Neural Networks I

Lecture 18

What’s the hype around neural networks?

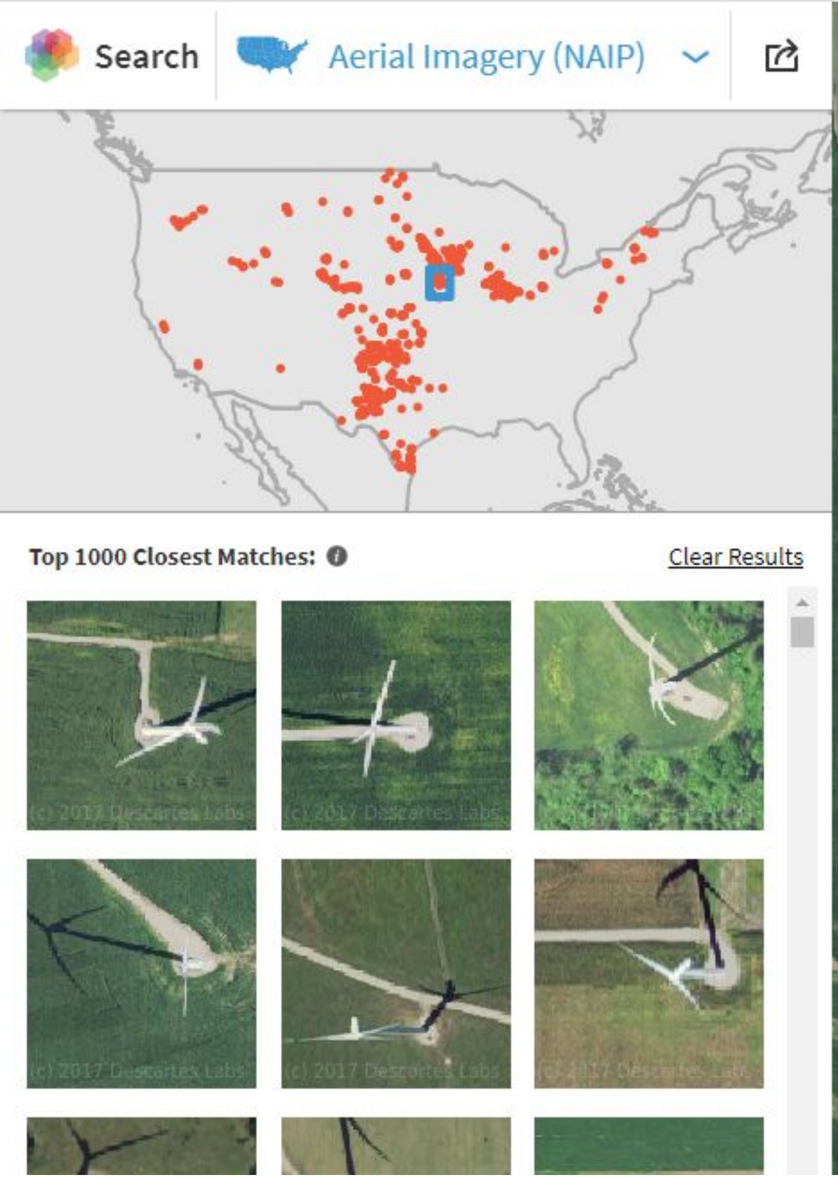
Character/handwriting recognition

Language translation

Medical diagnosis

Automated financial trading systems

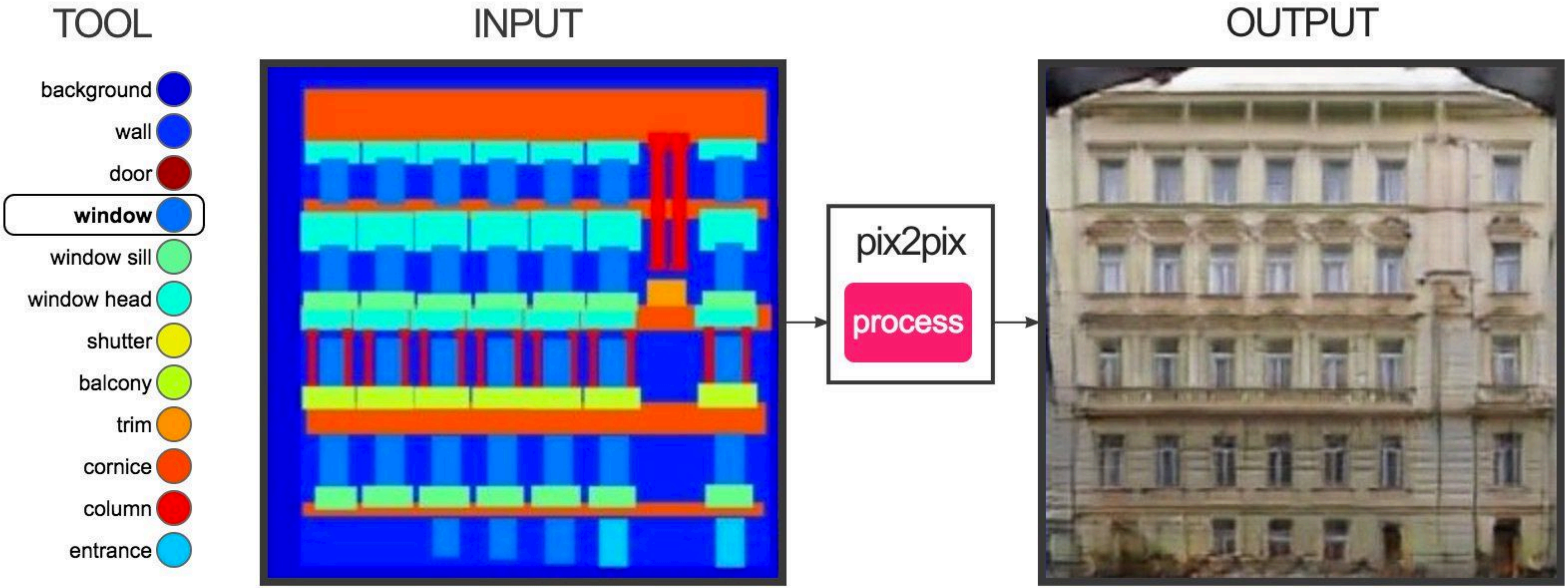
And some other interesting computer vision applications…



Geovisual Search

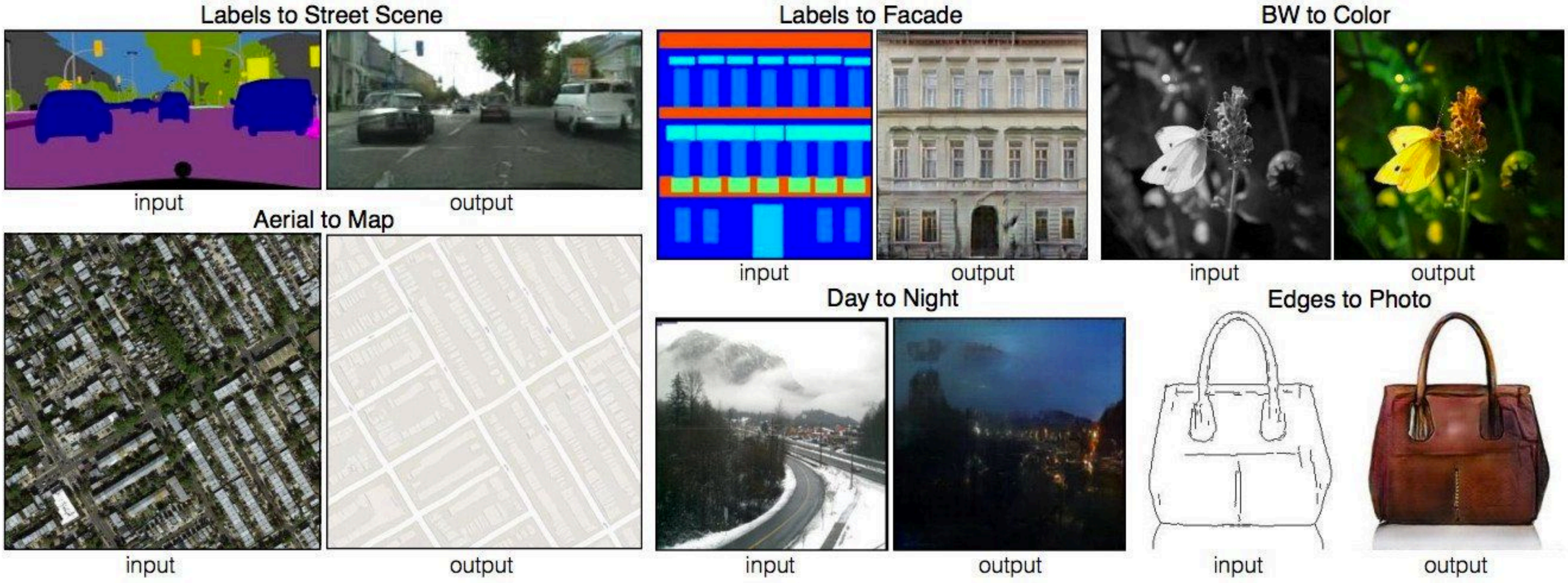
https://search.descarteslabs.com/

Image-to-image translation



Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).

Image-to-image translation



Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).

Image-to-image translation

Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).

Image Style Transfer

Dumoulin, Vincent, Jonathon Shlens, and Manjunath Kudlur. "A learned representation for artistic style." CoRR, abs/1610.07629 2.4 (2016): 5.

What makes neural networks special?

