

# Reinforcement Learning IV

# Reinforcement Learning Roadmap

Knowledge of **Environment**

## **Perfect knowledge**

Known Markov  
Decision Process



## **No knowledge**

Must learn from  
experience

## Dynamic Programming

What's a Markov Decision Process?  
How do we find optimal policies?

## Monte Carlo Control

How do we estimate our value functions?  
How do we use the value functions to choose actions?

# Markov Decision Process

## Components:

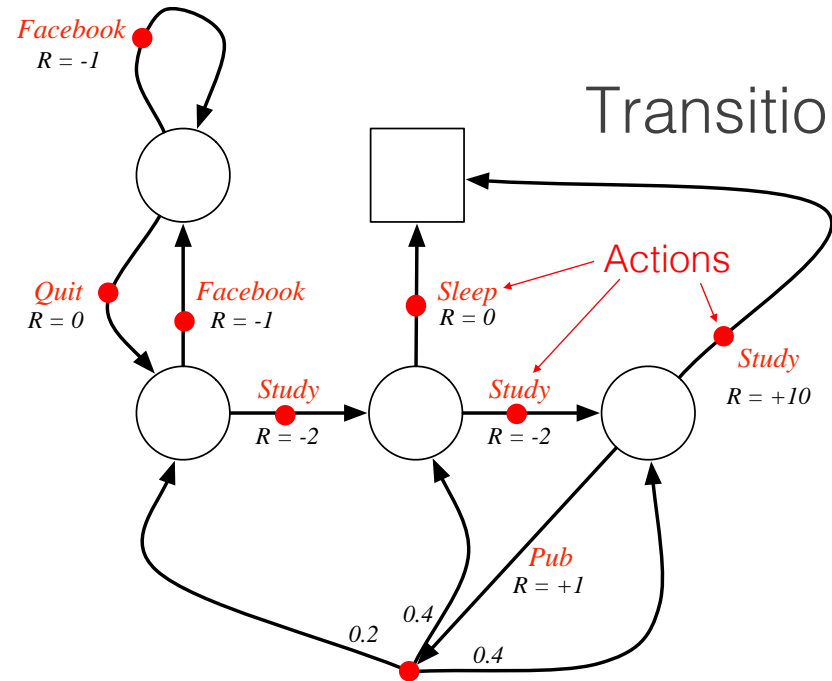
State space  $\mathcal{S}$

Transition probabilities,  $P$

Rewards,  $R$

Discount rate,  $\gamma$

Actions,  $\mathcal{A}$



## Returns (Expected future rewards)

(discount factor weights the the future)

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \dots$$

Policy (how we choose actions)

(can be stochastic or deterministic)

$$\pi(a|s) = P(a|s)$$

## State value function

(expected return from state  $s$ , and following policy  $\pi$ )

$$v_{\pi}(s) = E[G_t|s]$$

$$v_{\pi}(s) = E[R_s^a + \gamma v_{\pi}(s')|s]$$

## Action value function

(expected return from state  $s$ , taking action  $a$ , and following policy  $\pi$ )

$$q_{\pi}(s, a) = E[G_t|s, a]$$

$$q_{\pi}(s, a) = E[R_s^a + \gamma q_{\pi}(s', a')|s, a]$$

David Silver, UCL, 2015

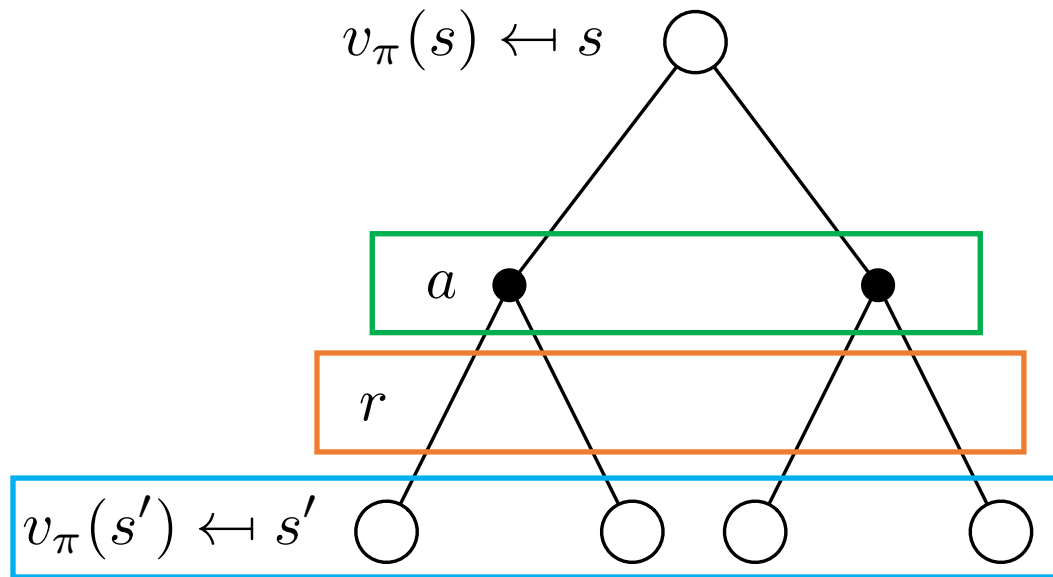
# Bellman Expectation Equations for the **state value** function

(expected return from state  $s$ , and following policy  $\pi$ )

$$v_{\pi}(s) = E[G_t | s]$$

$$v_{\pi}(s) = E[R_s^a + \gamma v_{\pi}(s') | s]$$

$$R_s^a = E[r_{t+1} | S_t = s, A_t = a]$$



Expectation over the possible actions

Expectation over the rewards

(based on state and choice of action)

Expectation over the next possible states

$$v_{\pi}(s) = \underbrace{\sum_a}_{\text{Expectation over actions}} \underbrace{\pi(a|s)}_{\text{Expectation over rewards}} \left( \underbrace{R_s^a + \gamma \sum_{s'} P_{ss'}^a v_{\pi}(s')}_{\text{Expectation over next possible states}} \right)$$

# Bellman Expectation Equations for the **action value** function

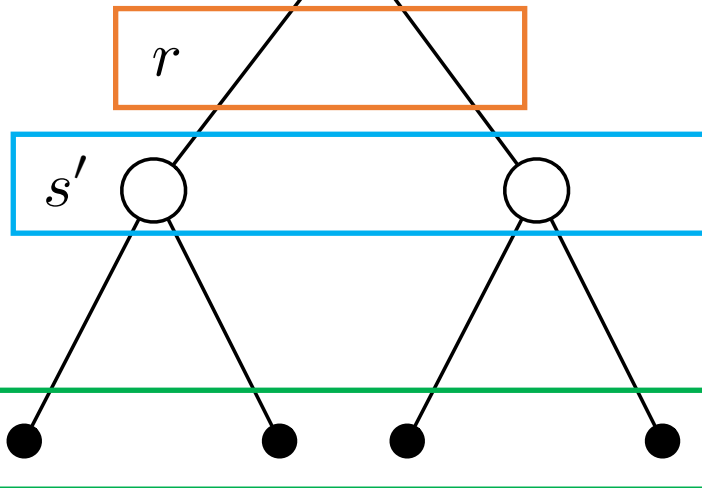
(expected return from state  $s$ , taking action  $a$ , then following policy  $\pi$ )

$$q_{\pi}(s, a) = E[G_t | s, a]$$

$$q_{\pi}(s, a) = E[R_s^a + \gamma q_{\pi}(s', a') | s, a]$$

$$R_s^a = E[r_{t+1} | S_t = s, A_t = a]$$

$$q_{\pi}(s, a) \leftarrow s, a$$



Expectation over the rewards

(based on state and choice of action)

Expectation over the next possible states

Expectation over the possible actions

$$q_{\pi}(s, a) = \underbrace{R_s^a}_{\text{orange}} + \gamma \underbrace{\sum_{s'} P_{ss'}^a}_{\text{blue}} \underbrace{\sum_{a'} \pi(a' | s') q_{\pi}(s', a')}_{\text{green}}$$

# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

1. Evaluate the returns a policy will yield? **Policy evaluation**

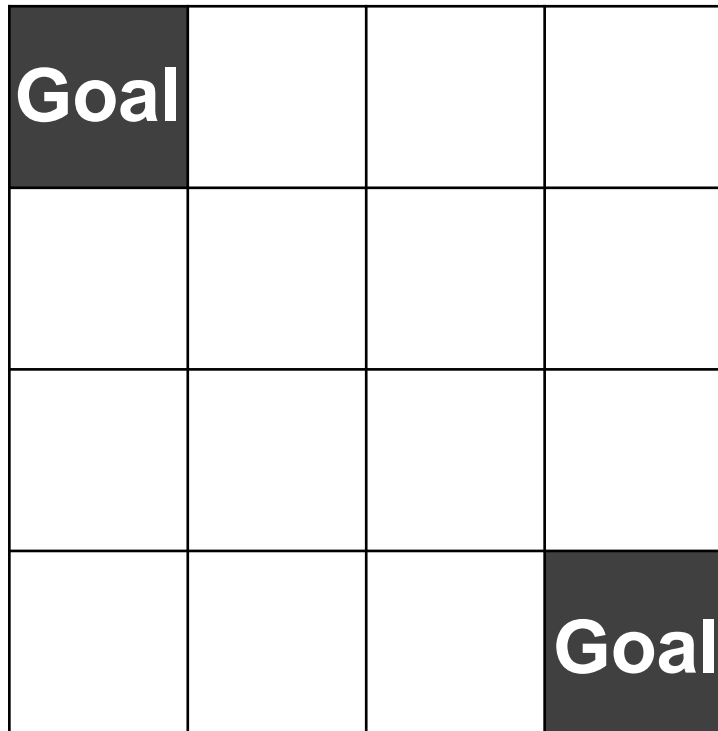
2. Find a **better** policy? **Policy improvement**

3. Find the **best** policy? **Policy iteration**

4. Find the best policy **faster**? **Value iteration**

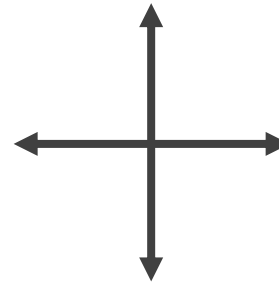
What if we don't have a fully known MDP? **Monte Carlo Methods**

# Running example: Gridworld



16 states, 2 of them terminal states labeled “goal”

Valid actions:  
(unless there is a wall)



Reward:

-1 for all transitions  
(until the terminal state has been reached)

Note: actions that would take the agent off the board are not allowed

Sutton and Barto, 2018

# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

1. Evaluate the returns a policy will yield? **Policy evaluation**

2. Find a **better** policy? **Policy improvement**

3. Find the **best** policy? **Policy iteration**

4. Find the best policy **faster**? **Value iteration**

What if we don't have a fully known MDP? **Monte Carlo Methods**

Dynamic Programming



# 1. Policy Evaluation

Evaluate the returns a policy will yield

Input: policy  $\pi(a|s)$

Output: value function  $v_\pi(s)$   
(unknown)

- 1 Select a policy function to evaluate (estimate the value function)
- 2 Start with a guess of the value function,  $v_0$  (often all zeros)
- 3 **Iteratively** apply the Bellman Expectation Equation to “backup” the values until they converge on the actual value function for the policy,  $v_\pi$

$$v_0 \rightarrow v_1 \rightarrow \dots \rightarrow v_\pi$$

Adapted from David Silver, 2015

# 1. Policy Evaluation

Evaluate the returns a policy will yield

Policy: 
$$\pi(a|s) = \frac{1}{N_{\text{valid\_actions}}}$$

Randomly go in any valid direction

Value function initialization:

$$v_0(s) = 0 \text{ (all zeros)}$$

$v_0(s)$   
(initialization)

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

We estimate the value function that corresponds to the policy:  $v_\pi(s)$

# 1. Policy Evaluation

Evaluate the returns a policy will yield

Policy:  $\pi(a|s) = 1/N_{\text{valid\_actions}}$   
(randomly go in any direction)

Bellman Expectation Equation:

$$v_{k+1}(s) = \sum_a \pi(a|s) \left( R_s^a + \gamma \sum_{s'} P_{ss'}^a v_k(s') \right)$$

In Gridworld:  $\frac{1}{N_a}$   $-1$   $1$  (once you pick an action there's no uncertainty as to which state you'll transition to)

$$v_{k+1}(s) = \sum_a \frac{1}{N_a} \left( -1 + \sum_{s'} v_k(s') \right) = -1 + \sum_a \frac{1}{N_a} \sum_{s'} v_k(s')$$

Each action leads to only one state, so the sum over states is not needed

$$= -1 + \sum_a \frac{1}{N_a} v_k(s')$$

Average of the value of the  $N_a$  neighboring states

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

# 1. Policy Evaluation

Evaluate the returns a policy will yield

$$v_{k+1}(s) = -1 + \sum_a \frac{1}{N_a} v_k(s')$$

$$v_1 = -1 + \sum_a \frac{1}{4} v_k(s') = -1$$

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

One neighborhood  
in  $v_0(s)$

	0		
0		0	
	0		

$v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

$v_0(s)$ 

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

 $v_1(s)$ 

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

 $v_2(s)$ 

0	-1.7	-2	-2
-1.7	-2	-2	-2
-2	-2	-2	-1.7
-2	-2	-1.7	0

 $v_3(s)$ 

0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0

 $v_{10}(s)$ 

0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0

 $v_\infty(s) = v_\pi(s)$ 

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	0

We've found the value function  
(expected returns) from our random  
movement policy

# 1. Policy Evaluation

Evaluate the returns a policy will yield

# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

1. Evaluate the returns a policy will yield? **Policy evaluation**

2. Find a **better** policy? **Policy improvement**

3. Find the **best** policy? **Policy iteration**

4. Find the best policy **faster**? **Value iteration**

What if we don't have a fully known MDP? **Monte Carlo Methods**

## 2. Policy Improvement

Find a **better** policy

Input: policy

$\pi(a|s)$

Output: better policy

$\pi'(a|s)$

Definition of better: has greater or equal expected return in all states:

$$v_{\pi'}(s) \geq v_{\pi}(s) \text{ for all states}$$

- 1 Select a policy function to improve
- 2 Evaluate the value function (our last discussion)
- 3 **Greedy** select a new policy,  $\pi'$ , that chooses actions that maximize value

$$\pi'(s) = \arg \max_a q_{\pi}(s, a)$$

$q_{\pi}(s, a)$  = expected return  
from state  $s$ , taking action  $a$ ,  
and following policy  $\pi$

i.e. pick the **action** that brings us to the state with **highest value** Adapted from David Silver, 2015

Value function:

In this case,  
 $q_{\pi}(s, \pi(s)) = v_{\pi}(s)$   
 since each action  
 leads to only one state

$v_0(s)$

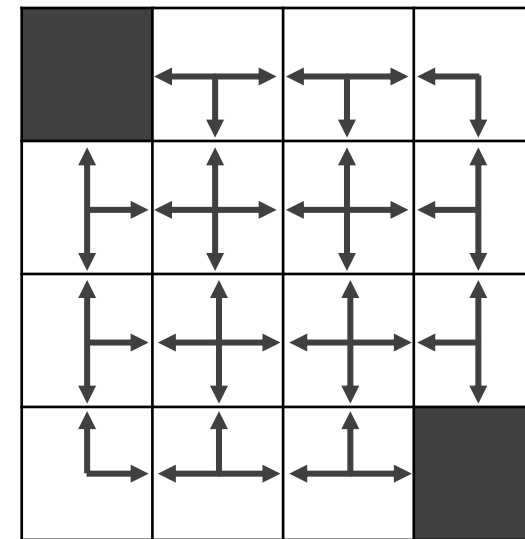
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>

$v_{\infty}(s) = v_{\pi}(s)$

<b>0</b>	<b>-14</b>	<b>-20</b>	<b>-22</b>
<b>-14</b>	<b>-18</b>	<b>-20</b>	<b>-20</b>
<b>-20</b>	<b>-20</b>	<b>-18</b>	<b>-14</b>
<b>-22</b>	<b>-20</b>	<b>-14</b>	

