

# Reinforcement Learning I

# Types of machine learning

	Supervised Learning	Unsupervised Learning	Reinforcement Learning
Goal	<b>Predict</b> ...from examples	<b>Describe</b> ...structure in data	<b>Strategize</b> learn by trial and error
Data	$(x, y)$	$x$	delayed feedback
Types	<ul style="list-style-type: none"><li>• Classification</li><li>• Regression</li></ul>	<ul style="list-style-type: none"><li>• Density estimation</li><li>• Clustering</li><li>• Dimensionality reduction</li><li>• Anomaly detection</li></ul>	<ul style="list-style-type: none"><li>• Model-free learning</li><li>• Model-based learning</li></ul>

# Resources

*This reinforcement learning series draws heavily on these resources*

Sutton and Barto, 1998

(2<sup>nd</sup> edition 2018)

Reinforcement Learning: An Introduction

Draft of updated edition available free online:

<http://www.incompleteideas.net/book/the-book-2nd.html>

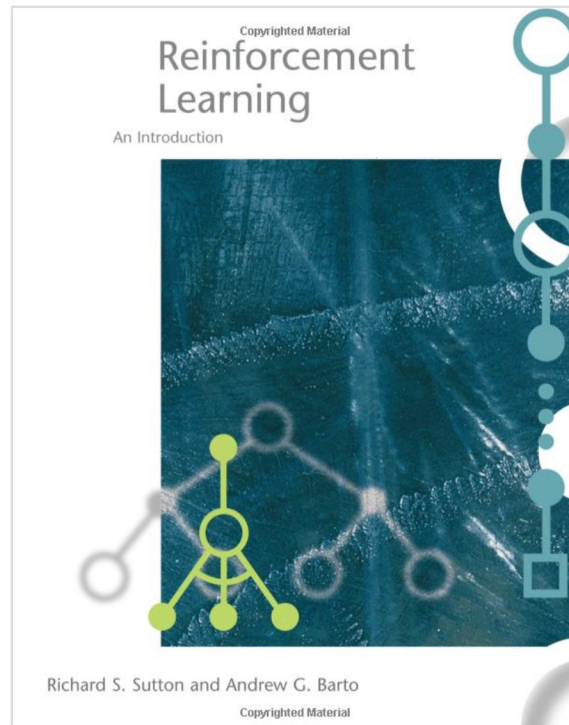


Image from Amazon.com (where the book may be purchased)

David Silver, 2015

University College London

Advanced Topics 2015 (COMPM050/COMPGI13)

Course website:

<http://www0.cs.ucl.ac.uk/staff/D.Silver/web/Teaching.html>

Video series:

<https://www.youtube.com/watch?v=2pWv7GOvuf0&list=PL7-jPKtc4r78-wCZcQn5lqyuWhBZ8fOxT>

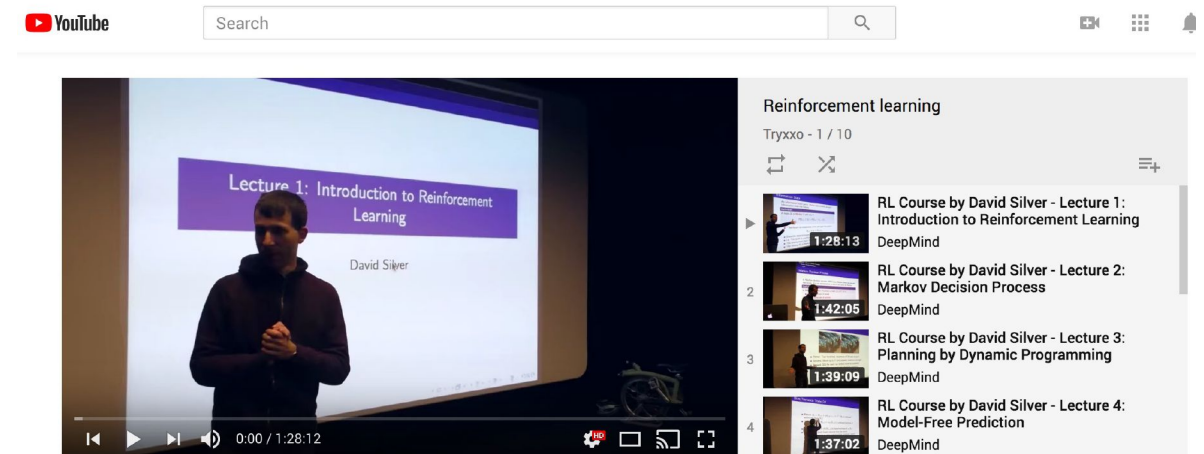
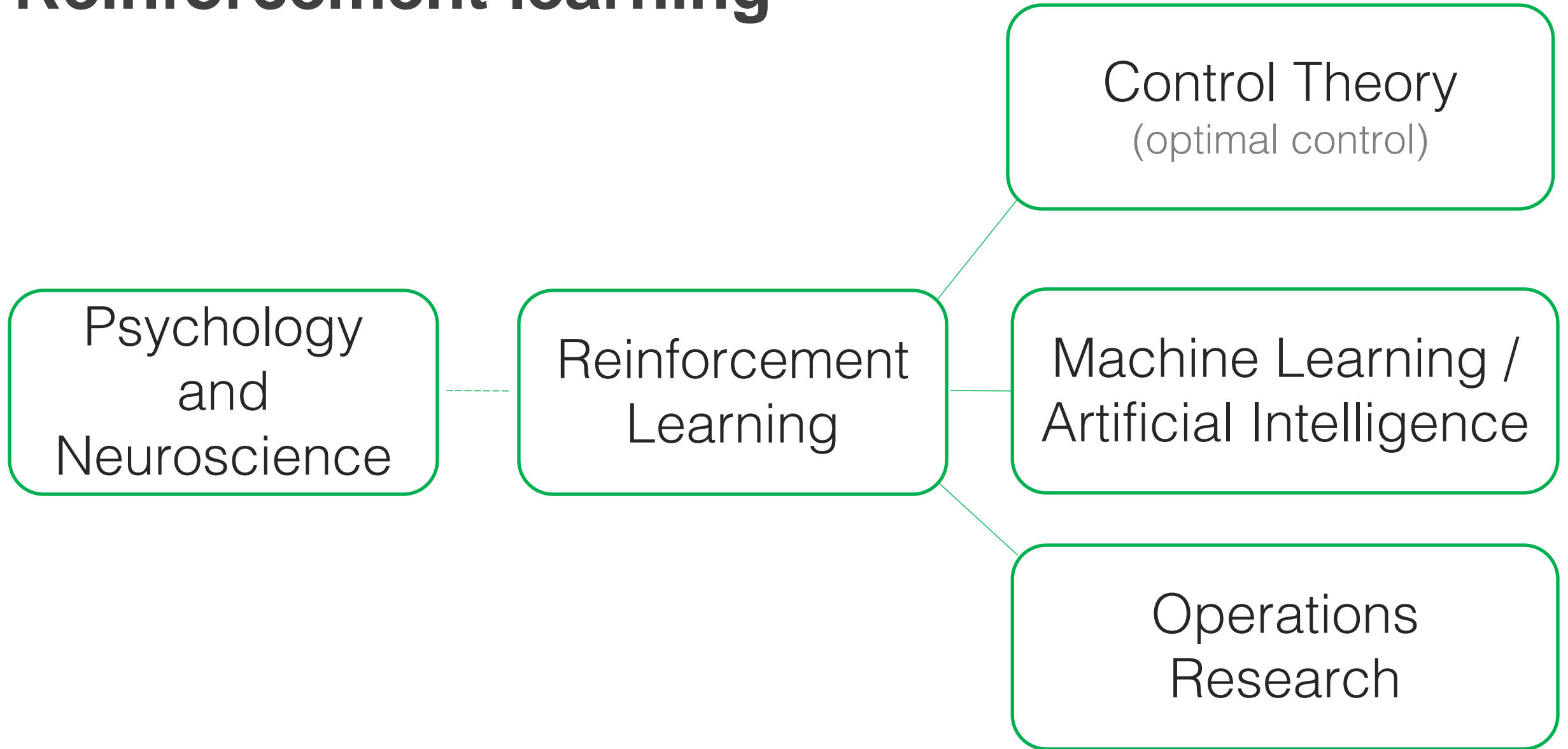