Neural network learning is **representation learning**

Previous ML algorithms we discussed required us to manually determine feature transformations

Neural networks **learn** feature transformations

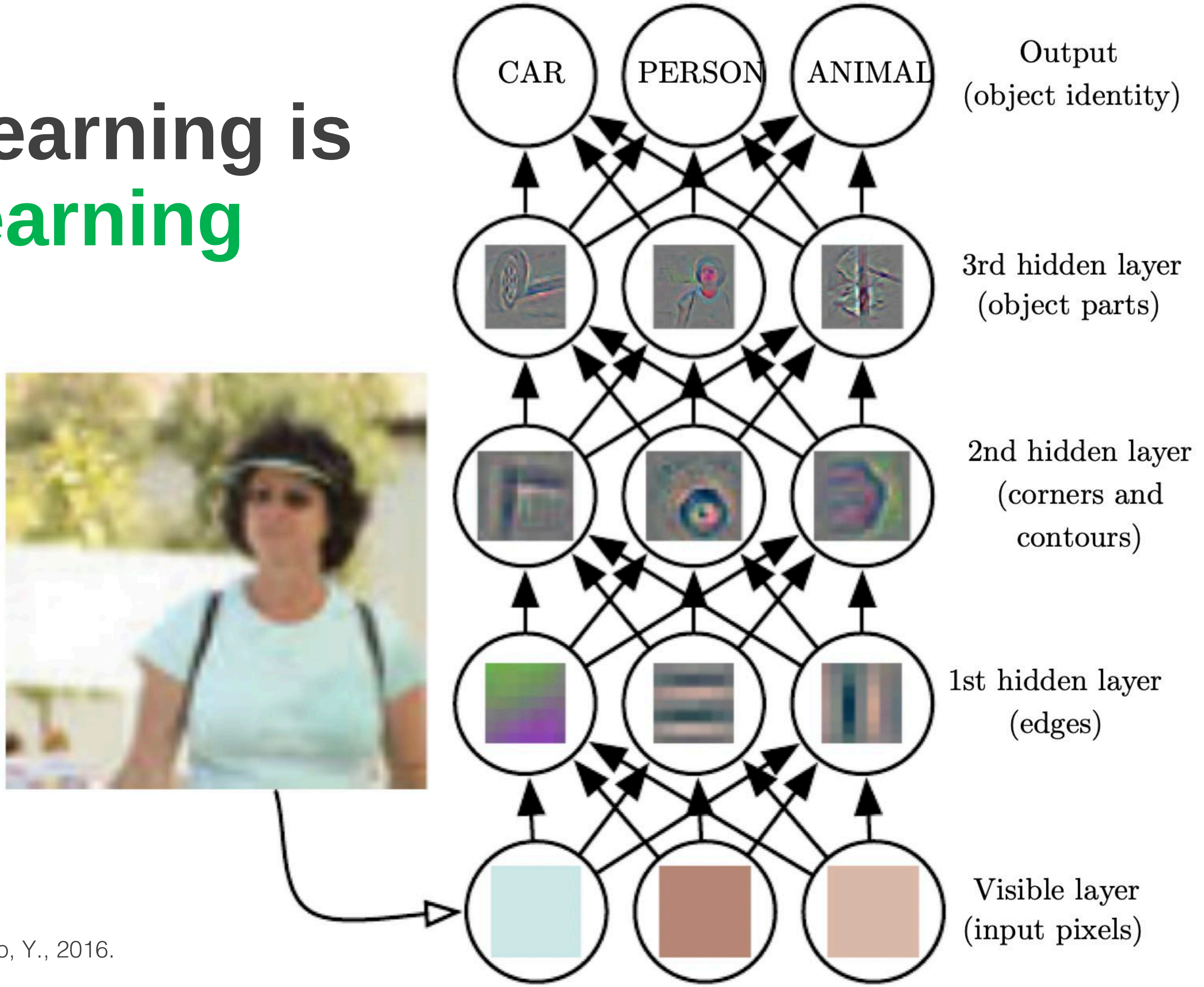


Image from Goodfellow, I., Bengio, Y., Courville, A. and Bengio, Y., 2016.
Deep learning (Vol. 1, No. 2). Cambridge: MIT press.

What is a neural network and **how does it work**?

How do we **optimize model weights**?
(i.e. how do we fit our model to data)

What are the challenges of using neural networks?

Recall our goal in supervised learning

$$y = f(\mathbf{x}, \mathbf{w})$$

Labels

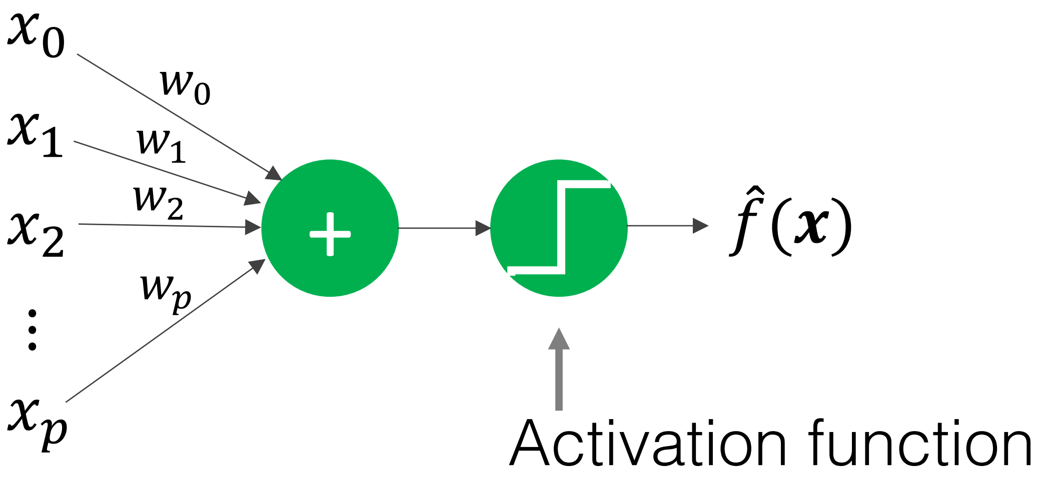
Model

Input Data

Parameter(s)

Perceptron

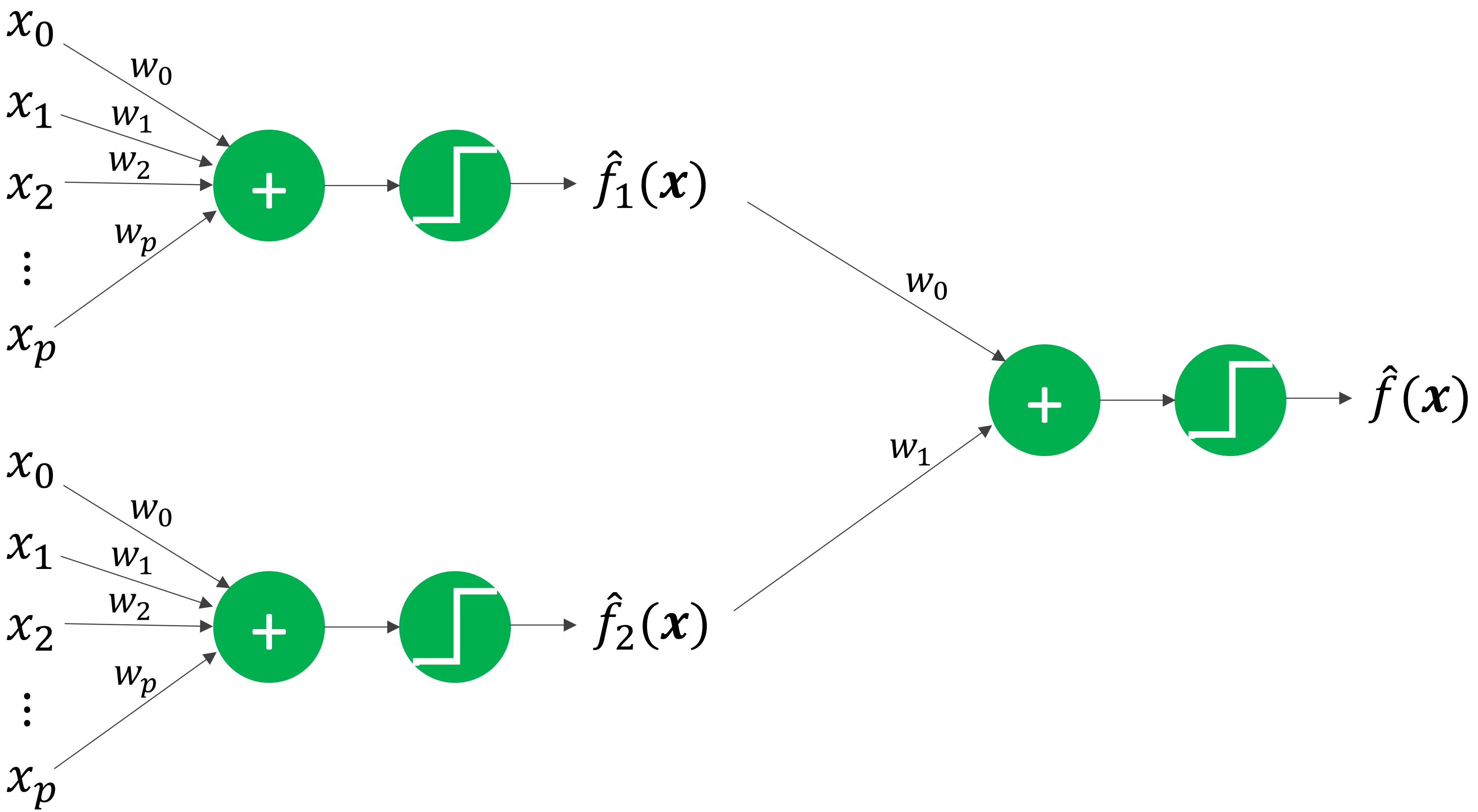
$$\hat{f}(\mathbf{x}) = sign\left(\sum_{i=0}^{p} w_i x_i\right)$$