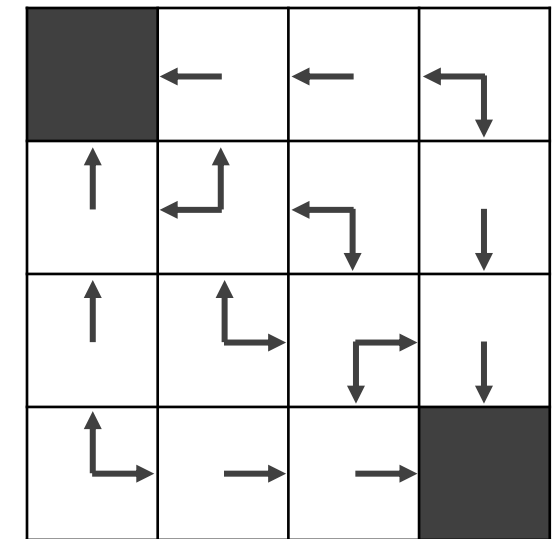
Initial policy:  $\pi(s)$

$\pi(a|s) =$  randomly go  
in any valid  
direction



Improved policy  
 (in this case this is an  
 optimal policy)

$\pi'(s)$



## 2. Policy Improvement

Find a **better** policy



# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

1. Evaluate the returns a policy will yield? **Policy evaluation**

2. Find a **better** policy? **Policy improvement**

3. Find the **best** policy? **Policy iteration**

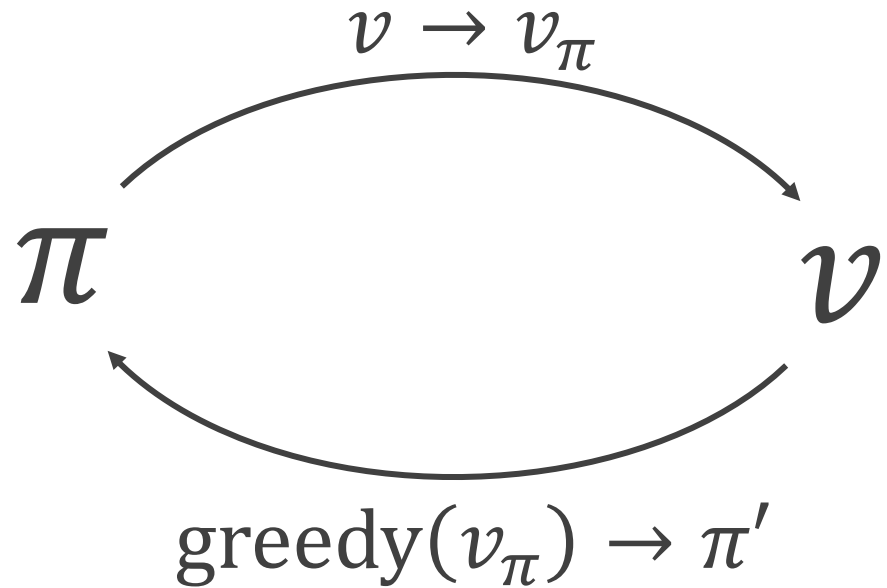
4. Find the best policy **faster**? **Value iteration**

What if we don't have a fully known MDP? **Monte Carlo Methods**

# 3. Policy Iteration

Find the **best** policy

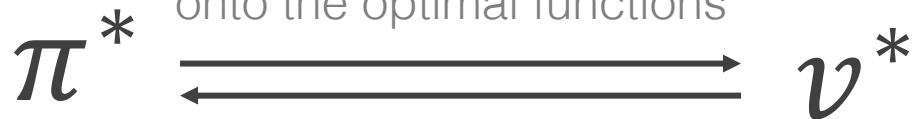
## Policy **Evaluation**



## Policy **Improvement**

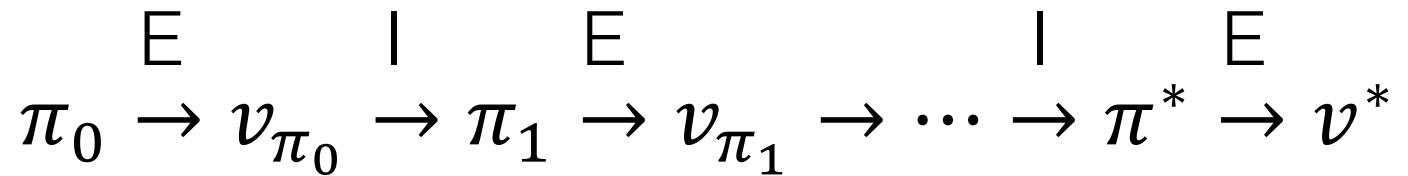
⋮

This process will converge onto the optimal functions



Input: policy  $\pi(a|s)$   
Output: **best** policy  $\pi^*(a|s)$

Best in the sense that:  $v_{\pi^*}(s) \geq v_\pi(s)$  for all states and for all **policies**



Adapted from David Silver, 2015 and Sutton and Barto, 1998

### 3. Policy Iteration

Find the **best** policy

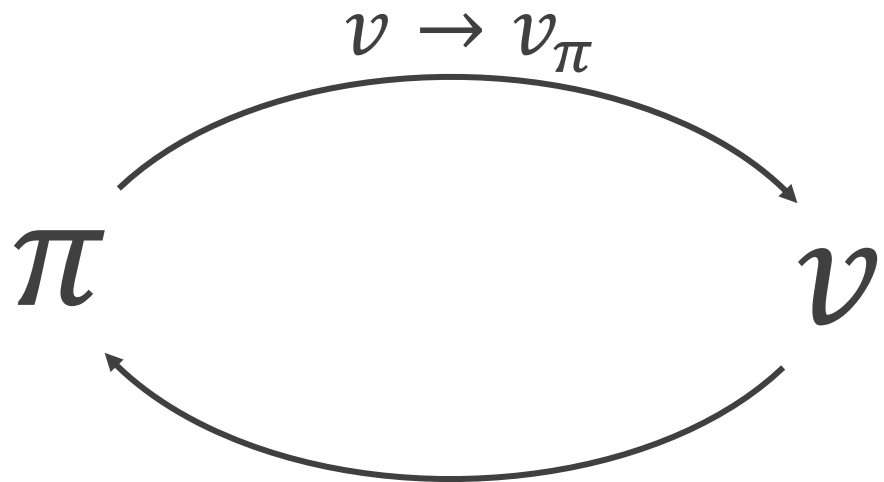
Input: policy

$\pi(a|s)$

Output: **best** policy

$\pi^*(a|s)$

Policy **Evaluation**

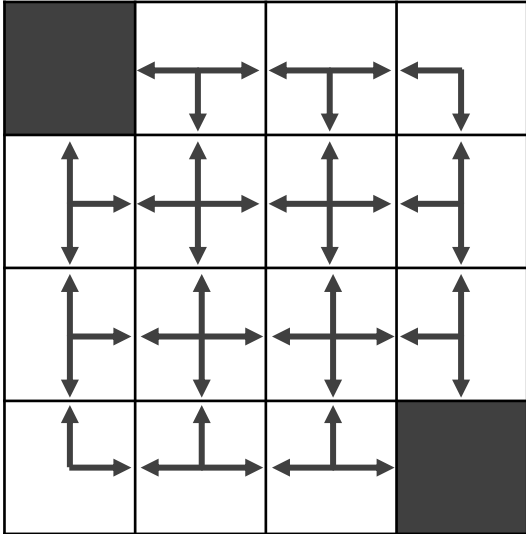


$\text{greedy}(v_\pi) \rightarrow \pi'$   
Policy **Improvement**

- 1 Policy Evaluation:** estimate  $v_\pi$   
Iterative policy evaluation  
Note: This is VERY slow
- 2 Policy Improvement:** generate  $\pi' \geq \pi$   
Greedy policy improvement
- 3** Iterate 1 and 2 until convergence

