In this lab, you will explore many properties of orbits, especially as it applies to a simulation of the solar system. The gravitational interaction of two objects is relatively simple to describe, but when many objects start interacting things get much more complicated. This is where computational models and simulations come in.

As you work through this lab, make sure you are answering the questions throughout the document. They are also included at the end to give you a concrete list of deliverables; but make sure you look at the question in the full instructions to get context.

Getting Python

If you have not used Python before (and I'm not expecting that you have) you can run Python on Google Collabs.

If you believe that you'll be using Python a lot (which is pretty likely if you're going to do science), I recommend that you obtain Python on your own computer by downloading Anaconda `https://www.anaconda.com/products/individual`. Follow the instructions on the website to download the most recent version of Python. This should work on any operating system. You can edit your code in a browser-based front end through Jupyter.

If you already have experience with Python, feel free to use whatever incarnation of it you like best.

Getting started with the simulation

The file `n_body_for_lab.ipynb` is a notebook containing a solar system simulation code which has (mostly) been written for you. Blanks have been left in the two functions (`get_potential` and `get_acceleration`) which you need to fill in before you can start using it. Write all your code between the pairs of comment characters (`##################################`). In each case, your goal is to make sure the function returns the proper thing; it already returns zeros if the inputted objects are the same, you need to set the return value in the case of distinct objects.

For the first function, you need to compute the gravitational binding energy between two objects. Mathematically, this formula should be

$$U = -G\frac{m_1 m_2}{|\vec{p}_1 - \vec{p}_2|}.$$

Question 1: Write down the code you used to compute $-G\frac{m_1 m_2}{|\vec{p}_1 - \vec{p}_2|}$.

For the second function, you need to compute the acceleration which an object undergoes due to its interaction with another object. Mathematically, this formula should be

$$\vec{a}_1 = \frac{\vec{F}_{2\text{ on }1}}{m_1} = -G\frac{m_2}{|\vec{p}_1 - \vec{p}_2|^3}\left(\vec{p}_1 - \vec{p}_2\right).$$

Question 2: Write down the code you used to compute $-G\frac{m_2}{|\vec{p}_1 - \vec{p}_2|^3}\left(\vec{p}_1 - \vec{p}_2\right)$.

You should run these functions multiple times in some simple cases to verify that they are working how you would expect.

Launching a simulation

In order to run a simulation (once you have the energy and acceleration functions working) all you need to do is choose values for `dt` and `tmax` and choose which objects you would like to include in your simulation. The objects you want to simulate should be in the list called `objects`; a few versions of this list have been defined to give you an idea of how to set things up.

The parameters already specified in the file when you get it (`dt=0.0002` and `tmax=0.2`) will simulate the Sun along with the eight planets for 0.2 years. This is a good starting point while troubleshooting the code. Once you have it working, extend the duration of your simulations with `tmax=5.0`.

For Jupyter notebooks: **With everything set up, it is recommended that you go to the `Kernel` menu and choose `Restart & Run All`**. Alternatively, you can hit `Shift-Enter` for each cell, but if you run things out of order or re-run things when you have previously run something you may get unexpected behavior (it's also just more work). Make sure the simulation works as expected. You may need to give it a few minutes to run. You should use this simulation as your baseline simulation (Sun and planets, `dt=0.0002`, `tmax=0.2`, and using first order time stepping [more on that later]); you will generally want to keep most of these as their default values and change only one thing to study its effect.

Once you've got it working, it's time to study a few different systems!

Conservation of energy

Look at the plot showing the percent change of energy as a function of time (for any simulation). Think about what this plot is telling you.

Question 3: Attach a plot of the percent change in energy over time (for any simulation). What does this plot tell you? Do you expect energy to be conserved? Why or why not?

The effects of `dt`

Now try varying the parameter `dt` found near the beginning of the code. Run the planets simulation for a few values of `dt`. Also run the Galilean satellites simulation for a few values of `dt`.

Question 4: What happens when you vary `dt`? How large can you make it? How small? What are advantages and disadvantages to making it larger or smaller? Does your answer depend on which simulation you are running? Why or why not?