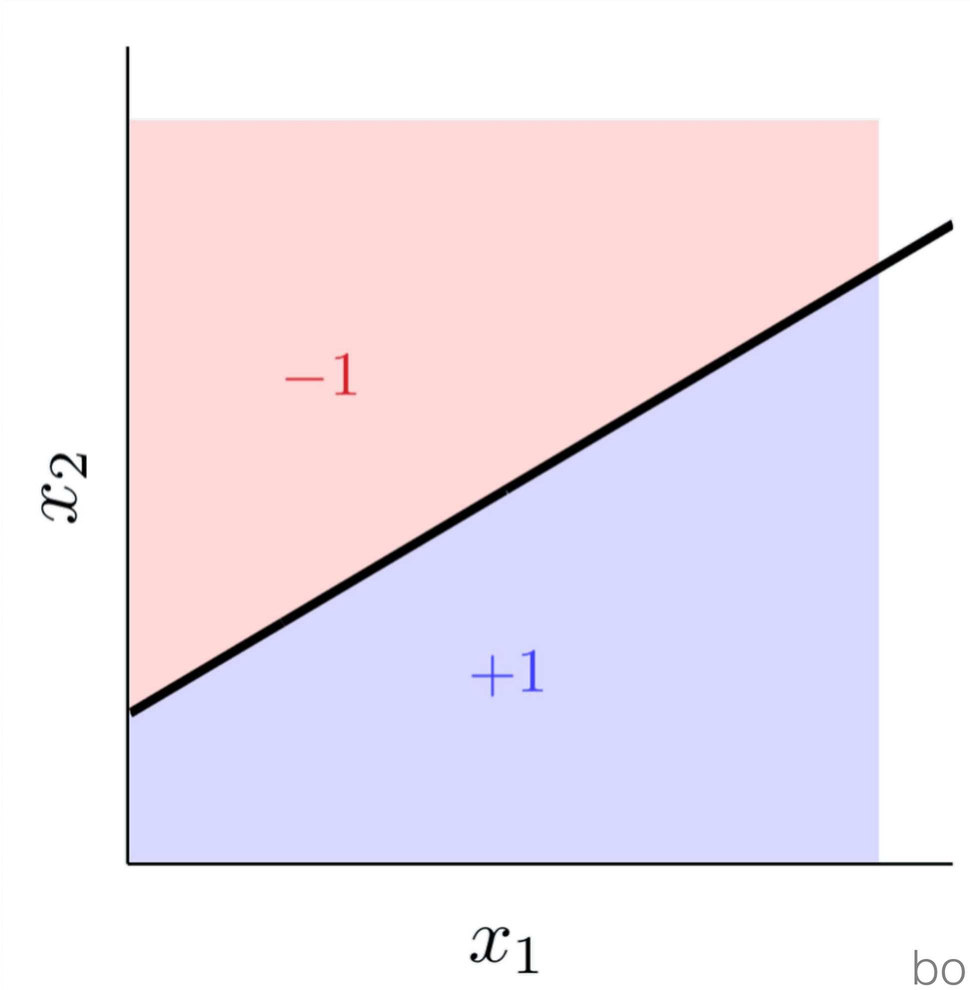
Source: Abu-Mostafa, Learning from Data, Caltech

Multilayer Perceptron

What if we stuck multiple perceptrons together?

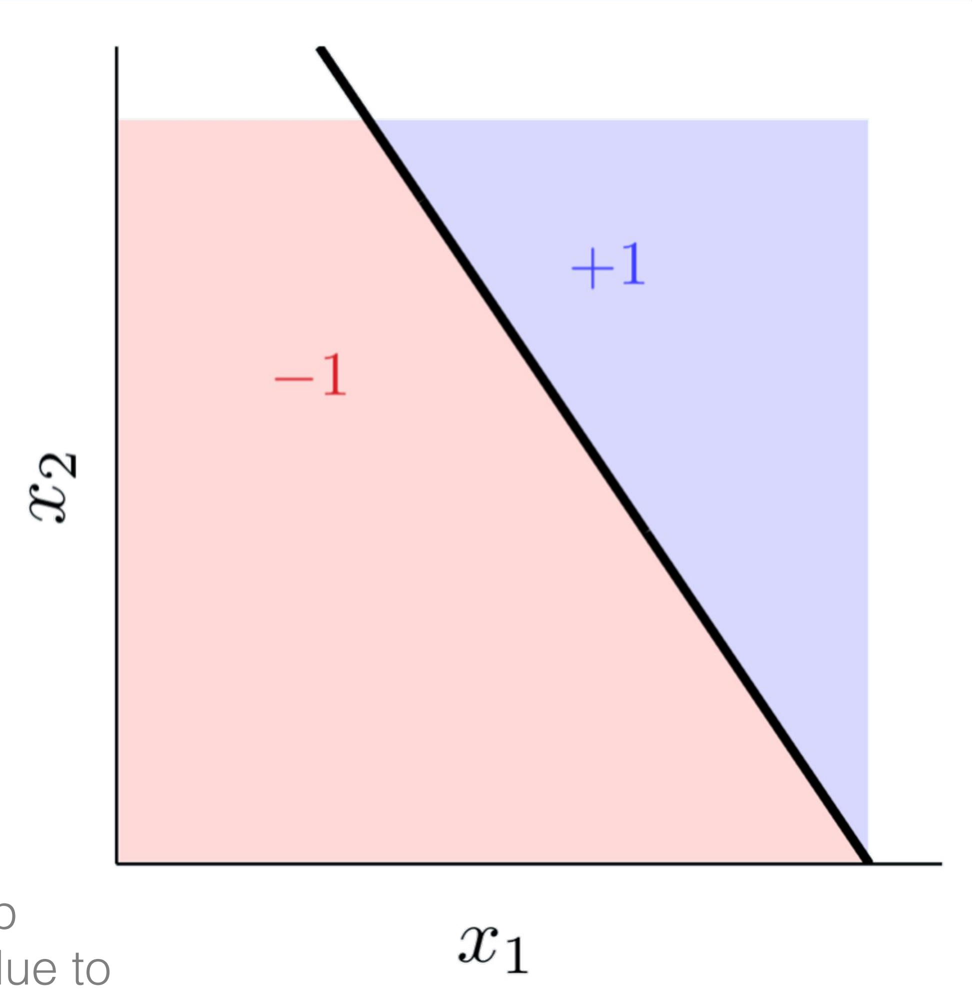


Perceptron #1



$$\hat{f}_1(\boldsymbol{x}) = sign(\boldsymbol{w}_1^T\boldsymbol{x})$$

Perceptron #2



$$\hat{f}_2(\boldsymbol{x}) = sign(\boldsymbol{w}_2^T\boldsymbol{x})$$

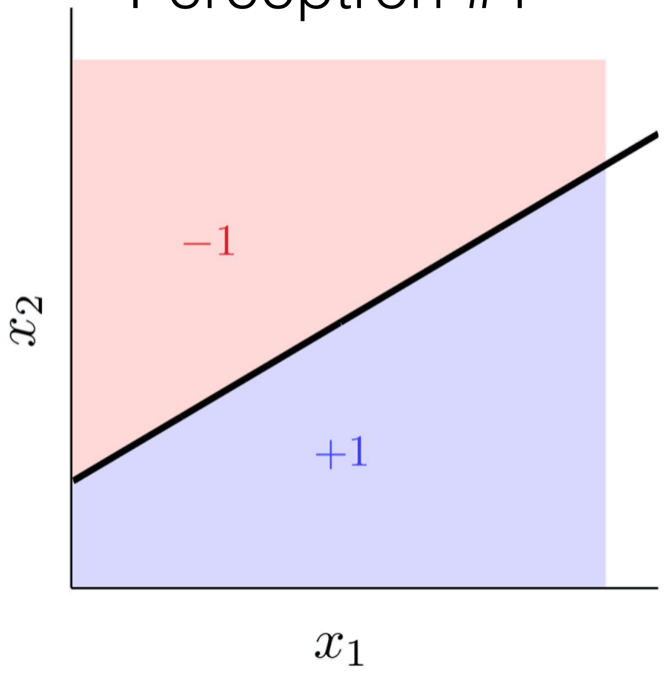
The sharp boundary is due to our sign function

Source: Abu-Mostafa, Learning from Data, Caltech

Multilayer perceptron:

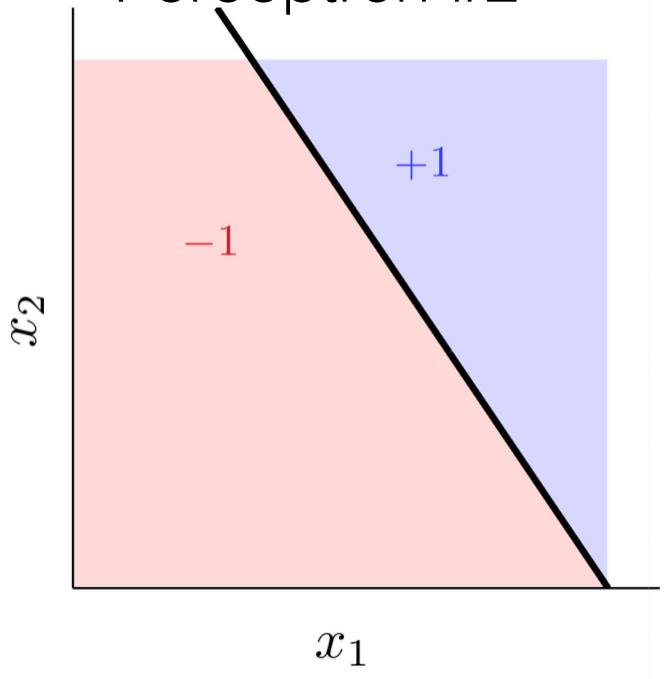
$$\hat{f}(\boldsymbol{x}) = \begin{cases} +1 & \hat{f}_1(\boldsymbol{x}) \neq \hat{f}_2(\boldsymbol{x}) \\ -1 & \hat{f}_1(\boldsymbol{x}) = \hat{f}_2(\boldsymbol{x}) \end{cases}$$

