

# Assignment 6

## Reinforcement Learning

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### Table of contents

#### Instructions

*Instructions for all assignments can be found [here](#). Note: this assignment falls under collaboration Mode 2: Individual Assignment – Collaboration Permitted. Please refer to the syllabus for additional information. Please be sure to list the names of any students that you worked with on this assignment. Total points in the assignment add up to 90; an additional 10 points are allocated to professionalism and presentation quality.*

#### Learning objectives

After completing this assignment, you will be able to... - Clearly articulate the role of the key components of reinforcement learning: the agent, actions, rewards, policies, state values, and action values. - Apply policy evaluation to a problem in practice - Use Monte Carlo control to determine and apply an optimal policy for a reinforcement learning problem and learn an optimal strategy from trial and error, alone

#### Background on our Blackjack variant

Your goal is to develop a reinforcement learning technique to learn the optimal policy for winning at blackjack through trial-and-error learning. Here, we're going to modify the rules from traditional blackjack a bit in a way that corresponds to the game presented in Sutton and Barto's *Reinforcement Learning: An Introduction* (Chapter 5, example 5.1). A full implementation of the game is provided and usage examples are detailed in the class header below.

The rules of this modified version of the game of blackjack are as follows:

- Blackjack is a card game where the goal is to obtain cards that sum to as near as possible to 21 without going over. We're playing against a fixed (autonomous) dealer.
- Face cards (Jack, Queen, King) have point value 10. Aces can either count as 11 or 1 (whichever is most advantageous to the player), and we refer to it as a 'usable' Ace if we're treating it as 11 (indicating that it could be used as a '1', instead, if need be). This game is played with a deck of cards sampled with replacement.
- The game starts with both the player and the dealer having one face up and one face down card.
- The player can request additional cards (known as taking a "hit", which we define as action "1") until either they decide to stop (known as "staying", which we define as action '0') or their cards exceed 21 (known as a "bust", at which time the game ends and player loses).
- If the player stays (and hasn't exceeded a score of 21), the dealer reveals their facedown card, and draws until their sum is 17 or greater. If the dealer busts the player wins. If neither player nor dealer busts, the outcome (win, lose, draw) is decided by whose sum is closer to 21. The reward for winning is +1, tying is 0, and losing is -1.

Over the course of these exercises, you will accomplish three things:

1. Try your hand at this game of blackjack and see what your human reinforcement learning system is able to achieve
2. Evaluate a naive policy using Monte Carlo policy evaluation
3. Determine an optimal policy using Monte Carlo control

*This problem is adapted from David Silver's [excellent series on Reinforcement Learning](#) at University College London*

## Exercise 1 - Human reinforcement learning

[5 points]

**1.1.** Using the code detailed below, play at least 20 hands of the modified blackjack game below, and record your returns (average cumulative reward) across all episodes. This will help you get accustomed with how the game works, the data structures involved with representing states, and what strategies are most effective. Since for this game, you only get a nonzero reward at the end of the episode, you can simply play the game and note the reward in a spreadsheet, then average across all plays of the game (of course, you're welcome to code up a way to do that automatically if you're so inspired).

```
import numpy as np

class Blackjack():
    """Simple blackjack environment adapted from OpenAI Gym:
```

[https://github.com/openai/gym/blob/master/gym/envs/toy\\_text/blackjack.py](https://github.com/openai/gym/blob/master/gym/envs/toy_text/blackjack.py)

Blackjack is a card game where the goal is to obtain cards that sum to as near as possible to 21 without going over. They're playing against a fixed dealer.

Face cards (Jack, Queen, King) have point value 10.  
Aces can either count as 11 or 1, and it's called 'usable' at 11.  
This game is played with a deck sampled with replacement.

The game starts with each (player and dealer) having one face up and one face down card.

The player can request additional cards (hit = 1) until they decide to stop (stay = 0) or exceed 21 (bust).

