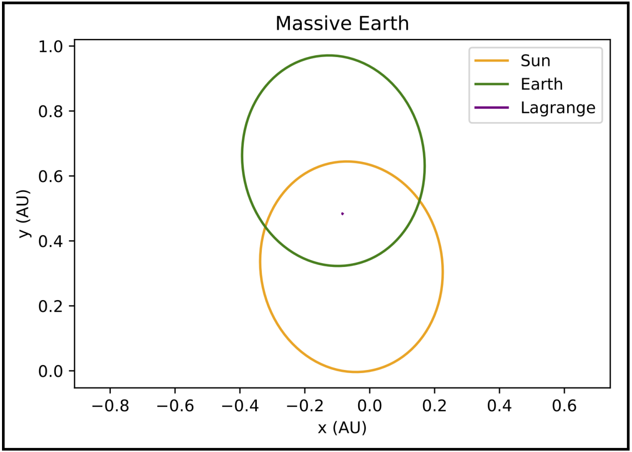
3-body effects: Sun-Earth-Moon system

- two time scales
  - one month period: elliptic nature of Moon's orbit around Earth
  - one year period: elliptic nature of Earth's orbit around Sun – how the Sun affects the Earth-Moon system changes throughout the year



(In)stability in the 3-body problem

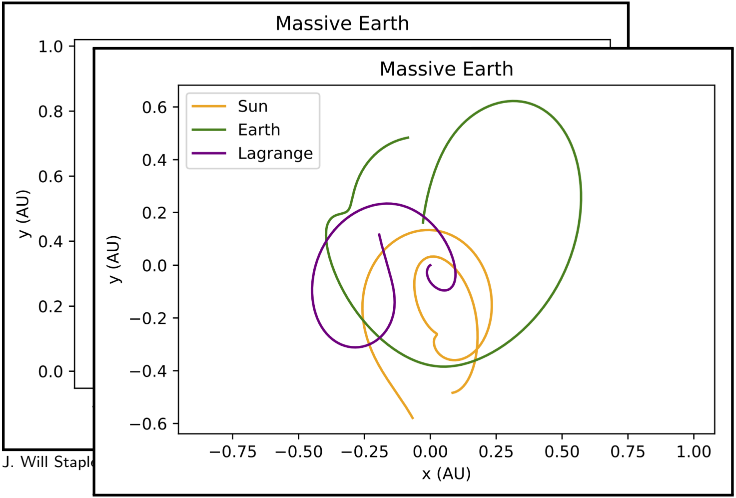
- stable configuration with one object at center of mass of other two



J. Will Staples, NCSSM ‘20

(In)stability in the 3-body problem

- one initial velocity changed by 25%



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Can a moon host a submoon in a stable orbit?

- innocuous question?

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Can Moons Have Moons?

Juna A. Kollmeier[1⋆] & Sean N. Raymond[2†]

[1] *Observatories of the Carnegie Institution of Washington, 813 Santa Barbara St., Pasadena, CA 91101*

[2] *Laboratoire d'Astrophysique de Bordeaux, Univ. Bordeaux, CNRS, B18N, allé Geoffroy Saint-Hilaire, 33615 Pessac, France*

Kollmeier & Raymond Feb 2019, MNRAS:L, V483, I1, pp. L80–L84; arXiv:1810.03304

Can a moon host a submoon in a stable orbit?

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  - submoon orbits too close: body disrupted by moon's tides
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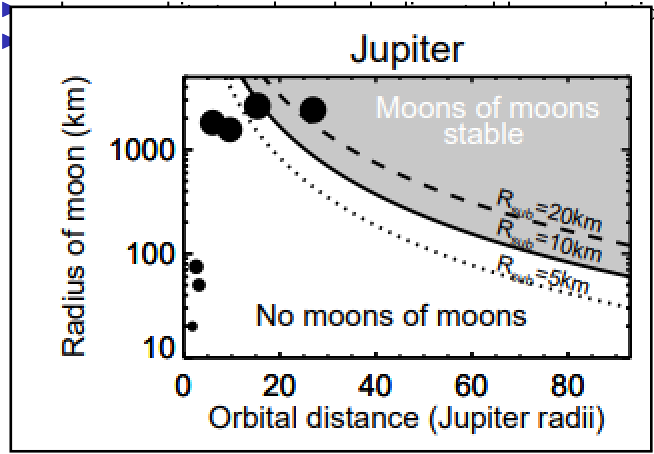

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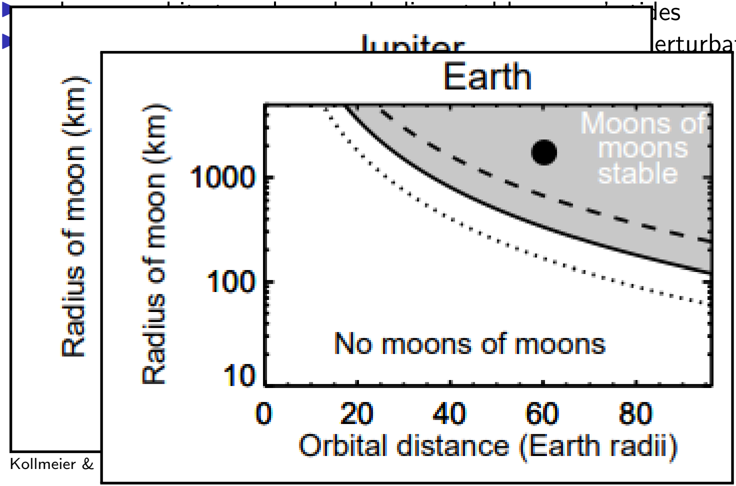