Adapted from David Silver, 2015 and Sutton and Barto, 1998

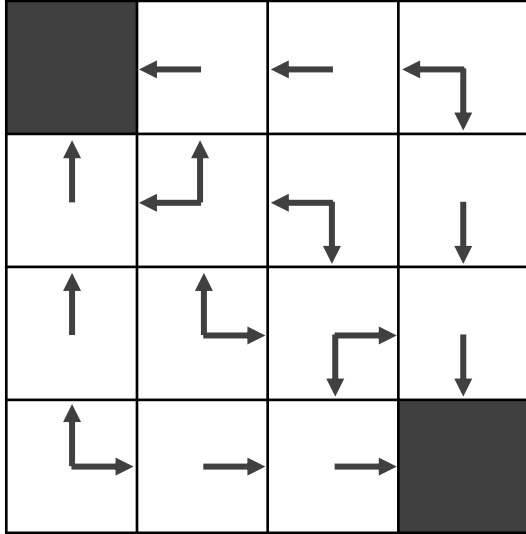
1  $\pi_0(s)$



$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

3  $\pi_1(s) = \pi^*(s)$



$v_0(s)$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

Policy Evaluation  
Policy Improvement

2  $v_\infty(s) \rightarrow v_{\pi_0}(s)$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

Policy Evaluation

4  $v_\infty(s) \rightarrow v_{\pi_0}(s)$

0	-1	-2	-3
-1	-2	-3	-2
-2	-3	-2	-1
-3	-2	-1	0

$v_{\pi^*}(s)$

# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

1. Evaluate the returns a policy will yield? **Policy evaluation**

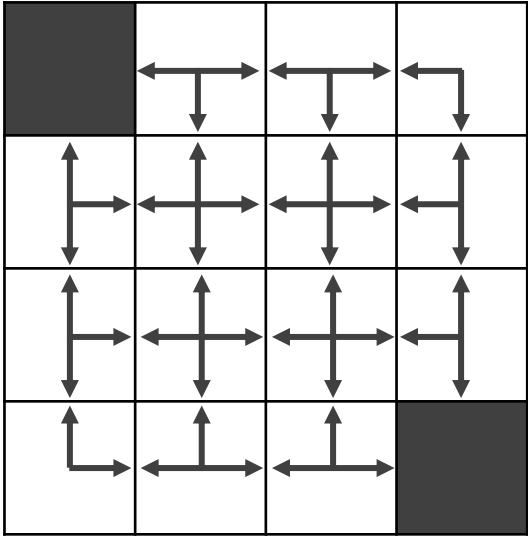
2. Find a **better** policy? **Policy improvement**

3. Find the **best** policy? **Policy iteration**

4. Find the best policy **faster**? **Value iteration**

What if we don't have a fully known MDP? **Monte Carlo Methods**

$$\pi_0(s)$$



$$v_0(s)$$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$$v_1(s)$$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

What if we stopped after one sweep. This is...

## 4. Value Iteration

Find the best policy **faster**

## 4. Value Iteration

Find the best policy **faster**

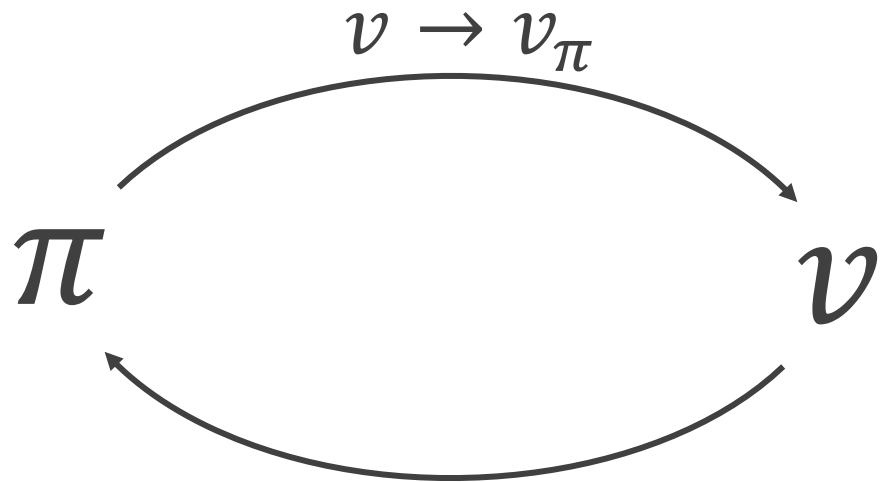
Input: policy

$\pi(a|s)$

Output: **best** policy

$\pi^*(a|s)$

Policy **Evaluation**



$\text{greedy}(v_\pi) \rightarrow \pi'$   
Policy **Improvement**

- 1 Policy Evaluation:** estimate  $v_\pi$   
**One-sweep** of policy evaluation
- 2 Policy Improvement:** generate  $\pi' \geq \pi$   
Greedy policy improvement
- 3** Iterate 1 and 2 until convergence

Adapted from David Silver, 2015 and Sutton and Barto, 1998

$v_0(s)$ 

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

 $v_1(s)$ 

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

 $v_2(s)$ 

0	-1.7	-2	-2
-1.7	-2	-2	-2
-2	-2	-2	-1.7
-2	-2	-1.7	0

 $v_3(s)$ 

0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0

 $v_{10}(s)$ 

0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0

 $v_\infty(s) = v_\pi(s)$ 

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

So far, we've run policy evaluation all the way to convergence (**this is slow**)



1  $v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

3  $v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

5  $v_2(s)$

0	-1	-2	-2
-1	-2	-2	-2
-2	-2	-2	-1
-2	-2	-1	0

7  $v_3(s) = v_{\pi^*}(s)$

0	-1	-2	-3
-1	-2	-3	-2
-2	-3	-2	-1
-3	-2	-1	0

2  $\pi_0(s)$

	↔	↔	↖
↑	↕	↕	↕
↑	↕	↕	↕
↙	↕	↕	

4  $\pi_1(s)$

	←	↔	↖
↑	↕	↕	↕
↑	↕	↕	↓
↙	↕	→	

6  $\pi_2(s)$

	←	←	↖
↑	↖	↕	↓
↑	↕	↘	↓
↙	→	→	

8  $\pi_3(s) = \pi^*(s)$

	←	←	↖
↑	↖	↕	↓
↑	↕	↘	↓
↙	→	→	

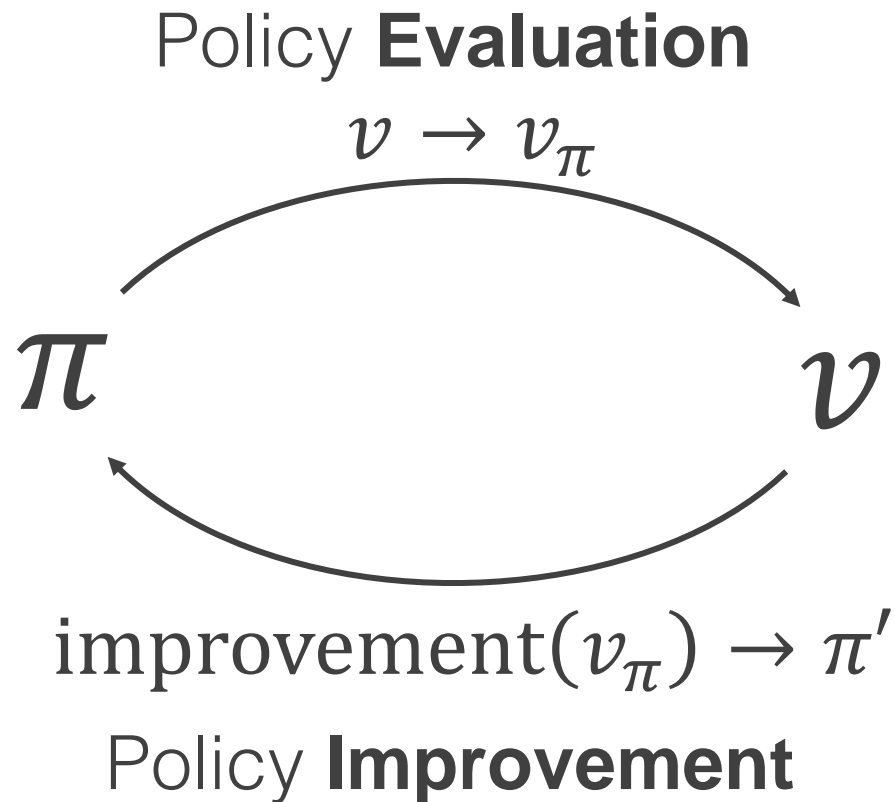
# Generalized Policy Iteration

Input: policy

$\pi(a|s)$

Output: **best** policy

$\pi^*(a|s)$



- 1 Policy Evaluation:** estimate  $v_\pi$   
**Any** policy evaluation algorithm
- 2 Policy Improvement:** generate  $\pi' \geq \pi$   
**Any** policy improvement algorithm
- 3** Iterate 1 and 2 until convergence

