[28 SEP, 2013]

$\boxed{\text{BELT DERIVATION}}$ 1

$$\int d\phi \; e^{-\frac{1}{2\sigma^2}[\phi - F]^2} \int d\rho_i \; e^{-\sum_i \lambda \rho_i \log(\rho_i/\rho_i^0)} \; \delta\Big[\sum_i f_i \rho_i - \phi\Big] \; \delta\Big[\sum_i \rho_i - 1\Big]$$

check Lagrange multipliers to get MAXENT –

MAXIMIZE: $A = -\sum_i \lambda \rho_i \log(\rho_i/\rho_i^0) + \alpha\Big[\sum_i f_i \rho_i - \phi\Big] + \beta\Big[\sum_i \rho_i - 1\Big]$

$$\frac{\partial A}{\partial \rho_i} = -\lambda \log(\rho_i/\rho_0) - \lambda + \alpha f_i + \beta = 0$$

$$\rho_i = \rho_0 \, e^{\alpha f_i} / Z$$

where α, Z is chosen so that $\sum f_i \rho_i = \phi$
$\sum \rho_i = 1.0$.

get second derivative:

$$\frac{\partial^2 A}{\partial \rho_i \partial \rho_j} = -\lambda/\rho_i \, \delta_{ij} .$$

$$A = S_{\substack{\text{max}\\\text{ent}}}(\phi) - \frac{1}{2}\sum_i \left[\frac{\lambda}{\rho_i}\right] \Delta\rho^2 + \ldots ,$$

with $\Delta\rho = [\rho_i - \rho_i^{\text{max ent}}]$.

2)

$$\int d\rho_i\; e^{A}$$

$$= \int d\Delta\rho_i\; e^{\left[S(\phi) - \frac{1}{2}\sum\left(\frac{\lambda}{\rho_i}\right)\Delta\rho_i^2\right]} + \delta\left(\sum\Delta\rho_i\right) + \delta\left(\sum f_i\Delta\rho_i\right)$$

$$= e^{S(\phi)}\int d\Delta\rho_i\, d\gamma\, d\chi\; e^{-\frac{1}{2}\sum\left(\frac{\lambda}{\rho_i^{ME}}\right)\Delta\rho_i^2 + i(\gamma\Delta\rho_i + \chi f_i\Delta\rho_i)}\left(\frac{1}{2\pi}\right)^2$$

$$= e^{S(\phi)}\left(\frac{1}{2\pi}\right)^2\int d\gamma\, d\chi\prod_i\left[\int d\Delta\rho_i\; e^{-\frac{1}{2}\frac{\lambda}{\rho_i^{ME}}\left[\Delta\rho + \left(\frac{\rho_i^{ME}}{\lambda}\right)i(\gamma+\chi f_i)\right]^2 - \frac{1}{2}\left(\frac{\rho_i}{\lambda}\right)(\gamma+\chi f_i)^2}\right]$$

$$= \frac{e^{S(\phi)}}{(2\pi)^2}\int d\gamma\, d\chi\prod_i(2\pi)^{1/2}\left(\frac{\rho_i^{ME}}{\lambda}\right)^{1/2} e^{-\frac{1}{2}\left(\frac{\rho_i^{ME}}{\lambda}\right)(\gamma+\chi f_i)^2}$$

$$= \frac{e^{S(\phi)}}{(2\pi)^2}\left(\frac{2\pi}{\lambda}\right)^{N/2}\sqrt{\prod_{i=1}^{N}\rho_i^{ME}}\int d\gamma\, d\chi\,\exp\left[-\frac{1}{2}\sum_i\left[\frac{\rho_i^{ME}}{\lambda}\right]\gamma^2 - \sum_i\left(\frac{\rho_i^{ME}}{\lambda}\right)\chi f_i\gamma + \frac{1}{2}\sum_i\frac{\rho_i^{ME}}{\lambda}\chi^2 f_i^2\right]$$

$$= \frac{e^{S}}{(2\pi)^2}\left(\frac{2\pi}{\lambda}\right)^{N/2}\sqrt{\prod_{i=1}^{N}\rho_i^{ME}}\int d\gamma\, d\chi\,\exp\left[-\frac{1}{2\lambda}\gamma^2 - \frac{\langle f\rangle}{\lambda}\chi\gamma + \frac{1}{2}\frac{\langle f^2\rangle^{ME}}{\lambda}\chi^2\right]$$

$$= \frac{e^{S}}{(2\pi)^2}\left(\frac{2\pi}{\lambda}\right)^{N}\sqrt{\prod_{i=1}^{N}\rho_i^{ME}}\int d\chi\,\sqrt{2\pi\lambda}\,\exp\left[-\left[\frac{\langle f^2\rangle^{ME} - \langle f\rangle^{ME\,2}}{2\lambda}\right]\chi^2\right]$$

$$= \frac{e^{S}}{2\pi}\lambda\cdot\left(\frac{2\pi}{\lambda}\right)^{N}\underbrace{\sqrt{\prod_{i=1}^{N}\rho_i^{ME}}}\;\underbrace{\frac{1}{\sqrt{\Delta f^2_{ME}}}}$$

$\boxed{\text{TWO DEPENDENCIES ON } \phi = \langle f\rangle.}$