Perceptron #1

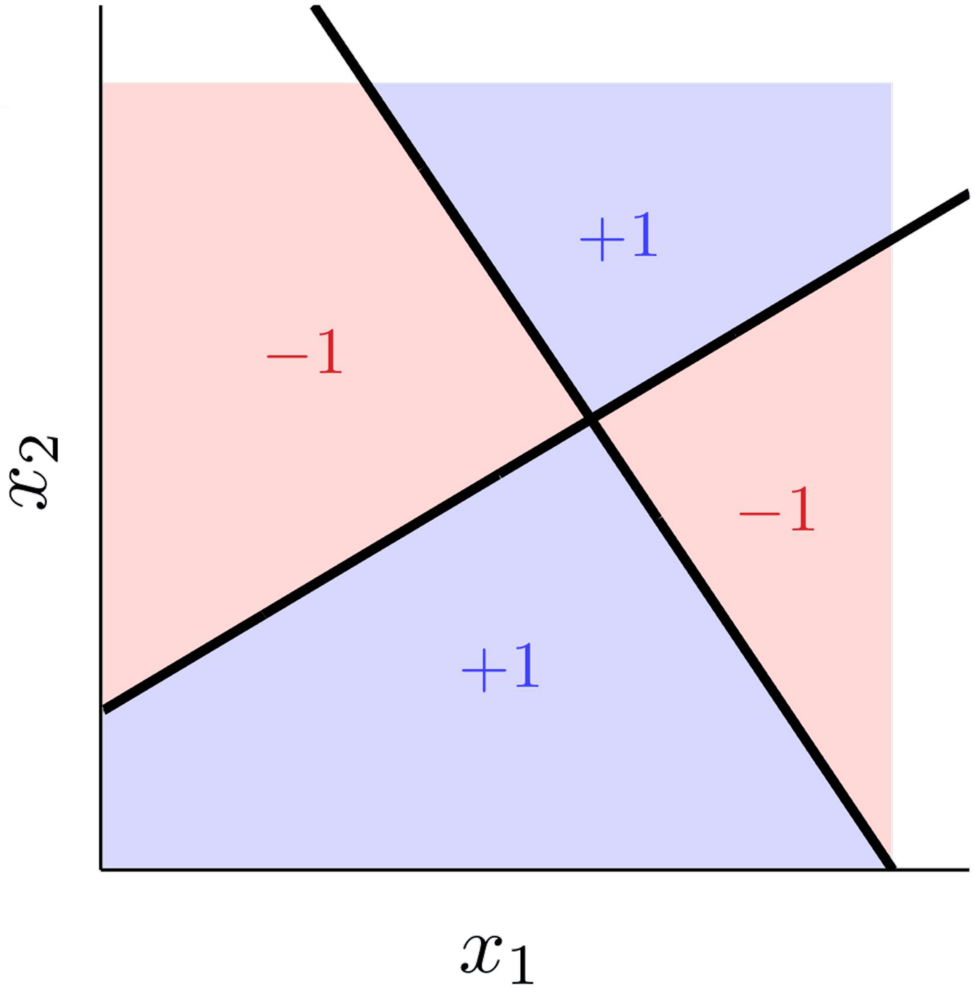


$$\hat{f}_1(\boldsymbol{x}) = sign(\boldsymbol{w}_1^T \boldsymbol{x})$$

Perceptron #2

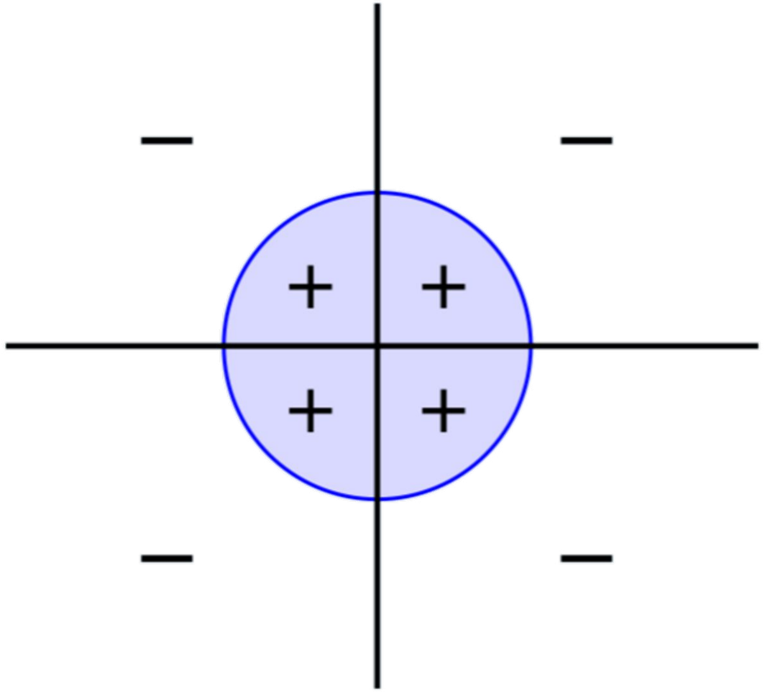


$$\hat{f}_2(\boldsymbol{x}) = sign(\boldsymbol{w}_2^T \boldsymbol{x})$$

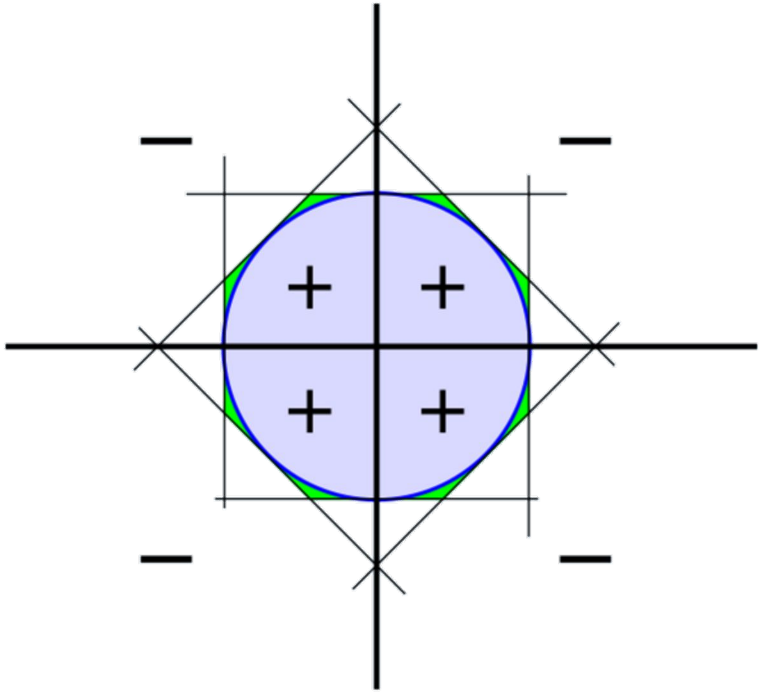


Source: Abu-Mostafa, Learning from Data, Caltech

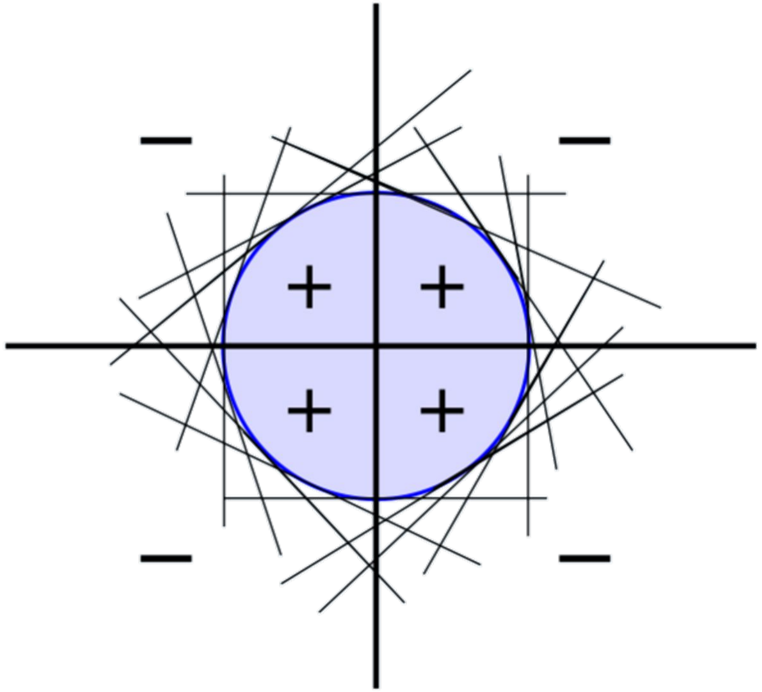
Multilayer Perceptron



Target



8 perceptrons



16 perceptrons

The more nodes/neurons, the more flexible is the model

Source: Abu-Mostafa, Learning from Data, Caltech

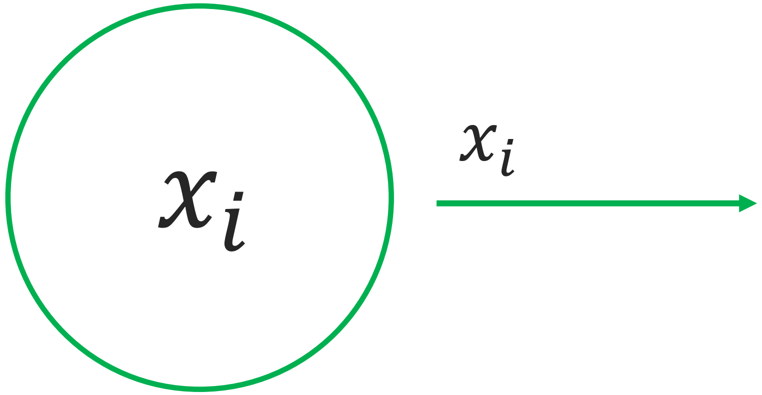
Universal function approximation

"A **feedforward network** with a single layer is sufficient to represent **any function**, but the layer may be infeasibly large and may fail to learn and generalize correctly."