Image from Youtube.com

# Reinforcement learning



# Reinforcement Learning

**Goal: select actions to maximize total long-term rewards**

Sequential decision making

Challenge: an action needs to be taken at each step

Evaluation of rewards versus instruction (examples of correct actions)

Challenge: this leads to a trial-and-error approach to learning

May be better to sacrifice immediate reward for long-term gains

Challenge: exploration (of untried actions) vs exploitation (of current knowledge)

Rewards may be delayed

Challenge: credit assignment: which action(s) led to the reward(s)?

# Reinforcement Learning Applications

- Self-driving cars ([link](#))
- Energy-efficient data center cooling control ([link](#))
- Financial trading ([link](#))
- Medical diagnosis and treatment ([link](#))
- Gaming ([AlphaGo](#), [Atari](#), [StarCraft](#))

Industry Leaders: Google Deepmind ([link](#))

# Reinforcement Learning Examples

Winning at Atari: <https://youtu.be/V1eYniJ0Rnk>

Balancing an inverted pendulum: [https://youtu.be/b1c0N\\_Fs9wc](https://youtu.be/b1c0N_Fs9wc)

Flipping pancakes: [https://youtu.be/W\\_gxLKSsSIE](https://youtu.be/W_gxLKSsSIE)

**RL is a unifying framework for a wide range of problems**



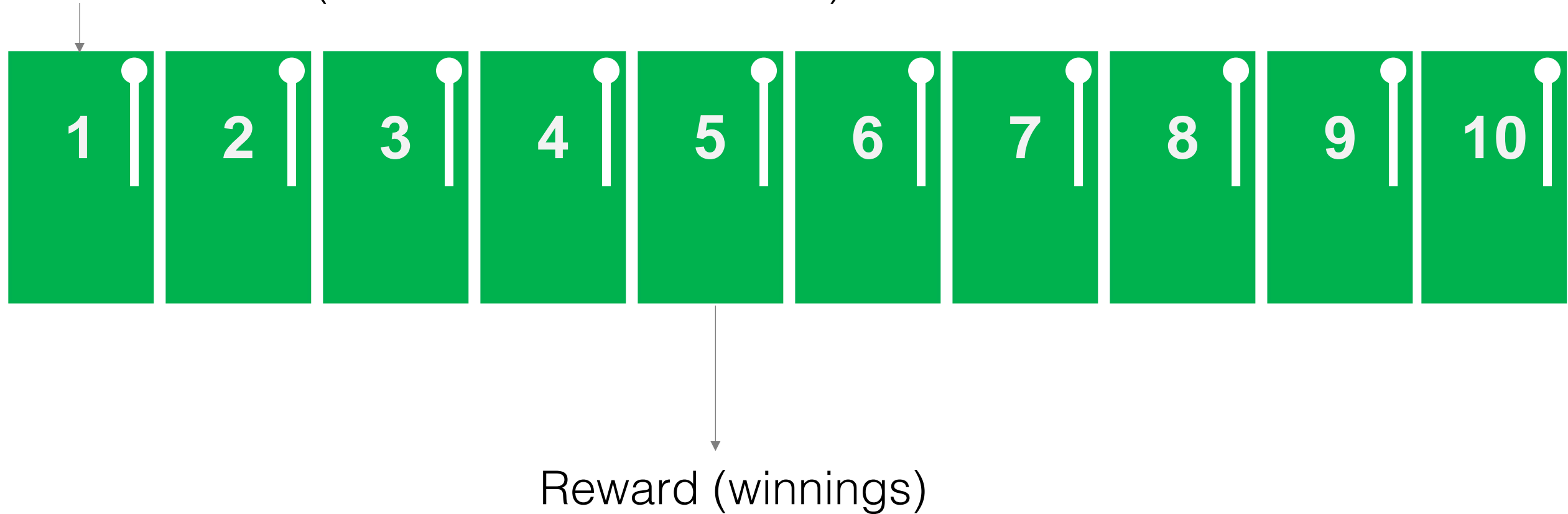
# Multi-armed Bandit



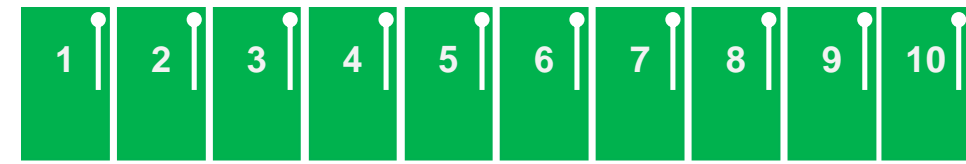


# You walk into a casino...

Slot Machine (a.k.a. one-armed bandit)