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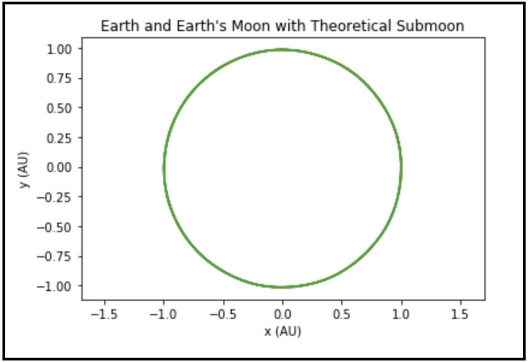

Mackenzie Savage, NCSSM '20

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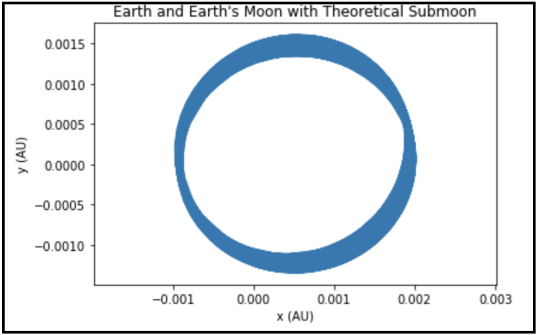

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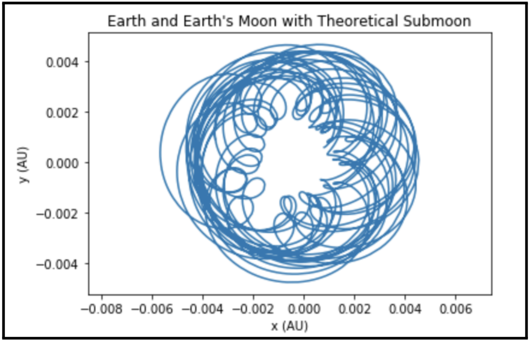

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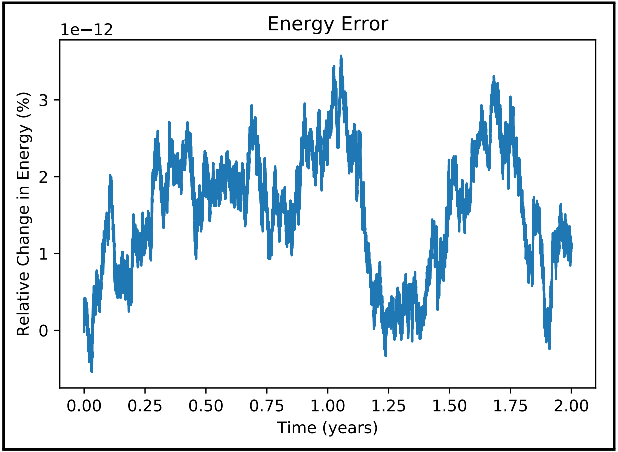
Assessing the accuracy of a computational model

- if you had some other way of knowing that the trajectories were accurate, you wouldn't need a computational model to begin with



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- "The percent error in the energy only gives a general indication of how well the simulation agrees with reality. Although low energy deviation does not necessarily imply correlation with reality, a high energy deviation almost certainly means that the simulation is too imprecise." – Andy Wang, NCSSM '20
- i.e., energy conservation is necessary but not sufficient

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On the accuracy of symplectic integrators for secularly evolving planetary systems

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[2] *Department of Astronomy and Astrophysics, University of Toronto, Toronto, Ontario, M5S 3H4, Canada*
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Rein, Brown, & Tamayo 2019, 10.1093/mnras/stz2942; arXiv:1908.03468

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Accepted 2019 Octo

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ABSTRACT

Symplectic integrators have made it possible to study the long-term evolution of planetary systems with direct N-body simulations. In this paper we reassess the accuracy of such simulations by running a convergence test on 20 Myr integrations of the Solar System using various symplectic integrators. We find that the specific choice of metric for determining a simulation's accuracy is important. Only looking at metrics related to integrals of motions such as the energy error can overestimate the accuracy of a method. As one specific example, we show that symplectic correctors do not improve the accuracy of secular frequencies compared to the standard Wisdom-Holman method without symplectic correctors, despite the fact that the energy error is three orders of magnitudes smaller. We present a framework to trace the origin of this apparent paradox to one term in the shadow Hamiltonian. Specifically, we find a term that leads to negligible contributions to the energy error but introduces non-oscillatory errors that result in artificial periastron precession. This term is the dominant error when determining secular frequencies of the system. We show that higher order symplectic methods such as the Wisdom-Holman method with a modified kernel or the SABAC family of integrators perform significantly better in secularly evolving systems because they remove this specific term.

Key words: methods: numerical — gravitation — planets and satellites: dynamical evolution and stability

Rein, Brown, & Tamayo 2019, 10.1093/mnras/stz2942; arXiv:1908.03468

Thank you!

Questions?

Feel free to send me any questions at kyle.slinker@ncssm.edu.

Materials can be found at https://github.com/kyle-slinker/teaching-resources.