Ian Goodfellow, Deep Learning

Creator of generative adversarial networks

Input nodes / neurons

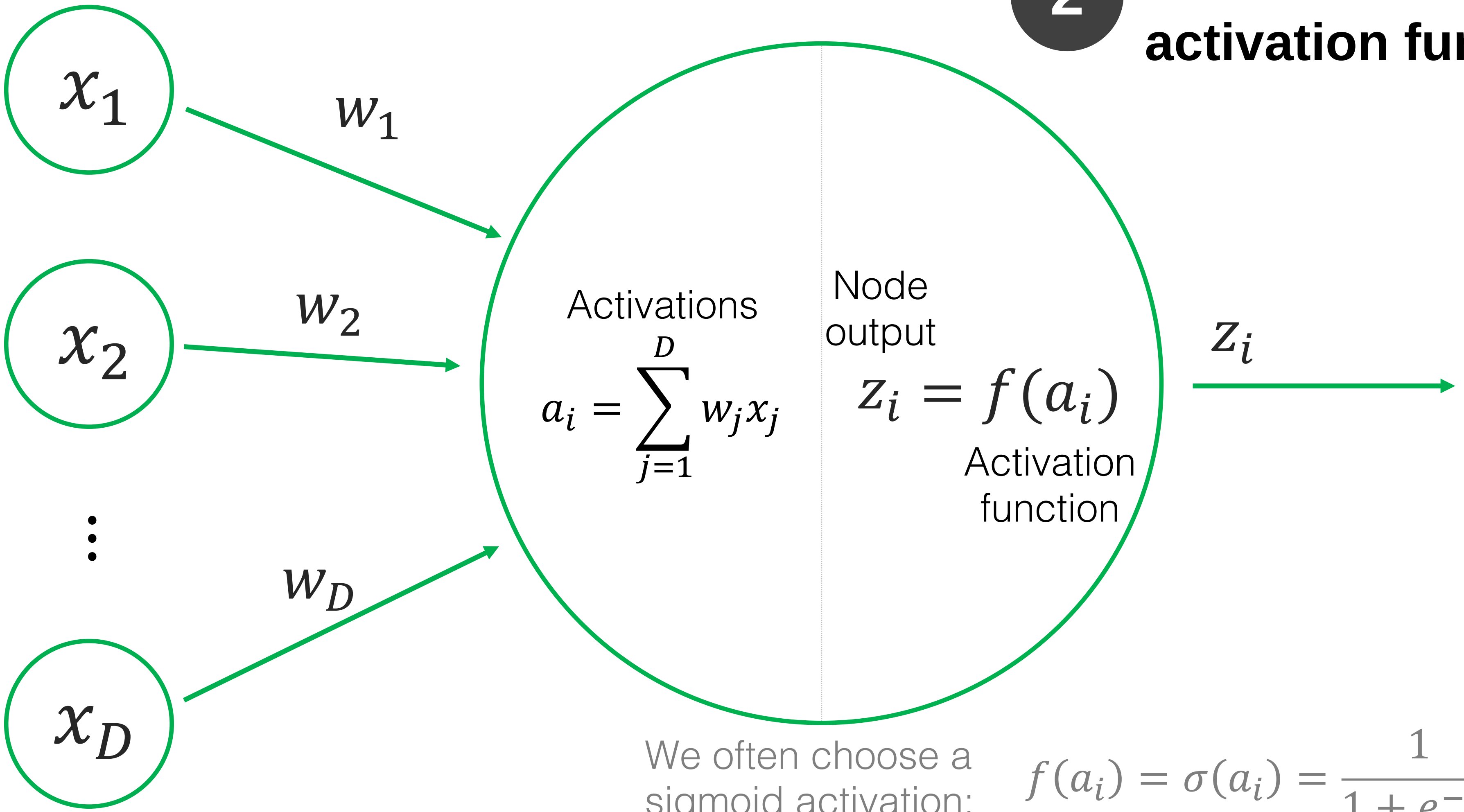


Simply passes the input value to the next layer

Hidden & output nodes

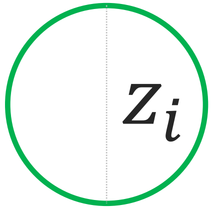
1 Calculate the **activations**: linear combinations of weights and the last layer's output

2 Calculate node output: apply the **activation function** to the activations

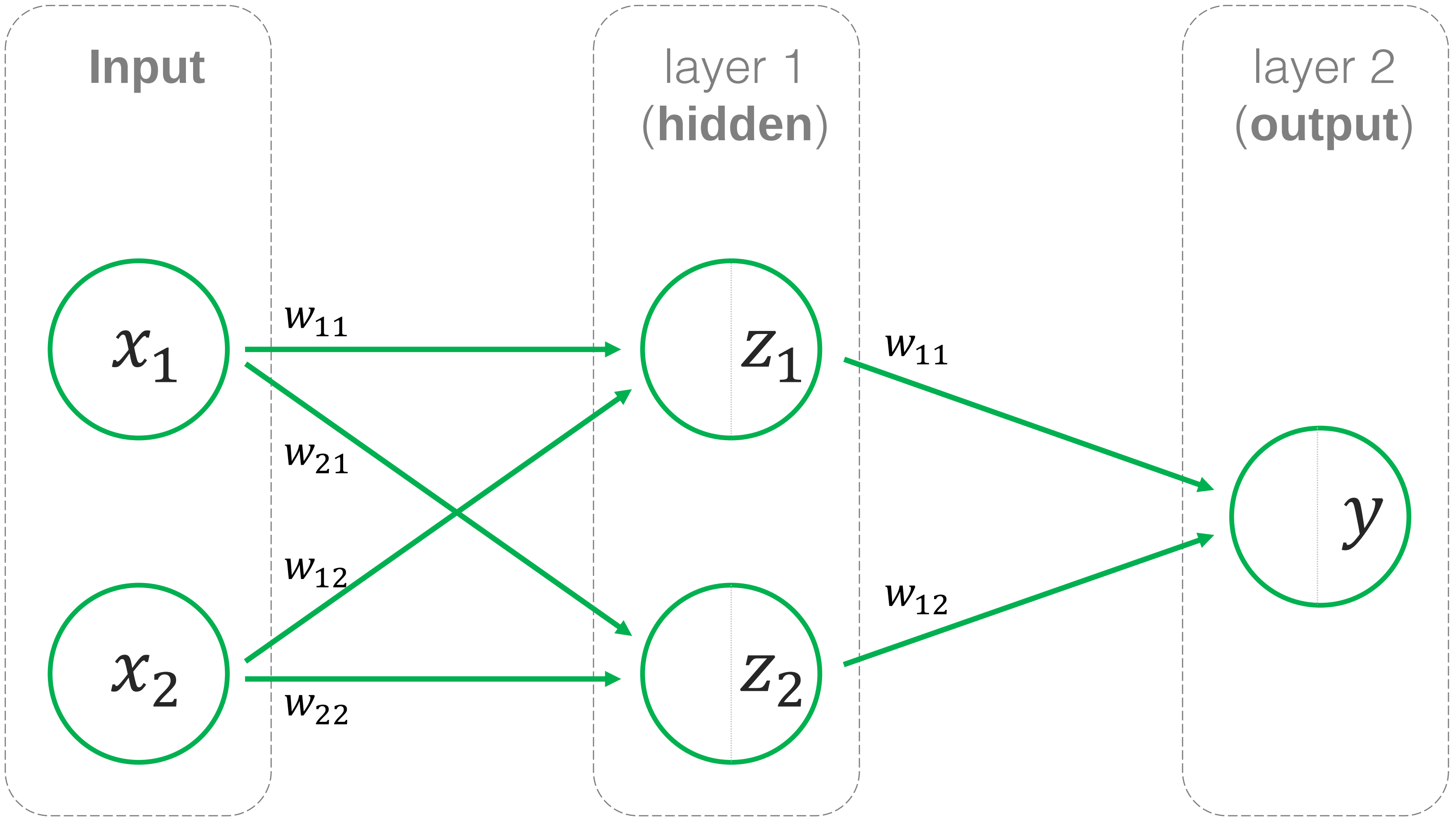


We often choose a sigmoid activation: $f(a_i) = \sigma(a_i) = \dfrac{1}{1 + e^{-a_i}}$

Represented as:



Simple Neural Network



$w_{ij}^{(k)}$
Layer k
From node j
(in the last layer)
to node i
(in the next layer)

Notational shorthand:
(a more precise alternative notation)

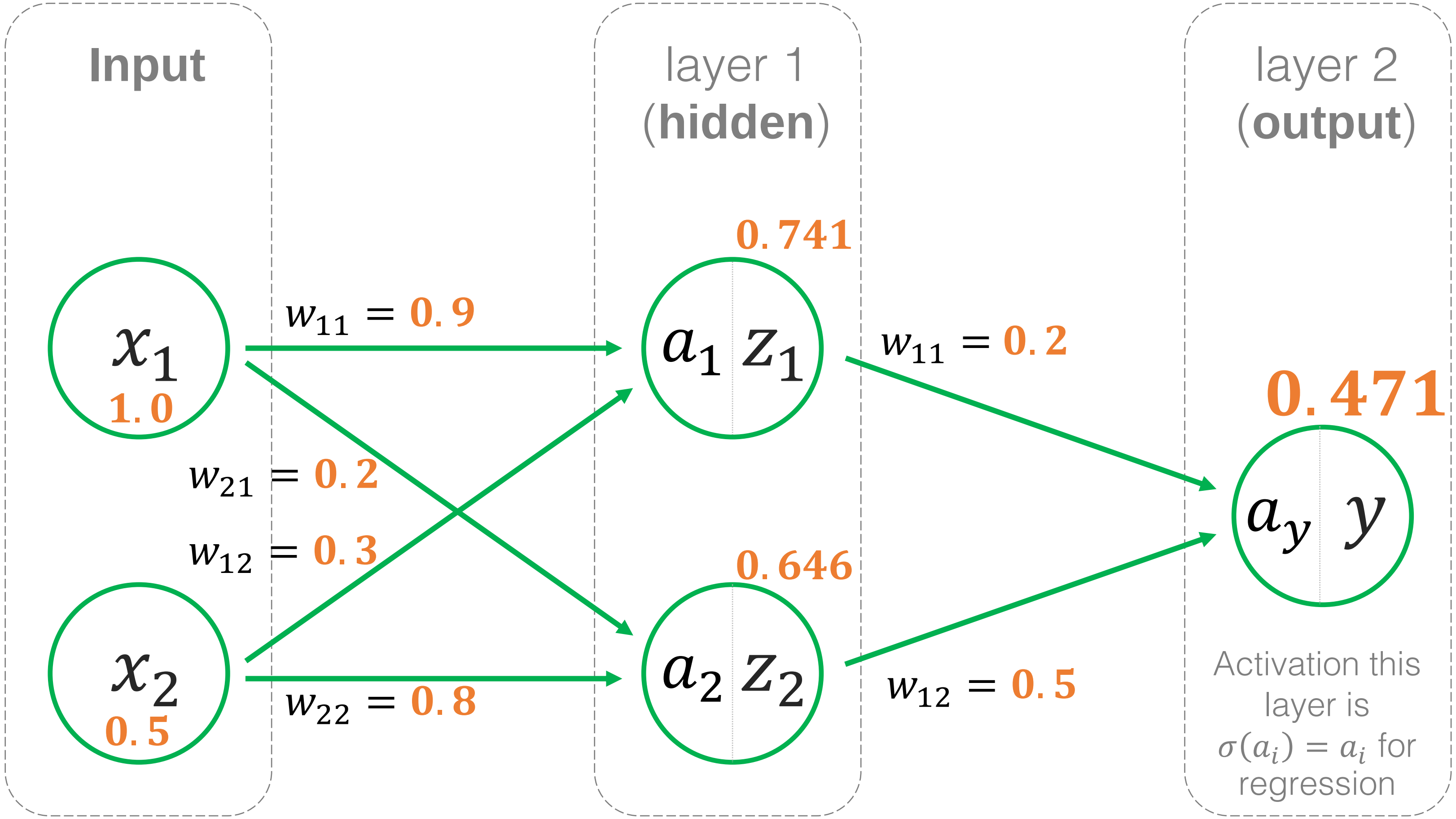
$w_{ij} = w_{ij}^{(1)}$ $z_i = z_i^{(1)}$ $w_{ij} = w_{ij}^{(2)}$ $y = z_1^{(2)}$