# Multi-armed bandit problem



**Trial/episode:** play one machine

**Action:** pick one machine to play (one action per trial/episode)

**Reward:** how much you win or lose

- Each machine has an unknown probability of payoff/reward
- The rewards are stochastic (their distributions are unknown)

**Action-Value:** expected reward for taking each action

**State:** only 1 “state” in this problem - our environment doesn’t change

**Policy:** create a policy How do we choose actions to maximize our total rewards?

- If we knew the best machine, we’d always pick it
- This is what we want to learn

# Multi-armed bandit

The *true* **action-value** of an action is  $q_*(a)$

Our *estimated* **action-value** at the  $t^{\text{th}}$  play is  $q_t(a)$

If action  $a$  has been chosen  $k_a$  times prior to  $t$ :

$$q_t(a) = \frac{r_1 + r_2 + \dots + r_{k_a}}{k_a}$$

As we take action  $a$  more, our action-value estimates improve

# Multi-armed bandit policies, $\pi(s)$

## Greedy action:

Select  $a^* = \arg \max_a q_t(a)$

**Problem:** if the initial rewards are not representative, this will be suboptimal

## $\epsilon$ -Greedy methods:

Select  $a^*$  with probability  $1 - \epsilon$ , otherwise, randomly select another option

**Problem:** in the long run, this will waste reward once the best action is known

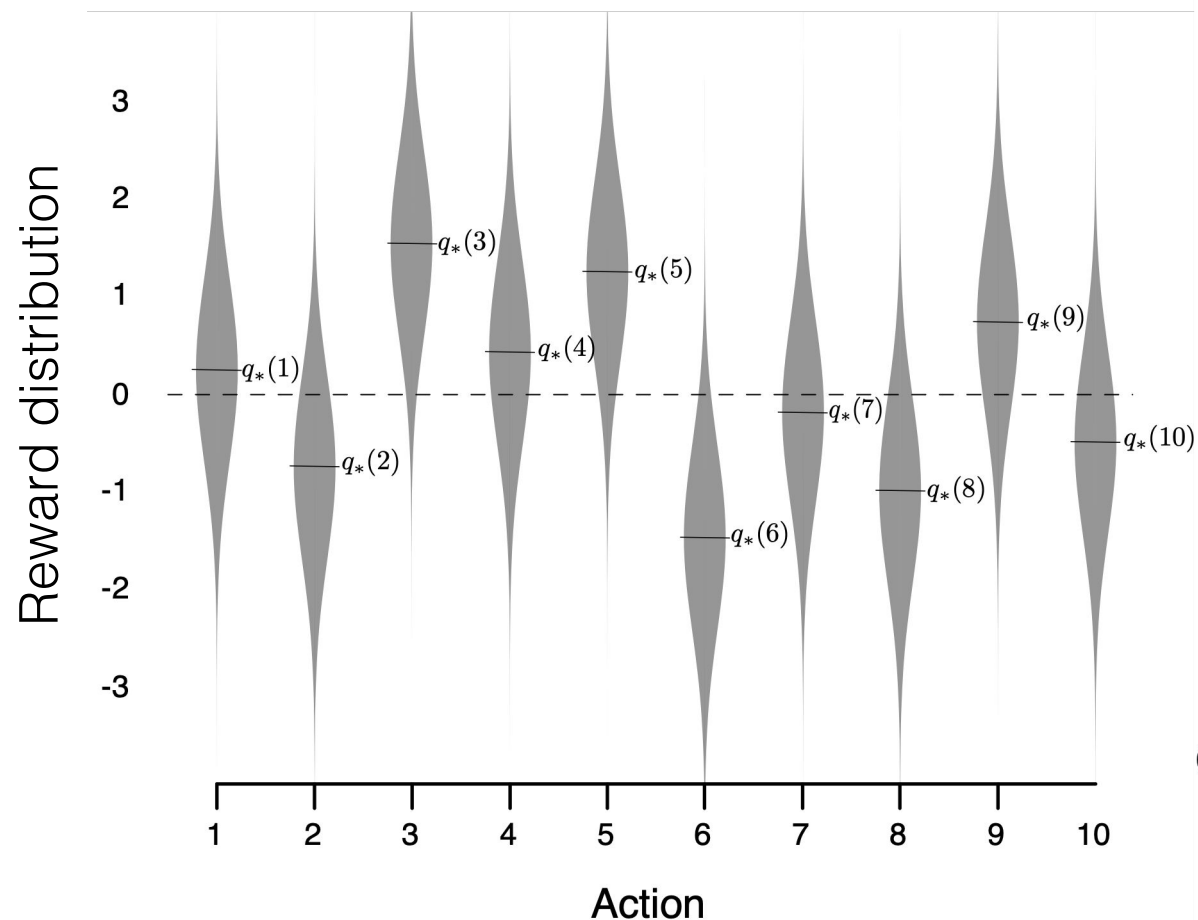
**Solution:** reduce  $\epsilon$  over time