The effects of order

Re-run the planets simulation using the fourth order timestep. To change the order, change the `order` parameter near the beginning of the code; it must either equal 1 or 4. What do you notice? Take a look at the code in the functions `first_order_step` and `fourth_order_step` and particularly pay attention to how they handle the acceleration. How do you think these two functions might be different? What you see here will likely convince you that you should use `fourth_order_step` for the rest of your project.

Question 5: In the function `first_order_step`, what do the following lines of code do? They should be familiar; where have you seen them before?

```
x[:,:,-1]=x[:,:,-2]+v[:,:,-2]*dt+0.5*a*dt**2
v[:,:,-1]=v[:,:,-2]+a*dt
```

Question 6: What changes when you change the order of the time step? What are the two functions `first_order_step` and `fourt_order_step` doing differently with the acceleration? Your findings should give you some indication if the DVAT equations are accurate. Are they? Do you expect them to be?

Sun-Earth-Moon

Set up a simulation which includes the Sun, the Earth, the Moon, and no other objects. Run it for at least 2 years. Make sure to look at the distance between the Moon and the Earth as a function of time (note that this is the distance between the Moon and the Earth, not the Sun). The plots included near the end of the code should give you an idea of how to do this. What choices do you need to make for `dt` and the order of the timestep?

Question 7: Attach a plot of the separation of the Earth and Moon over a period of 2 years (i.e., how the Earth-Moon distance varies over time). What do you notice in this plot? There should be two main features which happen on two different time scales. What did you choose for `dt` and the order of the timestep?

Galilean satellites

Set up a simulation which includes the Sun, Jupiter, the four Galilean satellites, and no other objects. What choices do you need to make for `dt` and the order of the timestep?

Question 8: Attach two plots showing the trajectories of the Galilean satellites orbiting around Jupiter, one with the Sun as the origin and one with Jupiter as the origin. What did you choose for `dt` and the order of the timestep?

Ceres

Set up a simulation for Ceres. You may be able to find more exact information about Ceres's orbit online, but for this activity **only** use the following information:

- Ceres has a semi-major axis of about 2.8 AU.
- Ceres has a mass of about $4.7 \times 10^{-10} M_\odot$.

Question 9: Attach a plot showing the orbit of Ceres along with the inner planets and Jupiter.

Question 10: How did you decide on the initial position and velocity of Ceres? Make sure you show mathematically how you chose any specific numbers you used.

Question 11: Clearly state any assumptions you must make in order to determine a starting position and velocity for Ceres based on the limited information provided.

Opposition of Mars

Determine when Mars is next in opposition with Earth. Opposition means that the Earth, Mars, and the Sun all fall on a straight line (ignoring their z-coordinates) with Earth in the middle. This is the closest Earth and Mars get on each of their orbits.[1] As with Ceres above, this is something you can find online; the point here is to determine it for yourself using the simulation. Keep in mind that the data for all the bodies in the simulation is from Jan 1, 2015.

Question 12: When is Mars next in opposition with Earth (after the current date)? How far apart are the two bodies at this time?

Question 13: Describe how you determined when Mars's opposition happened. What did you need to simulate to determine this? Include any relevant plots you used in your reasoning.

[1] It is also typically one of the best times to view Mars because it is highest in Earth's sky right at midnight.

Questions

Make sure all plots are clearly labeled!

Question 1: Write down the code you used to compute $-G\dfrac{m_1 m_2}{|\vec{p}_1 - \vec{p}_2|}$.

Question 2: Write down the code you used to compute $-G\dfrac{m_2}{|\vec{p}_1 - \vec{p}_2|^3}(\vec{p}_1 - \vec{p}_2)$.

Question 3: Attach a plot of the percent change in energy over time (for any simulation). What does this plot tell you? Do you expect energy to be conserved? Why or why not?

Question 4: What happens when you vary `dt`? How large can you make it? How small? What are advantages and disadvantages to making it larger or smaller? Does your answer depend on which simulation you are running? Why or why not?

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Question 8: Attach two plots showing the trajectories of the Galilean satellites orbiting around Jupiter, one with the Sun as the origin and one with Jupiter as the origin. What did you choose for `dt` and the order of the timestep?

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