Forward Propagation

Calculating the output from input

Hidden layer calculations

$$a_1 = (0.9)(1.0) + (0.3)(0.5) = 1.05$$
$$a_2 = (0.2)(1.0) + (0.8)(0.5) = 0.6$$
$$z_1 = \sigma(a_1) = \sigma(1.05) = 0.741$$
$$z_2 = \sigma(a_2) = \sigma(0.6) = 0.646$$



Output layer calculations

$$a_y = (0.2)(0.741) + (0.5)(0.646)$$
$$= 0.471$$
$$y = a_y = 0.471$$ Regression

Alternatively…

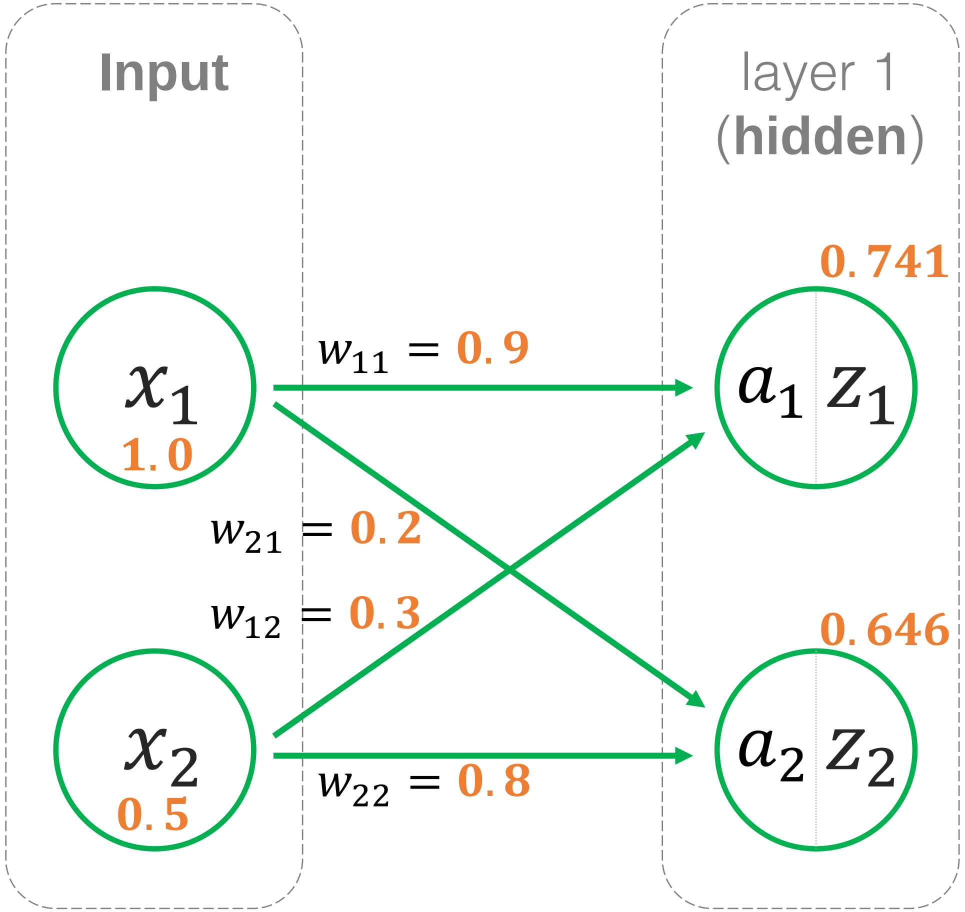
$$y = \sigma(a_y) = \sigma(0.471) = 0.616$$ Classification

$$\sigma(a_i) = \frac{1}{1 + e^{-a_i}}$$

Rashid, Make Your Own Neural Network

Forward Propagation

Calculating the output from input



Hidden layer matrix calculations

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \boldsymbol{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \boldsymbol{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\boldsymbol{W} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

→ The weights INTO node z_1

→ The weights INTO node z_2

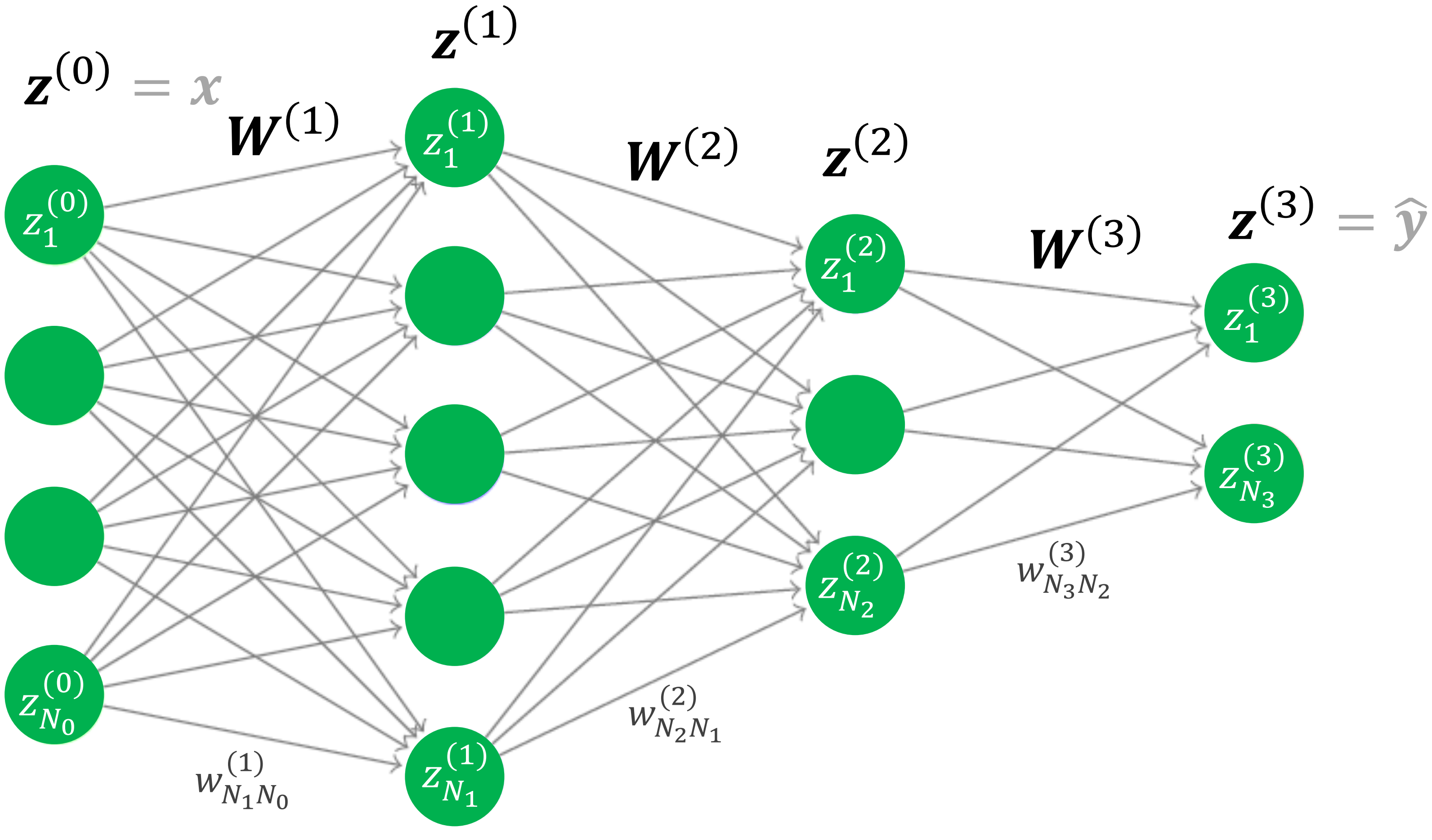
$$\boldsymbol{a} = \boldsymbol{W}\boldsymbol{x} \quad = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} w_{11}x_1 + w_{12}x_2 \\ w_{21}x_1 + w_{22}x_2 \end{bmatrix}$$

$$\boldsymbol{z} = \sigma(\boldsymbol{a}) \quad = \begin{bmatrix} \sigma(w_{11}x_1 + w_{12}x_2) \\ \sigma(w_{21}x_1 + w_{22}x_2) \end{bmatrix}$$

Forward Propagation

Example neural network with $L = 3$ layers and the ith layer has N_i nodes



Simple steps for forward propagation:

For $i = 1$ to $L - 1$:

$$\boldsymbol{z}^{(i)} = \sigma\left(\boldsymbol{W}^{(i)}\boldsymbol{z}^{(i-1)}\right)$$

Where:

$$\boldsymbol{z}^{(0)} = \boldsymbol{x}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{z}^{(L)}$$