## Alternative:

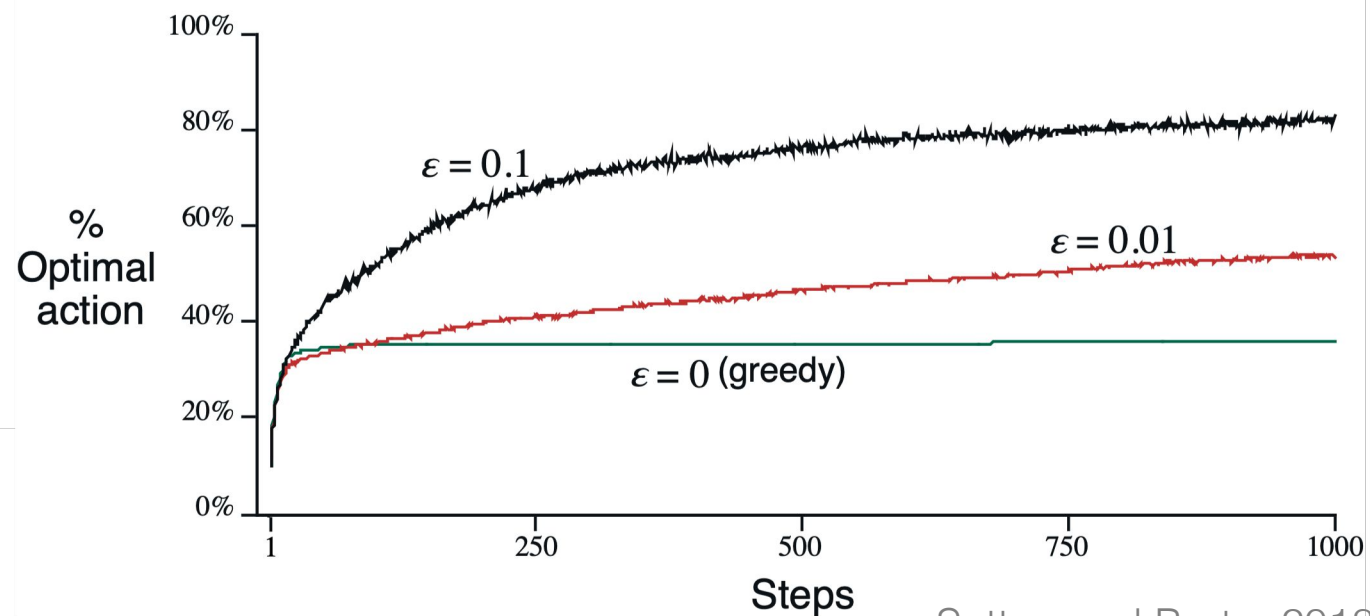
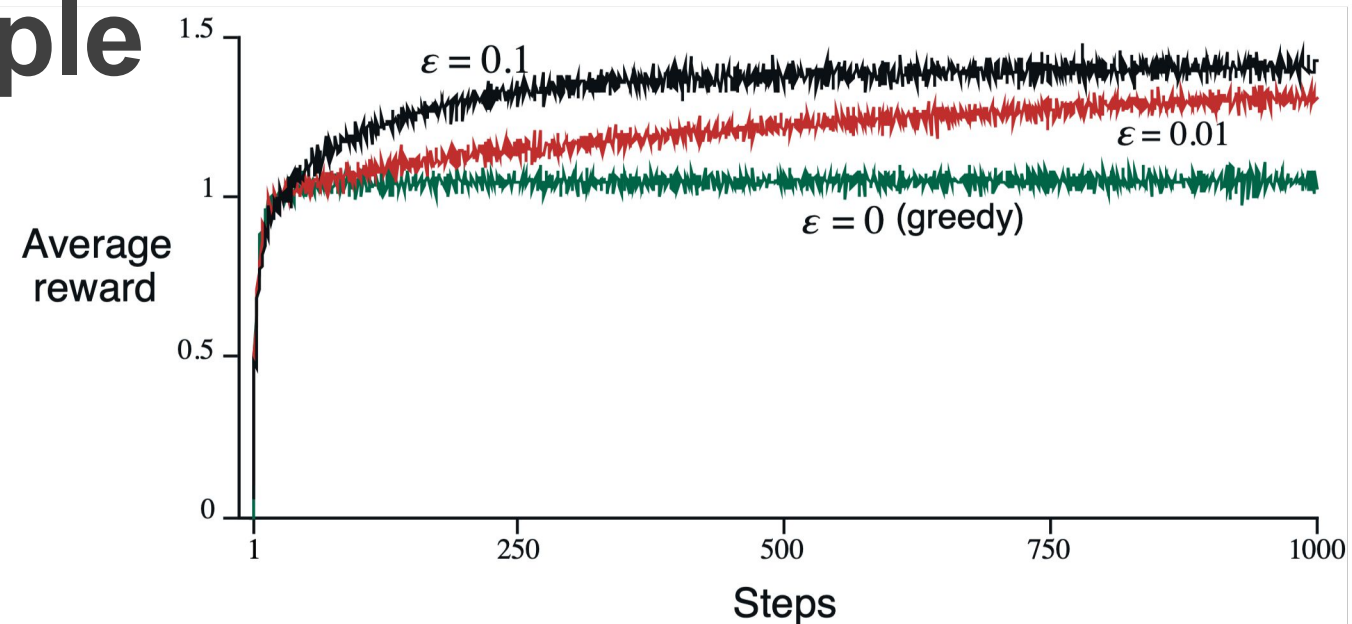
Select the action probabilities based on the expected value

Probability of selecting action  $P(a) = \frac{\exp(q_t(a))}{\sum_{b=1}^n \exp(q_t(b))}$