After the player stays, the dealer reveals their facedown card, and draws until their sum is 17 or greater. If the dealer goes bust the player wins. If neither player nor dealer busts, the outcome (win, lose, draw) is decided by whose sum is closer to 21. The reward for winning is +1, drawing is 0, and losing is -1.

The observation is a 3-tuple of: the player's current sum,  
the dealer's one showing card (1-10 where 1 is ace),  
and whether or not the player holds a usable ace (0 or 1).

This environment corresponds to the version of the blackjack problem described in Example 5.1 in Reinforcement Learning: An Introduction by Sutton and Barto (1998).

<http://incompleteideas.net/sutton/book/the-book.html>

Usage:

```
Initialize the class:  
game = Blackjack()
```

```
Deal the cards:  
game.deal()
```

```
(14, 3, False)
```

This is the agent's observation of the state of the game:

The first value is the sum of cards in your hand (14 in this case)  
 The second is the visible card in the dealer's hand (3 in this case)  
 The Boolean is a flag (False in this case) to indicate whether or not you have a usable Ace  
 (Note: if you have a usable ace, the sum will treat the ace as a value of '11' - this is the case if this Boolean flag is "true")

Take an action: Hit (1) or stay (0)

Take a hit: `game.step(1)`  
 To Stay: `game.step(0)`

The output summarizes the game status:

`((15, 3, False), 0, False)`

The first tuple (15, 3, False), is the agent's observation of the state of the game as described above.

The second value (0) indicates the rewards

The third value (False) indicates whether the game is finished

"""

```
def __init__(self):
    # 1 = Ace, 2-10 = Number cards, Jack/Queen/King = 10
    self.deck = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 10, 10, 10]
    self.dealer = []
    self.player = []
    self.deal()

def step(self, action):
    if action == 1: # hit: add a card to players hand and return
        self.player.append(self.draw_card())
        if self.is_bust(self.player):
            done = True
            reward = -1
        else:
            done = False
            reward = 0
    else: # stay: play out the dealers hand, and score
        done = True
        while self.sum_hand(self.dealer) < 17:
            self.dealer.append(self.draw_card())
```

```

        reward = self.cmp(self.score(self.player), self.score(self.dealer))
        return self._get_obs(), reward, done

def _get_obs(self):
    return (self.sum_hand(self.player), self.dealer[0], self.usable_ace(self.player))

def deal(self):
    self.dealer = self.draw_hand()
    self.player = self.draw_hand()
    return self._get_obs()

#-----
# Other helper functions
#-----
def cmp(self, a, b):
    return float(a > b) - float(a < b)

def draw_card(self):
    return int(np.random.choice(self.deck))

def draw_hand(self):
    return [self.draw_card(), self.draw_card()]

def usable_ace(self, hand): # Does this hand have a usable ace?
    return 1 in hand and sum(hand) + 10 <= 21

def sum_hand(self, hand): # Return current hand total
    if self.usable_ace(hand):
        return sum(hand) + 10
    return sum(hand)

def is_bust(self, hand): # Is this hand a bust?
    return self.sum_hand(hand) > 21

def score(self, hand): # What is the score of this hand (0 if bust)
    return 0 if self.is_bust(hand) else self.sum_hand(hand)

```

Here's an example of how it works to get you started:

```

import numpy as np

# Initialize the class:

```

```

game = Blackjack()

# Deal the cards:
s0 = game.deal()
print(s0)

# Take an action: Hit = 1 or stay = 0. Here's a hit:
s1 = game.step(1)
print(s1)

# If you wanted to stay:
# game.step(2)

# When it's gameover, just redeal:
# game.deal()

```

```

(12, 10, False)
((22, 10, False), -1, True)

```

### Sample Answer

My rewards for each episode played were: [??, ??, ??, ??, .....]

My average reward was [INSERT HERE] after playing ?? hands. [Note - you must play at least 20 hands]

## Exercise 2 - Perform Monte Carlo Policy Evaluation

[40 points]

Thinking that you want to make your millions playing this modified version of blackjack, you decide to test out a policy for playing this game. Your idea is an aggressive strategy: always hit unless the total of your cards adds up to 20 or 21, in which case you stay.