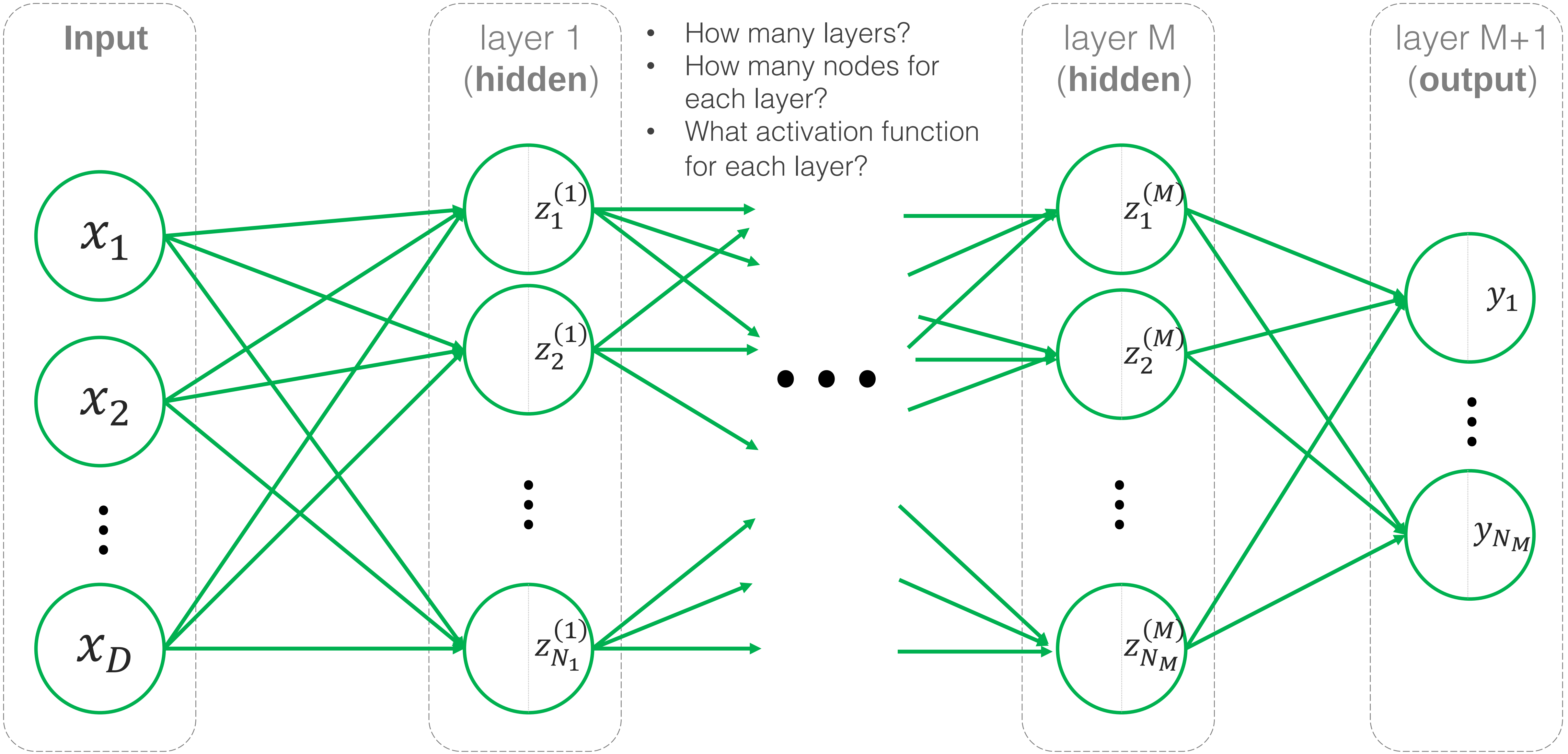
Prediction error is measured:

$$E_n = \frac{1}{2}(\hat{y}_n - y_n)^2$$

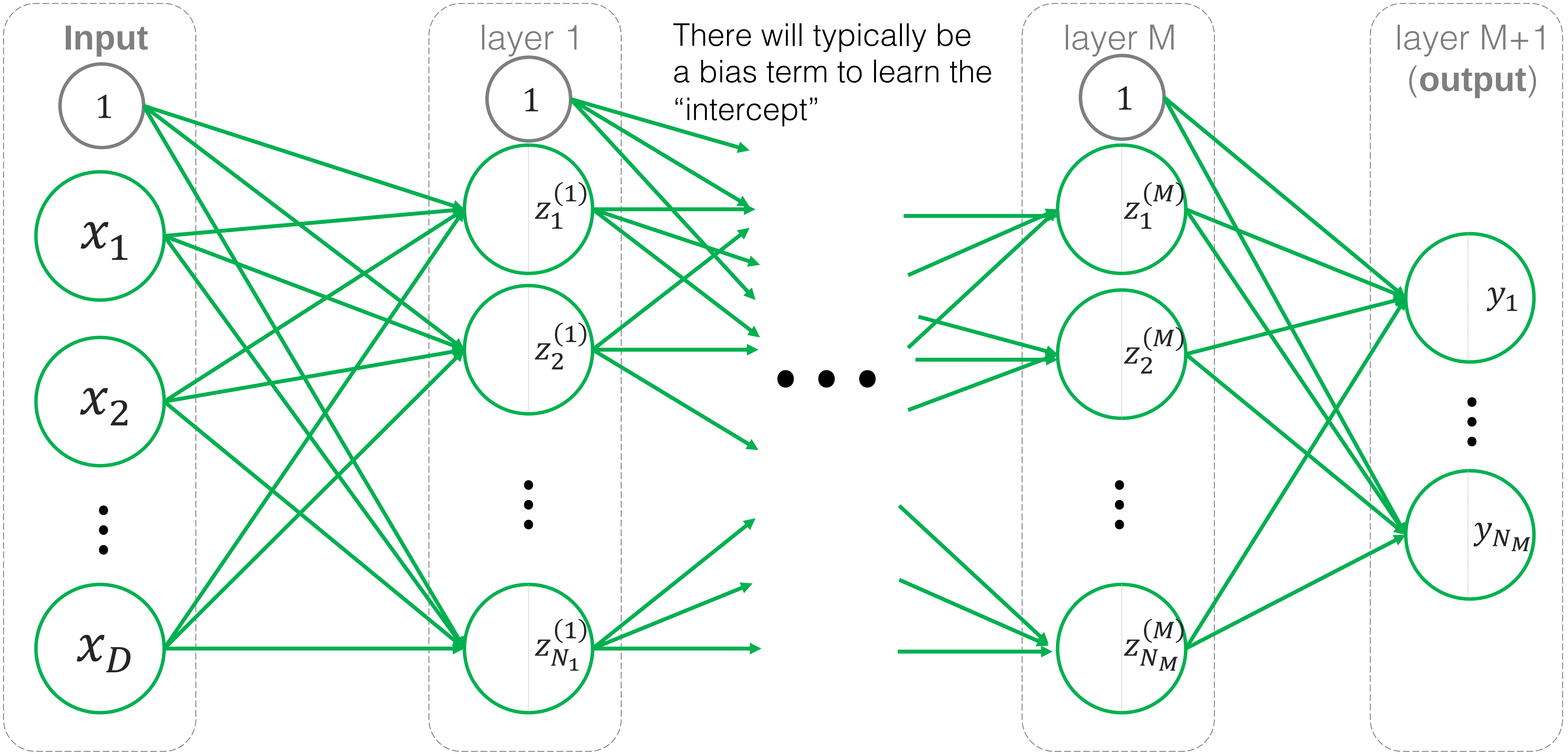
Sudeep Raja, A Derivation of Backpropagation in Matrix Form

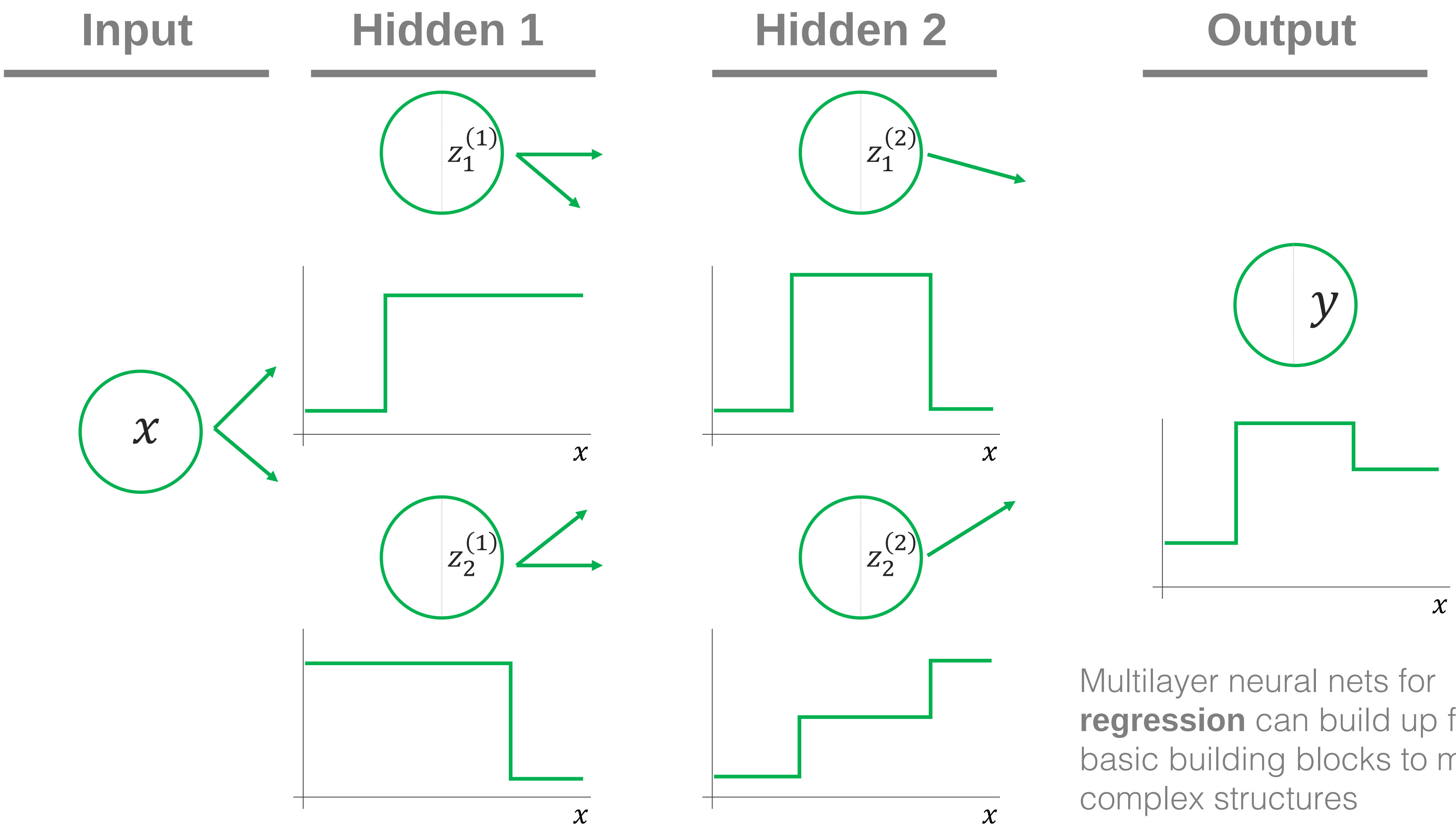
Neural networks can be customized

- How many layers?
- How many nodes for each layer?
- What activation function for each layer?



