# Creating a Portfolio Object with Portfolio Analytics

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## Abstract

The purpose of this vignette is to demonstrate the new interface in PortfolioAnalytics to specify a portfolio object, add constraints and objectis, and run optimizations.

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# 1 Getting Started

# 1.1 Load Packages

Load the necessary packages.

```
library(PortfolioAnalytics)

## Loading required package: zoo

##

## Attaching package: 'zoo'

## The following object(s) are masked from 'package:base':

##

## as.Date, as.Date.numeric

## Loading required package: xts

## Loading required package: PerformanceAnalytics
```

```
##
## Attaching package: 'PerformanceAnalytics'
## The following object(s) are masked from 'package:graphics':
##
## legend
```

#### 1.2 Data

The edhec data set from the PerformanceAnalytics package will be used as example data.

```
data(edhec)
# Use the first 4 columns in edhec for a returns object
returns <- edhec[, 1:4]
colnames(returns) <- c("CA", "CTAG", "DS", "EM")</pre>
print(head(returns, 5))
##
                  CA
                        CTAG
                                   DS
                                           EM
## 1997-01-31 0.0119 0.0393 0.0178
## 1997-02-28 0.0123 0.0298 0.0122 0.0525
## 1997-03-31 0.0078 -0.0021 -0.0012 -0.0120
## 1997-04-30 0.0086 -0.0170 0.0030 0.0119
## 1997-05-31 0.0156 -0.0015 0.0233
# Get a character vector of the fund names
fund.names <- colnames(returns)</pre>
```

# 2 Creating the Portfolio Object

The portfolio object is instantiated with the portfolio.spec function. The main argument to portfolio.spec is assets, this is a required argument. The assets argument can be a scalar value for the number of assets, a character vector of fund names, or a named vector of initial weights. If initial weights are not specified, an equal weight portfolio will be assumed.

The pspec object is an S3 object of class "portfolio". When first created, the portfolio object has an element named assets with the initial weights, an element named category\_labels, an

element named weight\_seq with sequence of weights if specified, an empty constraints list and an empty objectives list.

```
# Specify a portfolio object by passing a character vector for the assets
# argument.
pspec <- portfolio.spec(assets = fund.names)</pre>
print.default(pspec)
## $assets
    CA CTAG
##
               DS
                     EM
## 0.25 0.25 0.25 0.25
##
## $category_labels
## NULL
##
## $weight_seq
## NULL
##
## $constraints
## list()
##
## $objectives
## list()
##
## $call
## portfolio.spec(assets = fund.names)
##
## attr(,"class")
## [1] "portfolio.spec" "portfolio"
```

# 3 Adding Constraints to the Portfolio Object

Adding constraints to the portfolio object is done with add.constraint. The add.constraint function is the main interface for adding and/or updating constraints to the portfolio object. This function allows the user to specify the portfolio to add the constraints to, the type of constraints,

arguments for the constraint, and whether or not to enable the constraint (enabled=TRUE is the default). If updating an existing constraint, the indexnum argument can be specified.

### 3.1 Leverage Constraint

The leverage constraint specifies the constraint on the sum of the weights. Aliases for the leverage constraint type include weight\_sum, weight, and leverage. Here we add a constraint that the weights must sum to 1, or the full investment constraint.

There are two special cases for the leverage constraint:

- 1. The sum of the weights equal 1, i.e. the full investment constraint. The full investment constraint can be specified with type="full\_investment". This automatically sets min\_sum=1 and max\_sum=1.
- 2. The sum of the weights equal 0, i.e. the dollar neutral or active constraint. This constraint can be specified with type="dollar\_neutral" or type="active".

```
# The full investment constraint can also be specified with

# type='full_investment' pspec <- add.constraint(portfolio=pspec,

# type='full_investment')

# Another common constraint is that portfolio weights sum to 0. This can

# be specified any of the following ways pspec <-

# add.constraint(portfolio=pspec, type='weight_sum', min_sum=0, max_sum=0)

# pspec <- add.constraint(portfolio=pspec, type='dollar_neutral') pspec <-

# add.constraint(portfolio=pspec, type='dollar_neutral')</pre>
```

### 3.2 Box Constraint

Box constraints allows the user to specify upper and lower bounds on the weights of the assets. Here we add box constraints for the asset weights so that the minimum weight of any asset must be greater than or equal to 0.05 and the maximum weight of any asset must be less than or equal to 0.4. The values for min and max can be passed in as scalars or vectors. If min and max are scalars, the values for min and max will be replicated as vectors to the length of assets. If min and max are not specified, a minimum weight of 0 and maximum weight of 1 are assumed. Note that min and max can be specified as vectors with different weights for linear inequality constraints.

```
# Add box constraints
pspec <- add.constraint(portfolio = pspec, type = "box", min = 0.05, max = 0.4)

# min and max can also be specified per asset pspec <-
# add.constraint(portfolio=pspec, type='box', min=c(0.05, 0, 0.08, 0.1),
# max=c(0.4, 0.3, 0.7, 0.55))

# A special case of box constraints is long only where min=0 and max=1 The
# default action is long only if min and max are not specified pspec <-
# add.constraint(portfolio=pspec, type='box') pspec <-
# add.constraint(portfolio=pspec, type='long_only')</pre>
```

# 3.3 Group Constraint

Group constraints allow the user to specify the the sum of weights by group. Group constraints are currently supported by the ROI, DEoptim, and random portfolio solvers. The following code groups the assets such that the first 3 assets are grouped together labeled GroupA and the fourth asset is in its own group labeled GroupB. The group\_min argument specifies that the sum of the weights in GroupA must be greater than or equal to 0.1 and the sum of the weights in GroupB must be greater than or equal to 0.15. The group\_max argument specifies that the sum of the weights in GroupA must be less than or equal to 0.85 and the sum of the weights in GroupB must be less than or equal to 0.55. The group\_labels argument is optional and is useful if groups is not a named list for labeling groups in terms of market capitalization, sector, etc.

#### 3.4 Position Limit Constraint

The position limit constraint allows the user to specify limits on the number of assets with non-zero, long, or short positions. The ROI solver interfaces to the Rglpk package (i.e. using the glpk plugin) for solving maximizing return and ETL/ES/cVaR objectives. The Rglpk package supports integer programming and thus supports position limit constraints for the max\_pos argument. The quadprog package does not support integer programming, and therefore max\_pos is not supported for the ROI solver using the quadprog plugin. Note that max\_pos\_long and max\_pos\_short are not supported for either ROI solver. All position limit constraints are fully supported for DEoptim and random solvers.

```
# Add position limit constraint such that we have a maximum number of
# three assets with non-zero weights.
pspec <- add.constraint(portfolio = pspec, type = "position_limit", max_pos = 3)
# Can also specify maximum number of long positions and short positions
# pspec <- add.constraint(portfolio=pspec, type='position_limit',
# max_pos_long=3, max_pos_short=3)</pre>
```

#### 3.5 Diversification Constraint

The diversification constraint allows the user to target diversification. Diversification is defined as  $diversification = \sum_{i=1}^{N} w_i^2$  for N assets. The diversification constraint is implemented for the global optimizers by applying a penalty if the diversification value is more than 5% away from  $div_{target}$ . Note that diversification as a constraint is not supported for the ROI solvers, it is only supported for the global numeric solvers.

```
pspec <- add.constraint(portfolio = pspec, type = "diversification", div_target = 0.7)</pre>
```