# 10-Armed Bandit Example



Note: Each distribution has a mean  $q_*(a)$  with unit variance



Sutton and Barto, 2018



# Roadmap for this module

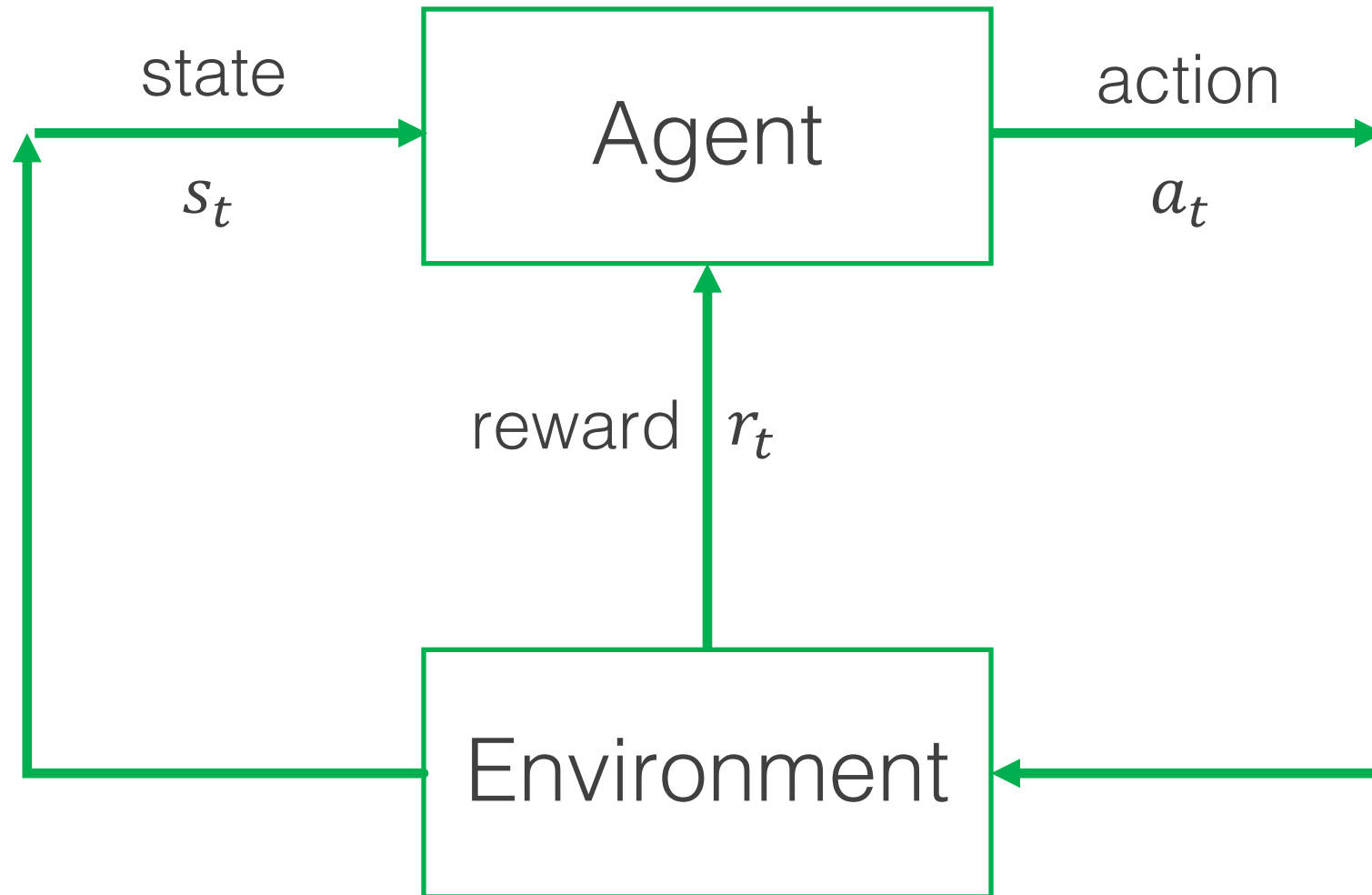
The multi-armed bandit only has 1 state, but the full RL problem learns policies when there are many states that the agent moves between

State representations and Markov decision processes (MDPs)  
(with a discussion of Markov processes)

Mathematically formulating the RL problem with MDPs

Methods for solving RL problems in practice  
(dynamic programming and Monte Carlo control)

# Agent-environment Interaction



**Agent** at each step  $t$ ...

Encounters state,  $s_t$   
Executes action  $a_t$   
Receives scalar reward,  $r_{t+1}$

**Environment** at each step  $t$ ...

Receives action  $a_t$   
Transitions to state,  $s_{t+1}$   
Emits scalar reward,  $r_{t+1}$