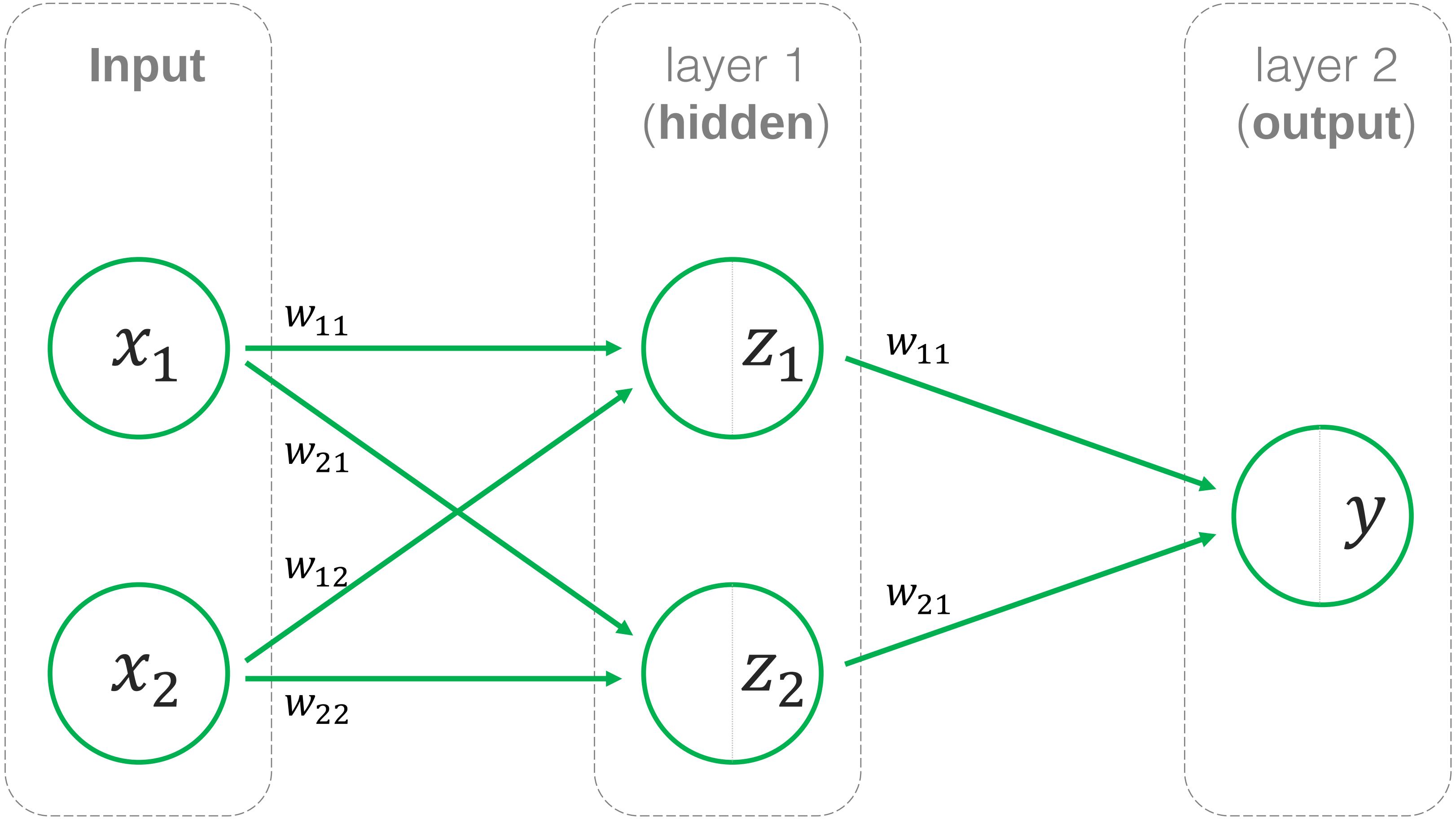
Neural networks can be customized





Multilayer neural nets for **regression** can build up from basic building blocks to more complex structures

From binary to multiclass classification



For **binary classification** with a sigmoid activation function, the output is between zero and one, so threshold this value to assign the class

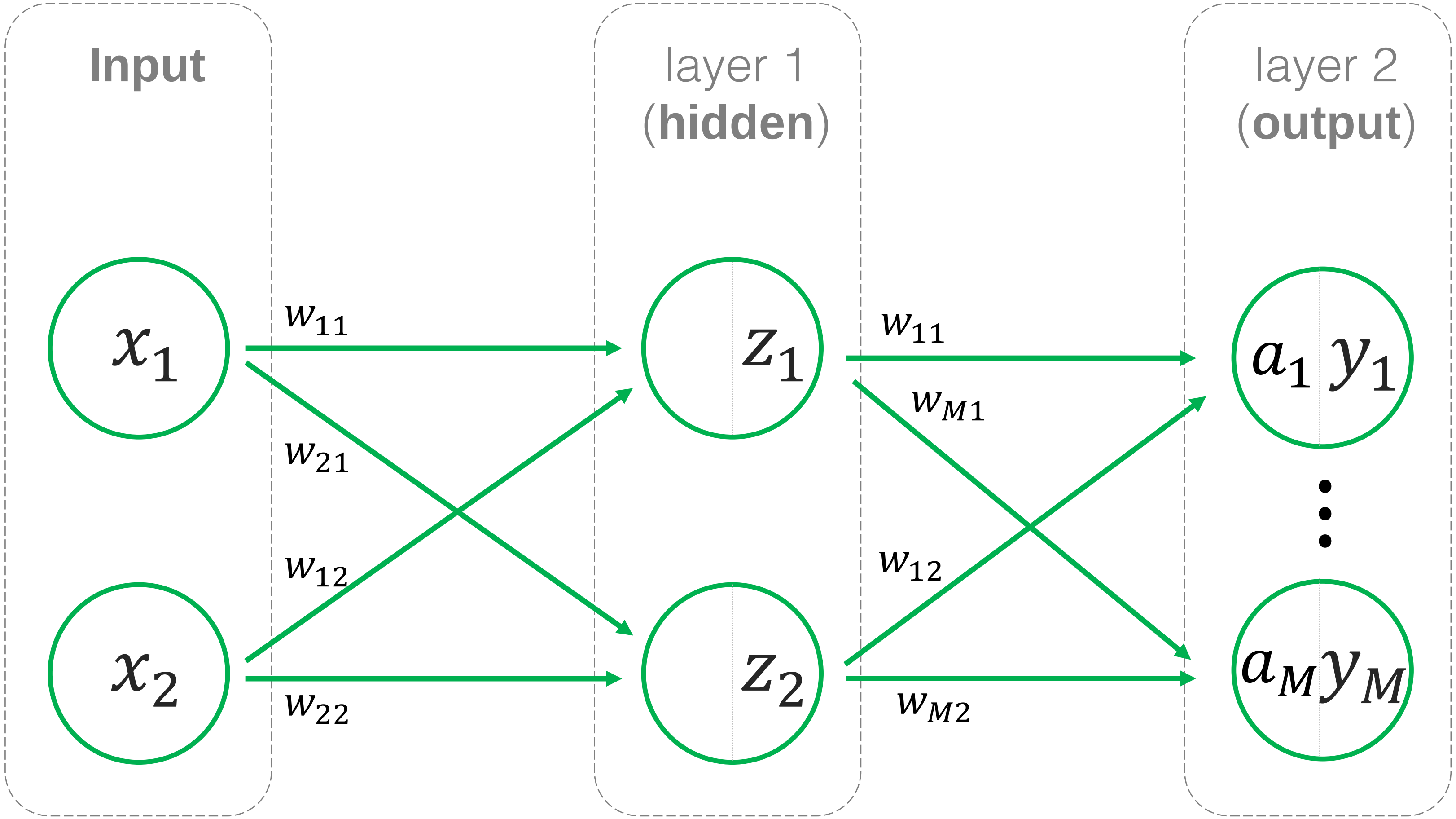
Softmax

Generalization of the logistic function to multiple dimensions

$$\begin{bmatrix} 5.1 \\ 4.2 \\ -3.1 \\ 0.7 \end{bmatrix} \rightarrow \boxed{\text{Softmax} \quad \frac{e^{a_i}}{\sum_{n=1}^{M} e^{a_n}}} \rightarrow \begin{bmatrix} 0.7046 \\ 0.2865 \\ 0.0002 \\ 0.0087 \end{bmatrix}$$

Always sums to 1
(normalizes to be a probability distribution)

From binary to multiclass classification



For **multiclass problems**, we can have multiple outputs and use a softmax function:

$$y_i = g(a_i) = \frac{e^{a_i}}{\sum_{n=1}^{M} e^{a_n}}$$

Choose the largest y value as the predicted class

As with many aspects of neural networks this is on of a number of approaches

Next time…

What is a neural network and **how does it work**?

How do we **optimize model weights**?
(i.e. how do we fit our model to data)

What are the challenges of using neural networks?