On the last page of these instructions are a series of questions to answer as you move through the activity.

Getting started with the simulation

The file `n_body_for_lab.ipynb` is a notebook containing a solar system simulation code which has (mostly) been written for you. Blanks have been left in two functions (`get_potential` and `get_acceleration`) which you need to fill in before you can start using it. Write all your code between the lines of comment characters (`#################################`). In each case, your goal is to make sure the function returns the proper thing; it already returns zeros if the inputted objects are the same, you need to set the return value in the case of distinct objects.

For the first function, you need to compute the gravitational binding energy between two objects. Mathematically, this formula should be

$$U = -G\frac{m_1 m_2}{|\vec{p}_1 - \vec{p}_2|}.$$

For the second function, you need to compute the acceleration which an object undergoes due to its interaction with another object. Mathematically, this formula should be

$$\vec{a}_1 = \frac{\vec{F}_{2\text{ on }1}}{m_1} = -G\frac{m_2}{|\vec{p}_1 - \vec{p}_2|^3}\left(\vec{p}_1 - \vec{p}_2\right).$$

You should run these functions multiple times in some simple cases to verify that they are working how you would expect. **When you think you have code written for these two functions, check with your instructor before moving on.**

Launching a simulation

In order to run a simulation (once you have the energy and acceleration functions working) all you need to do is choose values for `dt` and `tmax` and choose which objects you would like to include in your simulation. The objects you want to simulate should be in the list called `objects`; a few versions of this list have been defined to give you an idea of how to set things up.

The parameters already specified in the file when you get it (`dt=0.0002` and `tmax=0.2`) will simulate the Sun along with the eight planets for 0.2 years. This is a good starting point while troubleshooting the code. Once you have it working, extend the duration of your simulations with `tmax=5.0`.

With everything set up, it is recommended that you go to the `Kernel` menu and choose `Restart & Run All`. Alternatively, you can hit `Shift-Enter` for each cell, but if you run things out of order or re-run things when you have previously run something you may get unexpected behavior (it's also just more work). Make sure the simulation works as expected. You may need to give it a few minutes to run. You should use this simulation as your baseline simulation (Sun and planets, `dt=0.0002`, `tmax=0.2`, and using first order time stepping [more on that later]); you will generally want to keep most of these as their default values and change only one thing to study its effect.

Once you've got it working, it's time to study a few different systems! **As you do this, make sure you are answering the questions posed at the end of these instructions.**

Conservation of energy

Look at the plot showing the percent change of energy as a function of time (for any simulation). Think about what this plot is telling you.

The effects of `dt`

Now try varying the parameter `dt` found near the beginning of the code. Run the planets simulation for a few values of `dt`. What do you notice?

The effects of order

Look for these lines in the code:

```
#evolve the system; this is the part where the simulation is actually carried out
for t in range(0,(int)(tmax/dt)):
    x,v=first_order_step(x,v,m,nmax,newtong,t%10==0)
```

This is how the simulation moves each of the planets forward in time. Try replacing the function call with `first_order_step` with `fourth_order_step`. Re-run the planets simulation using the fourth order timestep. What do you notice? How do you think these two functions might be different? What you see here will likely convince you that you should use `fourth_order_step` for the rest of your project.

Sun-Earth-Moon

Set up a simulation which includes the Sun, the Earth, and the Moon (and no other objects). Run it for at least 2 years. Make sure to look at the distance between the Moon and the Earth as a function of time (note that this is the distance between the Moon and the Earth, not the Sun). The plots included near the end of the code should give you an idea of how to do this. What choices do you need to make for `dt` and the order of the timestep?

Galilean satellites

Set up a simulation which includes the Sun, Jupiter, and the four Galilean satellites (and no other objects). How long can you run it? Make sure to make a plot of the positions of the four moons centered around Jupiter. Again, the plots included near the end of the code should give you an idea of how to do this. What choices do you need to make for `dt` and the order of the timestep?

Ceres

Set up a simulation for Ceres. You may be able to find more exact information about Ceres's orbit online, but for this activity only use the following information:

- Ceres has a semi-major axis of about 2.8 AU.
- Ceres has a mass of about $4.7 \times 10^{-10} M_{\odot}$.

Clearly state any assumptions you must make in order to determine a starting position and velocity for Ceres based on the limited information provided.

Your own simulation

Pick your own set of objects to simulate. Data is included for a whole host of objects; you should be able to skim through the code to see which ones are available. Ask your instructor if you have questions about this. You can also create your own objects (a satellite, Vulcan, Planet Nine, Nemesis, etc.) or find other objects on the NASA Horizons database.

Think about values you want to choose for `dt` and `tmax` and if you want to use `first_order_step` or `fourth_order_step`. What factors go into your choice? What sorts of plots do you need to show what it is you have simulated?

After your simulation runs, look at the plot of energy variation. Were your choices for the parameters good ones? How does the energy variation inform your answer to this?

Questions

Make sure all the plots you include are clearly labeled and easy for your instructor to find.

1. Write down the code you used to compute $-G\dfrac{m_1 m_2}{|\vec{p}_1 - \vec{p}_2|}$.
2. Write down the code you used to compute $-G\dfrac{m_2}{|\vec{p}_1 - \vec{p}_2|^3}\,(\vec{p}_1 - \vec{p}_2)$.
3. You may notice that `get_potential` and `get_acceleration` both contain `if` statements. Computationally, these help to avoid dividing by zero, but they serve another purpose as well. Which pairs of objects is this skipping?
4. Attach a plot of the percent change in energy over time (for any simulation). What does this plot tell you? Do you expect energy to be conserved? Why or why not? Make sure you label this plot well so your instructor can figure out what it is you're showing them.
5. What happens when you vary `dt`? How large can you make it? How small? What are advantages and disadvantages to making it larger or smaller? Does your answer depend on which simulation you are running? Why or why not?
6. In the function `first_order_step`, what do the following lines of code do?

   ```
   x[:,:,-1]=x[:,:,-2]+v[:,:,-2]*dt+0.5*a*dt2
   v[:,:,-1]=v[:,:,-2]+a*dt
   ```

   They should be familiar; re-write the equations represented by this code in a more recognizable form.
7. What changes when you change the order of the time step? What are the two functions `first_order_step` and `fourth_order_step` doing differently? Your findings should give you some indication if the DVAT equations are accurate. Are they? Do you expect them to be?
8. Attach a plot of the separation of the Earth and Moon over a period of 2 years. What do you notice in this plot? There should be two main features which happen on two different time scales. Make sure you label this plot well so your instructor can figure out what it is you're showing them. What did you choose for `dt` and the order of the timestep?
9. Attach a plot of showing the Galilean satellites orbiting Jupiter. Do you have any comments about it? Make sure you label this plot well so your instructor can figure out what it is you're showing them. What did you choose for `dt` and the order of the timestep?
10. Attach a plot showing the orbit of Ceres along with the inner planets and Jupiter.
    (a) How did you decide on the initial position and velocity of Ceres? Make sure you show mathematically how you chose any specific numbers you used.
    (b) What assumptions did you need to make in order to choose these initial conditions?
11. Your own simulation:
    (a) Which objects did you simulate? Why did you pick these?
    (b) What did you choose for `dt` and `tmax`? Did you use `first_order_step` or `fourth_order_step`? What factors go into your choices?
    (c) Attach plots which show what you have simulated. Do you see anything interesting? Make sure you label these plots well so your instructor can figure out what it is you're showing them.
    (d) Attach a plot of energy variation for your simulation. Were your choices good ones? How does the energy variation inform your answer to this? Make sure you label this plot well so your instructor can figure out what it is you're showing them.