#### 3.6 Turnover Constraint

A target turnover can be specified as a constraint. The turnover is calculated from a set of initial weights. The initial weights can be specified, by default they are the initial weights in the portfolio object. The turnover constraint is implemented for the global optimizers by applying a penalty if the turnover value is more than 5% away from turnover\_target. Note that the turnover constraint is not currently supported for quadratic utility and minimum variance problems using the ROI solver.

```
pspec <- add.constraint(portfolio = pspec, type = "turnover", turnover_target = 0.2)</pre>
```

## 3.7 Target Return Constraint

The target return constraint allows the user to specify a target mean return.

```
pspec <- add.constraint(portfolio = pspec, type = "return", return_target = 0.007)</pre>
```

# 3.8 Factor Exposure Constraint

The factor exposure constraint allows the user to set upper and lower bounds on exposures to risk factors. The exposures can be passed in as a vector or matrix. Here we specify a vector for B with arbitrary values, e.g. betas of the assets, with a market risk exposure range of 0.6 to 0.9.

#### 3.9 Transaction Cost Constraint

The transaction cost constraint allows the user to specify proportional transaction costs. Proportional transaction cost constraints can be implemented for quadratic utility and minimum variance problems using the ROI solver. Transaction costs are supported as a penalty for the global numeric solvers. Here we add the transaction cost contraint with the proportional transaction cost value of 1%.

```
pspec <- add.constraint(portfolio = pspec, type = "transaction_cost", ptc = 0.01)</pre>
```

The print method for the portfolio object shows a concise view of the portfolio and the constraints that have been added.

```
## portfolio.spec(assets = fund.names)
##
## Assets
## Number of assets: 4
##
## Asset Names
## [1] "CA" "CTAG" "DS" "EM"
##
## Constraints
## Number of constraints: 9
## Number of enabled constraints: 9
## Enabled constraint types
## - weight_sum
## - box
## - group
## - position_limit
## - diversification
## - turnover
## - return
## - factor_exposure
## - transaction_cost
## Number of disabled constraints: 0
## Objectives
## Number of objectives: 0
## Number of enabled objectives: 0
## Number of disabled objectives: 0
```

The  $\operatorname{summary}$  function gives a more detailed view of the constraints.

```
## CA CTAG DS EM
## 0.25 0.25 0.25 0.25
##
## Constraints:
##
## ************
## weight_sum constraint
## ************
## $type
## [1] "weight_sum"
## $enabled
## [1] TRUE
##
## $message
## [1] FALSE
##
## $min_sum
## [1] 1
##
## $max_sum
## [1] 1
##
## $call
## add.constraint(portfolio = pspec, type = "weight_sum", min_sum = 1,
     max_sum = 1)
##
##
## attr(,"class")
## [1] "weight_sum_constraint" "constraint"
##
## ************
## box constraint
## ************
```

```
## $type
## [1] "box"
##
## $enabled
## [1] TRUE
##
## $min
## CA CTAG DS EM
## 0.05 0.05 0.05 0.05
##
## $max
## CA CTAG DS EM
## 0.4 0.4 0.4 0.4
##
## $call
## add.constraint(portfolio = pspec, type = "box", min = 0.05, max = 0.4)
##
## attr(,"class")
## [1] "box_constraint" "constraint"
##
##
## *************
## group constraint
## *************
## $type
## [1] "group"
##
## $enabled
## [1] TRUE
##
## $groups
## $groups$groupA
## [1] 1 2 3
##
```

```
## $groups$grouB
## [1] 4
##
##
## $group_labels
## [1] "groupA" "grouB"
##
## $cLO
## [1] 0.10 0.15
##
## $cUP
## [1] 0.85 0.55
##
## $call
\#\# add.constraint(portfolio = pspec, type = \#group, groups = \#1 c(1,
##
      2, 3), grouB = 4), group_min = c(0.1, 0.15), group_max = c(0.85, 0.15)
      0.55))
##
##
## attr(,"class")
## [1] "group_constraint" "constraint"
##
##
## position_limit constraint
## ************
## $type
## [1] "position_limit"
## $enabled
## [1] TRUE
##
## $max_pos
## [1] 3
##
```

```
## $call
## add.constraint(portfolio = pspec, type = "position_limit", max_pos = 3)
##
## attr(,"class")
## [1] "position_limit_constraint" "constraint"
##
##
## ***********
## diversification constraint
## ************
## $type
## [1] "diversification"
## $enabled
## [1] TRUE
##
## $div_target
## [1] 0.7
##
## $call
## add.constraint(portfolio = pspec, type = "diversification", div_target = 0.7)
##
## attr(,"class")
## [1] "diversification_constraint" "constraint"
##
##
## ***********
## turnover constraint
## *********************
## $type
## [1] "turnover"
##
## $enabled
## [1] TRUE
```

```
## $turnover_target
## [1] 0.2
##
## $call
## add.constraint(portfolio = pspec, type = "turnover", turnover_target = 0.2)
##
## attr(,"class")
## [1] "turnover_constraint" "constraint"
##
## ***********
## return constraint
## ************
## $type
## [1] "return"
##
## $enabled
## [1] TRUE
##
## $return_target
## [1] 0.007
##
## $call
## add.constraint(portfolio = pspec, type = "return", return_target = 0.007)
##
## attr(,"class")
## [1] "return_constraint" "constraint"
##
##
## *************
## factor_exposure constraint
## ************
## $type
```

```
## [1] "factor_exposure"
##
## $enabled
## [1] TRUE
##
## $B
##
      factor1
## CA
        -0.08
## CTAG
        0.37
## DS
       0.79
## EM
        1.43
##
## $lower
## [1] 0.6
##
## $upper
## [1] 0.9
##
## $call
## add.constraint(portfolio = pspec, type = "factor_exposure", B = c(-0.08,
##
      0.37, 0.79, 1.43), lower = 0.6, upper = 0.9)
##
## attr(,"class")
## [1] "factor_exposure_constraint" "constraint"
##
##
## ************
## transaction_cost constraint
## ************
## $type
## [1] "transaction_cost"
##
## $enabled
## [1] TRUE
```

```
##
## $ptc
## [1] 0.01 0.01 0.01 0.01
##
## $call
## add.constraint(portfolio = pspec, type = "transaction_cost",
## ptc = 0.01)
##
## attr(,"class")
## [1] "transaction_cost_constraint" "constraint"
##
## 0bjectives:
```

This demonstrates adding constraints to the portfolio object. As an alternative to adding constraints directly to the portfolio object, constraints can be specified as separate objects.

