Probabilistic Sharpe Ratio

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Abstract

This vignette gives an overview of the Probabilistic Sharpe Ratio , Minimum Track Record Length and the Probabilistic Sharpe Ratio Optimization technique used to find the optimal portfolio that maximizes the Probabilistic Sharpe Ratio. It gives an overview of the usability of the functions and its application"

1 Probabilistic Sharpe Ratio

Given a predefined benchmark Sharpe ratio SR^* , the observed Sharpe ratio $\hat{SR}$ can be expressed in probabilistic terms as

$$\hat{PSR}(SR^*) = Z\Big[\frac{(\hat{SR} - SR^*)\sqrt{n-1}}{\sqrt{1 - \hat{\gamma_3}SR^* + \frac{\hat{\gamma_4}-1}{4}\hat{SR^2}}}\Big]$$

Here n is the track record length or the number of data points. It can be daily,weekly or yearly depending on the input given

$\hat{\gamma_3}$ and $\hat{\gamma_4}$ are the skewness and kurtosis respectively. It is not unusual to find strategies with irregular trading frequencies, such as weekly strategies that may not trade for a month. This poses a problem when computing an annualized Sharpe ratio, and there is no consensus as how skill should be measured in the context of irregular bets. Because PSR measures skill in probabilistic terms, it is invariant to calendar conventions. All calculations are done in the original frequency of the data, and there is no annualization.

```
> data(edhec)
> ProbSharpeRatio(edhec,refSR = 0.28)

                                    Convertible Arbitrage CTA Global
Probabilistic Sharpe Ratio(p= 95 %):             0.6275225  0.3916662
                                    Distressed Securities Emerging Markets
```

```
Probabilistic Sharpe Ratio(p= 95 %):             0.9098406         0.239057
                                        Equity Market Neutral Event Driven
Probabilistic Sharpe Ratio(p= 95 %):             0.9861596    0.8841726
                                        Fixed Income Arbitrage Global Macro
Probabilistic Sharpe Ratio(p= 95 %):              0.5571144    0.9947881
                                        Long/Short Equity Merger Arbitrage
Probabilistic Sharpe Ratio(p= 95 %):         0.7895071        0.9949854
                                        Relative Value Short Selling
Probabilistic Sharpe Ratio(p= 95 %):      0.9589513   0.005129677
                                        Funds of Funds
Probabilistic Sharpe Ratio(p= 95 %):      0.6923129
```

2 Minimum Track Record Length

If a track record is shorter than Minimum Track Record Length(MinTRL), we do not have enough confidence that the observed $\hat{SR}$ is above the designated threshold SR^*. Minimum Track Record Length is given by the following expression.

$$MinTRL = n^* = 1 + \left[1 - \hat{\gamma_3}\hat{SR} + \frac{\hat{\gamma_4}}{4}\hat{SR^2}\right]\left(\frac{Z_\alpha}{\hat{SR} - SR^*}\right)^2$$

γ_3 and γ_4 are the skewness and kurtosis respectively. It is important to note that MinTRL is expressed in terms of number of observations, not annual or calendar terms.

```
> data(edhec)
> MinTrackRecord(edhec,refSR = 0.28)
```

```
                                        Convertible Arbitrage CTA Global
Minimum Track Record Length(p= 95 %):              3861.706   5403.966
                                        Distressed Securities Emerging Markets
Minimum Track Record Length(p= 95 %):               228.598         812.9394
                                        Equity Market Neutral Event Driven
Minimum Track Record Length(p= 95 %):                85.272     286.5561
                                        Fixed Income Arbitrage Global Macro
Minimum Track Record Length(p= 95 %):               19796.97     63.26741
                                        Long/Short Equity Merger Arbitrage
Minimum Track Record Length(p= 95 %):          631.8851         62.62216
                                        Relative Value Short Selling
Minimum Track Record Length(p= 95 %):       136.1482      63.00008
                                        Funds of Funds
Minimum Track Record Length(p= 95 %):       1619.463
```

3 Probabilistic Sharpe Ratio Optimal Portfolio

We would like to find the vector of weights that maximize the expression

$$P\hat{S}R(SR^*) = Z\Big[\frac{(\hat{SR} - SR^*)\sqrt{n-1}}{\sqrt{1 - \hat{\gamma_3}SR^* + \frac{\hat{\gamma_4}-1}{4}\hat{SR^2}}}\Big]$$

where $\sigma = \sqrt{E[(r-\mu)^2]}$,its standard deviation.$\gamma_3 = \frac{E\Big[(r-\mu)^3\Big]}{\sigma^3}$ its skewness,$\gamma_4 = \frac{E\Big[(r-\mu)^4\Big]}{\sigma^4}$ its kurtosis and $SR = \frac{\mu}{\sigma}$ its Sharpe Ratio.

Because $P\hat{S}R(SR^*) = Z[\hat{Z}^*]$ is a monotonic increasing function of $\hat{Z}^*$,it suffices to compute the vector that maximizes $\hat{Z}^*$ This optimal vector is invariant of the value adopted by the parameter SR^*.

```
> data(edhec)
> PsrPortfolio(edhec)
```

```
                           weight
Convertible Arbitrage  0.07775063
CTA Global             0.07775063
Distressed Securities  0.07775063
Emerging Markets       0.06699249
Equity Market Neutral  0.07775063
Event Driven           0.07775063
Fixed Income Arbitrage 0.07775063
Global Macro           0.07775063
Long/Short Equity      0.07775063
Merger Arbitrage       0.07775063
Relative Value         0.07775063
Short Selling          0.07775063
Funds of Funds         0.07775063
```