**2.1.** Use Monte Carlo policy evaluation to evaluate the expected returns from each state. Create plots for these similar to Sutton and Barto, Figure 5.1 where you plot the expected returns for each state. In this case create 2 plots (sample code is provided below):

1. When you have a usable ace, plot the value function showing the dealer's card on the x-axis, and the player's sum on the y-axis, and use a colormap to plot the state value corresponding to each state under the policy described above. The domain of your x and y axes should include all possible states (2 to 21 for the player sum, and 1 to 10 for the dealer's card). Show the estimated state value function after 10,000 episodes.

2. Repeat (1) for the states without a usable ace (code is also provided for this).
3. Repeat (1) after 500,000 episodes.
4. Repeat (2) after 500,000 episodes.

**2.2.** Show a plot of the cumulative average reward (returns) per episode vs the number of episodes.

- For this plot, make the x-axis log scale
- For both the 10,000 episode case and the 500,000 episode case, state the final value of that average returns for this policy in those two cases (these are just the values of the plot of cumulative average reward at iteration 10,000 and 500,000, respectively), you can write this in a line of text.

*Note on sample code: the code provided for these questions is meant to be a helpful starting point - you are not required to fill it out exactly or use all components, but it is meant to help you as you begin thinking about this problem.*

### Starter Code

```
def policy_evaluation(N_episodes):
    # Initialize variables
    avg_return = [] # Accumulator to store the average returns over all
                    # episodes
    episode_count = 0 # Number of episodes played (will increment as more
                     # episodes are played)
    N_player = 32 # Number of possible states of the players hand
                 # The max hand value is 21, but then if the player busts
                 # they could reach 32 (if they had 21 and took a hit)
    N_dealer = 10 # Maximum number of states the dealer's hand could take on
                 # 1 through 10 (Ace could be 1 or 11, but would still
                 # be a single card in hand)
    N_ace = 2 # There are two states for the ace:
              # (1) No usable ace, (2) Usable ace

    # Initialize the state value function (in this case, the value function is
    # the average return over episodes, so no need for an explicit returns
    # array)
    v = np.zeros((N_player, N_dealer, N_ace))

    # Initialize a variable counting the number of visits to each state
    n_visits = np.zeros((N_player, N_dealer, N_ace))

    # Initialize the policy that stays only if the player has 20 or 21,
    # otherwise hit
```

```

#####
# FILL IN THIS CODE
#####

# Load the game
B = Blackjack()

# HELPER FUNCTIONS
# Convert the current state into a set of indices for the value function
def state_to_index(s):
    ace = 0
    if s[2]:
        ace = 1
    index = [s[0]-1, s[1]-1, ace]
    return index

# Choose an action based on the policy, pi, and the current state, s
def choose_action(pi,s):
    #####
    # FILL IN THIS FUNCTION
    #####
    # Outputs a value of 0 or 1

# -----
# Run the policy evaluation
# -----

for episode in range(N_episodes):

    # Deal a hand of blackjack

    # Initialize a variable to hold the list of states visited and add to
    # the list the initial state

    # Play the hand through, following the policy

    # Update the average returns

    # Update your state value function

return (v, avg_return)