# 3.10 Specifying Constraints as Separate Objects

The following examples will demonstrate how to specify constraints as separate objects for all constraints types.

```
# full investment constraint
weight_constr <- weight_sum_constraint(min_sum = 1, max_sum = 1)

# box constraint
box_constr <- box_constraint(assets = pspec$assets, min = 0, max = 1)

# group constraint
group_constr <- group_constraint(assets = pspec$assets, groups = list(c(1, 2, 3), 4), group_min = c(0.1, 0.15), group_max = c(0.85, 0.55), group_labels = c("GroupA", "GroupB"))

# position limit constraint
poslimit_constr <- position_limit_constraint(assets = pspec$assets, max_pos = 3)</pre>
```

```
# diversification constraint
div_constr <- diversification_constraint(div_target = 0.7)

# turnover constraint
to_constr <- turnover_constraint(turnover_target = 0.2)

# target return constraint
ret_constr <- return_constraint(return_target = 0.007)

# factor exposure constraint
exp_constr <- factor_exposure_constraint(assets = pspec$assets, B = c(-0.08, 0.37, 0.79, 1.43), lower = 0.6, upper = 0.9)

# transaction cost constraint
ptc_constr <- transaction_cost_constraint(assets = pspec$assets, ptc = 0.01)</pre>
```

# 4 Adding Objectives

Objectives can be added to the portfolio object with add.objective. The add.objective function is the main function for adding and/or updating business objectives to the portfolio object. This function allows the user to specify the portfolio to add the objectives to, the type (currently 'return', 'risk', 'risk\_budget', or 'weight\_concentration'), name of the objective function, arguments to the objective function, and whether or not to enable the objective. If updating an existing constraint, the indexnum argument can be specified.

## 4.1 Portfolio Risk Objective

The portfolio risk objective allows the user to specify a risk function to minimize Here we add a risk objective to minimize portfolio expected tail loss with a confidence level of 0.95. Other default arguments to the function can be passed in as a named list to arguments. Note that the name of the function must correspond to a function in R. Many functions are available in the PerformanceAnalytics package or a user defined function.

```
pspec <- add.objective(portfolio = pspec, type = "risk", name = "ETL", arguments = list(p = 0.95)
```

### 4.2 Portfolio Return Objective

The return objective allows the user to specify a return function to maximize. Here we add a return objective to maximize the portfolio mean return.

```
pspec <- add.objective(portfolio = pspec, type = "return", name = "mean")</pre>
```

## 4.3 Portfolio Risk Budget Objective

The portfolio risk objective allows the user to specify constraints to minimize component contribution (i.e. equal risk contribution) or specify upper and lower bounds on percentage risk contribution. Here we specify that no asset can contribute more than 30% to total portfolio risk. See the risk budget optimization vignette for more detailed examples of portfolio optimizations with risk budgets.

### 4.4 Portfolio Weight Concentration Objective

The weight concentration objective allows the user to specify an objective to minimize concentration as measured by the Herfindahl-Hirschman Index. For otpimization problems solved with the global numeric optimizers, the portfolio HHI value is penalized using conc\_aversion value as the multiplier.

For quadratic utility problems with weight concentration as an objective using the ROI solver, this is implemented as a penalty to the objective function. The objective function is implemented as follows:

$$\underset{\boldsymbol{w}}{\text{maximize}} \boldsymbol{w}' \boldsymbol{\mu} - \frac{\lambda}{2} (\boldsymbol{w}' \boldsymbol{\Sigma} \boldsymbol{w} + \lambda_{hhi} * HHI)$$
 (1)

(2)

Where  $\mu$  is the estimated mean asset returns,  $\lambda$  is the risk aversion parameter,  $lambda_{hhi}$  is the concentration aversion parameter, HHI is the portfolio HHI,  $\Sigma$  is the estimated covariance matrix of asset returns and  $\boldsymbol{w}$  is the set of weights.

Here we add a weight concentration objective for the overall portfolio HHI.

The weight concentration aversion parameter by groups can also be specified. Here we add a weight concentration objective specifying groups and concentration aversion parameters by group.

The print method for the portfolio object will now show all the constraints and objectives that have been added.

```
print(pspec)
## *************
## PortfolioAnalytics Portfolio Specification
## **************
##
## Call:
## portfolio.spec(assets = fund.names)
##
## Assets
## Number of assets: 4
##
## Asset Names
## [1] "CA"
           "CTAG" "DS"
                      "EM"
##
## Constraints
## Number of constraints: 9
## Number of enabled constraints: 9
## Enabled constraint types
```

```
## - weight_sum
## - box
## - group
## - position_limit
## - diversification
## - turnover
## - return
## - factor_exposure
## - transaction_cost
## Number of disabled constraints: 0
##
## Objectives
## Number of objectives: 5
## Number of enabled objectives: 5
## Enabled objective names
## - ETL
## - mean
## - ETL
## - HHI
## - HHI
## Number of disabled objectives: 0
```

