We wish to invert the function $\Delta\Sigma(\rho)$, where $\Delta\Sigma(R) = \bar{\Sigma}(< R) - \Sigma(R)$ is the excess surface density (ESD), defined in terms of the surface density profile $\Sigma(R)$ and the mean enclosed surface density profile $\bar{\Sigma}(< R)$, where $R$ is the projected radial coordinate. $\rho(r)$ is the volume density profile, where $r$ is the spherical radial coordinate. The surface and volume density profiles are related as:

$$\Sigma(R) = 2\int_0^\infty \rho(R,z)dz \tag{1}$$

with $r^2 = R^2 + z^2$.

We assume a piecewise powerlaw volume density profile defined over a set of $N-1$ intervals $[r_n, r_{n+1})$:

$$\log\rho = a_n \log(r) + b_n \tag{2}$$

$$a_n = \frac{\log(\rho_{n+1}) - \log(\rho_n)}{\log(r_{n+1}) - \log(r_n)} \tag{3}$$

$$b_n = \log(\rho_n) - a_n \log(r_n) \tag{4}$$

$$(a_n, b_n) = \begin{cases} (a_0, b_0) & \text{if } r < r_0 \\ (a_n, b_n) & \text{if } r_n \le r < r_{n+1} \\ (a_{N-1}, b_{N-1}) & \text{if } r \ge r_N \end{cases} \tag{5}$$

(Throughout, log represents the natural logarithm.) Evaluating Eq. 1 for this volume density profile yields:

$$\Sigma(R) = \sum_{n=0}^{N-1} \begin{cases} 2e^{b_n} I_1(r_{n+1}, R, a_n) - I_1(r_n, R, a_n) & \text{if } R < r_n \\ 2e^{b_n} I_1(r_{n+1}, R, a_n) & \text{if } r_n \le R < r_{n+1} \\ 0 & \text{if } R \ge r_{n+1} \end{cases} \tag{6}$$

$$I_1(r, R, a) = \begin{cases} \sqrt{r^2 - R^2} R^a {}_2F_1\left(\frac{1}{2}, \frac{-a}{2}; \frac{3}{2}; 1 - \frac{r^2}{R^2}\right) & \text{if } r \text{ is finite} \\ \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(-\frac{a+1}{2}\right)}{\Gamma\left(-\frac{a}{2}\right)} R^{a+1} & \text{if } r = \infty \end{cases} \tag{7}$$