Adapted from David Silver, 2015 and Sutton and Barto, 1998

# Demo

[https://cs.stanford.edu/people/karpathy/reinforcejs/gridworld\\_dp.html](https://cs.stanford.edu/people/karpathy/reinforcejs/gridworld_dp.html)

So far, we've assumed full knowledge of the environment (MDP)

What if we **DO NOT assume full knowledge of the environment** (MDP)

This means we have to **learn by experience!**

# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

- |  |                           |
|--|---------------------------|
| 1. Evaluate the returns a policy will yield? | <b>Policy evaluation</b>  |
| 2. Find a <b>better</b> policy?              | <b>Policy improvement</b> |
| 3. Find the <b>best</b> policy?              | <b>Policy iteration</b>   |
| 4. Find the best policy <b>faster</b> ?      | <b>Value iteration</b>    |

Dynamic Programming

What if we don't have a fully known MDP? **Monte Carlo Methods**

# 1. Policy Evaluation

Evaluate the returns a policy will yield

Input: policy  $\pi(a|s)$

Output: value function  $v_\pi(s)$   
(unknown)

- 1 Select a policy function to evaluate (estimate the value function)
- 2 Start with a guess of the value function,  $v_0$  (often all zeros)
- 3 **Iteratively** apply the Bellman Expectation Equation to “backup” the values until they converge on the actual value function for the policy,  $v_\pi$

$$v_0 \rightarrow v_1 \rightarrow \dots \rightarrow v_\pi$$

## PREVIOUSLY

Adapted from David Silver, 2015

# Monte Carlo Policy Evaluation

For **state** values

Evaluate the returns a policy will yield

Input:	policy	$\pi(a s)$
Output:	state value	$v_{\pi}(s)$

- 1 Select a policy function to evaluate (estimate the value function)
- 2 Start with a guess of the value function,  $v_0$  (often all zeros)
- 3 Estimate the value function through experience by iterating:
  - A Generate an episode (take actions until a terminal state)
  - B Save the returns following the first occurrence of each state
  - C Assign  $\text{AVG}(\text{Returns}(s)) \rightarrow \hat{v}_{\pi}(s)$

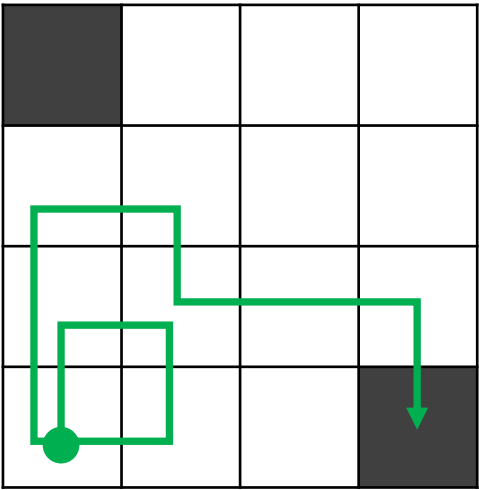
Sutton and Barto, 1998

# Monte Carlo Policy Evaluation

For **state** values  
“First Visit”

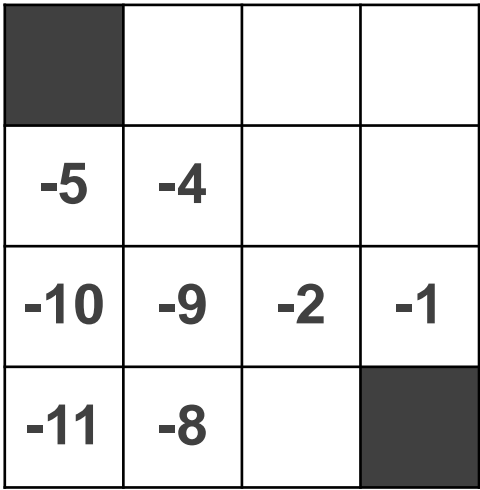
For each state, we  
store the running  
returns seen **after**  
the first visit to that  
state

**Episode 1**  
Total Reward: -11



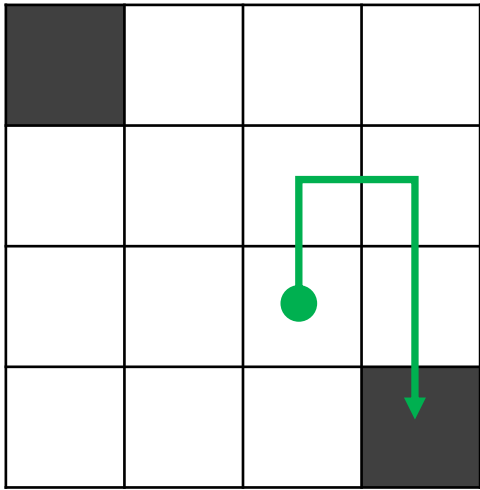
Episode 1 **returns** after  
the first visit of each state

$G^{(1)}$



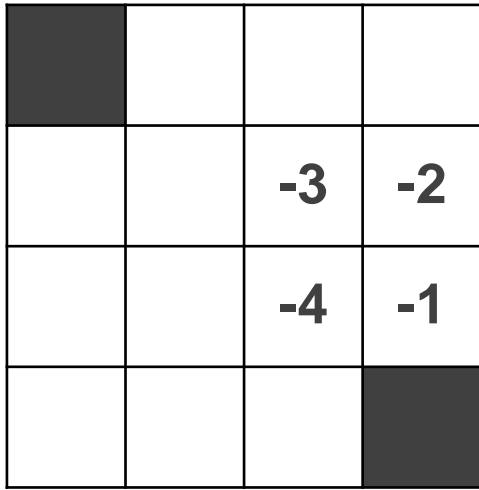
Discount rate:  $\gamma = 1$

**Episode 2**  
Total Reward: -4



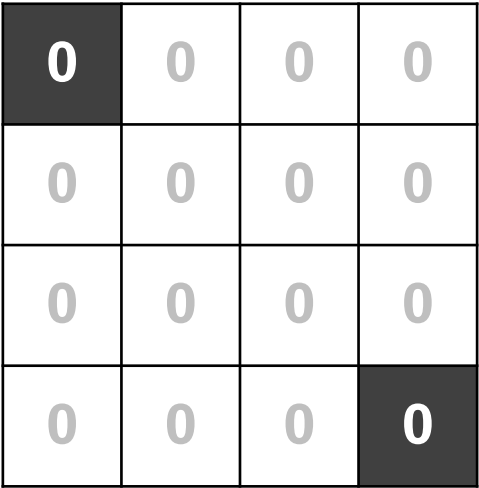
Episode 2 **returns** from  
the first visit of each state

$G^{(2)}$



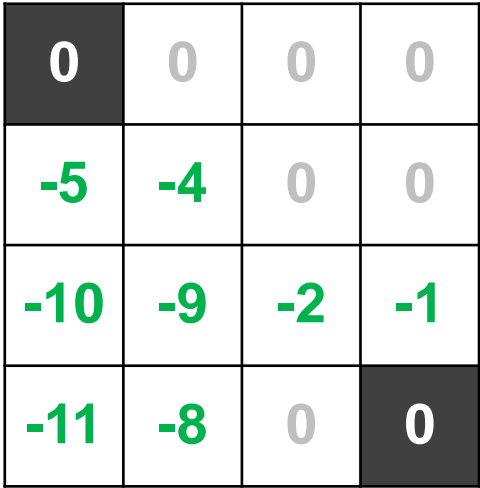
Discount rate:  $\gamma = 1$

$v_0(s)$



The value function is  
the **running average** of  
the returns after the visit  
to that state, averaged  
over episodes  
(only average over episodes  
when state is visited)

$v_1(s)$



$v_1$  is just the  
first visit  
returns,  $G^{(1)}$

$v_2(s)$



$v_2$  is the  
average first  
visit returns,  
 $G^{(1)}$  and  $G^{(2)}$ ,  
for those  
states visited



# State vs action value

The **state value function** doesn't tell us directly about actions

If we don't have a model, to pick a policy we need **action values**

# State vs action value

Greedy policy improvement over  $v(s)$  **requires a model of the MDP**

$$\pi'(s) = \operatorname{argmax}_a R_s^a + P_{ss'}^a v(s')$$

?      ?