The summary function gives a more detailed view.

```
## weight_sum constraint
## ************
## $type
## [1] "weight_sum"
##
## $enabled
## [1] TRUE
##
## $message
## [1] FALSE
## $min_sum
## [1] 1
##
## $max_sum
## [1] 1
##
## $call
## add.constraint(portfolio = pspec, type = "weight_sum", min_sum = 1,
##
     max_sum = 1)
##
## attr(,"class")
## [1] "weight_sum_constraint" "constraint"
##
##
## ************
## box constraint
## ************
## $type
## [1] "box"
##
## $enabled
## [1] TRUE
```

```
## $min
## CA CTAG DS EM
## 0.05 0.05 0.05 0.05
##
## $max
## CA CTAG DS EM
## 0.4 0.4 0.4 0.4
##
## $call
## add.constraint(portfolio = pspec, type = "box", min = 0.05, max = 0.4)
##
## attr(,"class")
## [1] "box_constraint" "constraint"
##
## ************
## group constraint
## *************
## $type
## [1] "group"
##
## $enabled
## [1] TRUE
##
## $groups
## $groups$groupA
## [1] 1 2 3
## $groups$grouB
## [1] 4
##
##
## $group_labels
## [1] "groupA" "grouB"
```

```
##
## $cLO
## [1] 0.10 0.15
##
## $cUP
## [1] 0.85 0.55
##
## $call
\#\# add.constraint(portfolio = pspec, type = "group", groups = list(groupA = c(1,
      2, 3), grouB = 4), group_min = c(0.1, 0.15), group_max = c(0.85, 0.15)
##
##
      0.55))
##
## attr(,"class")
## [1] "group_constraint" "constraint"
##
## ************
## position_limit constraint
## ************
## $type
## [1] "position_limit"
##
## $enabled
## [1] TRUE
##
## $max_pos
## [1] 3
##
## $call
## add.constraint(portfolio = pspec, type = "position_limit", max_pos = 3)
##
## attr(,"class")
## [1] "position_limit_constraint" "constraint"
```

```
## ***********
## diversification constraint
## ***********
## $type
## [1] "diversification"
##
## $enabled
## [1] TRUE
##
## $div_target
## [1] 0.7
##
## $call
## add.constraint(portfolio = pspec, type = "diversification", div_target = 0.7)
## attr(,"class")
## [1] "diversification_constraint" "constraint"
##
##
## ************
## turnover constraint
## ************
## $type
## [1] "turnover"
##
## $enabled
## [1] TRUE
## $turnover_target
## [1] 0.2
##
## $call
## add.constraint(portfolio = pspec, type = "turnover", turnover_target = 0.2)
```

```
## attr(,"class")
## [1] "turnover_constraint" "constraint"
##
##
## ************
## return constraint
## ************
## $type
## [1] "return"
## $enabled
## [1] TRUE
##
## $return_target
## [1] 0.007
##
## $call
## add.constraint(portfolio = pspec, type = "return", return_target = 0.007)
##
## attr(,"class")
## [1] "return_constraint" "constraint"
##
##
## ********************
## factor_exposure constraint
## ************
## $type
## [1] "factor_exposure"
##
## $enabled
## [1] TRUE
##
## $B
```

```
## factor1
## CA
        -0.08
## CTAG
        0.37
## DS
        0.79
      1.43
## EM
##
## $lower
## [1] 0.6
##
## $upper
## [1] 0.9
##
## $call
## add.constraint(portfolio = pspec, type = "factor_exposure", B = c(-0.08,
      0.37, 0.79, 1.43), lower = 0.6, upper = 0.9)
##
## attr(,"class")
## [1] "factor_exposure_constraint" "constraint"
##
## ***********
## transaction_cost constraint
## ********************
## $type
## [1] "transaction_cost"
##
## $enabled
## [1] TRUE
##
## $ptc
## [1] 0.01 0.01 0.01 0.01
##
## $call
## add.constraint(portfolio = pspec, type = "transaction_cost",
```

```
## ptc = 0.01)
##
## attr(,"class")
## [1] "transaction_cost_constraint" "constraint"
##
##
## Objectives:
## ************
## portfolio_risk_objective
## ************
## $name
## [1] "ETL"
##
## $target
## NULL
##
## $arguments
## $arguments$p
## [1] 0.95
##
## $arguments$portfolio_method
## [1] "single"
##
##
## $enabled
## [1] TRUE
## $multiplier
## [1] 1
##
## $call
## add.objective(portfolio = pspec, type = "risk", name = "ETL",
##
      arguments = list(p = 0.95))
```

```
## attr(,"class")
## [1] "portfolio_risk_objective" "objective"
##
##
## ************
## return_objective
## ************
## $name
## [1] "mean"
##
## $target
## NULL
##
## $arguments
## list()
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] -1
##
## $call
## add.objective(portfolio = pspec, type = "return", name = "mean")
##
## attr(,"class")
## [1] "return_objective" "objective"
##
##
## *************
## risk_budget_objective
## ************
## $name
```

```
## [1] "ETL"
##
## $target
## NULL
##
## $arguments
## $arguments$p
## [1] 0.95
##
## $arguments$portfolio_method
## [1] "component"
##
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] 1
##
## $max_prisk
## CA CTAG DS EM
## 0.3 0.3 0.3 0.3
## $min_concentration
## [1] FALSE
##
## $min_difference
## [1] FALSE
##
## $call
## add.objective(portfolio = pspec, type = "risk_budget", name = "ETL",
       arguments = list(p = 0.95), max_prisk = 0.3)
##
## attr(,"class")
```

```
## [1] "risk_budget_objective" "objective"
##
##
## ************
## weight_concentration_objective
## ************
## $name
## [1] "HHI"
##
## $target
## NULL
##
## $arguments
## list()
##
## $enabled
## [1] TRUE
## $multiplier
## [1] 1
##
## $conc_aversion
## [1] 0.1
##
## $call
## add.objective(portfolio = pspec, type = "weight_concentration",
      name = "HHI", conc_aversion = 0.1)
##
## attr(,"class")
## [1] "weight_concentration_objective" "objective"
##
##
## *************
## weight_concentration_objective
```

```
## ************
## $name
## [1] "HHI"
##
## $target
## NULL
##
## $arguments
## $arguments$groups
## $arguments$groups[[1]]
## [1] 1 2
##
## $arguments$groups[[2]]
## [1] 3 4
##
##
##
## $enabled
## [1] TRUE
##
## $multiplier
## [1] 1
##
## $conc_aversion
## [1] 0.03 0.06
##
## $conc_groups
## $conc_groups[[1]]
## [1] 1 2
##
## $conc_groups[[2]]
## [1] 3 4
##
##
```

```
## $call
## add.objective(portfolio = pspec, type = "weight_concentration",
## name = "HHI", conc_aversion = c(0.03, 0.06), conc_groups = list(c(1,
## 2), c(3, 4)))
##
## attr(,"class")
## [1] "weight_concentration_objective" "objective"
```

### 5 Solvers

The PortfolioAnalytics package currently supports random portfolios, DEoptim, pso, GenSA, and ROI as back ends. Note that some of the QP/LP problems are solved directly with Rglpk and quadprog. The solver can be specified with the optimize\_method argument in optimize.portfolio and optimize.portfolio.rebalancing.

### 5.1 DEoptim

Portfolio Analytics uses the DEoptim function from the R package DEoptim. Differential evolution is a stochastic global optimization algorithm. See ?DEoptim and the references contained therein for more information. See also Large scale portfolio optimization with DEoptim.

### 5.2 Random Portfolios

PortfolioAnalytics has three methods to generate random portfolios.

- 1. The 'sample' method to generate random portfolios is based on an idea by Pat Burns. This is the most flexible method, but also the slowest, and can generate portfolios to satisfy leverage, box, group, and position limit constraints.
- 2. The 'simplex' method to generate random portfolios is based on a paper by W. T. Shaw. The simplex method is useful to generate random portfolios with the full investment constraint, where the sum of the weights is equal to 1, and min box constraints. Values for min\_sum and max\_sum of the leverage constraint will be ignored, the sum of weights will equal 1. All other constraints such as the box constraint max, group and position limit constraints will be handled by elimination. If the constraints are very restrictive, this may result in very few feasible portfolios remaining.

3. The 'grid' method to generate random portfolios is based on the gridSearch function in package NMOF. The grid search method only satisfies the min and max box constraints. The min\_sum and max\_sum leverage constraints will likely be violated and the weights in the random portfolios should be normalized. Normalization may cause the box constraints to be violated and will be penalized in constrained\_objective.

The following plots illustrate the various methods to generate random portfolios.

```
R <- edhec[, 1:4]
# set up simple portfolio with leverage and box constraints
pspec <- portfolio.spec(assets = colnames(R))</pre>
pspec <- add.constraint(portfolio = pspec, type = "leverage", min_sum = 0.99,</pre>
   max_sum = 1.01)
pspec <- add.constraint(portfolio = pspec, type = "box", min = 0, max = 1)</pre>
# generate random portfolios using the 3 methods
rp1 <- random_portfolios(portfolio = pspec, permutations = 5000, rp_method = "sample")
## Warning: executing %dopar% sequentially: no parallel backend registered
rp2 <- random_portfolios(portfolio = pspec, permutations = 5000, rp_method = "simplex")
rp3 <- random_portfolios(portfolio = pspec, permutations = 5000, rp_method = "grid")
# show feasible portfolios in mean-StdDev space
tmp1.mean <- apply(rp1, 1, function(x) mean(R %*% x))</pre>
tmp1.StdDev <- apply(rp1, 1, function(x) StdDev(R = R, weights = x))</pre>
tmp2.mean <- apply(rp2, 1, function(x) mean(R %*% x))</pre>
tmp2.StdDev <- apply(rp2, 1, function(x) StdDev(R = R, weights = x))</pre>
tmp3.mean <- apply(rp3, 1, function(x) mean(R %*% x))</pre>
tmp3.StdDev <- apply(rp3, 1, function(x) StdDev(R = R, weights = x))</pre>
# plot feasible portfolios
plot(x = tmp1.StdDev, y = tmp1.mean, col = "gray", main = "Random Portfolio Methods")
points(x = tmp2.StdDev, y = tmp2.mean, col = "red", pch = 2)
points(x = tmp3.StdDev, y = tmp3.mean, col = "lightgreen", pch = 5)
legend("bottomright", legend = c("sample", "simplex", "grid"), col = c("gray",
```