```



```

# -----
# Sample plotting function for the state value function
# -----

import matplotlib.pyplot as plt
# Uncomment the line below on macs for clearer plots:
# %config InlineBackend.figure_format = 'retina'

# Plot the resulting state value function (expected returns from each state)
def plot_value(v):
    plt.figure(figsize=(10,7))
    drange = [1,10] # Dealer range
    prange = [4,21] # Player range
    axes = []
    axes.append(plt.subplot(1,2,1))
    plt.title('No Usable Ace')
    plt.imshow(v[prange[0]-1:prange[1],:,0],
               vmin=-1,vmax=1,
               origin='lower',
               extent=(drange[0]-0.5,
                       drange[1]+0.5,
                       prange[0]-0.5,
                       prange[1]+0.5),
               cmap = 'RdBu')
    plt.xticks(np.arange(drange[0],drange[1]+1,1))
    plt.yticks(np.arange(prange[0],prange[1]+1,1))
    plt.ylabel('Player Total')
    plt.xlabel('Dealer')

    axes.append(plt.subplot(1,2,2))
    plt.title('Usable Ace')
    im = plt.imshow(v[prange[0]-1:prange[1],:,1],
                    vmin=-1,vmax=1,
                    origin='lower',
                    extent=(drange[0]-0.5,
                            drange[1]+0.5,
                            prange[0]-0.5,
                            prange[1]+0.5),
                    cmap = 'RdBu')
    plt.xticks(np.arange(drange[0],drange[1]+1,1))
    plt.yticks(np.arange(prange[0],prange[1]+1,1))
    plt.ylabel('Player Total')

```

```
plt.xlabel('Dealer')
cbar = plt.colorbar(im, ax=axes)
cbar.set_label('Value (Expected Returns)')
```

### Exercise 3 - Perform Monte Carlo Control

[40 points]

Now it's time to actually implement a reinforcement learning strategy that learns to play this version of Blackjack well, only through trial-and-error learning. Here, you will develop your Monte Carlo Control algorithm and evaluate its performance for our Blackjack-like game.

**3.1.** Using Monte Carlo Control through policy iteration, estimate the optimal policy for playing our modified blackjack game to maximize rewards.

In doing this, use the following assumptions:

1. Initialize the state value function and the action value function to all zeros
2. Keep a running tally of the number of times the agent visited each state and chose an action.  $N(s_t, a_t)$  is the number of times action  $a$  has been selected from state  $s$ . You'll need this to compute the running average. You can implement an online average as:  

$$\bar{x}_t = \frac{1}{N}x_t + \frac{N-1}{N}\bar{x}_{t-1}$$
3. Use an  $\epsilon$ -greedy exploration strategy with  $\epsilon_t = \frac{N_0}{N_0 + N(s_t)}$ , where we define  $N_0 = 100$ . Vary  $N_0$  as needed. Varying  $0 \leq N_0 < \infty$  will determine the amount of exploration the algorithm performs where the lower  $N_0$  the less exploration and vice versa.

Show your results by plotting the optimal state value function:  $v^*(s) = \max_a q^*(s, a)$  and the optimal policy  $\pi^*(s)$ . Create plots for these similar to Sutton and Barto, Figure 5.2 in the 2018 edition (5.5 in the original edition) - sample code provided for the plots. Your results SHOULD be very similar to the plots in that text (although you will show your results with the player sum ranging from 4 to 21). For these plots include the following (note - code from the previous section of this assignment for state value function plotting and below for policy plotting are provided to help you to accomplish these):

1. When you have a useable ace, plot the value function with the dealer's card on the x-axis, and the player's sum on the y-axis, and use the colormap and `imshow` to plot the value function that corresponds with those states. Plot the state value corresponding to each state under the policy described above. The domain of your x and y axes should include all possible states (4 to 21 for the player sum, and 1 to 10 for the dealer's visible card).
2. Repeat (1) for the same states but without a usable ace.
3. Plot the optimal policy  $\pi^*(s)$  for the states with a usable ace (this plot can be an `imshow` plot with binary values - sample code provided).

4. Plot the optimal policy  $\pi^*(s)$  for the states without a usable ace (this plot can be an imshow plot with binary values - sample code provided).

**3.2.** Plot the cumulative average return per episode vs the number of episodes (your x-axis should be log-scaled to clearly see the trend). What is the average return your control strategy was able to achieve? You'll know your method is working if you see a steady rise in your average returns over time.