Greedy policy improvement over  $q(s, a)$  **requires no MDP knowledge**

$$\pi'(s) = \operatorname{argmax}_a q(s, a)$$

And the two value functions are related:  $v_\pi(s) = \sum_a \pi(a|s) q_\pi(s, a)$

# Monte Carlo Policy Evaluation

For **action** values

Input: policy  $\pi(a|s)$   
Output: **action value**  $q_\pi(s, a)$

Evaluate the returns a policy will yield

- 1 Select a policy function to evaluate (estimate its value function)
- 2 Start with a guess of the action value function,  $q_0$  (often all zeros)
- 3 Repeat forever:
  - A Generate an episode (take actions until a terminal state)
  - B Save returns following first occurrence of each state **& action**
  - C Assign  $\text{AVG}(\text{Returns}(s, a)) \rightarrow \hat{q}_\pi(s, a)$

Sutton and Barto, 1998

### 3. Policy Iteration

Find the **best** policy

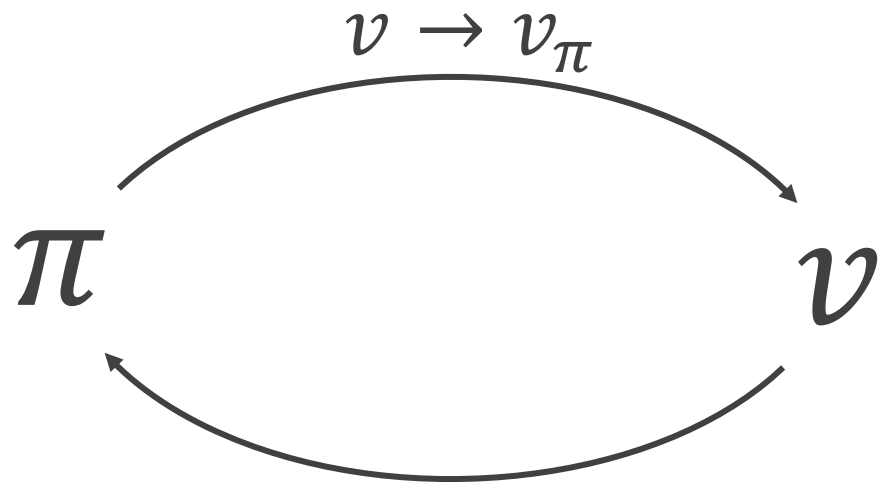
Input: policy

$\pi(a|s)$

Output: **best** policy

$\pi^*(a|s)$

Policy **Evaluation**



$\text{greedy}(v_\pi) \rightarrow \pi'$   
Policy **Improvement**

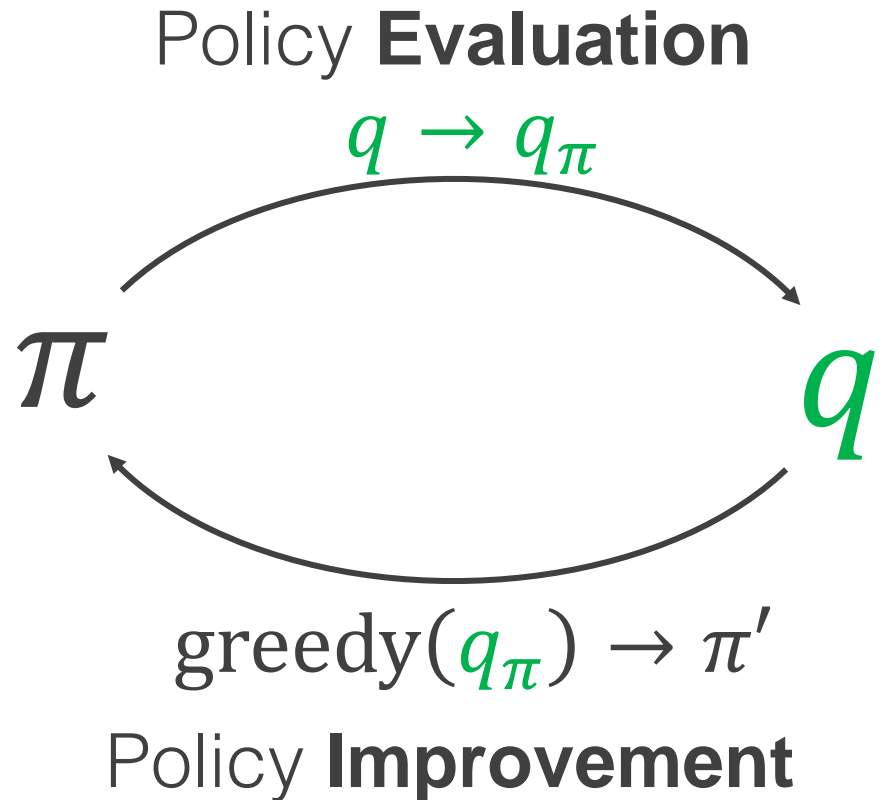
- 1 Policy Evaluation:** estimate  $v_\pi$   
Iterative policy evaluation  
Note: This is VERY slow
- 2 Policy Improvement:** generate  $\pi' \geq \pi$   
Greedy policy improvement
- 3** Iterate 1 and 2 until convergence

# PREVIOUSLY

Adapted from David Silver, 2015 and Sutton and Barto, 1998

# Monte Carlo Control

Find the **best** policy



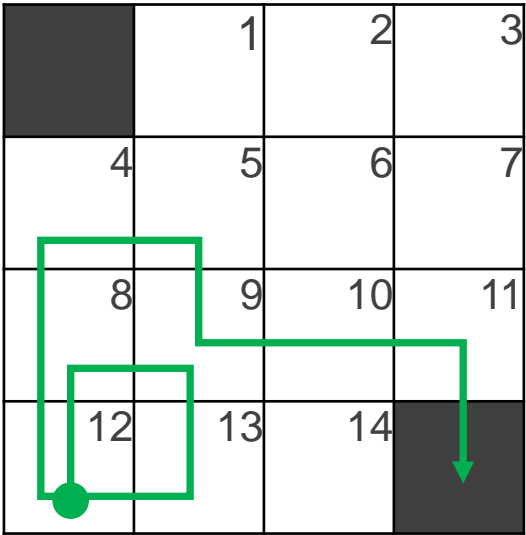
- 1 **Policy Evaluation:** estimate  $q_\pi$   
**Monte Carlo action policy evaluation**
- 2 **Policy Improvement:** generate  $\pi' \geq \pi$   
Greedy policy improvement
- 3 Iterate 1 and 2 until convergence

Sutton and Barto, 1998

# Monte Carlo Control

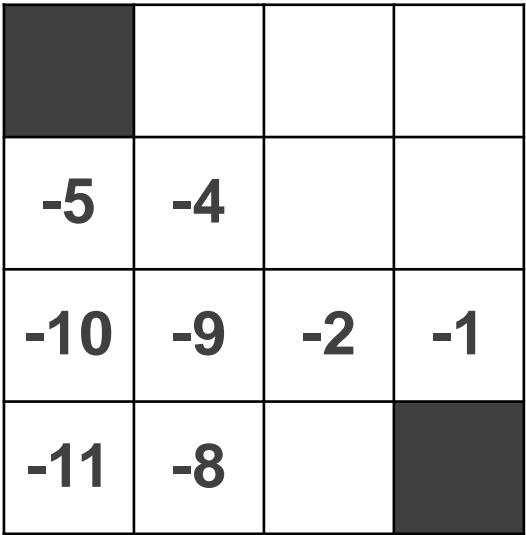
“First Visit” (of state AND action) is recorded