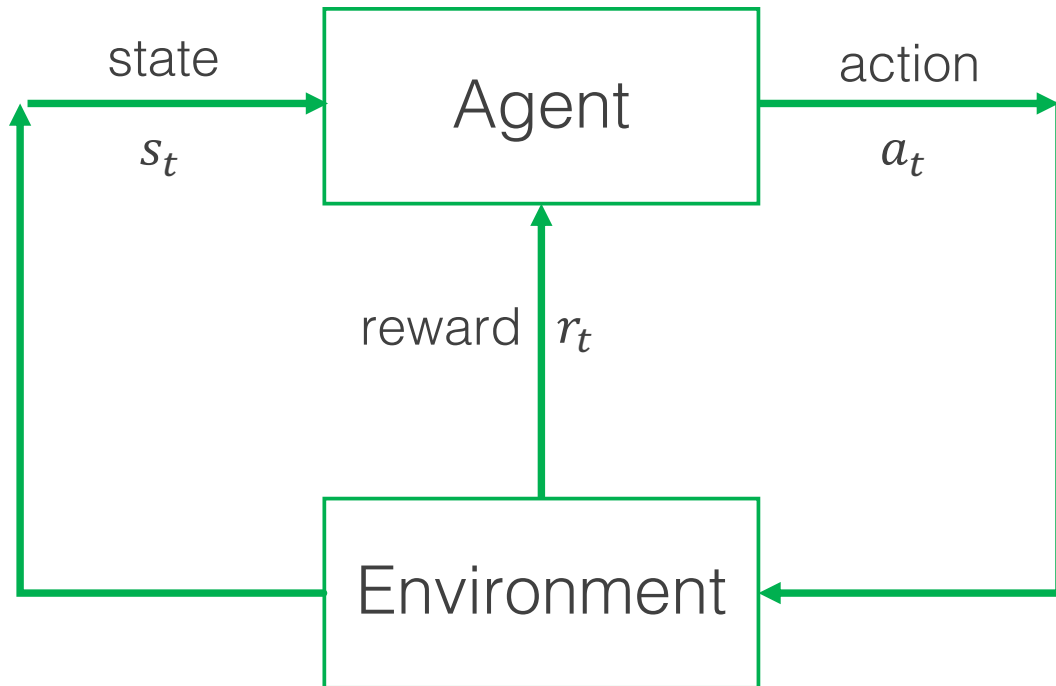
**Actions:** choices made by the agent

**States:** basis on which choices are made

**Rewards:** define the agent's goals

David Silver, 2015

# Reinforcement Learning Components

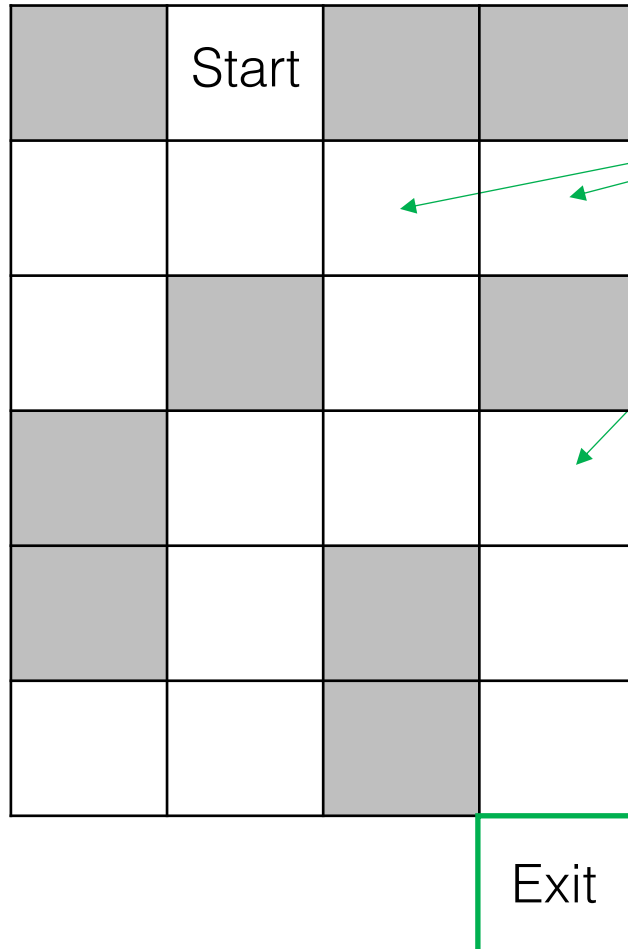


**Policy** (agent behavior),  $\pi(s)$

**Reward function** (the goal),  $r_t$

**Value functions** (expected returns),  
 $v(s)$  State value  
 $q(s, a)$  Action value

# Maze Example: Policy, Value, and Reward



Each location in the maze represents a **state**

The **reward** is -1 for each step the agent is in the maze

Available **actions**: move  $\uparrow, \downarrow, \leftarrow, \rightarrow$  (as long as that path is not blocked)

Adapted from David Silver, 2015

# Policy $\pi(s)$

(which actions to take in each state)

Start

	↓		
→	→	↓	←
↑		↓	
	→	→	↓
	↑		↓
→	↑		↓
Exit			

Adapted from David Silver, 2015



## Policy $\pi(s)$

(which actions to take in each state)

Start

	↓		
→	→	↓	←
↑		↓	
	→	→	↓
	↑		↓
→	↑		↓
Exit			

## Reward $r_t$

(rewards are received after actions are taken)

Start

	-1		
-1	-1	-1	-1
-1		-1	
	-1	-1	-1
	-1		-1
-1	-1		-1
Exit			

Adapted from David Silver, 2015

## Policy $\pi(s)$

(which actions to take in each state)

Start

	↓		
→	→	↓	←
↑		↓	
	→	→	↓
	↑		↓
→	↑		↓
Exit			

## Reward $r_t$

(rewards are received after actions are taken)

Start

	-1		
-1	-1	-1	-1
-1		-1	
	-1	-1	-1
	-1		-1
-1	-1		-1
Exit			

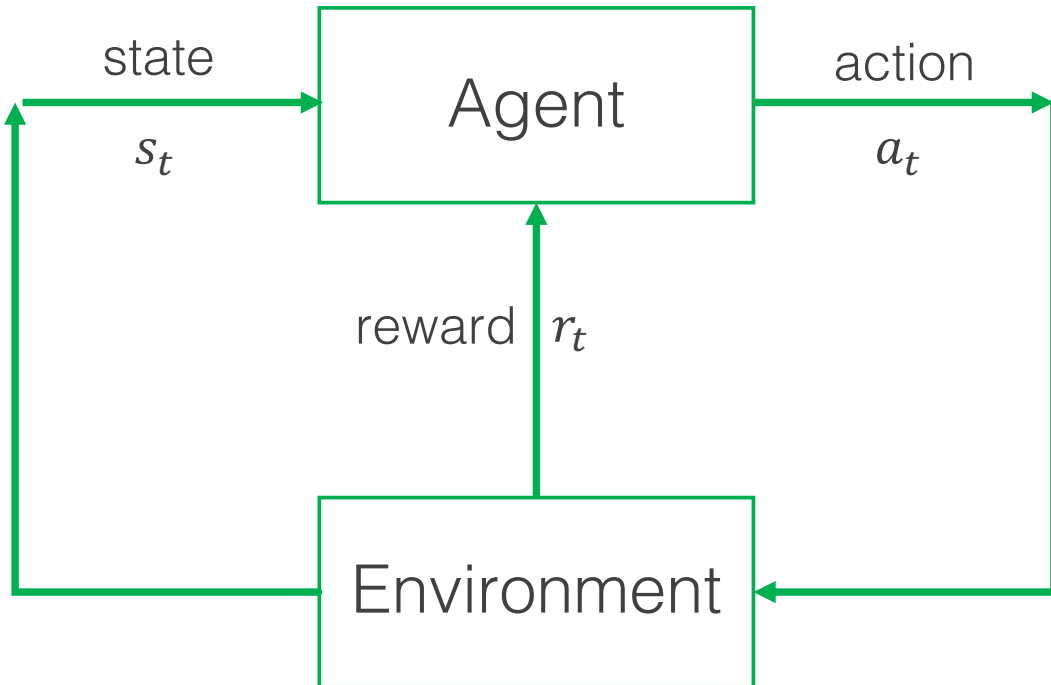
## State Value $v_\pi(s)$

(expected cumulative rewards starting from current state **if** we follow the policy)

Start

	-8		
-8	-7	-6	-7
-9		-5	
	-5	-4	-3
	-6		-2
-8	-7		-1
Exit			

# Policy



## Policy, $\pi(s)$

- Selects an action to choose based on the state
- Determines an agent's “behavior”

Deterministic policy:

$$a = \pi(s)$$

Stochastic policy:

$$\pi(a|s) = P(a_t = a | s_t = s)$$

Helps us “explore” the state space

RL tries to learn the “best” policy

# Goals and rewards

Rewards are the **only way** of communicating RL goals

Ex 1: Robot learning a maze

- 0 until it escapes, then +1 when it does
- -1 until it escapes (encourages it to escape quickly)

Ex 2: Robot collecting empty soda cans

- +1 for each empty soda can
- Negative rewards for bumping into things

Chess: what if we set +1 for capturing a piece?  
(it may not win the game and still maximize rewards)