## **Random Portfolio Methods**

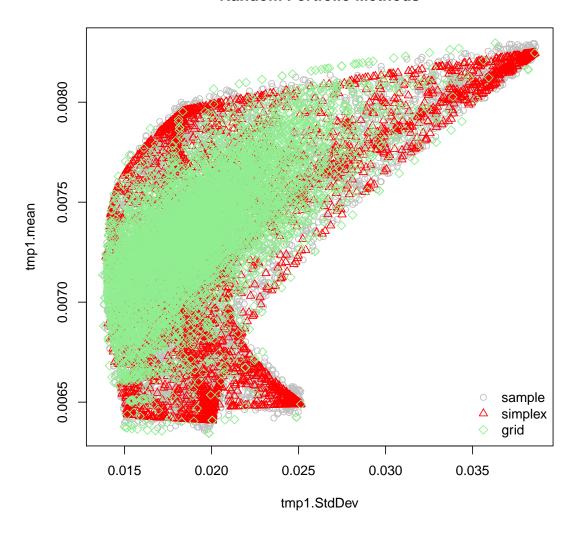


Figure 1: Random portfolio methods

```
"red", "lightgreen"), pch = c(1, 2, 5), bty = "n")
```

Figure 1 shows the feasible space using the different random portfolio methods. The 'sample' method has relatively even coverage of the feasible space. The 'simplex' method also has relatively even coverage of the space, but it is also more concentrated around the assets. The 'grid' method is pushed to the interior of the space due to the normalization.

The fev argument controls the face-edge-vertex biasing. Higher values for fev will result in the weights vector more concentrated on a single asset. This can be seen in the following charts.

```
par(mfrow = c(1, 1))
```

Figure 2 shows the feasible space varying the fev values.

The fev argument can be passed in as a vector for more even coverage of the feasible space. The default value is fev=0:5.

```
par(mfrow = c(1, 2))
# simplex

rp_simplex <- random_portfolios(portfolio = pspec, permutations = 2000, rp_method = "simplex",
    fev = 0:5)

tmp.mean <- apply(rp_simplex, 1, function(x) mean(R %*% x))

tmp.StdDev <- apply(rp_simplex, 1, function(x) StdDev(R = R, weights = x))

plot(x = tmp.StdDev, y = tmp.mean, main = "rp_method=simplex fev=0:5", ylab = "mean",
    xlab = "StdDev", col = rgb(0, 0, 100, 50, maxColorValue = 255))
# sample

rp_sample <- random_portfolios(portfolio = pspec, permutations = 2000, rp_method = "sample")

tmp.mean <- apply(rp_sample, 1, function(x) mean(R %*% x))

tmp.StdDev <- apply(rp_sample, 1, function(x) StdDev(R = R, weights = x))

plot(x = tmp.StdDev, y = tmp.mean, main = "rp_method=sample", ylab = "mean",
    xlab = "StdDev", col = rgb(0, 0, 100, 50, maxColorValue = 255))

par(mfrow = c(1, 1))</pre>
```

Figure 3 shows the feasible space comparing the 'sample' and 'simplex' methods to generate random portfolios.

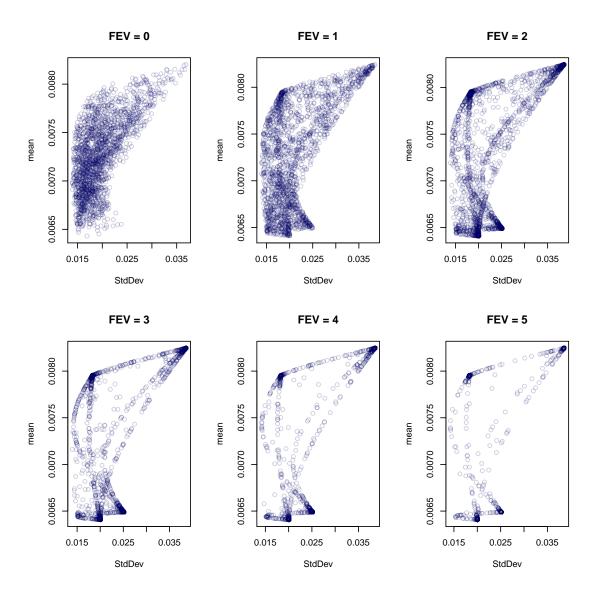


Figure 2: FEV biasing values

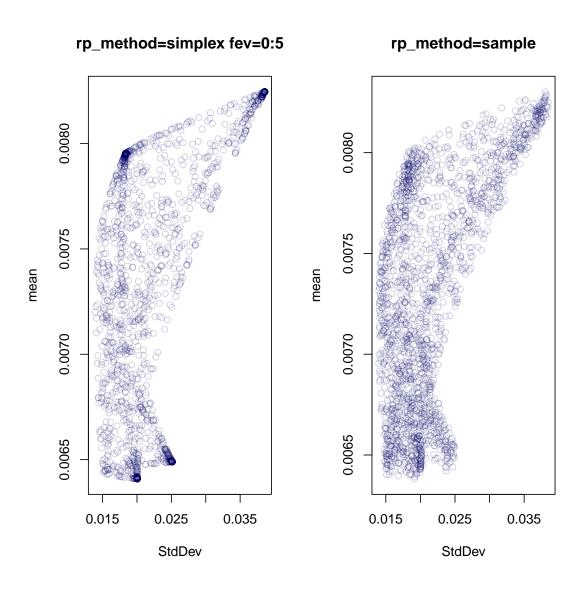


Figure 3: Random portfolio 'sample' and 'simplex' comparison

#### 5.3 pso

PortfolioAnalytics uses the psoptim function from the R package pso. Particle swarm optimization is a heuristic optimization algorithm. See ?psoptim and the references contained therein for more information.

#### 5.4 GenSA

PortfolioAnalytics uses the GenSA function from the R package GenSA. Generalized simmulated annealing is generic probabilistic heuristic optimization algorithm. See ?GenSA and the references contained therein for more information.

#### 5.5 ROI

The ROI package serves as an interface to the Rglpk package and the quadprog package to solve linear and quadratic programming problems. The interface to the ROI package solves a limited type of convex optimization problems:

- 1. Maxmimize portfolio return subject leverage, box, group, position limit, target mean return, and/or factor exposure constraints on weights.
- 2. Minimize portfolio variance subject to leverage, box, group, turnover, and/or factor exposure constraints (otherwise known as global minimum variance portfolio).
- 3. Minimize portfolio variance subject to leverage, box, group, and/or factor exposure constraints and a desired portfolio return.
- 4. Maximize quadratic utility subject to leverage, box, group, target mean return, turnover, and/or factor exposure constraints and risk aversion parameter. (The risk aversion parameter is passed into optimize.portfolio as an added argument to the portfolio object).
- 5. Minimize ETL subject to leverage, box, group, position limit, target mean return, and/or factor exposure constraints and target portfolio return.