**Episode 1**  
Total Reward: -11



**1** MC Policy Evaluation

Episode 1 **returns** after the first visit of each state



Discount rate:  $\gamma = 1$

$$q_{\pi}(s, a)$$

Action ( $a$ ):  $\uparrow$   $\rightarrow$   $\leftarrow$   $\downarrow$

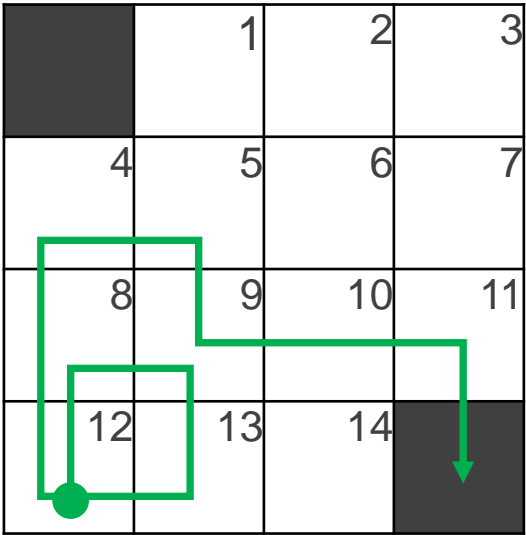
State ( $s$ )

1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				

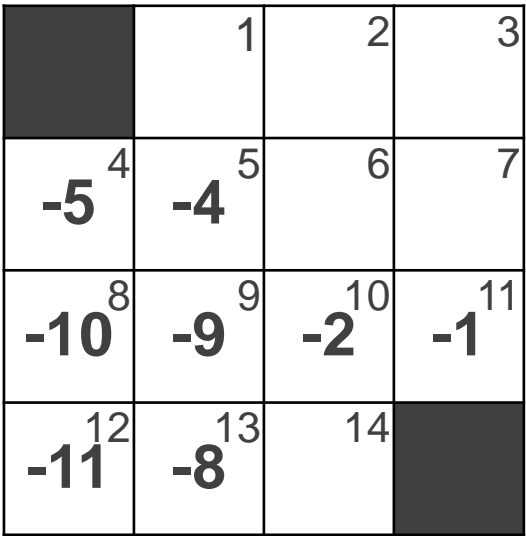
# Monte Carlo Control

“First Visit” (of state AND action) is recorded

**Episode 1**  
Total Reward: -11



Episode 1 **returns** after the first visit of each state



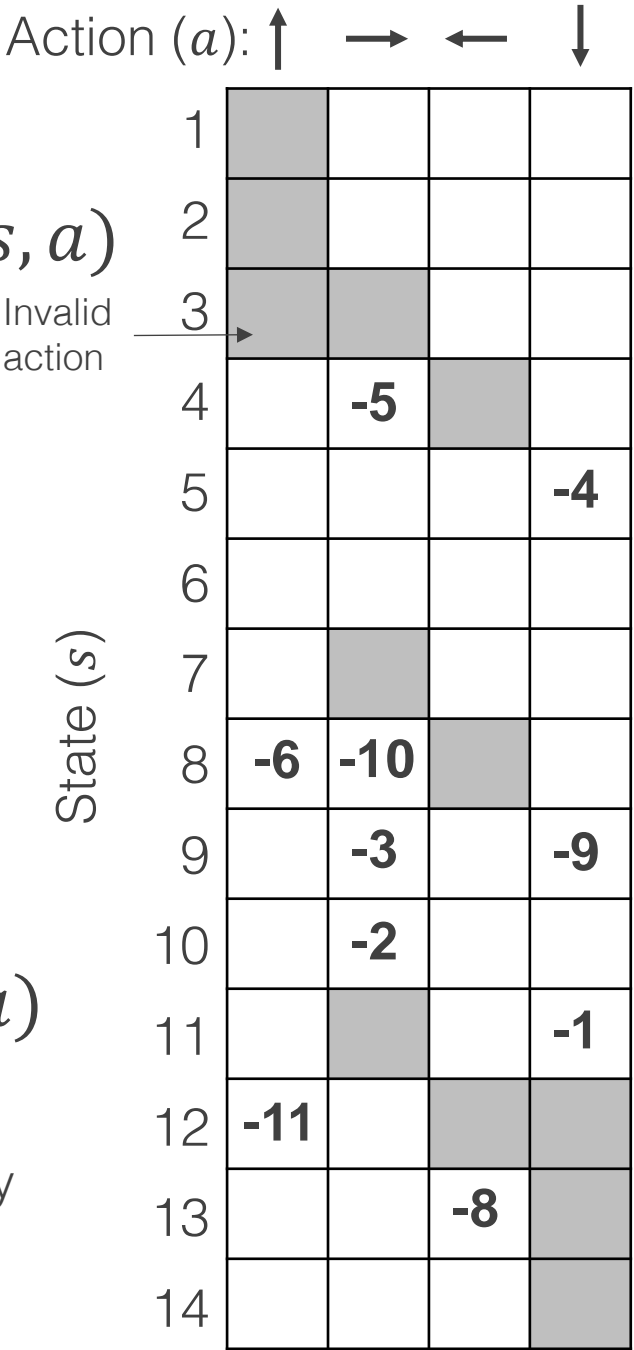
- 1 MC Policy Evaluation
- 2 MC Policy Improvement

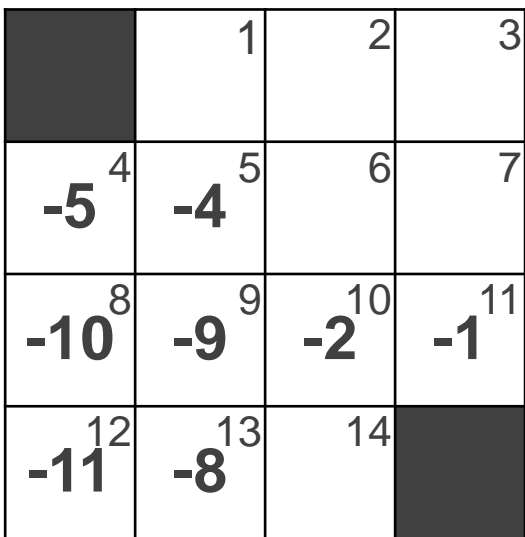
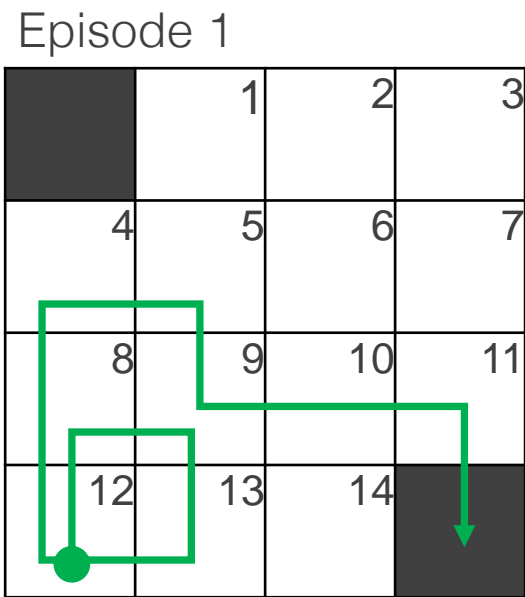
$$\pi'(s) = \operatorname{argmax}_a q_{\pi}(s, a)$$