**What** you want achieved not **how**

# Returns / cumulative reward

**Episodic** tasks (finite number,  $T$ , of steps, then reset)

$$G_t = r_{t+1} + r_{t+2} + \dots + r_T$$

**Continuing** tasks with discounting ( $T \rightarrow \infty$ )

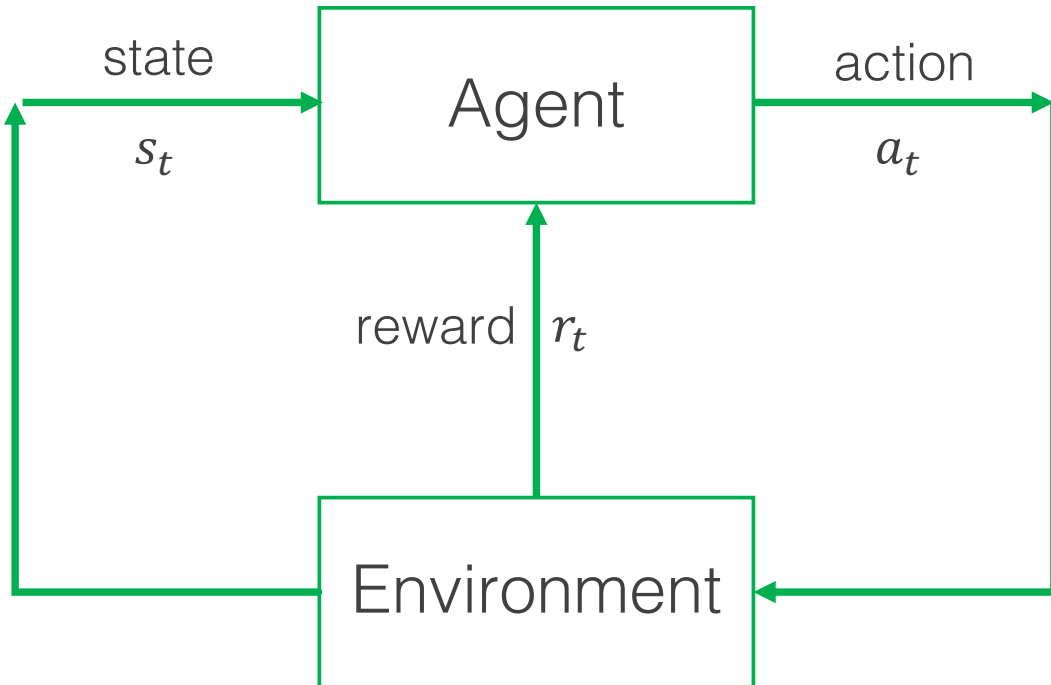
$$G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} \dots = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$$

where  $0 \leq \gamma \leq 1$  is the discount rate

This makes the agent care more about immediate rewards



# Value functions



## State Value function, $v_\pi(s)$

- How “good” is it to be in a state,  $s_t$  then follow policy  $\pi$  to choose actions
- Total expected rewards

$$v_\pi(s) = E_\pi[G_t | s_t = s]$$

## Action Value function, $q_\pi(s, a)$

- How “good” is it to be in a state,  $s$ , take action  $a$ , then follow policy  $\pi$  to choose actions
- Total expected rewards

$$q_\pi(s, a) = E_\pi[G_t | s_t = s, a_t = a]$$

Where  $G_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$

## Policy $\pi(s)$

(which actions to take in each state)

Start			
	↓		
→	→	↓	←
↑		↓	
	→	→	↓
	↑		↓
→	↑		↓
Exit			

## Reward $r_t$

(rewards are received after actions are taken)

Start			
	-1		
-1	-1	-1	-1
-1		-1	
	-1	-1	-1
	-1		-1
-1	-1		-1
Exit			

## State Value $v_\pi(s)$

(expected cumulative rewards starting from current state **if** we follow the policy)

Start			
	-8		
-8	-7	-6	-7
-9		-5	
	-5	-4	-3
	-6		-2
-8	-7		-1
Exit			

## Action Value $q_\pi(s, a)$

(expected cumulative rewards starting from current state **if** we take action  $a$  then follow the policy)

↑	-9
→	-7
←	-9

↑	-4
↓	-2

# Model

## Model (of the environment)

Transitions: predicts what state the environment will transition to next

$$P_{ss'}^a = P(s_{t+1} = s' | s_t = s, a_t = a)$$

Rewards: predicts the next reward given an action

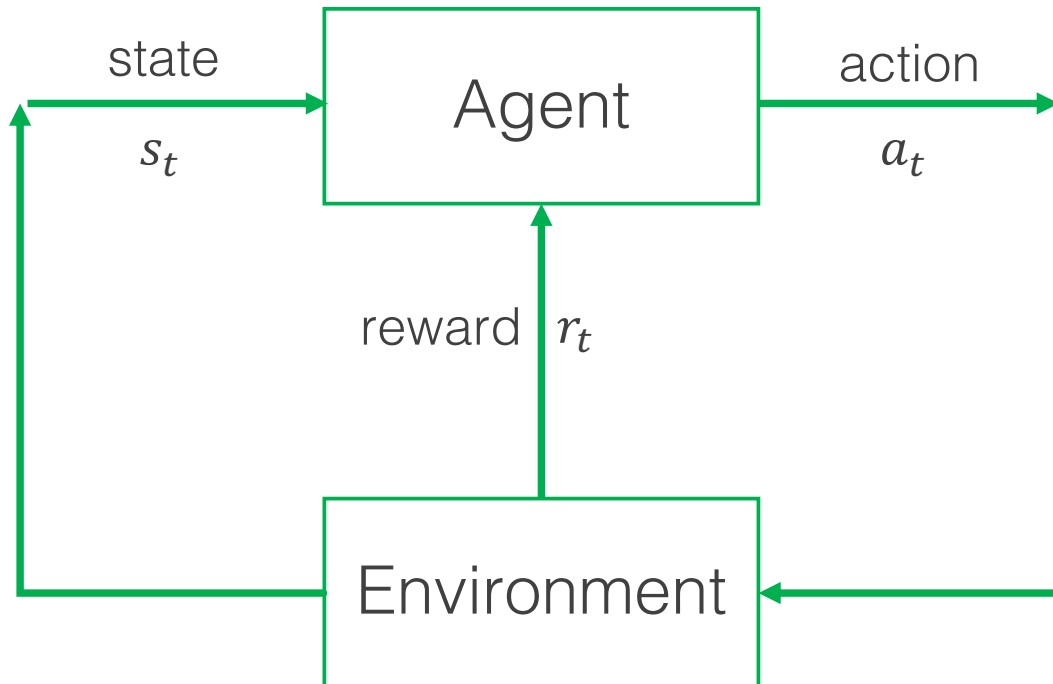
$$R_s^a = E[r_{t+1} | s_t = s, a_t = a]$$