*Note on convergence: convergence of this algorithm is extremely slow. You may need to let this run a few million episodes before the policy starts to converge. You're not expected to get EXACTLY the optimal policy, but it should be visibly close.*

*Note on sample code: the code provided for these questions is meant to be a helpful starting point - you are not required to fill it out exactly or use all components, but it is meant to help you as you begin thinking about this problem.*

### Sample Code

```
def mc_control(episodes):
    N_player = 32
    N_dealer = 10
    N_ace = 2
    N_actions = 2

    avg_return = []
    N = 0
    NO = 1 # Coefficient for controlling the level of exploration

    # Initialize the state value function, v, and the action value function, q
    v = np.zeros((N_player, N_dealer, N_ace))
    q = np.zeros((N_player, N_dealer, N_ace, N_actions))

    # Initialize a variable counting the number of visits to each state
    # For the state value function
    nv_visits = np.zeros((N_player, N_dealer, N_ace))
    # For the action value function
    nq_visits = np.zeros((N_player, N_dealer, N_ace, N_actions))

    # Initialize the policy to all always "stay" (all zeros)
    pi = np.zeros((N_player, N_dealer, N_ace))

    # Initialize the game
    B = Blackjack()
```

```

# HELPER FUNCTIONS
# Convert the current state into a set of indices for the
# state value function
def state_to_index_v(s):
    ace = 0
    if s[2]:
        ace = 1
    index = [s[0]-1, s[1]-1, ace]
    return index

# Convert the current state into a set of indices for the
# action value function
def state_to_index_av(s,a):
    ace = 0
    if s[2]:
        ace = 1
    index = [s[0]-1, s[1]-1, ace, int(a)]
    return index

# Choose an action based on the policy, pi, and the current state, s, with
# epsilon-greedy exploration
def choose_action(pi,s):
    #####
    # FILL IN THIS FUNCTION
    #####
    # Outputs an action

# -----
# Run episodes of MC Control
# -----
for episode in range(episodes):
    # Play a hand a blackjack

    # Update average return count

    # Update the state value function

    # Update the action value function

    # Update the policy

return (v, pi, avg_return)

```

```

#-----
# Sample code for plotting your policy
#-----
def plot_policy(pi):
    # Plot the policy
    drange = [1,10]
    prange = [4,21]
    plt.figure(figsize=(10,7))
    axes = []
    axes.append(plt.subplot(1,2,1))
    plt.title('No Usable Ace')
    plt.imshow(pi[prange[0]-1:prange[1],:,0],
               vmin=0,vmax=1,
               origin='lower',
               extent=(drange[0]-0.5,
                       drange[1]+0.5,
                       prange[0]-0.5,
                       prange[1]+0.5),
               cmap = 'binary_r')
    plt.xticks(np.arange(drange[0],drange[1]+1,1))
    plt.yticks(np.arange(prange[0],prange[1]+1,1))
    plt.ylabel('Player Total')
    plt.xlabel('Dealer')

    axes.append(plt.subplot(1,2,2))
    plt.title('Usable Ace')
    im = plt.imshow(pi[prange[0]-1:prange[1],:,1],
                    vmin=0,vmax=1,
                    origin='lower',
                    extent=(drange[0]-0.5,
                            drange[1]+0.5,
                            prange[0]-0.5,
                            prange[1]+0.5),
                    cmap = 'binary_r')
    plt.xticks(np.arange(drange[0],drange[1]+1,1))
    plt.yticks(np.arange(prange[0],prange[1]+1,1))
    plt.ylabel('Player Total')
    plt.xlabel('Dealer')
    cbar = plt.colorbar(im, ax=axes)
    cbar.set_label('Action (Stay = 0, Hit = 1)')

```

#### Exercise 4 - Discuss your findings

[5 points]

Compare the performance of your human control policy, in question 1, the naive policy from question 2, and the optimal control policy in question 3.

**4.1.** Which performs best? What was different about the policies developed for each and how may that have contributed to their comparative advantages?

**4.2.** Could you have created a better policy if you knew the full Markov Decision Process for this environment? Why or why not? (*assume the policy estimated from your MC control algorithm had **fully converged***)