Typically this is set to be  $\epsilon$ -greedy to better learn  $q(s, a)$

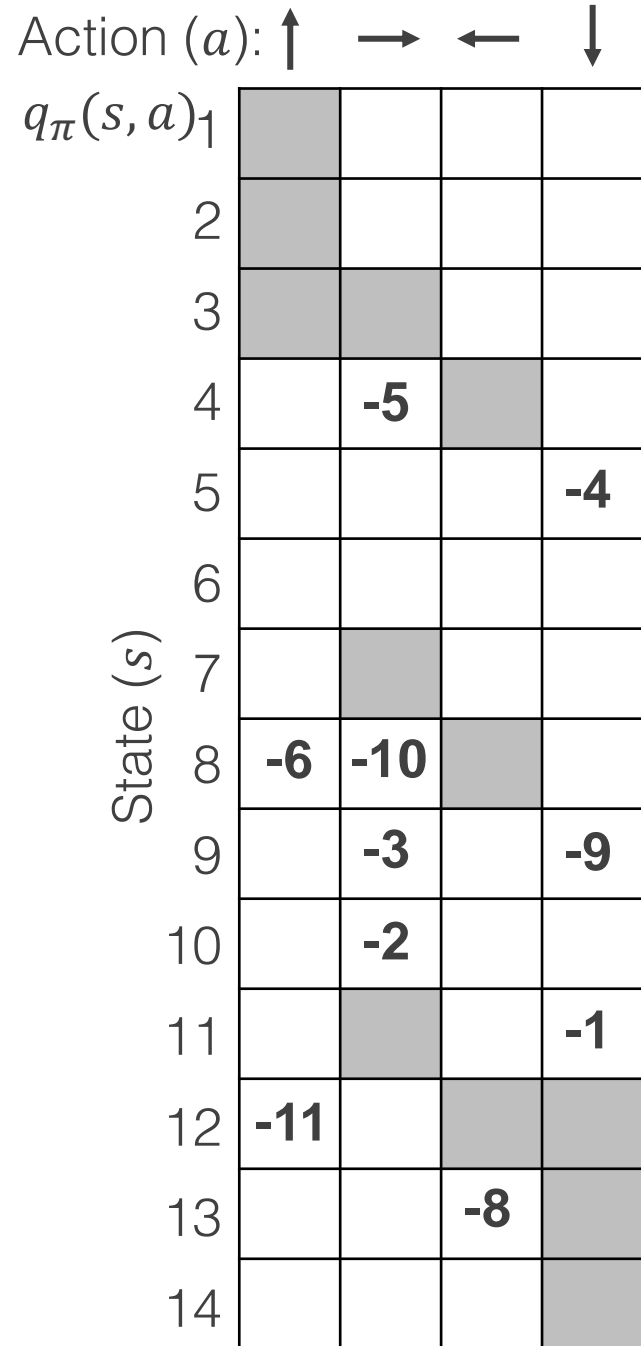
$$q_{\pi}(s, a)$$

Invalid action

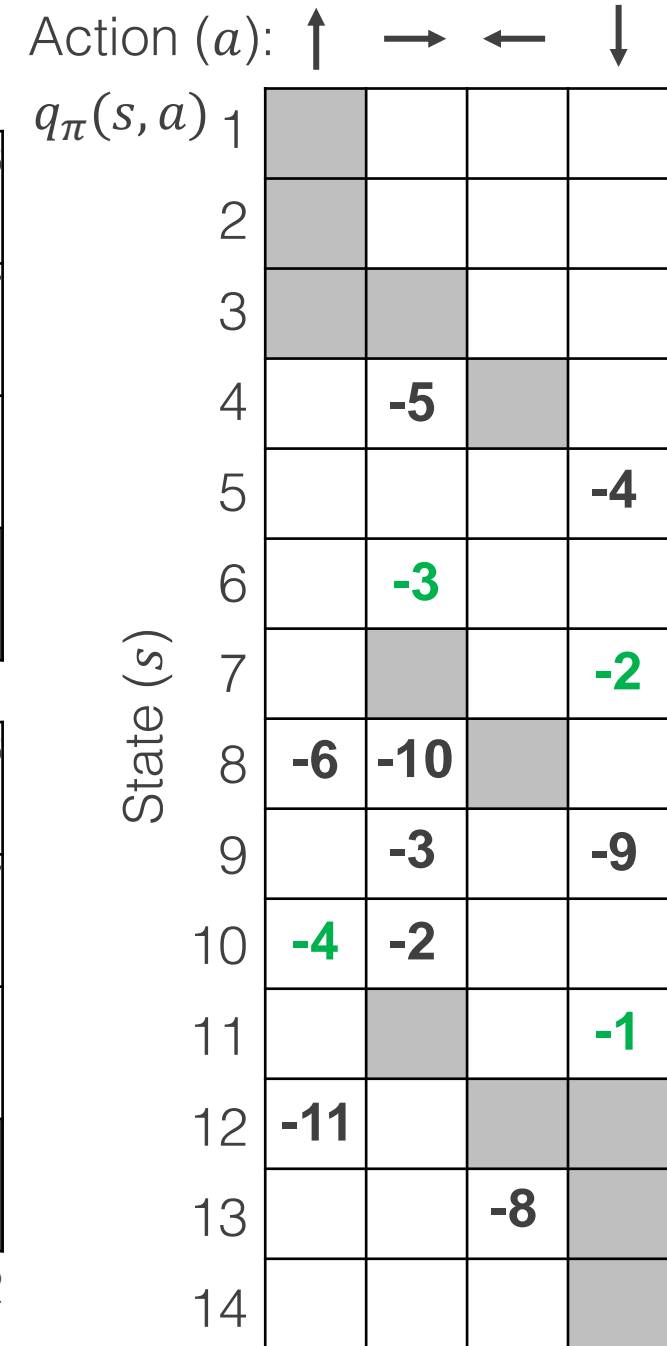




Returns from episode 1



Returns from episode 2





Action ( $a$ ):  $\uparrow \rightarrow \leftarrow \downarrow$

$q_{\pi^*}(s, a)$

State ( $s$ )	1		-3	-1	-3
	2		-4	-2	-3
	3			-3	-3
	4	-1	-3		-3
	5	-2	-4	-2	-4
	6	-3	-3	-3	-3
	7	-4		-4	-2
	8	-2	-4		-4
	9	-3	-3	-3	-3
	10	-4	-2	-4	-2
	11	-3		-3	-1
	12	-3	-3		
	13	-4	-2	-4	
	14	-3	-1	-3	

If we're in state 4,  
take the up action

If we know the  
optimal action  
value function,  
we also have our  
optimal policy



$v_{\pi^*}(s)$

<b>0</b>	<b>-1</b> <sup>1</sup>	<b>-2</b> <sup>2</sup>	<b>-3</b> <sup>3</sup>
<b>-1</b> <sup>4</sup>	<b>-2</b> <sup>5</sup>	<b>-3</b> <sup>6</sup>	<b>-2</b> <sup>7</sup>
<b>-2</b> <sup>8</sup>	<b>-3</b> <sup>9</sup>	<b>-2</b> <sup>10</sup>	<b>-1</b> <sup>11</sup>
<b>-3</b> <sup>12</sup>	<b>-2</b> <sup>13</sup>	<b>-1</b> <sup>14</sup>	<b>0</b>

$\pi^*(s)$

		<sup>1</sup>	<sup>2</sup>	<sup>3</sup>
		$\leftarrow$	$\leftarrow$	$\swarrow$
$\uparrow$	$\swarrow$	$\leftarrow$	$\updownarrow$	$\downarrow$
$\uparrow$	$\updownarrow$	$\updownarrow$	$\swarrow$	$\downarrow$
$\swarrow$	$\rightarrow$	$\rightarrow$		

# Extensions

Monte Carlo methods require that we finish each episode before updating

**Solution:** **Temporal Difference** (TD) methods

What if we want to learn about one policy while following or observing another?  
(e.g. evaluate a greedy policy while exploring the state space)

**Solution:** **Off-policy learning** instead of on-policy learning (e.g. Q-learning)

What if our state space has too many states that we can't build a table of values?

**Solution:** **Value function approximation** (involving supervised learning techniques)

How can we simulate what the environment might output for next states and rewards?

**Solution:** **Model-based learning**: simulate the environment and plan ahead

# Roadmap to optimal policies

If we assume a **fully known MDP environment**, how do we...  
(Markov Decision Process)

1. Evaluate the returns a policy will yield? **Policy evaluation**

2. Find a **better** policy? **Policy improvement**

3. Find the **best** policy? **Policy iteration**

4. Find the best policy **faster**? **Value iteration**

What if we don't have a fully known MDP? **Monte Carlo Methods**

# Reinforcement Learning Roadmap

Knowledge of **Environment**

## **Perfect knowledge**

Known Markov  
Decision Process



## **No knowledge**

Must learn from  
experience

## Dynamic Programming

What's a Markov Decision Process?  
How do we find optimal policies?

## Monte Carlo Control

How do we estimate our value functions?  
How do we use the value functions to choose actions?