“Planning” is the process of using a model to create or improve a policy

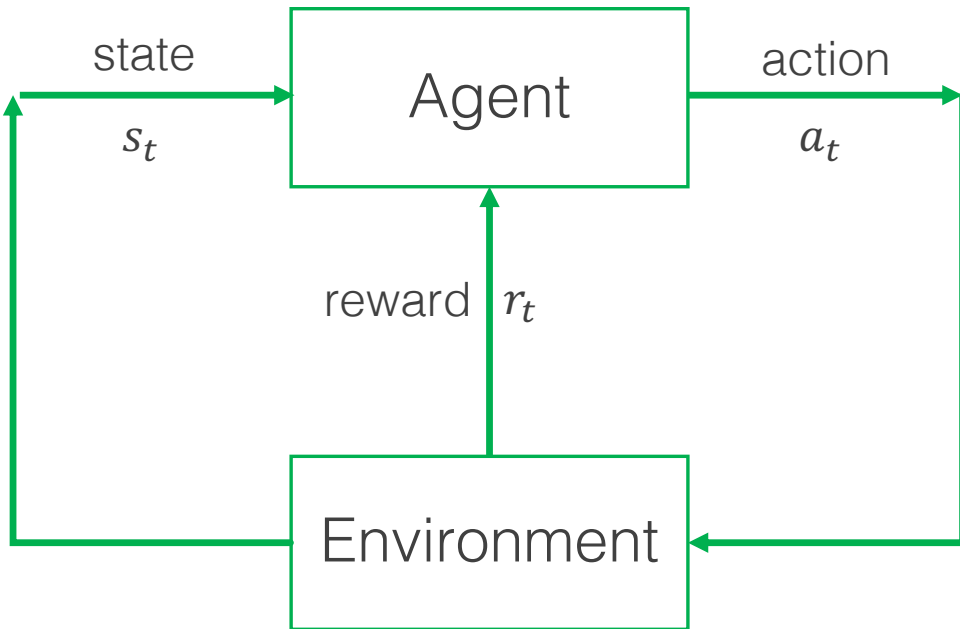
We don't always have a full model of the environment

**Model-based RL** uses a model

**Model-free RL** does not use a model



# Reinforcement Learning Components



**Policy** (determines agent behavior),  $\pi(s)$

- Determines action given current state
- Agent's way of behaving at a given time

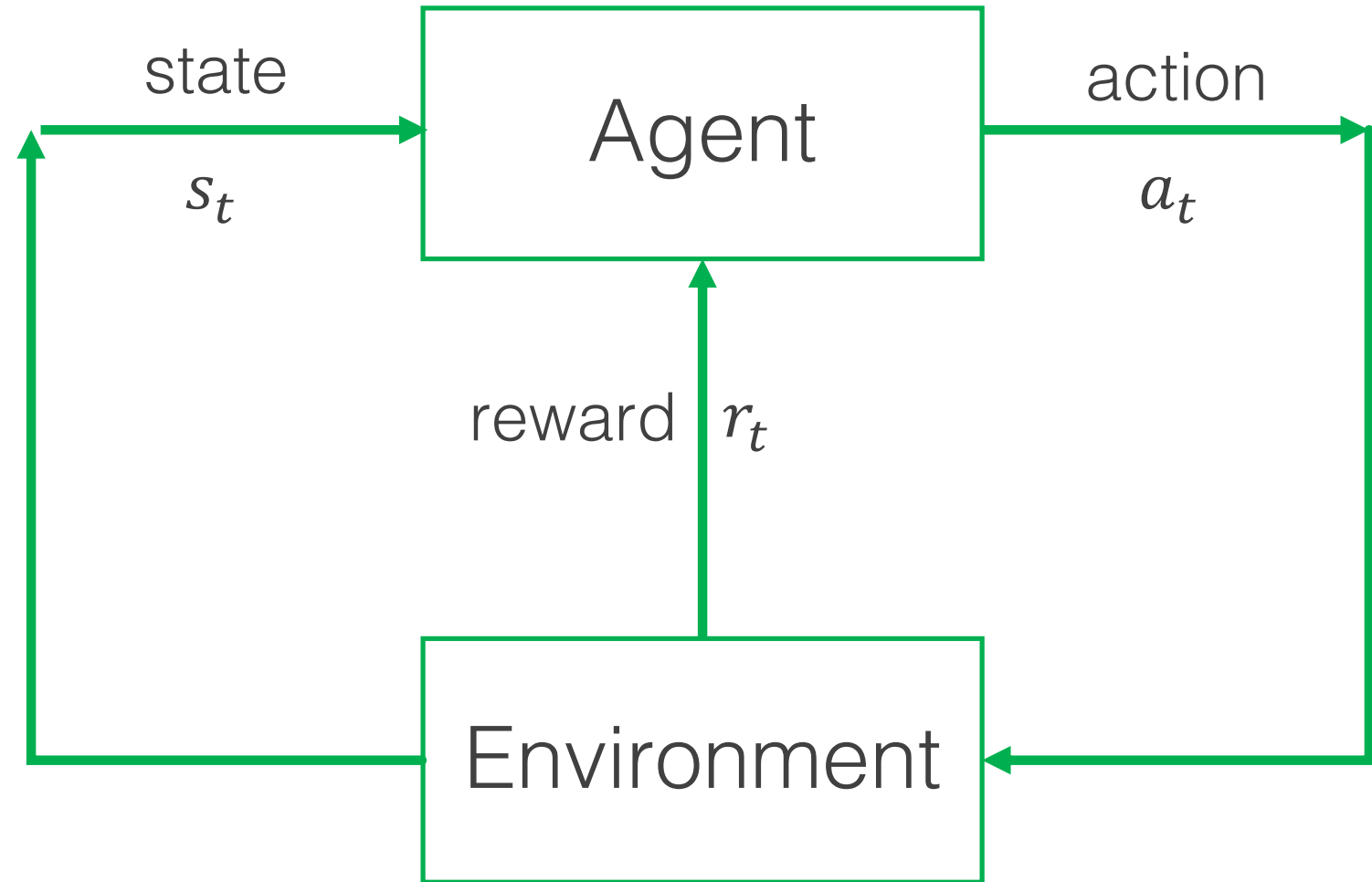
**Reward function** (sets the goal),  $r_t$

- Maps state of the environment to a reward that describes the state desirability
- Objective is to **maximize total rewards**

**Value** (estimates expected returns),  $v(s), q(s, a)$

- Expected returns from a state and following a specific policy
- How “good” is each state

# Environment



Markov Decision Process  
(assumed form for most RL problems)



# Goal

**Maximize returns (expected rewards)**

**Find the best policy to guide our actions in an environment**

Here, environment is modeled as a Markov Decision Process