### 6 Optimization

The previous sections demonstrated how to specify a portfolio object, add constraints, add objectives, and the solvers available. This section will demonstrate run the optimizations via optimize.portfolio. Only a small number of examples will be shown here, see the demos for several more examples.

### 6.1 Initial Portfolio Object

```
library(DEoptim)
##
## DEoptim package
## Differential Evolution algorithm in R
## Authors: D. Ardia, K. Mullen, B. Peterson and J. Ulrich
library(ROI)
## ROI: R Optimization Infrastructure
## Registered solver plugins: glpk, quadprog.
## Default solver: ROI_NULL.
##
## Attaching package: 'ROI'
## The following object(s) are masked from 'package:PortfolioAnalytics':
##
## objective
require(ROI.plugin.glpk)
## Loading required package: ROI.plugin.glpk
require(ROI.plugin.quadprog)
## Loading required package: ROI.plugin.quadprog
data(edhec)
R <- edhec[, 1:6]</pre>
colnames(R) <- c("CA", "CTAG", "DS", "EM", "EQMN", "ED")</pre>
funds <- colnames(R)</pre>
# Create an initial portfolio object with leverage and box constraints
init <- portfolio.spec(assets = funds)</pre>
init <- add.constraint(portfolio = init, type = "leverage", min_sum = 0.99,</pre>
   max_sum = 1.01)
init <- add.constraint(portfolio = init, type = "box", min = 0.05, max = 0.65)</pre>
```

#### 6.2 Maximize mean return with ROI

Add an objective to maximize mean return.

```
maxret <- add.objective(portfolio = init, type = "return", name = "mean")</pre>
  Run the optimization.
opt_maxret <- optimize.portfolio(R = R, portfolio = maxret, optimize_method = "ROI",
   trace = TRUE)
print(opt_maxret)
## ***********
## PortfolioAnalytics Optimization
## ***********
## Call:
## optimize.portfolio(R = R, portfolio = maxret, optimize_method = "ROI",
##
      trace = TRUE)
##
## Optimal Weights:
    CA CTAG DS
                 EM EQMN
## 0.05 0.05 0.16 0.65 0.05 0.05
##
## Objective Measure:
```

Chart the weights and optimal portfolio in risk-return space.

```
chart.Weights(opt_maxret)
```

#### 6.3 Minimize variance with ROI

mean

## 0.00796

Add an objective to minimize portfolio variance.

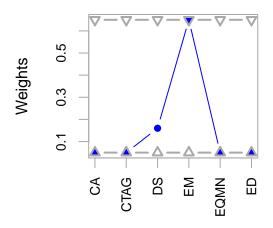


Figure 4: "Maximum Return Optimization

## **Maximum Return Optimizatio**

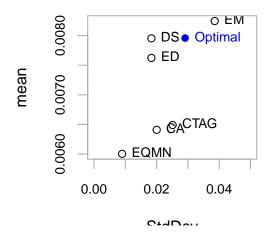


Figure 5: "Maximum Return Optimization

```
minvar <- add.objective(portfolio = init, type = "risk", name = "var")</pre>
```

Run the optimization. Note that although 'var' is the risk metric, 'StdDev' is returned as an objective measure.

```
opt_minvar <- optimize.portfolio(R = R, portfolio = minvar, optimize_method = "ROI",
   trace = TRUE)
print(opt_minvar)
## ***********
## PortfolioAnalytics Optimization
## ***********
##
## Call:
## optimize.portfolio(R = R, portfolio = minvar, optimize_method = "ROI",
##
      trace = TRUE)
##
## Optimal Weights:
   CA CTAG DS
                 EM EQMN
## 0.05 0.14 0.05 0.05 0.65 0.05
##
## Objective Measure:
## StdDev
## 0.0101
```

Chart the weights and optimal portfolio in risk-return space.

```
chart.Weights(opt_minvar)
```

### 6.4 Maximize quadratic utility with ROI

Add mean and var objectives for quadratic utility. Note that the risk aversion parameter for quadratic utility is specified in the objective as shown below.

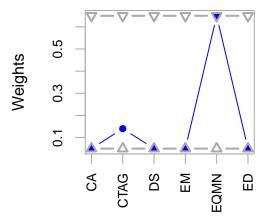


Figure 6: Minimum Variance Optimization

## **Minimum Variance Optimizatic**

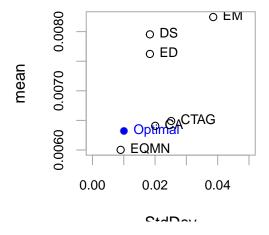


Figure 7: Minimum Variance Optimization

```
qu <- add.objective(portfolio = init, type = "return", name = "mean")
qu <- add.objective(portfolio = qu, type = "risk", name = "var", risk_aversion = 0.25)</pre>
```

Run the optimization.

```
opt_qu <- optimize.portfolio(R = R, portfolio = qu, optimize_method = "ROI",
   trace = TRUE)
print(opt_qu)
## **********
## PortfolioAnalytics Optimization
## ***********
##
## Call:
## optimize.portfolio(R = R, portfolio = qu, optimize_method = "ROI",
##
      trace = TRUE)
## Optimal Weights:
## CA CTAG DS EM EQMN
## 0.050 0.050 0.271 0.539 0.050 0.050
##
## Objective Measure:
##
     mean
## 0.00793
##
##
## StdDev
## 0.0266
```

```
chart.Weights(opt_qu)
```

```
chart.RiskReward(opt_qu, risk.col = "StdDev", return.col = "mean", main = "Quadratic Utility Option
chart.assets = TRUE, xlim = c(0, 0.05))
```

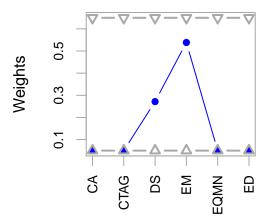


Figure 8: Maximum Quadratic Utility Optimization

## **Quadratic Utility Optimization**

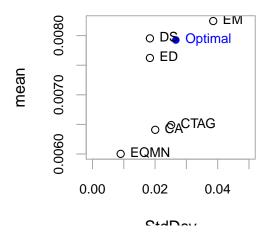


Figure 9: Maximum Quadratic Utility Optimization

### 6.5 Minimize expected tail loss with ROI

Add ETL objective.

```
etl <- add.objective(portfolio = init, type = "risk", name = "ETL")
```

Run the optimization.

```
opt_etl <- optimize.portfolio(R = R, portfolio = etl, optimize_method = "ROI",</pre>
   trace = TRUE)
print(opt_etl)
## ***********
## PortfolioAnalytics Optimization
## **********
##
## Call:
## optimize.portfolio(R = R, portfolio = etl, optimize_method = "ROI",
      trace = TRUE)
##
## Optimal Weights:
     CA CTAG
##
                DS
                     EM EQMN
## 0.050 0.297 0.050 0.050 0.493 0.050
##
## Objective Measure:
     ETL
## 0.0196
```

```
chart.Weights(opt_etl)
```

#### 6.6 Maximize mean return per unit ETL with random portfolios

Add mean and ETL objectives.

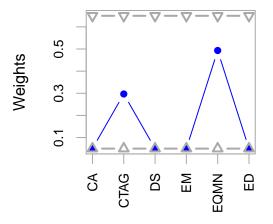


Figure 10: Minimum ETL Optimization

# **ETL Optimization**

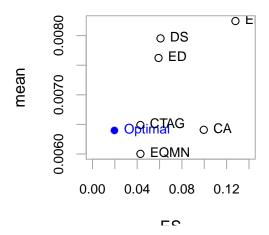


Figure 11: Minimum ETL Optimization

```
meanETL <- add.objective(portfolio = init, type = "return", name = "mean")
meanETL <- add.objective(portfolio = meanETL, type = "risk", name = "ETL", arguments = list(p = 0)</pre>
```

Run the optimization. The default random portfolio method is 'sample'.

```
opt_meanETL <- optimize.portfolio(R = R, portfolio = meanETL, optimize_method = "random",
   trace = TRUE, search_size = 2000)
print(opt_meanETL)
## **********
## PortfolioAnalytics Optimization
## ***********
##
## Call:
## optimize.portfolio(R = R, portfolio = meanETL, optimize_method = "random",
##
      search_size = 2000, trace = TRUE)
##
## Optimal Weights:
     CA CTAG
                      EM EQMN
                                 ED
## 0.050 0.328 0.066 0.058 0.432 0.066
##
## Objective Measures:
##
      mean
## 0.006548
##
##
##
      ETL
## 0.01983
```

The optimization was run with trace=TRUE so that iterations and other output from random portfolios is stored in the opt\_meanETL object. The extractStats function can be used to get a matrix of the weights and objective measures at each iteration.

```
stats_meanETL <- extractStats(opt_meanETL)
dim(stats_meanETL)
## [1] 2000 9</pre>
```

```
head(stats_meanETL)
                                                                 w.EM w.EQMN
##
                    mean
                             ETI.
                                      0111
                                           w.CA w.CTAG
                                                          w.DS
## .rnd.portf.1 0.007120 0.04512 0.03800 0.1667 0.1667 0.1667 0.1667 0.1667
  .rnd.portf.2 0.007418 0.05029 0.04288 0.0660 0.0800 0.1120 0.0540 0.0740
  .rnd.portf.3 0.006950 0.03358 0.02663 0.1600 0.2120 0.1940 0.0600 0.2000
  .rnd.portf.4 0.007564 0.05826 0.05070 0.0640 0.0860 0.4920 0.1820 0.0800
  .rnd.portf.5 0.007045 0.02711 0.02006 0.0920 0.4680 0.0700 0.1720 0.0500
  .rnd.portf.6 0.007486 0.05013 0.04264 0.0540 0.1320 0.2100 0.1200 0.0500
##
                  w.ED
  .rnd.portf.1 0.1667
##
  .rnd.portf.2 0.6160
## .rnd.portf.3 0.1720
## .rnd.portf.4 0.0920
## .rnd.portf.5 0.1500
## .rnd.portf.6 0.4360
```

Chart the optimal weights and optimal portfolio in risk-return space. Because the optimization was run with trace=TRUE, the chart of the optimal portfolio also includes the trace portfolios of the optimization. This is usefule to visualize the feasible space of the portfolios. The 'neighbor' portfolios relative to the optimal portfolio weights can be included the chart of the optimal weights.

```
chart.Weights(opt_meanETL, neighbors = 25)

chart.RiskReward(opt_meanETL, risk.col = "ETL", return.col = "mean", main = "mean-ETL Optimization")
```

Calculate and plot the portfolio component ETL contribution.

```
pct_contrib <- ES(R = R, p = 0.95, portfolio_method = "component", weights = extractWeights(opt_method)
barplot(pct_contribspct_contrib_MES, cex.names = 0.8, las = 3)</pre>
```

This figure shows that the Equity Market Nuetral strategy has greater than 50% risk contribution. A risk budget objective can be added to limit risk contribution percentage to 40%.

### 6.7 Maximize mean return per unit ETL with ETL risk budgets

Add objectives.

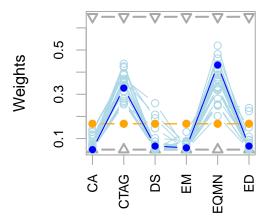


Figure 12: mean-ETL Optimization

# mean-ETL Optimization

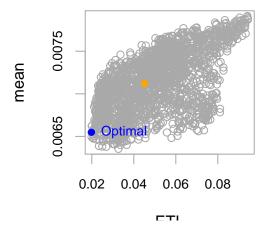


Figure 13: mean-ETL Optimization

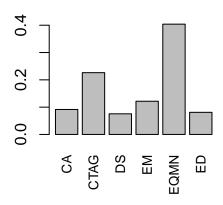


Figure 14: mean-ETL risk contribution

Run the optimization. Set traceDE=5 so that every fifth iteration is printed. The default is to print every iteration.

```
## ***********
## PortfolioAnalytics Optimization
## ***********
##
## Call:
## optimize.portfolio(R = R, portfolio = rb_meanETL, optimize_method = "DEoptim",
      search_size = 2000, trace = TRUE, traceDE = 5)
##
## Optimal Weights:
     CA CTAG
                DS
                     EM EQMN
                                ED
## 0.008 0.348 0.141 0.002 0.354 0.150
##
## Objective Measures:
##
      mean
## 0.006716
##
##
##
      ETL
## 0.01881
##
## contribution :
        CA
              CTAG DS EM
                                           EQMN
## 2.407e-04 6.965e-03 2.784e-03 6.963e-05 5.532e-03 3.218e-03
##
## pct_contrib_MES :
        CA
              CTAG
                        DS
                                EM
                                       EQMN
## 0.012798 0.370274 0.148020 0.003702 0.294117 0.171089
```

```
chart.Weights(opt_rb_meanETL)
```

Chart the contribution to risk in percentage terms.

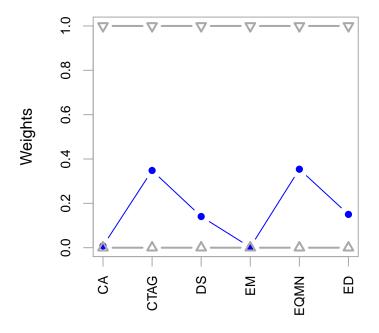


Figure 15: mean=ETL Optimization with Risk Budget

# Risk Budget mean-ETL Optimization

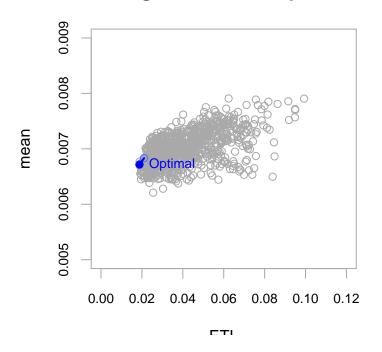


Figure 16: mean=ETL Optimization with Risk Budget

### **Risk Contribution**

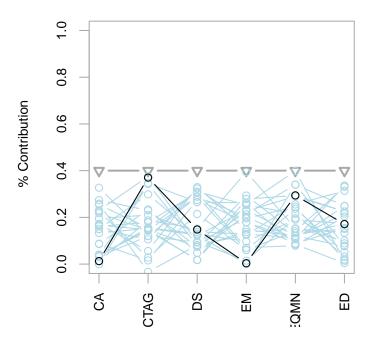


Figure 17: mean-ETL with Risk Budget Objective

```
chart.RiskBudget(opt_rb_meanETL, risk.type = "percentage", neighbors = 25)
```

# 6.8 Maximize mean return per unit ETL with ETL equal contribution to risk

Add objectives.

Run the optimization. Set traceDE=5 so that every fifth iteration is printed. The default is to print every iteration.

```
opt_eq_meanETL <- optimize.portfolio(R = R, portfolio = eq_meanETL, optimize_method = "DEoptim",
   search_size = 2000, trace = TRUE, traceDE = 5)
## Iteration: 5 bestvalit: 2406.986543 bestmemit:
                                                 0.054027
                                                             0.004000
                                                                        0.135273
## Iteration: 10 bestvalit: 1900.234832 bestmemit:
                                                              0.338000
                                                 0.036000
                                                                         0.156000
## Iteration: 15 bestvalit: 1830.058573 bestmemit:
                                                 0.047324
                                                              0.358199
                                                                         0.145757
## Iteration: 20 bestvalit: 1830.058573 bestmemit:
                                                  0.047324
                                                              0.358199
                                                                         0.145757
## [1] 0.04732 0.35820 0.14576 0.08484 0.21418 0.15200
print(opt_eq_meanETL)
## ***********
## PortfolioAnalytics Optimization
## ***********
##
## Call:
## optimize.portfolio(R = R, portfolio = eq_meanETL, optimize_method = "DEoptim",
      search_size = 2000, trace = TRUE, traceDE = 5)
##
## Optimal Weights:
                    EM EQMN
     CA CTAG DS
## 0.047 0.358 0.146 0.085 0.214 0.152
## Objective Measures:
##
      mean
## 0.006931
##
##
##
     ETL
## 0.0226
##
## contribution :
        CA
             CTAG
                       DS
                                EM
                                        EQMN
## 0.002164 0.002643 0.004568 0.005308 0.002891 0.005023
##
## pct_contrib_MES :
```

0.071243

0.05400

0.08484

0.08484

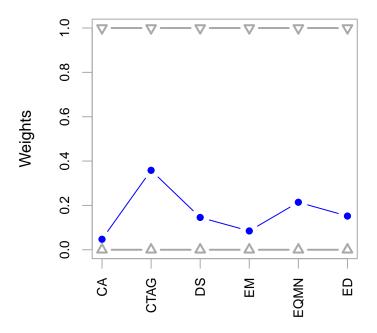


Figure 18: mean-ETL with ETL Equal Risk Contribution

```
## CA CTAG DS EM EQMN ED
## 0.09578 0.11696 0.20215 0.23491 0.12794 0.22227
```

Chart the optimal weights and optimal portfolio in risk-return space.

```
chart.Weights(opt_eq_meanETL)
```

```
chart.RiskReward(opt_eq_meanETL, risk.col = "ETL", return.col = "mean", main = "Risk Budget mean-lumber c(0, 0.12), ylim = c(0.005, 0.009))
```

Chart the contribution to risk in percentage terms. It is clear in this chart that the optimization results in a near equal risk contribution portfolio.

```
chart.RiskBudget(opt_eq_meanETL, risk.type = "percentage", neighbors = 25)
```

The opt\_meanETL, opt\_rb\_meanETL, and opt\_eq\_meanETL optimizations are similar and can be easily compared.

# Risk Budget mean-ETL Optimization

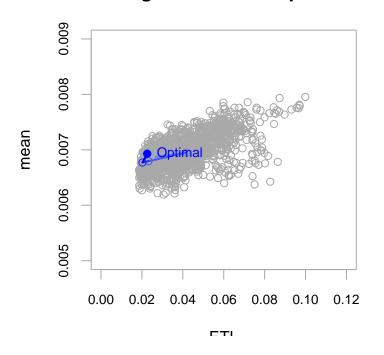


Figure 19: mean-ETL with ETL Equal Risk Contribution

# **Risk Contribution**

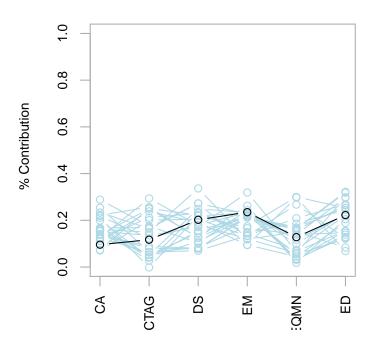


Figure 20: Percentage Contibution to Risk

- opt\_meanETL Objective to maximize mean return per unit ETL. The constraints are full investment and box constraints such that the minimum weight of any asset is 0.05 and maximum weight of any asset is 0.65.
- opt\_rb\_meanETL Objective to maximize mean return per unit ETL with risk budget objective to limit maximum percent risk 40%. The constraints are full investment and long only constraints.
- opt\_eq\_meanETL Objective to maximize mean return per unit ETL with equal contribution to risk. The constraints are full investment and long only constraints.

Combine the optimizations for easy comparison.

```
opt_combine <- optimizations.combine(list(meanETL = opt_meanETL, rbmeanETL = opt_rb_meanETL,</pre>
    eqmeanETL = opt_eq_meanETL))
# View the weights and objective measures of each optimization
extractWeights(opt_combine)
##
                        CTAG
                                 DS
                                               EQMN
                                                        F.D
                  CA
                                          EM
             0.05000 0.3280 0.0660 0.05800 0.4320 0.0660
## meanETL
## rbmeanETL 0.00800 0.3482 0.1407 0.00200 0.3540 0.1502
## eqmeanETL 0.04732 0.3582 0.1458 0.08484 0.2142 0.1520
obj_combine <- extractObjectiveMeasures(opt_combine)</pre>
chart.Weights(opt_combine, plot.type = "bar", legend.loc = "topleft", ylim = c(0,
```

1))

```
Chart the optimal portfolios of each optimization in risk-return space.
```

Calculate the STARR of each optimization

```
STARR <- obj_combine[, "mean"]/obj_combine[, "ETL"]
barplot(STARR, col = "blue", cex.names = 0.8, cex.axis = 0.8, las = 3, main = "STARR",
   ylim = c(0, 1))</pre>
```

# 

Figure 21: Optimal Weights of Optimizations

## **ETL Optimization Comparison**

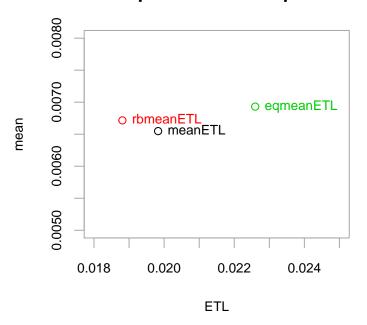


Figure 22: Optimal Portfolios in Risk-Return Space

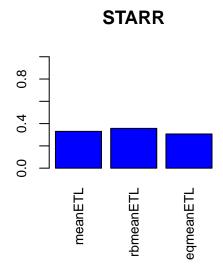


Figure 23: STARR of Optimizations

## **Risk Budget**

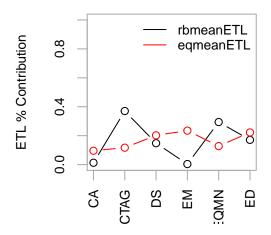


Figure 24: Percentage Contribution to Risk of Optimizations